



International Journal of Mathematics and Computer in Engineering

<https://reference-global.com/journal/IJMCE>

Original Study

Experimentally optimizing a spinning disk by manipulating its mass distribution and radius

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Communicated by Haci Mehmet Baskonus; Received: 14.01.2025; Accepted: 20.12.2025; Online: 02.02.2026

Abstract

The scientific method enables the experimental study of complex phenomena by isolating key variables. This work explores the significant properties of spinning bodies. Optimizing spinning disks is the primary aim of this work. Optimization is achieved by manipulating the moment of inertia (MOI) of the disk, allowing a longer duration of spin and lowering the rate of energy dissipation. Experiments are designed and conducted to explore the relationship between the radius and mass distribution of the disk and the angular deceleration experienced by it. Effects of the same on energy retention is analyzed. Empirical data is interpreted graphically while accounting for systematic and random uncertainties. Percentage change in duration of spin as a result of percentage change in physical quantities is studied. Moving mass away from the central axis of the spinning disk increases its duration of spin from a constant initial angular velocity. Energy retention is also improved. Increasing the radius of the disk increases the duration of spin and reduces the rate of energy dissipation. The above conclusions are drawn from experiments where the mass and thickness of the disk are controlled along with other necessary factors that can influence the results. The experiments confirm the existing theory relating to the moment of inertia, angular quantities, resistive torques and kinetic energy of spinning disks. The experiments provide insights into the behavior of spinning disks in practical situations, especially in problems concerned with optimization in the field of mechanical engineering.

Keywords: Spinning disk, angular quantities, moment of inertia, torque, kinetic energy.

AMS 2020 codes: 70B05; 70B10; 70E17; 03A99.

1 Introduction

The study of the physical world has long aimed to understand the dynamics of objects around us. Mathematics supports this effort through abstraction and logical analysis. *Mass* is one of the most fundamental concepts in physics [1]. In classical mechanics, mass is conserved and intrinsic to matter. Newton described it as *quantity of matter*, a core property introduced into physics to supplement size and shape-concepts Descartes had considered *sufficient* on their own [2]. This leads naturally to the study of *inertia*. Newton's first law states that *Every body preserves in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed thereon*. This paper focuses on MOI of spinning disks. In the *Principia Mathematica*, Newton illustrates this with the example of a top: *A top does not cease in its rotation, otherwise*

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than it is retarded by the air. He also notes that planetary bodies maintain their motion longer due to lower resistance in space [3]. This highlights the link between inertia, spinning motion, and resistive forces.

From electrons to galaxies, *spinning* appears to be a fundamental property of nature. In modern particle physics, the *spin* of fundamental particles remains an active area of study [4, 5]. The fascination with spinning tops from an early age, combined with their clear role in engineering, forms the motivation for this work. Physical and mechanical systems are often too complex to be studied in their entirety [6], but the scientific method allows to isolate key variables and study them in detail [7]. These investigations build on earlier work while developing new insights [8]. While the theory of rotational dynamics is well established, there are fewer controlled experiments that directly test how changes in radius and mass distribution affect spin behavior. This study aims to fill that gap by using a simple and repeatable setup to analyze these effects.

This study focuses on optimizing spinning disks by increasing their MOI to reduce angular deceleration and slow down energy loss. The radius and mass distribution of the disk are varied to study their influence on spin time. The effects on kinetic energy decay are also analyzed. The main sources of resistive torque are identified as bearing friction and drag. The experiment follows a structured method and yields results that can be applied in mechanical systems, especially those involving rotational dynamics [9, 10].

The contribution of this study lies in its detailed experimental analysis of how mass distribution and radius affect spin behavior and energy loss in rotating disks. While theoretical studies on MOI and angular deceleration exist, this work provides a simple, physical setup to test those ideas directly. It combines torque modeling with kinetic energy analysis, using both to validate classical results and extend them into real design use-cases. The findings offer practical insights into optimizing rotating systems in engineering.

The rest of this paper is organized as follows. Section 2 covers the theoretical background related to spinning disks. Section 3 outlines the experimental methodology based on the scientific method. Section 4 presents the experiment on mass distribution and its effect on angular deceleration and energy loss. Section 5 focuses on how changing the radius influences the same. Section 6 discusses general sources of error. Finally, Section 7 concludes the paper and highlights its applications.

2 Background

A *rigid body* is defined as a system of particles in which the distance between the particles remain unchanged. All *solids* change shape to some extent as a result of heat, air pressure or an external force. However, it is conventional to assume rigidity if the movements associated with the change in shape are very small when compared to the movements of the body as a whole [11]. The object studied in this paper is a rigid disk rotating about a fixed axis and executing plane motion. All particles of this rigid body move in circular paths about the axis of rotation, and all lines in the body which are perpendicular to the axis of rotation rotate through the same angle in the same time period. The angular velocity (ω) and angular acceleration (α) of the spinning disk are defined using Figure 1.

Consider a fixed plane P passing through the axis in Figure 1. If, in time t the plane moves from P_1 to P_2 and sweeps an angle of θ . Then, the angular velocity (ω) of the disk is given by:

$$\omega = \frac{d\theta}{dt}. \quad (1)$$

The angular acceleration (α) is given by

$$\alpha = \frac{d\omega}{dt}. \quad (2)$$

When force is applied to a body at rest the resistance offered by the body is due to the inertia of its mass. In the experiments that follow, the disk is rotated using the fixed axis through its center. This force, or torque (τ) provides the required change in angular velocity. Torque is directly proportional to the moment of inertia (I)

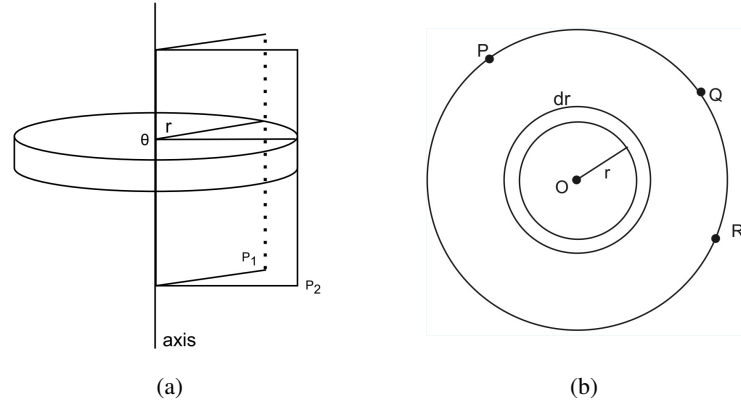


Fig. 1 Spinning disk with a fixed central axis (a) defining angular quantities, (b) cross section of the disk.

and is related to it in the following way [12]:

$$\tau = I\alpha. \quad (3)$$

The MOI of an object depends on the configuration of particles that form the object. In spinning disks, the distance of these particles from the axis of rotation heavily influences the MOI [13]. Consider a thin circular section PQR (Figure 1 (b)) with mass dM of the disk in Figure 1 (a). Let σ represent this section per unit area. Divide the section further into thin rings, one of which is represented in Figure 1 (b) with radius r and thickness dr . The mass of this thin disk is $\sigma 2\pi r dr$. Every particle of the ring is at a distance r from the axis. Thus, the MOI of the ring is

$$\sigma 2\pi r dr \cdot r^2 \quad \text{or} \quad \sigma 2\pi r^3 dr.$$

Thus, the MOI of this section PQR is

$$I = \int_0^h \sigma 2\pi r^3 dr = \sigma \frac{\pi b^2}{2} = \frac{dM}{2} b^2. \quad (4)$$

Now, if we consider the disk to be made up of an infinite number of sections like PQR, each having the MOI $\frac{dM \cdot b^2}{2}$ about the axis. Then for the whole disk made up of an infinite number of particles each of mass m , the MOI is [14]

$$\begin{aligned} \sum mr^2 &= \int dM \frac{b^2}{2} = \frac{Mb^2}{2} \\ I &= \frac{1}{2} Mb^2. \end{aligned} \quad (5)$$

The MOI of a compound object can be calculated using the parallel axis theorem. The MOI of an object about an axis through its center of mass represents the minimum MOI for any axis in that spatial direction. The MOI about an axis parallel to this through the center of mass is given by

$$I = I_{\text{cm}} + Md^2, \quad (6)$$

where I_{cm} is the MOI about the center of mass axis, M is the mass of the object, and d is the distance between the axes. This additional term corresponds to the MOI of a point mass located at the center of mass [15]. A disk rotated on its central axis has rotational kinetic energy by virtue of its motion. Formulating the empirical formula. Consider a point mass m_i at a distance r_i from the center of the disk. The kinetic energy of m_i is given by:

$$\text{KE}_i = \frac{1}{2} m_i v_i^2. \quad (7)$$

We know that:

$$v_i = r_i \omega, \quad (8)$$

hence,

$$KE_i = \frac{1}{2} m_i r_i^2 \omega^2. \quad (9)$$

The disk can be considered as a collection of point masses. Hence, the rotational kinetic energy (KE_{rot}) of the disk can be expressed using equation (9) as:

$$KE_{\text{rot}} = \frac{1}{2} \omega^2 \sum m_i r_i^2. \quad (10)$$

Kinetic energy can be expressed clearly in terms of the MOI using equation (5),

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2. \quad (11)$$

The disk has maximum Kinetic Energy when it's angular velocity is the highest. From that point on it is allowed to rotate freely and come to rest. It loses kinetic energy in pure passive deceleration, and the system is non-conservative. The disk loses energy in the form of bearing friction due to the contact between the axis and its mounting on the motor. The resultant torque produces a linear damping:

$$\tau_{\text{friction}} = -b\omega. \quad (12)$$

Energy is also lost due to air drag or aerodynamic resistance. Effects are significant at high speed. The resultant torque produces quadratic damping:

$$\tau_{\text{drag}} = -k\omega^2. \quad (13)$$

The total resisting torque can be modeled as:

$$\tau_{\text{total}} = -b\omega - k\omega^2. \quad (14)$$

3 Experimental methodology

The heart of scientific inquiry lies in its distinct and robust methodology. The scientific method provides a reliable process to frame and explore hypotheses. Moreover, results produced via the scientific method are accepted throughout the scientific community and beyond. A consistent and uniform body of knowledge. Experimentation is an integral tool in the physical sciences. It provides crucial evidence to support or refute a hypothesis, providing a firm basis for scientific knowledge. Controlled and repeatable experiments that produce empirical data can be used to study the relationship between physical quantities. It is also important to keep in mind the logical structure of scientific inquiry; and the connections between human reasoning, theories and experimentation [16, 17]. Criticisms of the process have emerged over the years, raising pertinent questions. On the subject of distinguishing between results produced by an apparatus and the disturbance created by the same: Hacking, in his 1981 paper wrote, "Do we see through a microscope?" [18]. Peter Galison raised an important concern about different members of a research group finding different pieces of evidence more convincing, based on their personal biases in his *How experiments End*. While examining the interactions between theory and experiment it is sometimes difficult to differentiate between relevant results and background noise, making it difficult to establish causal links [19]. The process of experimentation goes through continuous evaluation and subsequent changes. Despite its acknowledged limitations, the scientific method along with experimentation provides an effective way to study physical phenomenon.

Table 1 Description of variables for Experiment 1.

Independent Variable	Mass Distribution with respect to vertical axis [I]
Dependent Variable	Angular deceleration [$\theta \text{ s}^{-2}$]
Controlled Variables	Mass of the disk [g], Voltage supplied [V] Radius of disk [mm], Thickness of disk [mm]

Table 2 Physical quantities studied in Experiment 2.

Independent Variable	Radius of Disk [mm]
Dependent Variable	Angular deceleration [$\theta \text{ s}^{-2}$]
Controlled Variables	Mass of the disk [g], Voltage supplied [V] Radius of disk [mm], Thickness of disk [mm]

3.1 Physical quantities being studied

This paper focuses on optimizing spinning disks by reducing angular deceleration experienced by the disk. In order to study the same, the physical attributes of the disk were chosen while keeping in mind the feasibility of the experiment. The physical quantities chosen are well-suited for the relationships that are to be studied. The independent variable and dependent variable for experiment 1 and experiment 2 are outlined in Table 1 and Table 2, respectively. In Experiment 1 studies the effect of distribution of mass on the angular deceleration of the disk. In Experiment 2 the effect of radius on the angular deceleration of the disk is studied. It is necessary to outline control variables in order to establish an effective causal relationship between the dependent and the independent variables, information on the control variable is also provided in Table 1 and Table 2.

3.2 Requirements

The experiment was designed such that the required relationships between physical quantities could be established in the most accurate and precise manner possible. The apparatus and materials used to conduct the experiments were selected such that the concerned experiment could be conducted feasibly while keeping in mind the necessary control variables. The apparatus used along with their respective least counts and uncertainties are outlined in Table 3. It is essential to recognize the uncertainty of the apparatus in order to conduct an effective error analysis of the experiments. These uncertainties will determine the extent of the applicability of the results of the experiments. The materials used in the experiment are outlined in Table 4.

3.3 Experimental setup

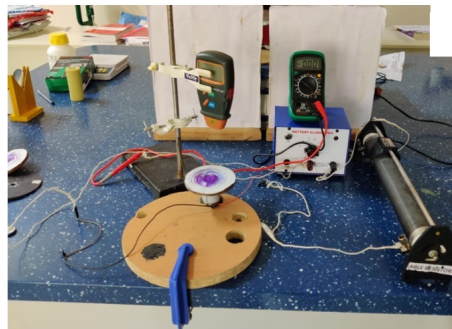
The experiment was set up on a table with a horizontal surface in the Physics Lab. The DC Motor was mounted on a wooden object with a horizontal surface using an adhesive. This was done to avoid fixing the motor directly on the table. It would also provide additional rigidity to the setup. The surface with the motor was fixed on the table using metal clamps. The required rigid metal disk was mounted on the motor using a plastic connector that was permanently fixed to the disk. A strip of reflective tape was fixed to the edge of the disk. The digital Tachometer was fixed at the required height: not more than 15 cm away from the disk, in

Table 3 Description of apparatus.

S. No.	Apparatus	Quantity	Least Count	Uncertainty
1	Scale (30 cm)	1	1 mm	0.5 mm
2	Digital Tachometer	1	1 RPM	1 RPM
3	Digital Weighing Scale	1	0.1 g	0.1 g
4	Digital Voltmeter	1	0.01 V	0.01 V

Table 4 Description of materials.

Material	Description
DC Motor	10 V with a flat base can be mounted on any flat surface. Thin metal axis.
Battery	2 V to 12 V .
Connecting wires	Made up of copper and insulated with thread.
Rheostat	A device with variable resistance.
Reflective tape	Stuck on the disk. Helps the tachometer record the RPM.
Metal disk	Mass - 175 ± 1 g ; Radius - 50 ± 1 mm ; Thickness - 0.3 ± 0.1 mm
Iron disk	Mass - 248 ± 1 g ; Radius - 91 ± 1 mm ; Thickness - 6 ± 1 mm
Copper disk	Mass - 245 ± 1 g ; Radius - 78 ± 1 mm ; Thickness - 6 ± 1 mm
Aluminum disk	Mass - 249 ± 1 g ; Radius - 141 ± 1 mm ; Thickness - 6 ± 1 mm
Small stainless-steel disk $\times 4$	Mass 11 ± 1 g each. Used to add mass to the metal disk.
Stand	Used to position the Digital Tachometer.
Metal clamps $\times 2$	Used to firmly attach the material on which the motor is mounted to the tabletop.
Blu-Tact	A rubbery material used to add mass to the disk.
M-Seal	An adhesive paste used to mound the motor onto any required surface.
Fevi-Quick	A very strong liquid adhesive.

**Fig. 2** Visual representation of experimental setup.

order to ensure accuracy of the readings. It was positioned such that the beam aligned with the reflective tape. The motor was connected to the battery eliminator and the rheostat in series and the voltmeter in parallel using insulated copper wires. The experiment setup is displayed in Figure 2.

3.4 Experimental procedure

Following the setting up of the experiment with the required configurations is given. The stopwatch was started simultaneously along with the battery. The battery was set at a fixed voltage for Experiment 1 and it was varied in Experiment 2. The reading from the tachometer was noted down every 10 seconds as the top experienced angular acceleration. The stopwatch was stopped when the top reached a constant RPM. The constant RPM was identified visually. The disk was allowed to spin for at-least 15 seconds at the constant RPM to ensure that the angular velocity has stabilized. This time is noted down along with the voltage reading. The stopwatch was then reset. The stopwatch was started once again, and the battery was turned off simultaneously. The reading from the tachometer was noted down every 10 seconds as the top experienced angular deceleration. The stopwatch was stopped exactly when the top came to rest and this time was noted down. 5 trials were conducted for each setup of each experiment. Repeated trials were conducted to reduce the effect of significant random errors on the collected data. It was ensured that the temperature of the room remained constant at 24°C . The fans in the room were always turned off to avoid interference in the form of drag. Between trials, it

was ensured that the motor is kept off for a while to avoid wear from constant use. The battery was turned off to allow the wires and motor to cool down and reduce the effects of heating. All trials were started in similar thermal conditions. All electrical connections were re-checked after each trial.

3.5 Risk assessment

The experiment involved a fairly heavy metal disk rotating at high speeds. It was ensured that the disk was mounted firmly on the motor in order to avoid it from becoming loose while in motion, as it could cause injury. Protective eye-wear was used at all times. Gloves were used while mixing M-seal (an industrial adhesive paste). The battery was turned on only after making sure all connections were completed in order to avoid electric shock. An additional precaution was required for Experiment 1 as there was a possibility that the attached masses would detach while the top was in motion. In such an incidence they would fly at a very high velocity in a tangential path. To avoid this, the additional masses were firmly fixed onto the disk using a very strong adhesive.

3.6 Data collection and processing

The data was collected in RPM or Rotations Per Minute. This was converted into angular velocity in the following way:

$$\text{rotations per second} = \frac{RPM}{60}, \quad (15)$$

$$\text{angular velocity} = \frac{RPM}{60} \times 2\pi. \quad (16)$$

As 5 trials were conducted the average angular velocity ($\bar{\omega}$) was calculated in the following way:

$$\bar{\omega} = \frac{\omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5}{5},$$

where ω_1 , ω_2 , ω_3 , ω_4 , and ω_5 are the values of angular velocity from Trial 1, Trial 2, Trial 3, Trial 4, and Trial 5 respectively. The Uncertainty in $\bar{\omega}$ was calculated in the following way:

$$\Delta\bar{\omega} = \frac{\omega_{Max} - \omega_{Min}}{2},$$

where ω_{Max} is the largest value from the 5 conducted trials and ω_{Min} is the smallest. The uncertainty in average voltage ($\Delta\bar{V}$), average time taken to reach constant angular velocity ($\Delta\bar{T}_1$) and the average time taken to come to rest ($\Delta\bar{T}_2$) was calculated using the same method. This method of calculating uncertainty was used as it is highly suitable for an experiment involving repeated trials [20]. As multiple trends had to be graphed together, it was not feasible to add error bars. The results of the experiments conducted were analyzed using graphs. All graphs were plotted using Python 3.9.12. [21–25].

4 Effect of mass distribution on angular deceleration

This experiment aims to establish a relationship between the distribution of mass relative to the central axis of a spinning disk and the angular deceleration it experiences. Rate and nature of kinetic energy dissipation due to resistive torques is also explored. It can be hypothesized that moving mass away from the central axis of the disk will increase the MOI and subsequently reduce angular deceleration while increasing energy retention. This is confirmed by Equation 5. A Metal disk (Mass is 175 ± 1 g; Radius is 50 ± 1 mm; Thickness is 0.3 ± 0.1 mm) and 4 smaller stainless-steel disks (each of mass -11 g) used as additional masses were used in this experiment. The smaller disks were fixed on the top of the metal disk at varying distances from its center. They were positioned such that the distance (d) between the center of the metal disk and the 4 smaller disks

was first 0 ± 0.01 cm (d_1) and then progressively increased to 1.15 ± 0.01 cm (d_2), 2.5 ± 0.01 cm (d_3) and finally 3.85 ± 0.01 cm (d_4). These specific values were chosen to make full use of the 5 cm radius of the main disk, while leaving enough space between each configuration to observe distinct changes in moment of inertia. The largest value, 3.85 cm, was selected as the safe upper limit beyond which placing the masses would risk contact with the disk edge or detachment during high-speed rotation. This also allowed for clear and measurable variation across setups without compromising symmetry or safety. The small disks were fixed such that they were equidistant from each other, this is important to ensure that the additional mass is distributed evenly. This was achieved by drawing two mutually perpendicular diameters on the top of the metal disk, outlining the required position and then finally fixing the disk using an adhesive of negligible mass. The setup is the same as described in *experimental setup*. The *experimental procedure* was used to carry out the experiment.

4.1 Calculating moment of inertia

The setups include a central disk (Mass- 175 ± 1 g; Radius- 50 ± 1 mm; Thickness - 0.3 ± 0.1 cm) and 4 additional smaller disk-shaped masses (Mass- 11 ± 1 g; Radius 0.7 cm). The calculation for the MOI of the above described compound object (which will be subsequently referred to as a "disk") is carried out below. Using Equation 5, MOI of central disk (I_c) :

$$I_c = \frac{1}{2} \times 175 \text{ g} \times (5 \text{ cm})^2 = 2189 \text{ g cm}^2.$$

The MOI of individual smaller disk (I_s):

$$I_s = \frac{1}{2} \times 11 \text{ g} \times (0.7 \text{ cm})^2 = 2.7 \text{ g cm}^2.$$

MOI of Setup 1 (I_1) :

$$I_1 = I_c + 4I_s = 2211 \text{ g cm}^2.$$

Using the *Parallel Axis Theorem*, MOI of setup 2 (I_2) can be calculated by

$$I_n = I_c + 4(I_s + md^2),$$

where m is the mass of the additional object placed and d is the distance that mass is placed at from the axis of the disk. Hence

$$I_2 = I_c + 4(I_s + 11 \text{ g} \times (1.15 \text{ cm})^2) = 2189 + 4(2.7 + 11 \text{ g} \times (1.15 \text{ cm})^2) = 2258 \text{ g cm}^2.$$

MOI of Setup 3 (I_3) :

$$I_3 = I_c + 4(I_s + 11 \text{ g} \times (2.5 \text{ cm})^2) = 2189 + 4(2.7 + 11 \text{ g} \times (2.5 \text{ cm})^2) = 2476 \text{ g cm}^2.$$

MOI of Setup 4 (I_4) :

$$I_4 = I_c + 4(I_s + 11 \text{ g} \times (3.85 \text{ cm})^2) = 2189 + 4(2.7 + 11 \text{ g} \times (3.85 \text{ cm})^2) = 2852 \text{ g cm}^2.$$

4.2 Results

Angular velocity ($\bar{\omega} \pm \Delta\bar{\omega}$) is calculated as stated in *data collection and Processing* and is presented in Table 5 against its respective value of time. Figure 3 shows that as we move from Setup 1 to Setup 4 the duration of spin of the disk increases consistently. Comparing the percentage change in MOI with the percentage change in duration of spin. From Setup 1 to Setup 2 there is a 2 % increase in MOI and a resultant 19 % increase in the

duration of the spin. From Setup 1 to Setup 3 there is a 12 % increase in MOI and a resulting 27 % increase in the duration of the spin. Lastly, from Setup 1 to Setup 4 there is a 29 % increase in MOI and a resulting 42 % increase in time of spin. The rate of change (c) in duration of spin as a result of the change in MOI is calculated as the ratio of the percentage change in spin duration to the percentage change in MOI.

From Setup 1 to Setup 2

$$c_{1 \rightarrow 2}(\text{MOI}) = \frac{19\%}{2\%} = 9.50,$$

from Setup 1 to Setup 3

$$c_{1 \rightarrow 3}(\text{MOI}) = \frac{27\%}{12\%} = 2.25,$$

from Setup 1 to Setup 4

$$c_{1 \rightarrow 4}(\text{MOI}) = \frac{42\%}{29\%} = 1.45.$$

A 1 % increase in MOI as a result of the change in the distribution of mass, an average causes a 4.4 % increase in the duration of the spin. As the maximum angular velocity for each setup differs minimally it can be stated that as the additional mass moves away from the center, the angular deceleration decreases. There is a negative correlation between the distance of mass from the axis and angular deceleration. This conclusion can help in the optimization of disks in the following way: if they are designed such that most of the mass is concentrated away from the axis, it will spin for a longer time for a given initial angular velocity. The collected data was used to plot Angular Acceleration versus Time using Equation 2. Figure 4 shows continuous non-uniform angular deceleration that only decreases with time, this is expected in the situation at hand: a rotating disk coming to rest. The experiment is also consistent with the fact that rotating bodies lose angular velocity quickly initially and then slowly thereon [26], this can be seen in Figure 3: the plots move upwards (corresponding to lower angular deceleration) as the disk continues to spin.

The Kinetic energy of the disk is calculated using equation (11) and is plotted in Figure 5. The graphs are monotonically decreasing as is the case in a system under pure passive deceleration. In all setups, the rate of energy decay is initially steep and later gradually levels off. This is indicative of the fact that at higher angular velocities more energy is lost due to damping. Increasing the radial distance of mass increases the moment of inertia. It is observed that the disks with their mass pushed outwards have a higher initial Kinetic Energy and requires more time for the Kinetic Energy to reach zero. Graphical analysis yields that outward mass distribution minimizes energy loss rates and enables optimization with a longer duration of spin. To better understand the components of energy dissipation, the total resistive torque was modeled using equation (14). The models for Setup 1–4 are as follows:

$$\begin{aligned}\tau_{(1)} &= -1.0447 \times 10^{-5} \omega + 1.5248 \times 10^{-8} \omega^2, \\ \tau_{(2)} &= -7.8670 \times 10^{-6} \omega + 4.7184 \times 10^{-9} \omega^2, \\ \tau_{(3)} &= -7.0533 \times 10^{-6} \omega - 2.7894 \times 10^{-9} \omega^2, \\ \tau_{(4)} &= -7.8818 \times 10^{-6} \omega - 2.4683 \times 10^{-9} \omega^2.\end{aligned}$$

The positive coefficient of linear damping (b) value across all setups indicates that bearing friction is the dominant source of energy loss. In Setup 1 and Setup 2 the coefficient of quadratic damping (k) is negative, indicating that the effect of drag is not abundant when the additional masses are placed close to the center of the disk and the MOI is smaller. A negative value is also not physically possible, indicating that there is noise. Potentially, it is induced by mechanical disturbances. In Setups 3 and 4 k is positive, hence, the torque produced due to drag resistance is relatively significant. However, bearing friction is the main source of resistive torque. The damping model indicates that effect of drag resistance increases when mass is pushed away from the center. Torque vs time graphs for all setups is plotted in Figure 6. The blue line shows the actual torque:

Table 5 Angular deceleration vs time.

S. No.	Setup 1	Setup 2	Setup 3	Setup 4
d (cm)	$(d_1) = 0$	$(d_2) = 1.15 \pm 0.01$	$(d_3) = 2.5 \pm 0.01$	$(d_4) = 3.85 \pm 0.01$
$\bar{V} \pm \Delta \bar{V}$	$(7.60 \pm 6.06) V$	$(7.64 \pm 0.07) V$	$(7.66 \pm 0.08) V$	$(7.64 \pm 0.12) V$
Time(s)	$(\bar{\omega} \pm \Delta \bar{\omega}) \text{ rad/s}$	$(\bar{\omega} \pm \Delta \bar{\omega}) \text{ rad/s}$	$(\bar{\omega} \pm \Delta \bar{\omega}) \text{ rad/s}$	$(\bar{\omega} \pm \Delta \bar{\omega}) \text{ rad/s}$
0	207.5 ± 1.2	235.9 ± 2.1	231.5 ± 1.9	228.1 ± 0.7
10	143.1 ± 38.9	165.6 ± 3.7	158.4 ± 4.3	160.1 ± 5.6
20	78.7 ± 3.3	112.1 ± 5.9	112.6 ± 3.9	113.9 ± 3.7
30	52.0 ± 2.3	77.6 ± 1.9	79.0 ± 3.6	81.5 ± 4.0
40	34.8 ± 1.9	52.4 ± 1.7	60.1 ± 12.2	61.6 ± 5.1
50	20.3 ± 1.1	35.9 ± 1.8	38.5 ± 2.1	44.1 ± 1.6
60	7.4 ± 2.1	22.8 ± 1.7	25.7 ± 1.3	29.9 ± 2.4
70	-	10.6 ± 1.6	14.3 ± 1.8	20.6 ± 2.1
80	-	-	-	12.0 ± 0.4
90	-	-	-	5.1 ± 2.1

Table 6 Time rise and time fall.

	Time Rise	Time Fall
S. No.	$\bar{T}_1 \pm \Delta \bar{T}_1 (s)$	$\bar{T}_1 \pm \Delta \bar{T}_1 (s)$
Setup 1	41.23 ± 2.69	64.10 ± 0.95
Setup 2	53.75 ± 2.19	75.84 ± 1.80
Setup 3	53.66 ± 2.84	80.87 ± 1.20
Setup 4	58.85 ± 2.74	90.52 ± 0.60

$$\tau_{\text{actual}} = I \cdot \alpha,$$

whereas, the red line shows the predicted torque

$$\tau_{\text{predicted}} = -b\omega - k\omega^2.$$

In all setups predicted torque closely follows the overall trend of the actual torque. The damping model fits the experimental data. As we move from Setup 1 to Setup 4 the energy dissipation becomes more gradual, hence, the system is able to store energy for a longer time. Validating the model ensures that the energy dissipation of the disks closely follow physical laws discussed in Section 2. Rotating disks can be optimized by choosing appropriate b and k values that meet the expectations of energy retention.

This experiment shows that moving mass outward improves spin duration. However, real-world machines cannot always follow this design. In clutch plates, brake rotors, or spinning sensors, added mass at the outer edge can cause imbalance or wear at high speeds. Applications like handheld tools or electric fans have weight limits that restrict how mass can be distributed. In these cases, the benefit of longer spin time must be weighed against the cost of extra load on the motor or bearings. The results in this paper can help estimate how far the mass can be moved from the center before other effects become a problem. Engineers can use this to build parts that spin longer, but still meet safety and weight needs.

4.3 Error analysis

There were small changes in the voltage due to inconsistencies in the voltage supplied by the battery eliminator, thus proving to be a physical constraint inducing systematic error. The accuracy of the data collected can be improved by using a more consistent battery eliminator. Another systematic error was due to

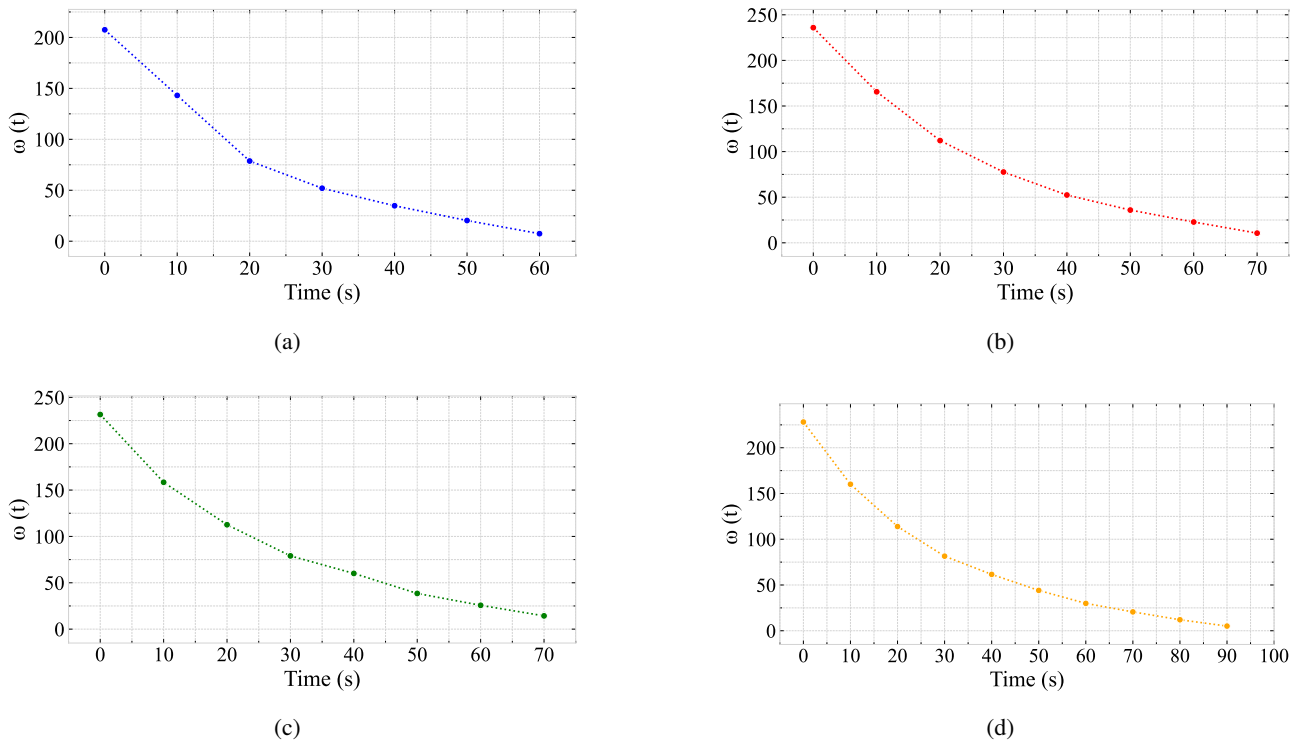


Fig. 3 Angular velocity versus time (a) 0 ± 0.01 cm (d_1) , (b) 1.15 ± 0.01 cm (d_2) , (c) 2.5 ± 0.01 cm (d_3) and (d) 3.85 ± 0.01 cm (d_4).

the presence of air resistance or drag force. The shape of the disk was not uniform after attaching the additional masses, increasing the influence of air resistance on the accuracy of the readings [27]. This explains the overlapping of the trends in Setup 2 and Setup 3 and the noticeable difference in the final constant velocity reached in Setup 1 from the rest of the setups in Figure 3.

5 Effect of radius on angular deceleration

The aim of this experiment is to establish a relationship between the radius of the disk and the extent of angular deceleration experienced by it. Rate and nature of kinetic energy dissipation due to resistive torques is also explored. This is being studied in the context of optimization or minimizing the angular deceleration experienced by the spinning disk and increasing energy retention. It can be hypothesized using equation (5) that an increase in radius will result in an increase in MOI and further an increase in the duration of spin of the disk. Three different disks of the same mass and thickness but different radii are used. Disk 1: Copper (Mass - 245 ± 1 g; Radius - 78 ± 1 mm; Thickness - 6 ± 1 mm), Disk 2: Iron (Mass - 248 ± 1 g; Radius - 91 ± 1 mm; Thickness - 6 ± 1 mm) and Disk 3: Aluminum (Mass - 249 ± 1 g; Radius - 141 ± 1 mm; Thickness - 6 ± 1 mm). The experiment is set up as described in experimental setup. The MOI of Disk 1, Disk 2 and Disk 3, with radii $r_1 = 7.8$ cm, $r_2 = 9.1$ cm, $r_3 = 14.1$ cm is calculated using equation (5) to be $I_1 = 7,452$ g cm², $I_2 = 10,268$ g cm², $I_3 = 24,751$ g cm² respectively. All disks must have a final constant RPM of 2000. To achieve this the rheostat was set at the lowest possible resistance and the battery was turned on. The slider of the rheostat was then moved slowly until the disk reached a constant velocity of about 2000 RPM. There is a maximum percentage change of only 0.4% in the final constant RPM of the disks, indicating that the process was carried out with precision. The methodology developed to achieve this result is positively evaluated. This process was repeated for all disks and their respective voltages were noted down. The experiment was then conducted as

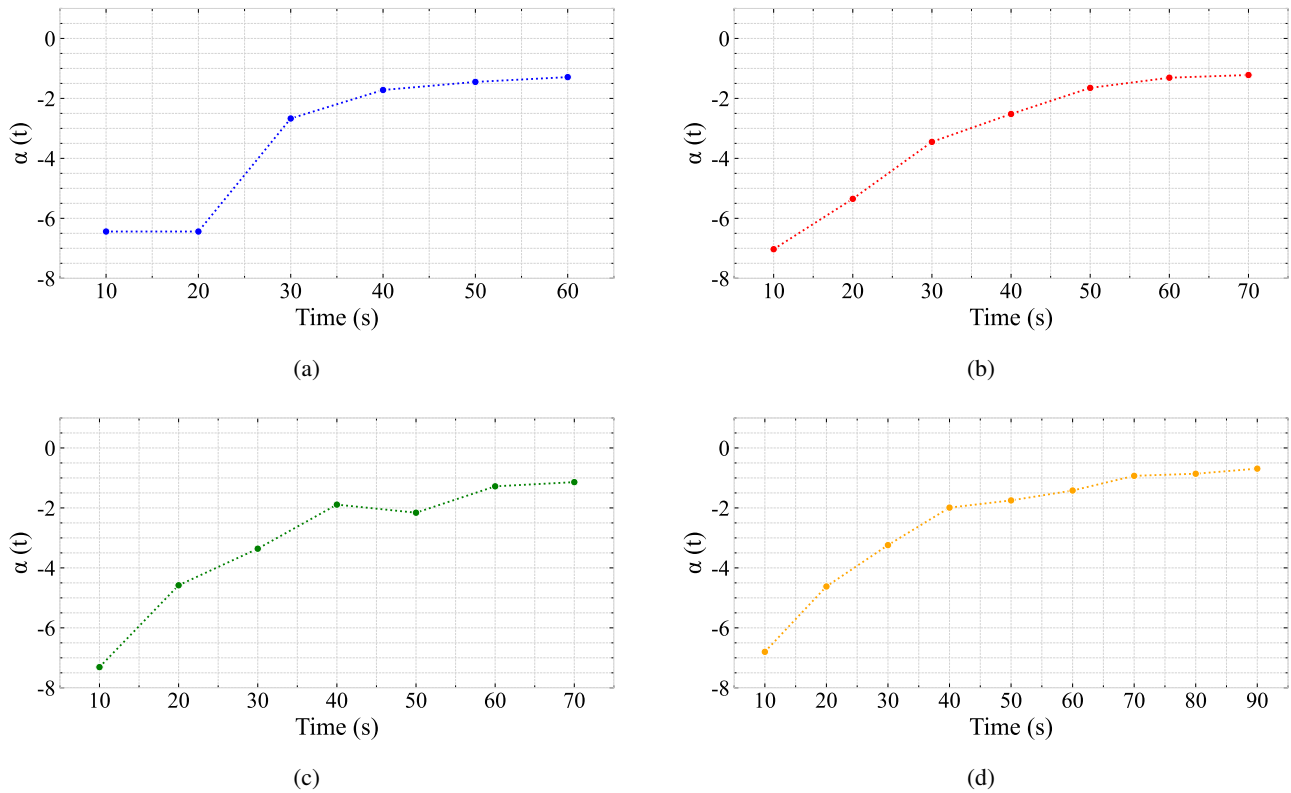


Fig. 4 Angular acceleration versus time (a) 0 ± 0.01 cm (d_1) , (b) 1.15 ± 0.01 cm (d_2) , (c) 2.5 ± 0.01 cm (d_3) and (d) 3.85 ± 0.01 cm (d_4).

described in Subsection 3.4 with the respective voltages of each disk found as discussed.

5.1 Results

Angular Velocity ($\bar{\omega} \pm \Delta\bar{\omega}$) is calculated as stated in Subsection 3.6 and is presented in Table 6. A noticeable trend can be observed in Figure 7, there is an increase in the spin time of the disk as we move from Figure 7(a) to Figure 7(b) and then to Figure 7(c). Total time taken to reach maximum angular velocity and to come to rest is presented in Table 8. This shows that a disk with a larger radius experiences less angular deceleration thus effectively increasing the duration of spin. Comparing the percentage change in radius and MOI with the percentage change in duration of spin. As we move from Setup 1 to Setup 2, the radius and MOI increase by 17 % and 37 % respectively, while the duration of spin increases by 48 %. Further, as we move from Setup 1 to Setup 3, the radius and MOI increase by 81 % and 232 % respectively, while the duration of spin increases by 177 %. Lastly, as we move from Setup 2 to Setup 3, the radius and MOI increase by 54 % and 141 % respectively, while the duration of spin increases by 88 %. The rate of change (c) in duration of spin is calculated as the ratio of the percentage change in spin duration to the percentage change in the radius and in the MOI separately.

From Setup 1 to Setup 2

$$c_{1 \rightarrow 2}(\text{Radius}) = \frac{48\%}{17\%} = 2.82;$$

$$c_{1 \rightarrow 2}(\text{MOI}) = \frac{48\%}{37\%} = 1.30;$$

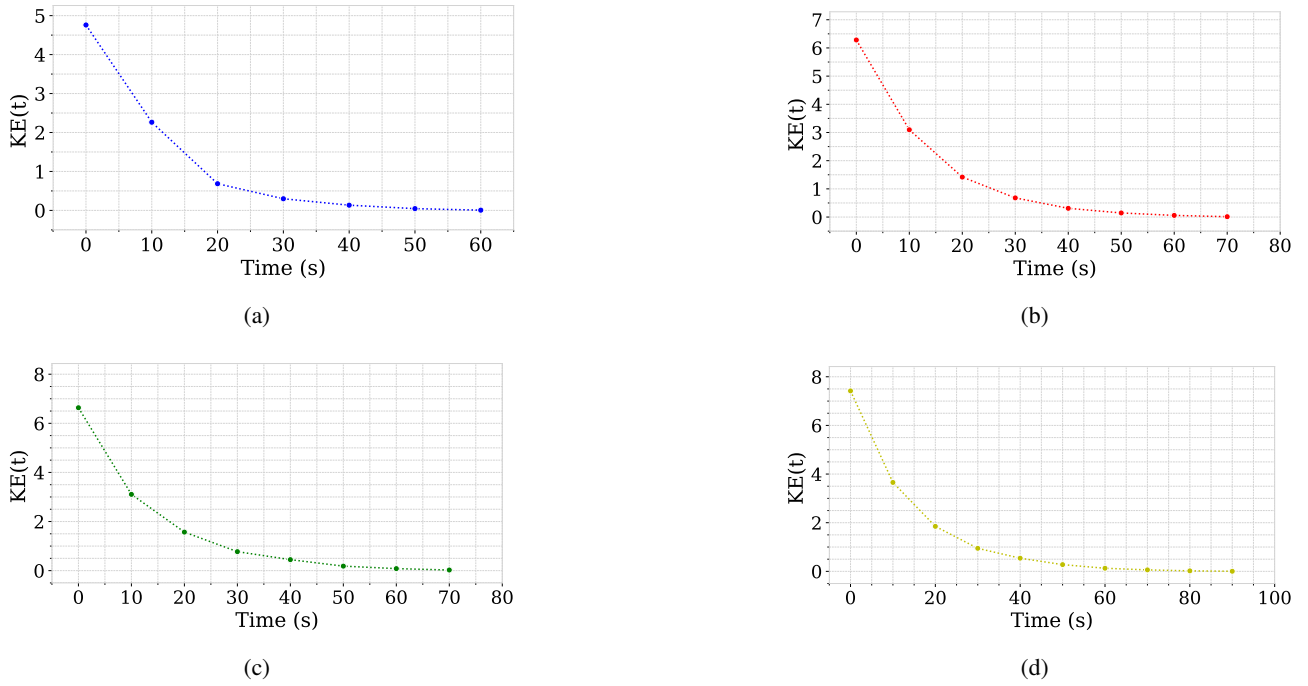


Fig. 5 Kinetic Energy versus Time (a) 0 ± 0.01 cm (d_1), (b) 1.15 ± 0.01 cm (d_2), (c) 2.5 ± 0.01 cm (d_3), and (d) 3.85 ± 0.01 cm (d_4).

from Setup 1 to Setup 3

$$c_{1 \rightarrow 3}(\text{Radius}) = \frac{177\%}{81\%} = 2.19;$$

$$c_{1 \rightarrow 3}(\text{MOI}) = \frac{177\%}{232\%} = 0.76;$$

from Setup 2 to Setup 3

$$c_{2 \rightarrow 3}(\text{Radius}) = \frac{88\%}{54\%} = 1.63;$$

$$c_{2 \rightarrow 3}(\text{MOI}) = \frac{88\%}{141\%} = 0.62.$$

From graphical and empirical results, it can be concluded that there is a negative correlation between the radius of a disk and the angular deceleration it experiences. A 1 % increase in radius, on average causes a 2.5 % increase in the duration of the spin. Further, a 1 % increase in moment of inertia, as a result of increase in radius, causes a 1 % increase in duration of spin. Thus, to optimize a spinning disk, the largest practical radius must be considered. This experiment is also consistent with the fact that rotating bodies lose angular velocity quickly initially and then slowly thereon, this can be seen in Figure 8: the plots move upwards (corresponding to lower angular deceleration) as the disk continues to spin.

The kinetic energy of the disk is evaluated using equation (11) and shown in Figure 9. As in the previous case, the kinetic energy decreases monotonically, confirming passive deceleration of the system. Here, the observed variations arise from changes in disk radius, which directly influence the moment of inertia, as summarized in Table 7. Disks with larger radii exhibit higher initial kinetic energy and require a longer time for the kinetic energy to decay to zero. This indicates that increasing the disk radius reduces the rate of energy dissipation and enables sustained rotational motion for a longer duration of spin. To better understand the

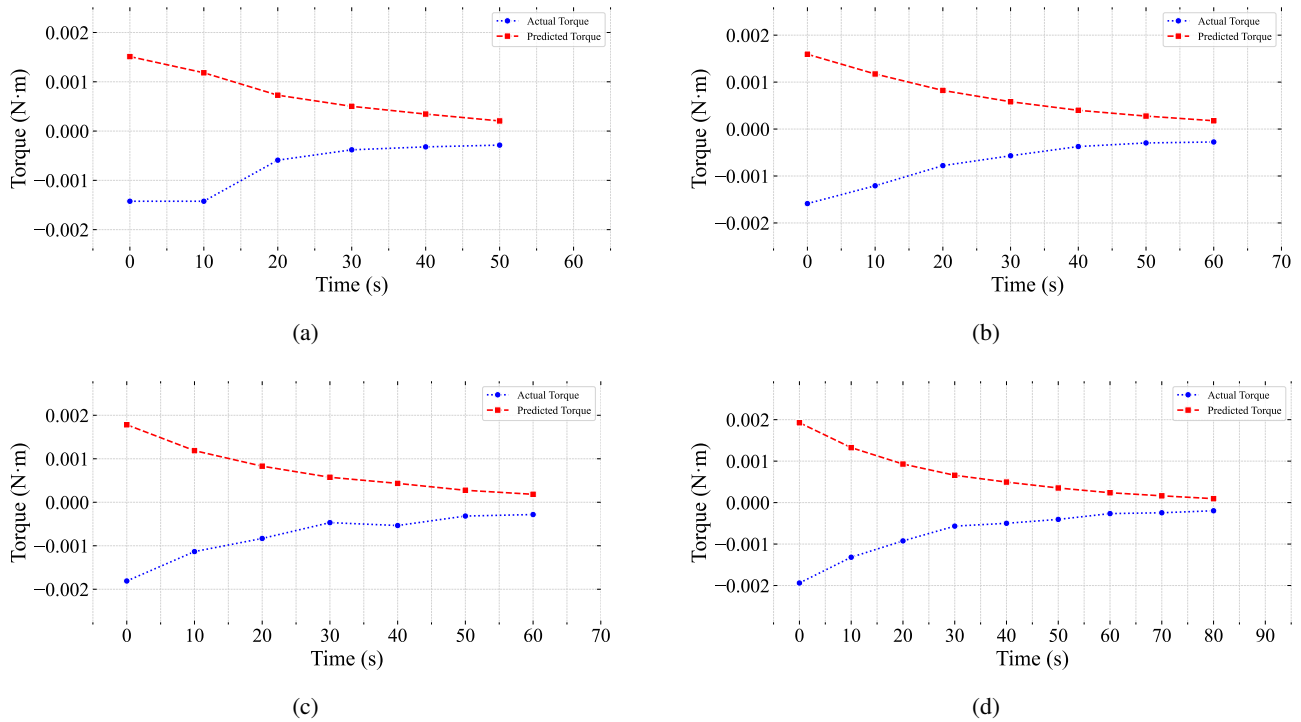


Fig. 6 Torque versus Time for Setup (a) 0 ± 0.01 cm (d_1), (b) 1.15 ± 0.01 cm (d_2), (c) 2.5 ± 0.01 cm (d_3), and (d) 3.85 ± 0.01 cm (d_4).

components of energy dissipation, the total resistive torque was modeled using equation (14). The models for Setup 1–3 are as follows:

$$\begin{aligned}\tau_{(1)} &= -2.24171 \times 10^{-5} \omega + 2.01483 \times 10^{-8} \omega^2, \\ \tau_{(2)} &= -2.24078 \times 10^{-5} \omega + 1.57791 \times 10^{-8} \omega^2, \\ \tau_{(3)} &= -2.32271 \times 10^{-5} \omega - 4.08344 \times 10^{-8} \omega^2.\end{aligned}$$

The positive coefficient of linear damping (b) value across all setups indicates that bearing friction is the dominant source of energy loss. Also, the value of the b remains constant in all setups. In Setup 1 and Setup 2 the coefficient of quadratic damping (k) is negative, which is not physically possible, indicating that there is noise in the data. In Setup 3 the value of k is positive and significantly large. Hence, at higher radii the influence of drag resistance is higher. This is due to the fact that a larger surface area sweeps more air as the disk rotates. It is important to note that although a larger radius increases the MOI and increases the time of spin, it also increases the rate of energy dissipation through drag resistance. Torque vs time graphs for all setups is plotted in Figure 10. The blue line shows the actual torque whereas the red line shows predicted torque. Figure 10 (a) shows significant deviation from the predicted torque. This is due to minimal air drag at small radius and random mechanical errors. In Figure 10 (b), the actual torque follows resembles predicted torque more closely. In Figure 10 the actual torque follows the predicted torque almost exactly. The model is a perfect fit when the effect of air drag is significant.

The results of this experiment conclusively show that increasing the radius of a disk reduces increases spin time and reduces the rate of energy loss. In real-world engineering, however, there are practical constraints that need to be considered. Devices like flywheels, turbines, and gyroscopes cannot be made as large as desired. Engineers keep into account size, weight, material strength, and cost. As observed, a larger radius improves spin time but also increases surface area. This leads to more air drag and greater stress at the edges of the

Table 7 Angular velocity versus time.

S. No.	Setup 1	Setup 2	Setup 3
Material	Copper	Iron	Aluminum
$\bar{V} \pm \Delta \bar{V}$	$(9.32 \pm 0.06) V$	$(9.51 \pm 0.04) V$	$(9.72 \pm 0.03) V$
Time (s)	$(\bar{\omega} \pm \Delta \bar{\omega}) \text{ rad/s}$	$(\bar{\omega} \pm \Delta \bar{\omega}) \text{ rad/s}$	$(\bar{\omega} \pm \Delta \bar{\omega}) \text{ rad/s}$
0.0	209.4 ± 1.4	209.0 ± 0.9	209.8 ± 0.5
10.0	154.8 ± 4.8	167.0 ± 7.6	-
20.0	117.9 ± 0.8	137.1 ± 4.0	152.4 ± 6.9
30.0	106.6 ± 56.5	112.0 ± 3.0	-
40.0	60.4 ± 3.3	92.8 ± 2.2	121.6 ± 14.2
50.0	43.0 ± 0.5	76.0 ± 2.7	-
60.0	25.9 ± 2.7	58.4 ± 3.2	96.7 ± 14.7
70.0	13.4 ± 1.8	45.9 ± 3.1	-
80.0	-	34.3 ± 1.3	75.9 ± 12.2
90.0	-	24.1 ± 2.4	-
100.0	-	13.5 ± 2.6	59.3 ± 9.5
110.0	-	4.4 ± 4.7	-
120.0	-	-	46.1 ± 7.7
130.0	-	-	-
140.0	-	-	33.7 ± 2.3
150.0	-	-	-
160.0	-	-	24.9 ± 5.8
170.0	-	-	-
180.0	-	-	16.5 ± 5.9
190.0	-	-	-
200.0	-	-	7.6 ± 5.4

Table 8 Time rise and time fall.

-	Time Rise	Time Fall
	$\bar{T}_1 \pm \Delta \bar{T}_1 (s)$	$\bar{T}_1 \pm \Delta \bar{T}_1 (s)$
Copper	124.32 ± 2.69	76.33 ± 0.95
Iron	163.75 ± 2.19	113.63 ± 1.80
Aluminium	214.28 ± 2.85	212.46 ± 1.25

disk, especially at high speeds. In rotating machines, these effects may increase mechanical stress. In portable systems compact size is a priority, making large radius designs less practical. The findings of this study help quantify how much the radius can be increased before negative effects outweigh the gains. The torque model developed here can help engineers simulate these trade-offs and design disks that are both efficient and realistic to build.

5.2 Error analysis

An instance of oscillatory angular deceleration is observed in Figure 6 (a), and, this is not usual in the current context and is caused by a random error in the experiment. Such a disturbance was only noticed in isolated trials in Setup 1 of experiment 2. The error was not systematic and was induced by random mechanical disturbances. As the disks were freshly made, they had a reflective finish. As a result, light reflecting from the disk was interfering with the reading of the tachometer. A white paper was cut in the shape of each disk and pasted on

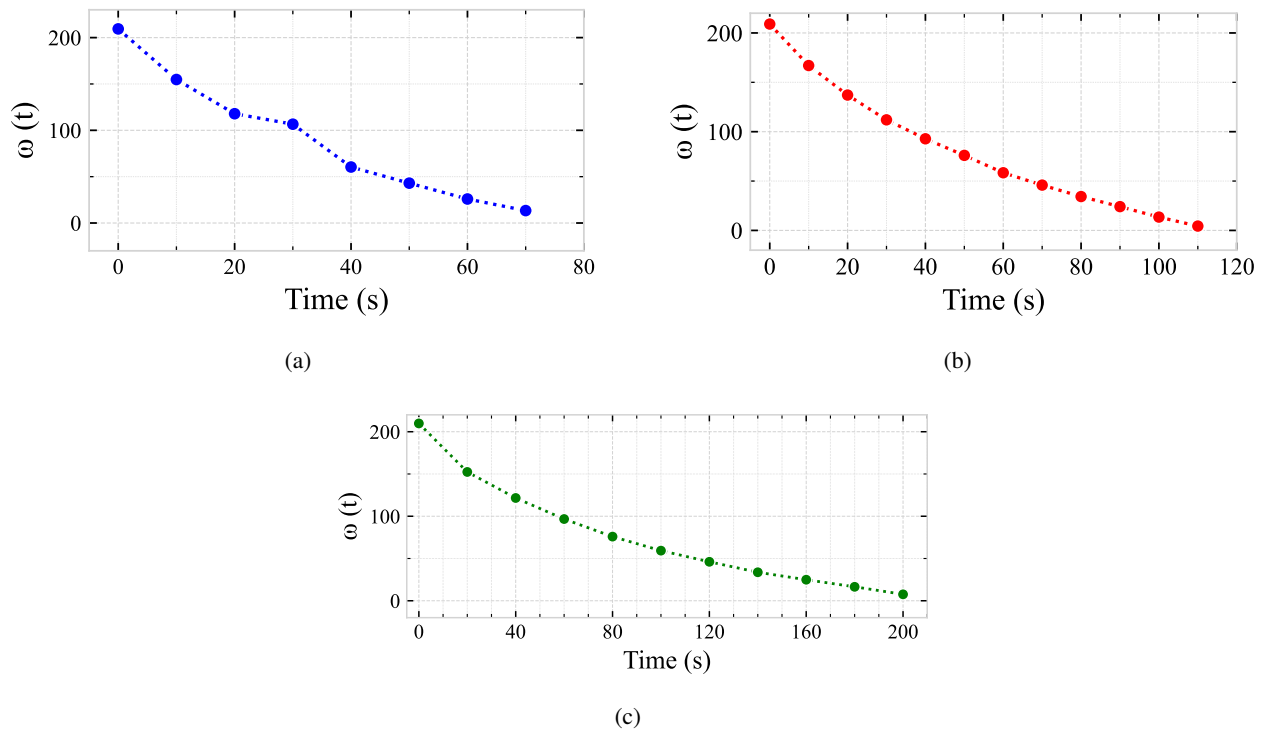


Fig. 7 Angular velocity versus time (a) Radius - 78 ± 1 mm, (b) Radius - 91 ± 1 mm, and (c) Radius - 141 ± 1 mm.

it; the additional mass was negligible. The reflective sticker was then pasted on this paper. This significantly increased the accuracy of readings. The disks were not made with high-precision hardware, due to which the masses were not accurate to the gram. Additional mass was added using Blu-tack in order to make the mass of each disk exactly the same (250 g). The blue tack was flattened out on the disk to ensure the mass is spread out. The mass of the disk is an essential controlled variable. The magnitude of error is high in some readings in Table 6. Possible systematic and random errors can explain these inconsistencies. The plastic connector had to be placed exactly at the center of each disk. The centers of the disks were found using basic geometrical tools such as pencil and ruler. However, as the area of contact of the connector was about 1 cm^2 the connector could not be placed exactly at the center. This affected the data collected as it could have slightly changed the axis of rotation. Measures were taken to ensure that adding extra mass, adhesives and positioning of the axis does not affect the center of mass of the disk. The disk was spun manually to observe any wobbles or lateral deviations. The disks were adjusted until the center of mass was aligned. Moreover, the disks themselves had imperfections as they were not of uniform thickness [28]. These systematic errors can be avoided by using high-precision tools to make the disks and find their respective centers.

6 General sources of error

All the experiments required the reading from the tachometer to be observed and noted down manually every 10 seconds. As the readings were changing very quickly, the task was mentally challenging. This constraint in methodology induced random errors and affected the accuracy of the data immensely [29, 30]. This explains the stark increase in the magnitude of error for a few readings in Experiment 1 and Experiment 2 which are presented in Table 5 and Table 6, respectively. All the experiments relied heavily on the digital tachometer. Digital devices occasionally experience quantisation errors. This could have influenced the readings of the digital tachometer and the digital voltmeter [31]. As copper wires get heated up upon use, it could change the resistance of the

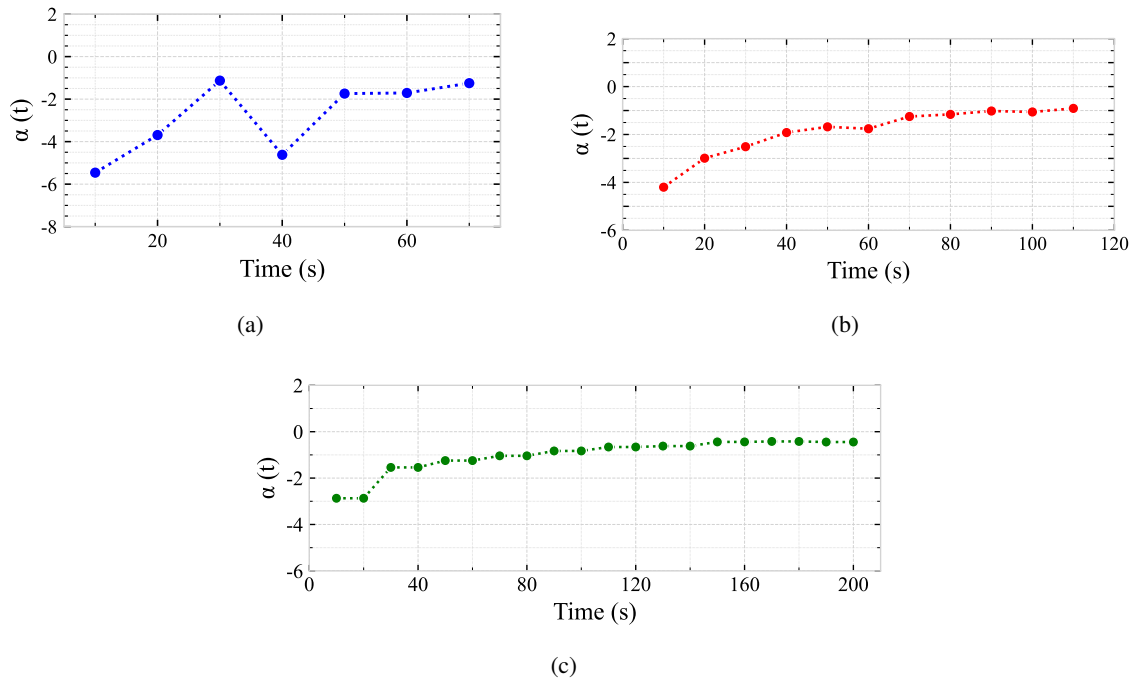


Fig. 8 Angular acceleration versus time (a) Radius - 78 ± 1 mm, (b) Radius - 91 ± 1 mm, and (c) Radius - 141 ± 1 mm.

circuit. This influenced the effective potential difference supplied to the motor. Heating could have also affected the performance of the motor upon repeated use [32]. Moreover, wear and tear of the motor upon repeated use may have affected the consistency of its performance. Systematic error due to heating can be avoided by waiting for the setup to cool down before conducting further trials, however, this was not possible due to time constraints. Lastly, multiple motors can be used to avoid the influence of wear and tear.

7 Conclusion

The experiments established clear cause-effect relationships between physical quantities and the motion of a spinning top, helping in design decisions for optimization. A spinning disk can be optimized by placing most of the mass away from the central axis and by increasing its radius. A 1% increase in MOI from mass distribution gives an average 4.4% increase in spin time, while the same change from radius gives a 1% increase. This contrast shows that both methods must be considered based on use-case. The experiment also studied how energy is lost as the disk spins. Disks with more outward mass and larger radii had higher initial energy and took longer to lose it. This means they dissipate energy more slowly. The torque model using friction and air drag matched the data, especially in high-inertia setups. Drag was less significant when inertia was low but became important as radius increased. These results explain energy loss in spinning systems and are useful for designing efficient mechanical components.

The study offers an experimental evaluation of core ideas in rotational dynamics, especially those related to angular velocity, angular acceleration, and moment of inertia. Their link to angular deceleration is tested using spinning disks, and observed patterns align with theoretical expectations. The role of these theories in optimization is also explored. The torque model combining friction and air drag was tested against experimental data and matched well, particularly in setups with outward mass. This supports its ability to describe real energy loss in spinning disks. The study shows that increasing MOI reduces angular deceleration, but disks with higher inertia take longer to reach constant angular velocity. This must be considered while optimizing designs. The findings apply to real-world rotating systems like wheels, turbines, and engines. They can help

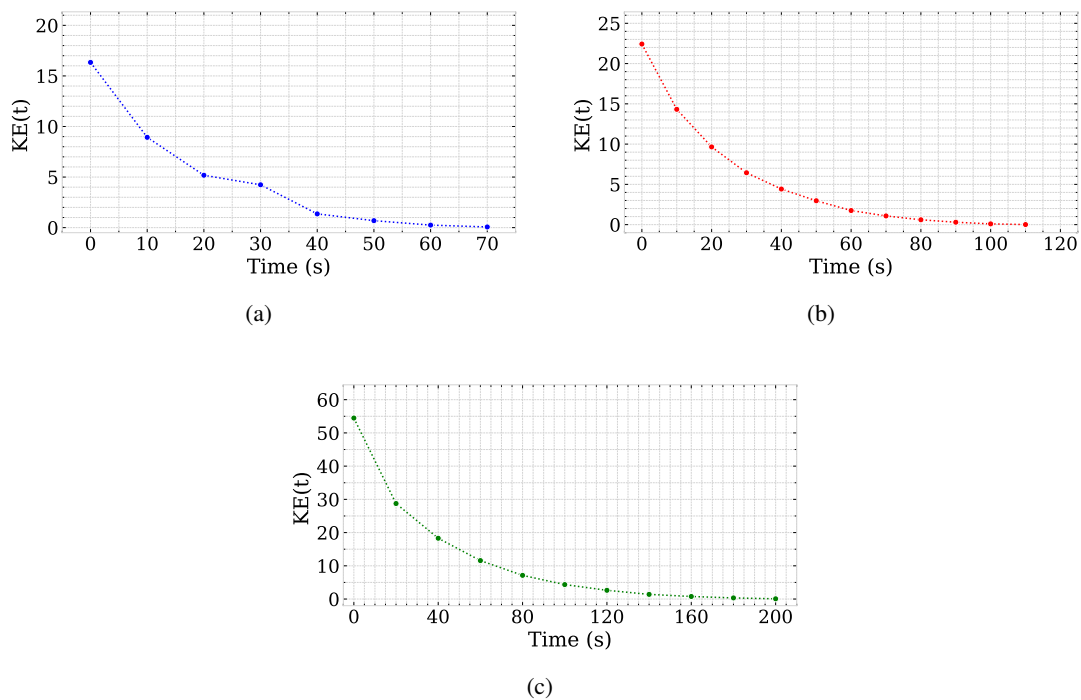


Fig. 9 Kinetic Energy versus time (a) Radius - 78 ± 1 mm, (b) Radius - 91 ± 1 mm, and (c) Radius - 141 ± 1 mm.

improve machine efficiency by guiding how mass is distributed. Since most machines run on non-renewable energy, any improvement in efficiency supports sustainability. In this context, the results are both practically and environmentally relevant. The novelty of this work lies in its use of simple, controlled experiments to explore how radius and mass distribution affect spin duration and energy loss. Unlike purely theoretical studies, this paper uses measured data to validate torque models and analyze damping behavior. The findings are practically useful in optimizing real-world rotating systems where both spin time and energy efficiency are important.

One limitation of this study is that the damping coefficients b and k were calculated from curve fitting and not verified through independent measurements. Although the model showed a good fit, further work is needed to isolate the effects of friction and air drag more precisely. The experiment was also conducted at moderate angular speeds, and higher-speed behavior was not examined. At greater velocities, the role of air resistance could become more complex. Additionally, the disks were assumed to be ideal in shape and symmetry, but small imperfections may have affected the accuracy of the results. Future studies could explore the effect of different materials, shapes, or spin speeds. Internal Measurement Unit (IMU) sensors placed on the disk itself would yield much more precise and significant data as it would minimize the dependence on the experimenter thus effectively avoiding many random errors caused by delay in reaction time. These improvements would help extend the application of the findings to real-world mechanical systems. Despite limitations, it can be stated with reasonable accuracy that moving the mass away from the center and increasing the radius of the top will minimize angular deceleration. Therefore, this exploration yields the result that a disk can be optimized by pushing the mass away from the center and increasing its radius.

8 Declarations

8.1 Conflict of interest:

Not applicable.

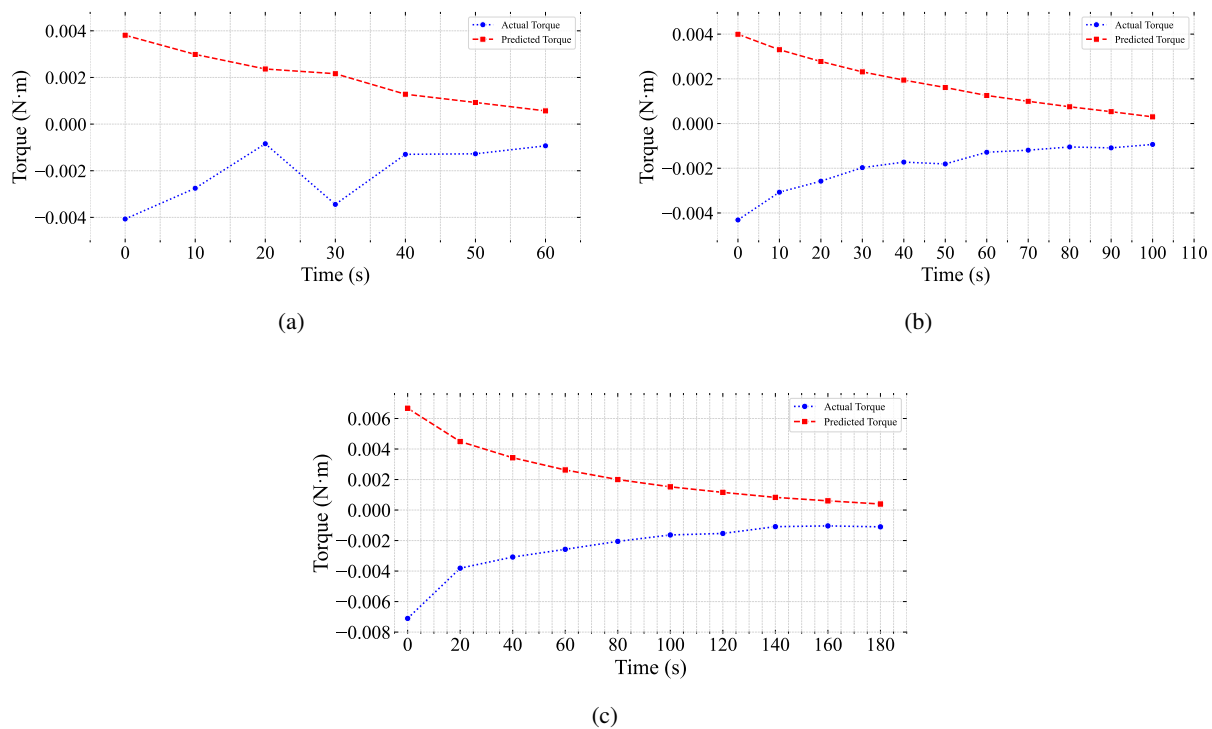


Fig. 10 Torque versus time for disks of varying radii: (a) Radius - 78 ± 1 mm, (b) Radius - 91 ± 1 mm, and (c) Radius - 141 ± 1 mm.

8.2 Funding:

Not applicable.

8.3 Author's contribution:

A.N.-Validation, Investigation, Formal Analysis, Writing-Original Draft. P.V.-Validation, Conceptualization, Writing-Review & Editing. V.G.P.-Validation, Investigation, Conceptualization. This paper has been submitted with the knowledge and consent of all authors.

8.4 Acknowledgement:

Not applicable.

8.5 Data availability statement:

All data that support the findings of this study are included within the article.

8.6 Usage of AI tools:

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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