

An effective method for determination of elastic constants of materials using multiresolution analysis

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Abstract

This study introduces a novel algorithm for determining elastic constants in materials using a multiresolution approach. This innovative method accurately estimates the natural frequency of the tested material, a crucial parameter for evaluating material constants such as Young's modulus and stiffness. This algorithm has substantial significance in the field of materials characterization. The experimental setup involves a specialized mechanical assembly designed for testing metal plates. Vibrational signals are acquired during experimentation. These raw signals undergo a multi-resolution process, leading to the extraction of the material specimen's fundamental frequency. Using this estimated fundamental frequency, essential elastic constants such as Young's modulus and stiffness are calculated with high precision. To validate the experimental findings, a thorough comparison is made using the finite element method (FEM), specifically utilizing ANSYS software. The experimental procedure is conducted on two different materials: stainless steel SA 240 Gr 304 and copper. Additionally, various testing conditions, including changes in ball weight and ball release height, are employed to comprehensively assess the algorithm's performance across diverse scenarios. The experimental outcomes highlight the algorithm's accuracy. Notably, the average percentage errors for estimating the fundamental frequency are impressively low, standing at 1.30% for stainless steel (SS) and 1.19% for copper. Similarly, the algorithm demonstrates excellence in calculating the modulus of elasticity, resulting in average percentage errors of 1.97% for SS and 2.33% for copper.

Keywords

Young Modules, FEA, stiffness

1. Introduction

Various materials find applications in industries such as industrial, automotive, and civil engineering. Leiss outlined an investigation into plate crack identification using wavelet analysis [1]. Deobald and Gamson [2] introduced the Rayleigh–Ritz method for computing the orthotropic rectangular plate model. Hwang

and Chang [3] proposed a finite element analysis approach, combined with optimal design techniques, for determining elastic constants. Maletta and Page put forth another method, utilizing the genetic algorithm (GA) in conjunction with the finite element method (FEM), for estimating elastic constants [4]. It is worth noting that the GA method's computational complexity is a recognized limitation.

Alfano and Pagnotta [5] presented a technique for determining Poisson's ratio and the dynamic modulus of elasticity for thin square materials, specifically focusing on Giraudeau et al. [6]. They proposed an approach to ascertain stiffness and damping parameters for thin vibrating plates. Additionally, Ablitzera and Pezerat [7] elaborated on a method for determining stiffness and damping characteristics of vibrating structures, known as the force analysis technique (FAT).

The vibration modes of the plate are analyzed using the continuous wavelet transform (CWT), and both the location and depth of the crack are estimated and presented by Douka et al. [8]. Daubechies scaling wavelet elements (DSWN) used to solve plate bending problems are presented by AlvarezDíaz et al. [9]. In this paper, a semi-analytical 2D model is proposed for the vibration analyses of rectangular plates. Daubechies wavelets, including both highly

localized and highly smooth members, are compactly supported and orthogonal.[10].

The mentioned study introduced a novel perspective, casting the spectrum into a series expansion, which encompasses the square moduli of wavelet Fourier transforms at varying scales. The selection of these scales is determined based on the frequency content of the process. The study conducted a comparative assessment that encompassed the evaluation of eigenvalue cumulative index and correlation coefficients across different filters: the Daubechies filter, Least asymmetric filter, and Coiflet filters. Importantly, the findings from the wavelet analysis showed similar eigenvalue cumulative index values, with the Coiflet filter standing out for its exceptional effectiveness in handling correlation coefficients. Consequently, the Coiflet filter was designated for utilization in the subsequent wavelet packet analysis.

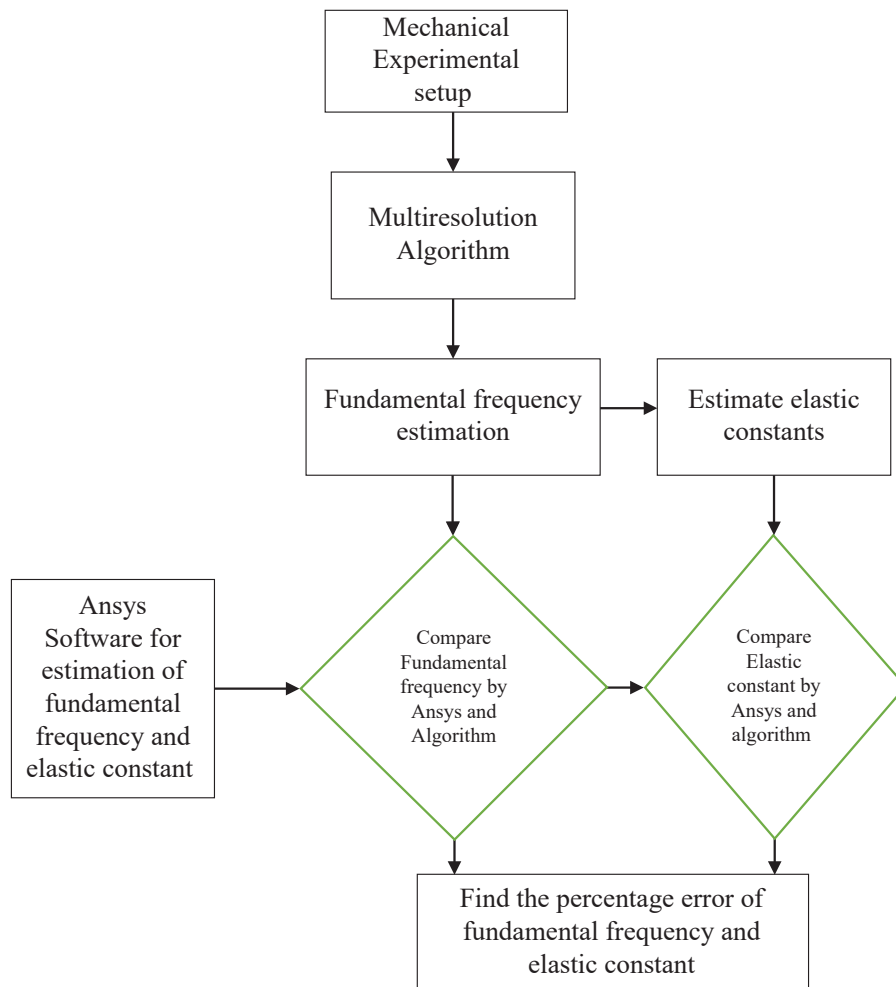


Figure 1: Flow diagram of the experimental setup.

II. Impact Test Set Up

a. Impact testing set up

The experimental arrangement for the impact test is depicted in Figure 3. This setup involves securely affixing a metal sheet specimen onto a frame. Positioned at a distance of 37.24 mm from the central point of the metal plate is the placement of the sensor, as illustrated in Figure 10. Throughout the experimental procedure, balls are released from varying heights, ranging from 50 to 150 cm. The test specimens utilized are composed of SS SA 240 material and copper, featuring dimensions of 220 mm × 220 mm and thicknesses of 1.3 mm and 1 mm, respectively.

In order to capture the vibrations generated by the impact of the falling ball on the test specimen sheet, an acceleration sensor is deployed. The data captured by this sensor are subsequently collected via a data card with a 24-bit resolution. The vibration data are analyzed using a multiresolution approach, which facilitates the determination of the fundamental frequency.

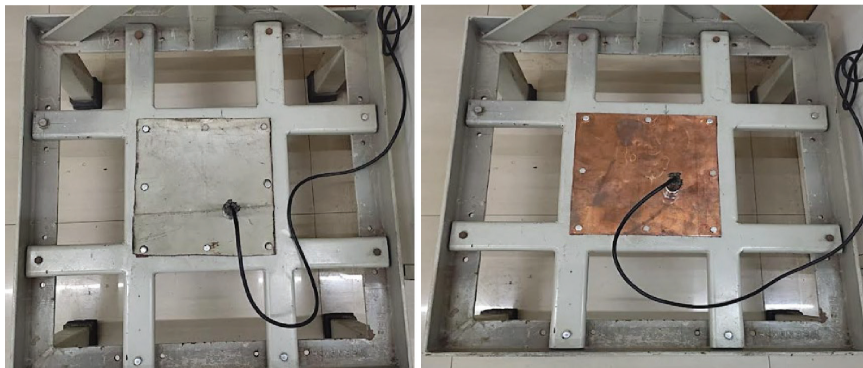
III. Proposed Method for Determination of Elastic Constant

a. Multiresolution analysis

Wavelets are mathematical functions that have become an essential tool in various fields of science



(A) Mechanical Experimental setup



(B)

(C)

Figure 2: (A) Experimental setup, (B) SS SA 240 Gr 304 with eight clamps at corners, and (C) copper plate with eight clamps at corner.

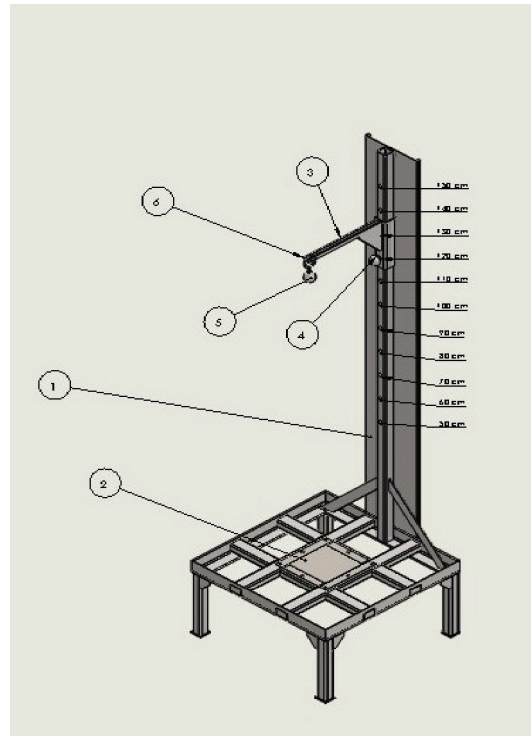


Figure 3: Free ball impact test setup.

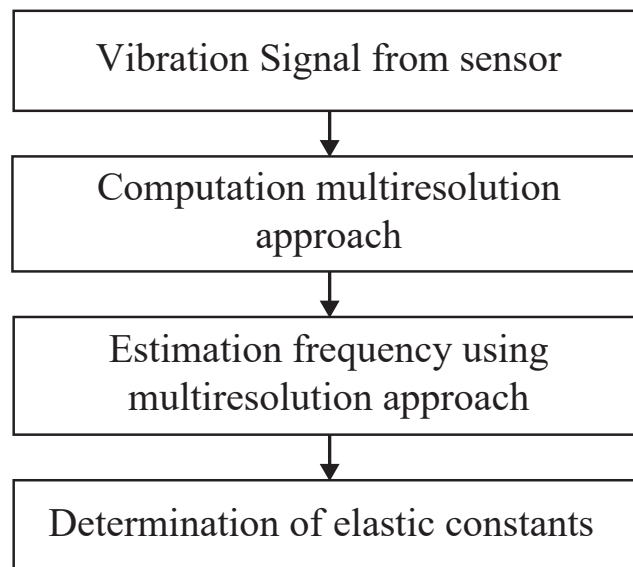


Figure 4: Proposed techniques.

and engineering for analyzing and processing signals and data. They are particularly useful for representing signals with both time and frequency information, making them well-suited for tasks such as

compression, noise reduction, feature extraction, and more. There are several types of wavelets, each with its own characteristics and applications. Here are some common types and their uses.

a.i Daubechies wavelets

These are a family of wavelets named after Ingrid Daubechies. They have compact support (meaning they are non-zero over a finite interval) and excellent approximation properties. They are widely used in image compression, signal denoising, and solving differential equations.

a.ii Morlet wavelet

The Morlet wavelet is used in CWT for analyzing time-frequency content of signals. It is especially useful in analyzing non-stationary signals such as EEG and seismic data.

a.iii Biorthogonal wavelets

These wavelets come in pairs, a wavelet and a scaling function, which are used in wavelet transforms to decompose signals. They are used in image compression, feature extraction, and denoising.

a.iv Coiflet wavelets

These are a family of wavelets with properties similar to Daubechies wavelets. They provide a balance between regularity and smoothness and are often used in image compression, denoising, and feature extraction.

a.v Symlet wavelets

Symlet wavelets are designed to have more symmetry than Daubechies wavelets. They are used in signal and image compression, denoising, and feature extraction.

The vibration signal is acquired from a vibrating sensor. The input signal passes into different wavelet transforms. The wavelet function is designed to strike a balance between the time domain (finite length) and frequency domain (finite bandwidth).

The discrete wavelet transform (DWT) functions as a filter, effectively breaking down the input signal into its approximation and detail coefficients through the application of two distinct filters: a low-pass filter for the approximation coefficients and a high-pass filter for the detail coefficients. In simpler terms, the primary signal can be constructed following the formulation outlined in Eq. (1).

$$X(t) = A_i + \sum D_i$$

Within this equation, the variables A_i and D_i represent the approximation (associated with low-frequency content) and detail (pertaining to high-frequency content) signals, with the parameter i indicating the specific level of decomposition. This signifies that the resultant approximation coefficients stemming from each decomposition can, in turn, be subjected to further decomposition into both approximation and detail components. By iteratively repeating this process, multiple layers of signal decomposition can be attained. While the decomposition process theoretically has the potential for indefinite iterations, the practical determination of the appropriate number of decomposition stages relies on the inherent characteristics of the given signal [13]. A schematic depiction of the DWT procedure is provided in Figure 5.

Coefficients of DWT have different applications depending on the nature of the signal [13].

Wavelet analysis emerges as a notably effective and straightforward approach to address such scenarios. Furthermore, it introduces novel techniques for mitigating signal noise, thereby complementing the conventional methodologies offered by Fourier analysis. Unlike Fourier analysis, which excels in approximating signals featuring abrupt spikes or discontinuities in a scale-independent manner, wavelet analysis excels in adapting to variations in scale or resolution, rendering it particularly suited for such cases.

b. Selection of wavelet

The signal-to-noise ratio (SNR) is a widely used metric for evaluating the effectiveness of different wavelet families in denoising signals. SNR measures the ratio of the signal power to the noise power and can be used to quantitatively compare denoised signals produced by various wavelet types. Here is how SNR can be used for wavelet selection:

c. Calculate initial SNR

Calculate the SNR of the noisy signal before any denoising is applied. This serves as a reference point for assessing the initial noise level.

d. Apply wavelet denoising

Apply wavelet denoising using the wavelet family to evaluate (e.g., Coiflet). Use appropriate thresholding methods to remove noise while preserving important signal features.

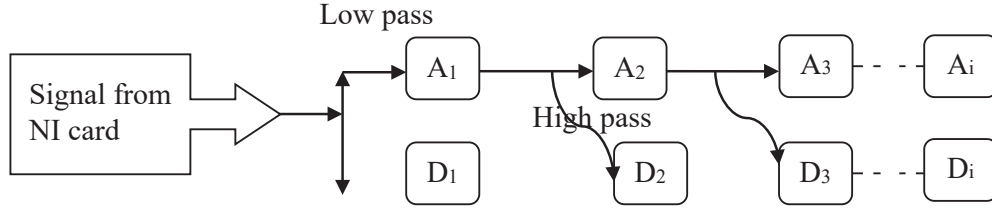


Figure 5: Decomposition process of a signal using DWT [13]. DWT, discrete wavelet transform.

e. Calculate denoised SNR

Calculate the SNR of the denoised signal produced by the chosen wavelet family.

f. Compare SNR improvement

Compare the improvement in SNR between the noisy signal and the denoised signal. A higher increase in SNR indicates better noise reduction.

g. Repeat for different wavelet families

Repeat steps 2–4 for other wavelet families to compare (e.g., Daubechies, Symlet). Calculate the SNR improvement for each wavelet family.

h. Choose the wavelet with highest SNR improvement

Select the wavelet family that yields the highest increase in SNR. This suggests that the chosen wavelet effectively removes noise while retaining the signal's integrity.

SNR used to evaluate the denoising effect, and the definition formula

$$SNR = 10 \lg \left[\frac{\sum_{t=1}^k s^2(t)}{\sum_{t=1}^k [s'(t) - s(t)]^2} \right]. \quad (3)$$

The SNR comparison table after denoising with db4, coif4, sym3, and bior2.2 wavelets is shown in Table 1.

It can be seen from Table 1 that SNR with the **coif4** wavelet is higher than that of db4, sym3, and bior2.2 wavelets. The larger the SNR, the better the denoising effect. The experimental results show that the **coif4** wavelet which meets the wavelet selection for the denoising effect.

Table 1: Average estimated error value of SS specimen sheet.

SNR input signal (dB)	db4	coif4	sym3	bior2.2
80	8.2710	13.3138	5.2091	6.6665

SNR, signal-to-noise ratio.

a. Determination of elastic constant

The frequency estimated using the multiresolution is further used to compute the elastic constants.

a.i Young's modulus

Young's modulus (E) of the test specimen can be determined by Eq. (4) as mentioned in [27, 24].

$$E = \frac{48}{\pi^2} \left(\frac{f}{\lambda(\nu)} \right)^2 \frac{ma^3(1-\nu^2)}{bt^3} \quad (4)$$

Where,

λ = non-dimensional frequency factor

ν = Poisson's ratio

m = mass of specimen

f = natural frequency of specimen

a = length of specimen

b = width of specimen

t = thickness of specimen.

a.ii. Stiffness

Stiffness is determine by It is given by Eq. (5). [28]

$$K = \frac{E^* A}{L} \quad (5)$$

Where, E = modulus of elasticity

A = area

L = length of material

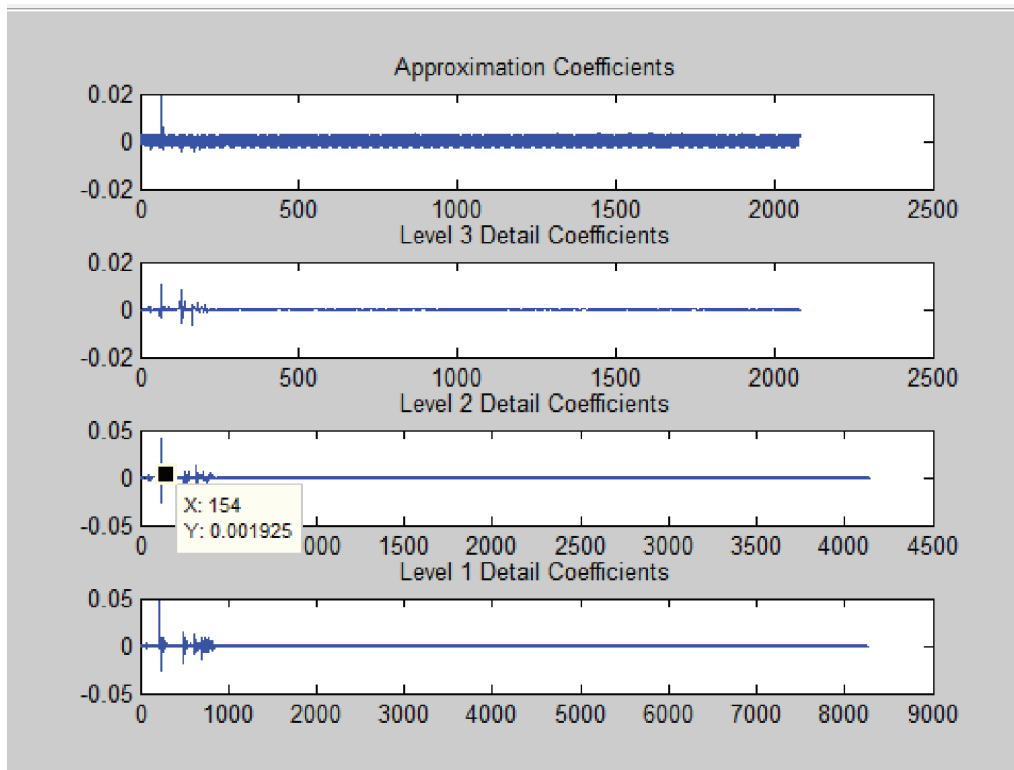


Figure 6: SS plate Level 2. SS, stainless steel.

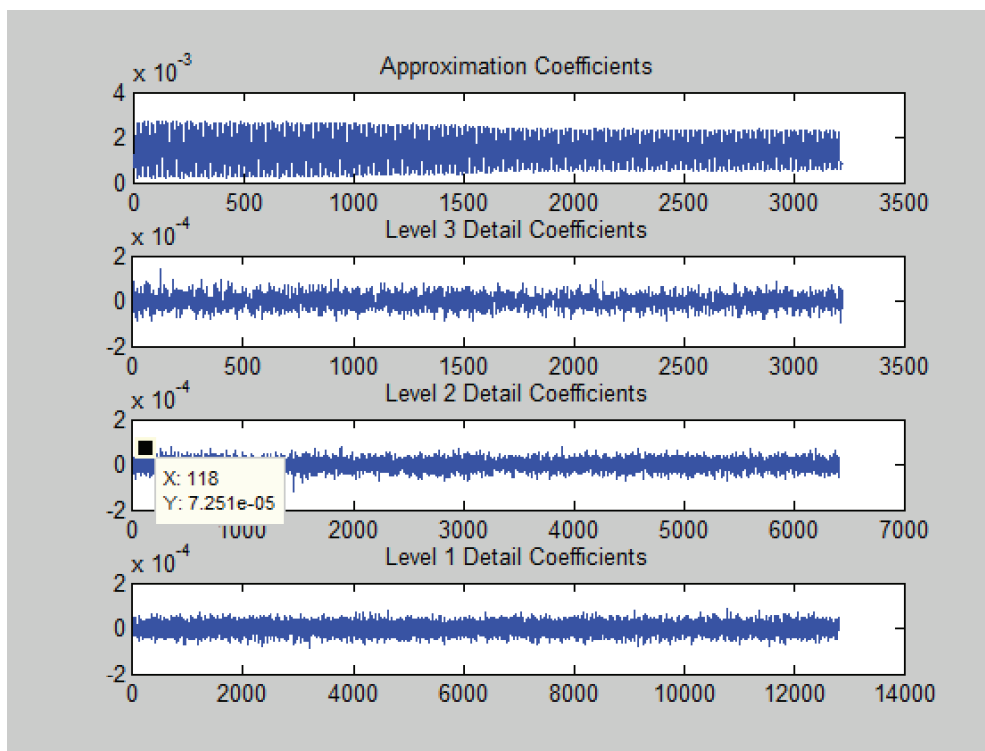
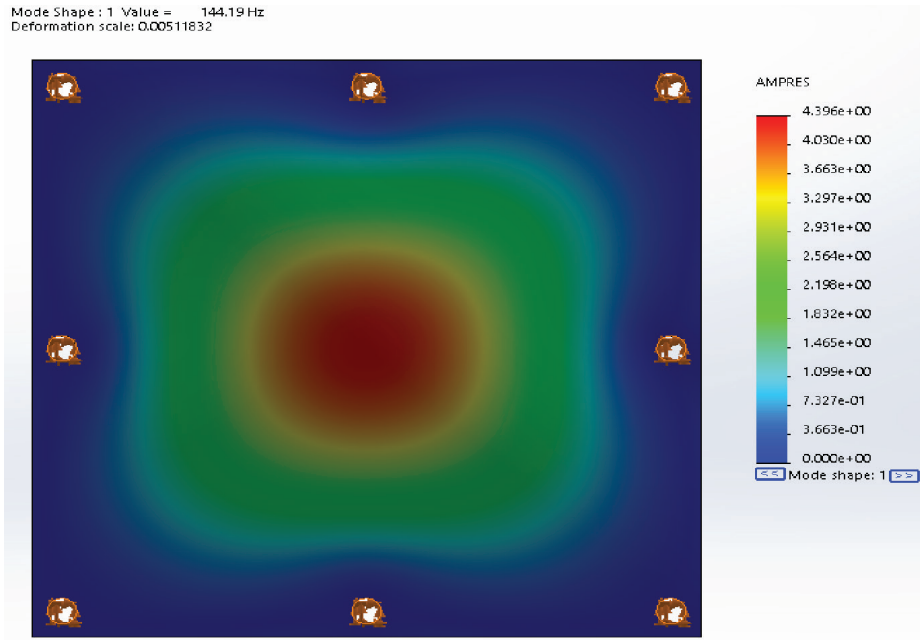
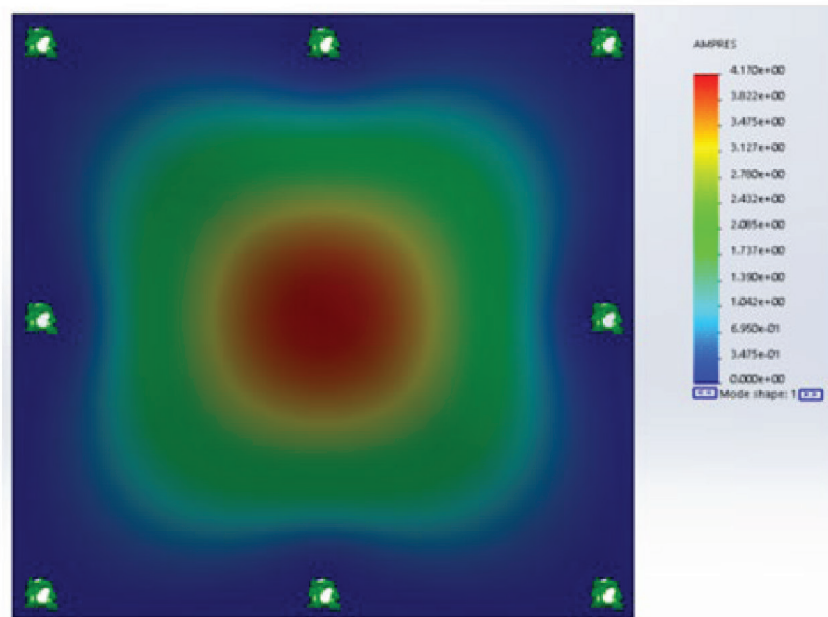


Figure 7: Copper plate level 2.



(A)

Mode shape: value = 107.04Hz



(B)

Figure 7: (A) The SS specimen sheet with dimensions 220 mm × 220 mm × 1.3 mm. Through ANSYS analysis, the fundamental frequency for the first modal vibration is determined to be 144.19 Hz, considering clamping along eight edges. (B) The copper specimen sheet measuring 220 mm × 220 mm × 1 mm. Utilizing ANSYS, the natural frequency of the first modal vibration is identified as 107.04 Hz when considering clamping along eight edges. SS, stainless steel.

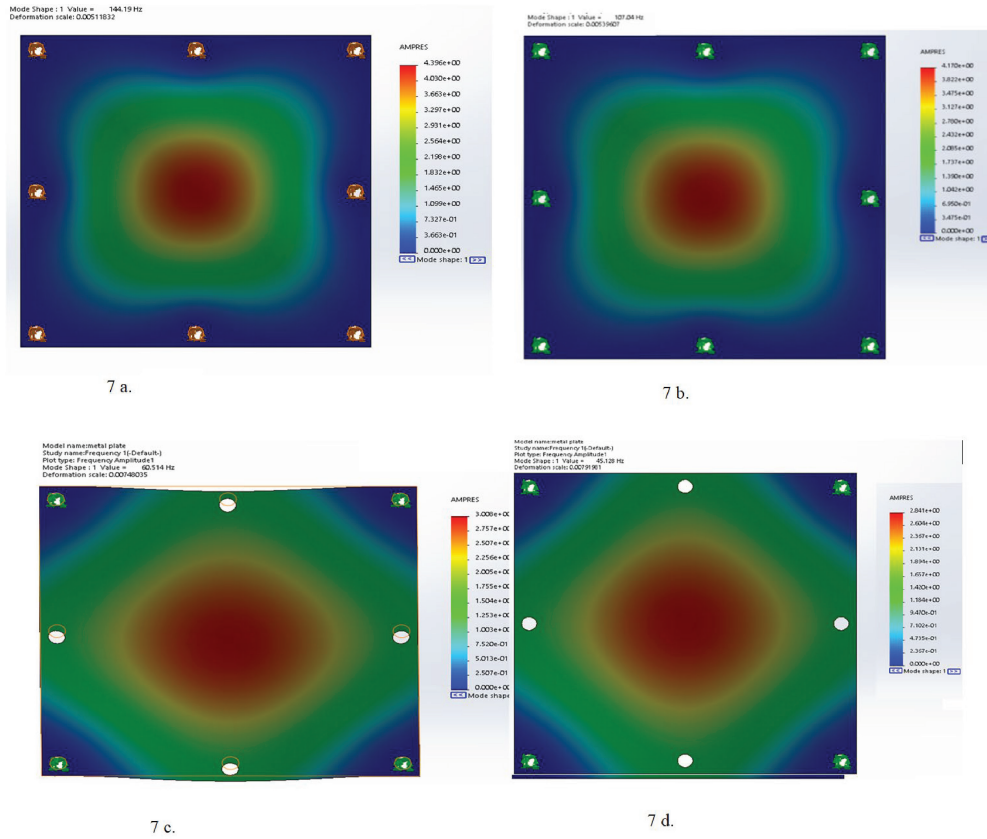


Figure 7: (A) The stainless steel specimen sheet measuring 220 mm × 220 mm × 1.3 mm. In ANSYS analysis, the first modal natural frequency is found to be 144.19 Hz when considering clamping along eight edges. (B) The copper specimen sheet sized 220 mm × 220 mm × 1 mm, with the ANSYS analysis revealing a first modal natural frequency of 107.04 Hz for eight clamp edges. (C) The stainless steel specimen sheet with the same dimensions (220 mm × 220 mm × 1.3 mm), clamping along four edges. 60.51 Hz. (D) The copper specimen sheet of dimensions 220 mm × 220 mm × 1 mm, similarly clamped along four edges, with ANSYS indicating a first modal natural frequency of 45.12 Hz.

b. Proposed method

The elastic constants of the plate, including Young's Modulus and stiffness, are inferred from the fundamental frequency. This paper employs a multiresolution approach to accurately estimate this fundamental frequency. Subsequently, this estimated frequency is harnessed to calculate the elastic constants. The methodology for determining these elastic constants is illustrated in Figure 4.

IV. Experimentation and Results

The experiment involved dropping a ball from varying heights onto different metal plates. Specifically, metal sheets of stainless steel (SS) and copper with dimensions of 220 mm × 220 mm × 1.3 mm and 220 mm × 220 mm × 1 mm, respectively, were

used for the experiment. To estimate the fundamental frequency of both the SS and copper plates, the obtained values were compared with the results from the FEM, employing ANSYS software. FEM, a robust and widely utilized numerical approximation technique, has demonstrated its efficacy in performance evaluations, as acknowledged by numerous researchers [17, 21]. The metal sheets of SS SA 240 Gr 304 and copper, sized 220 mm × 220 mm × 1.3 mm and 220 mm × 220 mm × 1 mm, were simulated within ANSYS to determine their fundamental frequencies, as depicted in Figures 7A and 7B. The natural frequencies were calculated as 144.19 Hz for stainless steel SA 240 Gr 304 and 107.04 Hz for copper, respectively.

Plots in Figures 8A–8D illustrate the power spectral density derived through wavelet transform. The

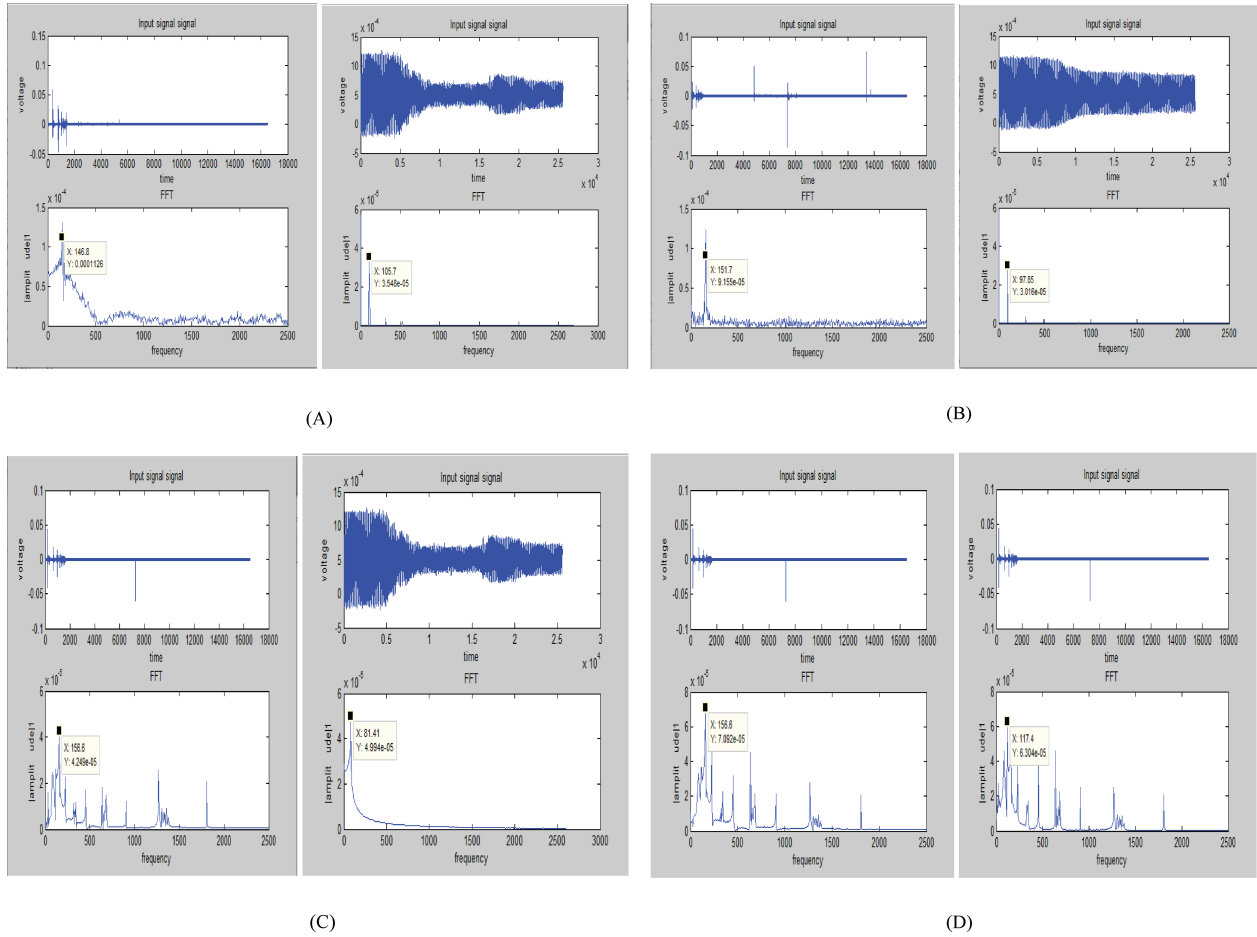


Figure 8: (A) The power spectral density of SS specimen sheet and copper specimen sheet, both measuring 220 mm $\times$ 220 mm $\times$ 1.3 mm, obtained using Coiflet wavelets. (B) The power spectral density of SS and Copper specimen sheets is depicted, with the analysis performed using Daubechies Wavelets. (C) A typical power spectral plot for SS and copper specimen sheets, each sized 220 mm $\times$ 220 mm $\times$ 1.3 mm, employing Symlets wavelets. (D) A similar depiction showcases the typical power spectral plot for SS and Copper specimen sheets, again measuring 220 mm $\times$ 220 mm $\times$ 1.3 mm, but utilizing BiorSplines wavelets. SS, stainless steel.

application of wavelet transform addresses the inherent resolution constraints of the Short Time Fourier Transform (STFT). A fundamental distinction between the wavelet transform and the STFT lies in their treatment of window lengths; the former exhibits variable window lengths and represents the signal as a summation of wavelets across distinct scales. The vibration signal originates from the release of a metallic ball weighing 200 g from a height of 90 cm, applied to both SS and copper specimen sheets. This depiction ensures originality while conveying the pertinent information.

Tables 2 and 3 provide the fundamental frequency values along with the corresponding average

percentage errors for both SS and copper specimen sheets. These values are obtained through experimentation involving metallic balls weighing 300 g and 200 g to generate impact. The average percentage error is calculated in relation to the fundamental frequency acquired via the FEM using ANSYS.

Table 2 presents the estimated frequencies along with the corresponding average errors for the specimen sheets of SS and copper. These sheets have dimensions of 220 mm $\times$ 220 mm $\times$ 1.3 mm and 220 mm $\times$ 220 mm $\times$ 1 mm, respectively. The impact is generated using a 300 g ball.

Table 3 showcases the estimated frequencies along with the corresponding

Table 2: Average estimated error value of copper specimen sheet.

Sr. no.	SS specimen plate					Copper specimen plate			
	Height of impact in cm	Frequency obtained by FEM (ANSYS software)	Estimated frequency using proposed method	Error (%)	Average error (%)	Frequency obtained by FEM (ANSYS software)	Estimated frequency using proposed method	Error (%)	Average error (%)
1	50	144.19	146.60	1.64	1.30%	107.04	105.7	1.26	1.18%
2	70		146.02	1.25			105.8	1.17	
3	90		146.80	1.77			105.6	1.36	
4	110		145.60	0.96			105.4	1.55	
5	130		145.80	1.10			106.2	0.80	
6	150		145.80	1.10			106.0	0.98	
Average frequency and error			146.10	1.30			105.78	1.18	

FEM, finite element method; SS, stainless steel.

Table 3: Elastic constant for SS specimen sheet.

Sr no.	SS specimen plate					Copper specwimen plate			
	Height of impact in cm	Frequency obtained by FEM (ANSYS software)	Estimated frequency using proposed method	Error (%)	Average error (%)	Frequency obtained by FEM (ANSYS software)	Estimated frequency using proposed method	Error (%)	Average error (%)
1	50	144.19	146.60	1.64	1.30%	107.04	105.7	1.26	1.18%
2	70		146.02	1.25			105.8	1.17	
3	90		146.80	1.77			105.6	1.36	
4	110		145.60	0.96			105.4	1.55	
5	130		145.80	1.10			106.2	0.80	
6	150		145.80	1.10			106.0	0.98	
Average frequency and error			146.10	1.30			105.78	1.18	

FEM, finite element method; SS, stainless steel.

average errors for the specimen sheets composed of SS and copper. These sheets possess dimensions of 220 mm × 220 mm × 1.3 mm and 220 mm × 220 mm × 1 mm, respectively. The impact is initiated using a 200 g ball.

V. Results

The stiffness of the SS specimen sheet and the value of Young's modulus are shown in Table 5. The calculated error in the FEM is compared with the value obtained from the suggested method of Young's modulus and stiffness. The average error for stiffness and Young's modulus is 0.14% and 1.97%, respectively.

The stiffness of the copper specimen sheet and the value of Young's modulus are shown in Table 6. The calculated error in the FEM is compared with the value obtained from the suggested method of Young's modulus and stiffness. The average error for stiffness and Young's modulus is 0.21% and 2.33%, respectively.

VI. Conclusion

The estimation of the vibration signal's fundamental frequency has been effectively accomplished through the use of the multiresolution Wavelet transform approach. Using the suggested method, the fundamental frequencies of copper and SS are estimated.

Table 4: Different wavelet estimated frequency and average error for specimen sheets of SS and copper with dimensions 220 mm × 220 mm × 1.3 mm and 220 mm × 220 mm × 1 mm using 200 g ball

Name of wavelet family	SS specimen plate		Copper specimen plate	
	Frequency obtained by FEM (ANSYS software)	Estimated frequency using proposed method	Frequency obtained by FEM (ANSYS software)	Estimated frequency using proposed method
Daubechies wavelets	144.19	151.7	107.04	97.80
Coiflet wavelet		146.10		105.7
Symlets wavelet		156.6		81.41
BiorSplines wavelet		156.8		117.6

FEM, finite element method; SS, stainless steel.

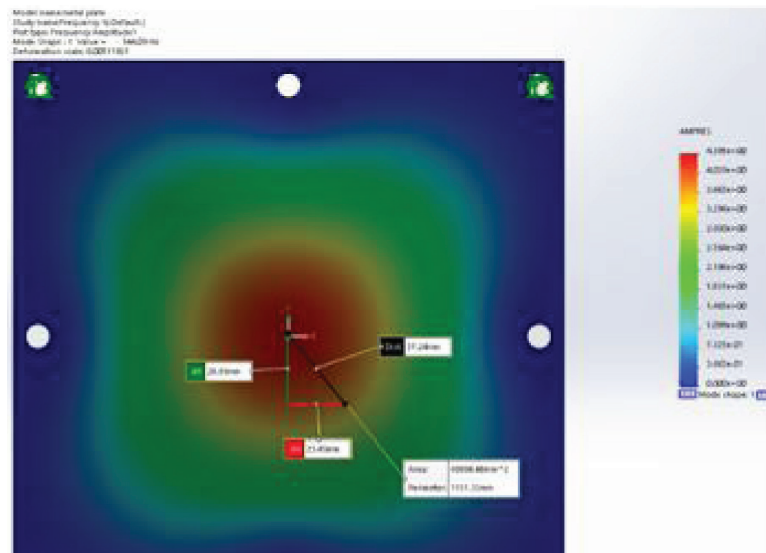


Figure 10: Location of the acceleration sensor on specimen sheets, 37.24 mm from center.

Table 5: The calculated error copper and SS plate frequency

SS plate size of 220 mm × 220 mm × 1.3 mm			
Parameter	Fundamental frequency (Hz)	Young's modules (N/m ²)	Stiffness (N/m ²)
FEM by ANSYS	144.19	187.69	17.06
Multiresolution method	146.10	191.39	17.20
Average error (%)	1.30	1.97	0.14

FEM, finite element method; SS, stainless steel.

Table 6: Elastic constant for copper specimen sheet

Copper plate size of 220 mm × 220 mm × 1 mm			
Parameter	Fundamental frequency (Hz)	Young's modules (N/m ²)	Stiffness (N/m ²)
FEM by ANSYS	107.04	133.82	12.55
Multiresolution method	105.78	129.49	12.34
Average error (%)	1.19	2.33	0.21

FEM, finite element method.

This algorithm has estimated the fundamental frequency with minimal error for coifelt 4 at the third level of decomposition, resulting in a more accurate estimate of the material's elastic constant due to the multiresolution nature of wavelet filters.

Fundamental frequency estimation has been conducted under a variety of test conditions, including changing the ball's weight and realizing height. The experimental outcomes are compared with those obtained using the FEM (ANSYS). For copper and stainless steel 304, the average frequency estimation errors are 1.15% and 1.36%, respectively. Copper and SS 304 Young's modulus average error.

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