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Original Study

Dynamics of new truncated M-fractional derivative wave structures to the nonlinear Zhiber-Shabat equation arising in variety of fields

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Abstract

In this work, we explore the different wave structures and the effect of fractional parameter on the nonlinear partial differential equation known as the nonlinear Zhiber-Shabat equation (ZSE). This model has a variety of applications in the mathematical community, including fluid dynamics, integral quantum field theory, nonlinear optics. The recently developed integration techniques known as generalized Riccati equation mapping method, the Kumar-Malik method (KMM) and multivariate generalized exponential integral function approach are adopted. The suggested model is transformed into a nonlinear ordinary differential equation with the application of the truncated M-fractional derivative in order to get the intended results. The obtained structures are novel and expressed in the form of solitary wave solutions including hyperbolic, periodic as well as exponential function solutions under certain conditions. Various combinations and magnitudes of the physical parameters are employed to investigate the soliton solutions of the resultant system. Graphs are constructed by plotting the final solutions with the appropriate parameter values to elucidate the scientific interpretation and physical importance of the analytical findings.

Keywords: Modified generalized Riccati mapping method, KMM, Generalized exponential rational integral function approach, solitons, Zhiber-Shabat equation.
AMS 2020 codes: 35C08; 35Dxx; 45Kxx; 65Mxx; 45K05.

1 Introduction

Nonlinearity in nature is fascinating, and many researchers see nonlinear science as the most promising field for enhancing our understanding of the universe [1, 2]. The mathematical characterization of complex systems with time-varying parameters necessitates the examination of a variety of nonlinear partial differential equations (NLPDEs). Since the mid-18th century, scholars have endeavored to simplify intricate physical phenomena by employing NLPDEs. Researchers frequently use NLPDEs to reconstruct various dynamic phenomena [3]. As a result, the study of NLPDEs has continued to attract a substantial quantity of attention in recent years. To explore the exact solutions to nonlinear equations is a critical component of the investigation of nonlinear physical phenomena [4]. The significance of obtaining exact solutions for nonlinear equations is paramount, as these solutions are essential for numerical solution verification and stability analysis. A mathematical structure must be used to express the physical characteristics of any nonlinear physical system in order to obtain an

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exact solution. This mathematical structure is frequently represented by NLPDEs. Therefore, the investigation of NLPDEs is essential; these equations provide a fundamental framework that facilitates the highly precise understanding of a diverse array of nonlinear physical phenomena. NLPDEs are basic mathematical models that help us understand many different types of phenomena in physics, photonics, optics, fluid dynamics and nonlinear fiber optics. They are used in fields like plasma physics and nonlinear optics [5].

The soliton theory has garnered significant attention in trial designs due to its function as a research domain in the disciplines of media transmission, numerical physical science, designing, and various nonlinear sciences [6]. Specifically, the current period has seen a concentration of optical solitons. Optical solitons are waves that possess the ability to generate waves without dispersing over a significant distance, i.e., they maintain their shape over a significant distance [7, 8]. The basic role of solitons in the media communications society underscores their significance in nonlinear optics. In the system of single wave-based interchanges, optical pulse generators, fiber-optic amplifiers, and many others, solitons models are extensively useful. Examining the motion of solitons through nonlinear optical fibers, extreme laser radiation into plasmas, and the hypothesis of optical solitons are among the most interesting topics [9]. The scholars have adopted a variety of nonlinear models to predict and comprehend the nature of soliton waves.

Researchers have developed a number of approaches and methods to deal with these problems and get solutions in a wide range of systems. Among the most well-known and widely used techniques developed recently are: the Darboux transformation [10], the enhanced modified extended tanh expansion method [11], the advanced F-expansion function approach [12], the truncated Painlevé approach [13], the modified F-expansion method [14], the modified simple equation method [15], the iterative transform method [16], the Lie classical approach [17], the new sub equation approach [18], the Adomian decomposition technique [19], the simplest equation technique [20], the $\tan(\frac{\phi}{2})$ technique [21], the multiple exp-function approach [22], the Bernoulli $\frac{G'}{G}$-expansion method [23], $(\frac{G'}{G})$-expansion method [24], the tanh-coth method [25], the inverse scattering approach [26], the Bäcklund transformation [27], the Riccati equation mapping technique [28], the bifurcation analysis [29, 30] and so on [31–36].

Moreover, the model under consideration has not been investigated using these novel sophisticated methodologies in the existing literature. So, this paper has the aim to study various kinds of soliton solutions of the M-fractional nonlinear ZSE by applying the new integration methods known as modified generalized Riccati equation mapping method (MGREMM) [37], KMM [38] and multivariate generalized exponential rational integral function method (MGERIFM) [39]. These methods can be employed to obtain novel and exact solutions, including exponential, trigonometric, and rational functions. The proposed solutions will markedly improve the comprehension of diverse physical processes in associated domains.

The rest of the article is organized as follows: The definition of the new truncated M-fractional derivative of $\hbar$ is presented in Section 2. The applications of methods together with graphs are presented in Section 3. The conclusion obtained in this paper is reported in Section 4.

2 Preliminaries

In order to explain the complex nonlinear systems and provide a more accurate representation of system dynamics, fractional derivatives are a more sophisticated approach. Compared to their integer-order counterparts, the fractional models demonstrate superior precision and accuracy, closely aligning with the experimental data. Additionally, the system's overall efficacy is improved by the implementation of sophisticated optimization and control strategies, which are made possible by the fractional form. The truncated M-fractional derivatives, a novel idea of fractional order derivatives, is presented contemporaneously with the development of this process. Scholars have given these fractional-order derivatives a lot of attention.

Definition 1. Let $\hbar : [0,\infty) \to \mathbb{R}$, then, the new truncated M-fractional derivative of $\hbar$ is defined [40] as:

$$\mathscr{D}_M^{\alpha,\varsigma}\{(\hbar)(t)\} = \lim_{\varepsilon\to 0}\frac{\hbar(t\mathbb{E}_\varsigma(\varepsilon t^{1-\alpha})) - \hbar(t)}{\varepsilon}, \quad \varsigma > 0,\ \alpha \in (0,1),\ \forall t > 0, \tag{1}$$

with $\mathbb{E}_\varsigma(.)$ being the one parameter truncated Mittag-Leffler function [40].

Let $\varsigma > 0$, $0 < \alpha \leq 1$, p, $s \in \mathbb{R}$, and $\hbar$, β differentiable at a point $t > 0$. Then,

1. $\mathscr{D}_M^{\alpha,\varsigma}\{(p\beta + s\hbar)(t)\} = p\mathscr{D}_M^{\alpha,\varsigma}\{\beta(t)\} + s\mathscr{D}_M^{\alpha,\varsigma}\{\hbar(t)\}$.
2. $\mathscr{D}_M^{\alpha,\varsigma}\{(\beta\,.\,\hbar)(t)\} = \beta(t)\mathscr{D}_M^{\alpha,\varsigma}\{\hbar(t)\} + \hbar(t)\mathscr{D}_M^{\alpha,\varsigma}\{\beta(t)\}$.
3. $\mathscr{D}_M^{\alpha,\varsigma}\{\frac{\beta}{\hbar}(t)\} = \frac{\hbar(t)\mathscr{D}_M^{\alpha,\varsigma}\{\beta(t)\} - \beta(t)\mathscr{D}_M^{\alpha,\varsigma}\{\hbar(t)\}}{[\hbar(t)]^2}$.
4. $\mathscr{D}_M^{\alpha,\varsigma}\{c\} = 0$, where $\beta(t) = c$ is a constant.
5. If β is differentiable, then, $\mathscr{D}_M^{\alpha,\varsigma}\{\beta(t)\} = \frac{t^{1-\alpha}}{\Gamma(\varsigma+1)}\frac{d\beta(t)}{dt}$.

3 Applications

In this section, we apply the powerful methods such as MGREMM, KMM, and MGERIFM to obtain desired solutions for the nonlinear ZSE [41–43] in truncated M-fractional derivative given as:

$$\mathscr{D}_{M,xt}^{2\varepsilon,\beta}u + pe^{u} + qe^{-u} + re^{-2u} = 0, \tag{2}$$

where $u = u(x,t)$ is an unknown function, while the parameters $p,\ q,\ r$ are real constants. Eq.(2) transforms to sinh-Gordon equation by taking $r = 0$, while for $q = r = 0$ it transforms to the Liouville equation. Similarly, for $q = 0$, Eq.(2) transforms to the Dodd-Bullough-Mikhailov equation and for $p = 0,\ q = -1,\ r = 1$, it converts to the form of Tzitzeica-Dodd-Bullough equation. Moreover, the subscripts x and t are the partial derivatives and $\mathscr{D}_M^{\varepsilon,\varsigma}$ denote the M-fractional derivative with $\beta \in (0,1)$ and $\varsigma > 0$. Furthermore, the suggested model has been examined in the literature from different perspectives. In [41], various solitary wave solutions have been extracted by using tanh method, while a variety of new exact solutions have recovered in [42] by applying $\left(\frac{1}{G'}\right)$ expansion method, $\left(\frac{G'}{G}\right)$ and $\left(\frac{1}{G}\right)$ methods. In [43], the existence of soliton solutions are derived by employing ansatz approach.

In this work, we study the exact solutions of the proposed model using the suggested techniques with truncated M-fractional derivatives. First of all, let's consider the following transformation as:

$$u(x,t) = \Phi(\xi), \quad \xi = \frac{\Gamma(\beta+1)}{\varepsilon}\left(x^{\varepsilon} - ct^{\varepsilon}\right), \tag{3}$$

where c is the wave speed. On manipulating the Eq.(3) into Eq.(2) provides

$$pe^{u} + qe^{-u} + re^{-2u} - cu'' = 0. \tag{4}$$

Furthermore, for solving the Eq.(4), the following assumption is used as:

$$v = e^{u} \quad \text{or } u = \ln v. \tag{5}$$

Manipulating Eq.(5) into Eq.(4), we get

$$c\left((v')^2 - vv''\right) + pv^3 + qv + r = 0. \tag{6}$$

Now, applying the homogeneous balance principle between the terms v^3 and vv'' in Eq.(6) gives $n = 2$.

3.1 Application of the MGREMM

The solution for MGREMM [37] is expressed as:

$$v(\xi)=\phi_0+\sum_{r=1}^{n}\phi_r\Omega^r(\xi)+\sum_{r=1}^{n}\psi_r(\xi)\left(\frac{\Omega'(\xi)}{\Omega(\xi)}\right)^r. \tag{7}$$

For $n=2$, we get

$$v(\xi)=\phi_0+\phi_1\Omega(\xi)+\phi_2\left(\Omega(\xi)\right)^2+\psi_1(\xi)\frac{\Omega'(\xi)}{\Omega(\xi)}+\psi_2(\xi)\left(\frac{\Omega'(\xi)}{\Omega(\xi)}\right)^2, \tag{8}$$

and $\Omega'(\xi)=\delta_0+\delta_1\Omega(\xi)+\delta_2\Omega(\xi)^2$. On putting Eq.(8) into Eq.(6), the general solutions are given as:

(I): When $\Delta=\delta_1^2-4\delta_0\delta_2>0$, $\delta_1\delta_2\neq 0$, $\delta_0\delta_2\neq 0$ and $\phi_1=-\delta_1\delta_2\psi_2$, $p=\frac{2c}{\psi_2}$, $q=\frac{c(\delta_0\delta_2\psi_2+\phi_0)((\delta_1^2-10\delta_0\delta_2)\psi_2-6\phi_0)}{\psi_2}$, $r=\frac{c(\delta_0\delta_2\psi_2+\phi_0)^2(4\phi_0-(\delta_1^2-8\delta_0\delta_2)\psi_2)}{\psi_2}$, $\phi_2=-\delta_2^2\psi_2$, $\psi_1=-\delta_1\psi_2$, the soliton solutions are

$$u_1(x,t)=\ln\left(\frac{\delta_0\delta_2\psi_2\operatorname{sech}^2\left(\frac{\sqrt{\Delta}\xi}{2}\right)\left(\delta_1\sqrt{\Delta}\sinh\left(\sqrt{\Delta}\xi\right)+\left(\delta_1^2-2\delta_0\delta_2\right)\cosh\left(\sqrt{\Delta}\xi\right)-\delta_1^2+6\delta_0\delta_2\right)}{\left(\delta_1+\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}\xi}{2}\right)\right)^2}+\phi_0\right), \tag{9}$$

$$u_2(x,t)=\ln\left(\frac{\delta_0\delta_2\psi_2\operatorname{csch}^2\left(\frac{\sqrt{\Delta}\xi}{2}\right)\left(\delta_1\sqrt{\Delta}\sinh\left(\sqrt{\Delta}\xi\right)+\left(\delta_1^2-2\delta_0\delta_2\right)\cosh\left(\sqrt{\Delta}\xi\right)+\delta_1^2-6\delta_0\delta_2\right)}{\left(\delta_1+\Delta\coth\left(\frac{\sqrt{\Delta}\xi}{2}\right)\right)^2}+\phi_0\right), \tag{10}$$

$$u_3(x,t)=\ln\left(\phi_0+\delta_0\delta_2\psi_2\left(1+\frac{i\left(\delta_1^2-4\delta_0\delta_2\right)}{\delta_1^2\sinh\left(\sqrt{\Delta}\xi\right)-2\delta_0\delta_2\left(\sinh\left(\sqrt{\Delta}\xi\right)+i\right)+\delta_1\sqrt{\Delta}\cosh\left(\sqrt{\Delta}\xi\right)}\right)\right), \tag{11}$$

$$u_4(x,t)=\ln\left(\frac{\delta_1\sqrt{\Delta}(\delta_0\delta_2\psi_2+\phi_0)\sinh\left(\sqrt{\Delta}\xi\right)+\left(\delta_1^2-2\delta_0\delta_2\right)(\delta_0\delta_2\psi_2+\phi_0)\cosh\left(\sqrt{\Delta}\xi\right)+\delta_0\delta_2\left(\left(\delta_1^2-6\delta_0\delta_2\right)\psi_2-2\phi_0\right)}{\left(\delta_1\sinh\left(\frac{\sqrt{\Delta}\xi}{2}\right)+\sqrt{\Delta}\cosh\left(\frac{\sqrt{\Delta}\xi}{2}\right)\right)^2}\right). \tag{12}$$

When $\phi_1=\frac{2c\delta_1\delta_2}{p}$, $q=-\frac{(c\delta_1^2+2c\delta_0\delta_2-3p\phi_0)(2c\delta_0\delta_2-p\phi_0)}{p}$, $r=\frac{(p\phi_0-2c\delta_0\delta_2)^2(2p\phi_0-c\delta_1^2)}{p^2}$, $\psi_2=0$, $\phi_2=\frac{2c\delta_2^2}{p}$, $\psi_1=0$, we have the solitary wave solutions:

$$u_5(x,t)=\ln\left(\frac{-4c\delta_0\delta_2+c\Delta\operatorname{csch}^2\left(\frac{\sqrt{\Delta}\xi}{2}\right)+2p\phi_0}{2p}\right), \tag{13}$$

$$u_6(x,t)=\ln\left(\frac{1}{p\left(d+e\sinh\left(\sqrt{\Delta}\xi\right)\right)^2}\left(c\delta_1^2e\left(e-d\sinh\left(\sqrt{\Delta}\xi\right)\right)-c\sqrt{\Delta}e\sqrt{\Delta\left(d^2+e^2\right)}\cosh\left(\sqrt{\Delta}\xi\right)\right.\right.$$
$$\left.\left.p\phi_0\left(d+e\sinh\left(\sqrt{\Delta}\xi\right)\right)^2-c\delta_0\delta_2\left(2d^2+e^2\cosh\left(2\sqrt{\Delta}\xi\right)+3e^2\right)\right)\right), \tag{14}$$

$$u_7(x,t)=\ln\left(-\frac{4c\delta_0\delta_1\delta_2}{\delta_1p-\sqrt{\Delta}p\tanh\left(\frac{\sqrt{\Delta}\xi}{2}\right)}+\frac{4c\delta_0^2\delta_2^2\left(\cosh\left(\sqrt{\Delta}\xi\right)+1\right)}{p\left(\delta_1\cosh\left(\frac{\sqrt{\Delta}\xi}{2}\right)-\sqrt{\Delta}\sinh\left(\frac{\sqrt{\Delta}\xi}{2}\right)\right)^2}+\phi_0\right). \tag{15}$$

When $\phi_1=-\delta_1\delta_2\psi_2,\ q=-\frac{1}{2}p\left(\delta_0\delta_2\psi_2+\phi_0\right)\left(6\phi_0-\left(\delta_1^2-10\delta_0\delta_2\right)\psi_2\right),$
$r=\frac{1}{2}p\left(\delta_0\delta_2\psi_2+\phi_0\right)^2\left(4\phi_0-\left(\delta_1^2-8\delta_0\delta_2\right)\psi_2\right), c=\frac{p\psi_2}{2},\ \phi_2=-\delta_2^2\psi_2,\ \psi_1=-\delta_1\psi_2$, we have the following solutions:

$$u_8(x,t)=\ln\left(\frac{1}{4}\psi_2\left(4\delta_0\delta_2+\Delta\operatorname{csch}^2\left(\frac{\sqrt{\Delta}\xi}{2}\right)\right)+\phi_0\right), \tag{16}$$

$$u_9(x,t)=\ln\left(\delta_0\delta_2\psi_2-\frac{\Delta\psi_2}{2+2\sinh\left(\sqrt{\Delta}\xi\right)}+\phi_0\right), \tag{17}$$

$$u_{10}(x,t)=\ln\left(\frac{1}{4}\psi_2\left(4\delta_0\delta_2+\Delta\operatorname{csch}^2\left(\frac{\sqrt{\Delta}\xi}{2}\right)\right)+\phi_0\right). \tag{18}$$

(II): When $\Delta=\delta_1^2-4\delta_0\delta_2<0$, $\delta_1\delta_2\neq 0$, $\delta_0\delta_2\neq 0$ and $\phi_1=-\delta_1\delta_2\psi_2,\ q=-\frac{1}{2}p\left(\delta_0\delta_2\psi_2+\phi_0\right)\left(6\phi_0-\left(\delta_1^2-10\delta_0\delta_2\right)\psi_2\right),$
$r=\frac{1}{2}p\left(\delta_0\delta_2\psi_2+\phi_0\right)^2\left(4\phi_0-\left(\delta_1^2-8\delta_0\delta_2\right)\psi_2\right), c=\frac{p\psi_2}{2},\ \phi_2=-\delta_2^2\psi_2,\ \psi_1=-\delta_1\psi_2$, we have the following periodic solutions:

$$u_{11}(x,t)=\ln\left(\frac{\phi_0\left(\sqrt{-\Delta}\tan\left(\frac{\sqrt{-\Delta}\xi}{2}\right)-\delta_1\right)^2}{4\delta_0\delta_2}+\frac{\delta_1\phi_0\left(\sqrt{-\Delta}\tan\left(\frac{\sqrt{-\Delta}\xi}{2}\right)-\delta_1\right)}{2\delta_0\delta_2}+\phi_0\right), \tag{19}$$

$$u_{12}(x,t)=\ln\left(\frac{\delta_0\delta_2\psi_2\csc^2\left(\frac{\sqrt{-\Delta}\xi}{2}\right)\left(\delta_1\sqrt{-\Delta}\sin\left(\sqrt{-\Delta}\xi\right)-\delta_1^2+6\delta_0\delta_2-\Delta\cos\left(\sqrt{-\Delta}\xi\right)\right)}{\left(\delta_1+\sqrt{-\Delta}\cot\left(\frac{\sqrt{-\Delta}\xi}{2}\right)\right)^2}+\phi_0\right), \tag{20}$$

$$\begin{aligned}u_{13}(x,t)=\ln\Bigg(&\frac{\delta_0\delta_1\delta_2\sqrt{-\Delta}\psi_2\cosh\left(\sqrt{\Delta}\xi\right)+\delta_1\sqrt{-\Delta}\phi_0\cos\left(\sqrt{-\Delta}\xi\right)+\sin\left(\sqrt{-\Delta}\xi\right)}{\left(\delta_1^2-2\delta_0\delta_2\right)\sin\left(\sqrt{-\Delta}\xi\right)+\delta_1\sqrt{-\Delta}\cos\left(\sqrt{-\Delta}\xi\right)-2\delta_0\delta_2}\\&\left(\delta_1^2-2\delta_0\delta_2\right)\left(\delta_0\delta_2\psi_2+\phi_0\right)+\delta_0\delta_2\left(\left(\delta_1^2-6\delta_0\delta_2\right)\psi_2-2\phi_0\right)\Bigg),\end{aligned} \tag{21}$$

$$u_{14}(x,t)=\ln\left(\frac{\delta_1\sqrt{\Delta}\left(\delta_0\delta_2\psi_2+\phi_0\right)\sin\left(\sqrt{-\Delta}\xi\right)+\left(\delta_1^2-2\delta_0\delta_2\right)\left(\delta_0\delta_2\psi_2+\phi_0\right)\cosh\left(\sqrt{\Delta}\xi\right)+\delta_0\delta_2\left(\left(\delta_1^2-6\delta_0\delta_2\right)\psi_2-2\phi_0\right)}{\left(\delta_1\sin\left(\frac{\sqrt{-\Delta}\xi}{2}\right)+\sqrt{\Delta}\cosh\left(\frac{\sqrt{\Delta}\xi}{2}\right)\right)^2}\right). \tag{22}$$

When $\phi_1=\frac{2c\delta_1\delta_2}{p},\ q=-\frac{\left(c\delta_1^2+2c\delta_0\delta_2-3p\phi_0\right)\left(2c\delta_0\delta_2-p\phi_0\right)}{p},\ r=\frac{\left(p\phi_0-2c\delta_0\delta_2\right)^2\left(2p\phi_0-c\delta_1^2\right)}{p^2},\ \psi_2=0,\ \phi_2=\frac{2c\delta_2^2}{p},\ \psi_1=0,$ we have:

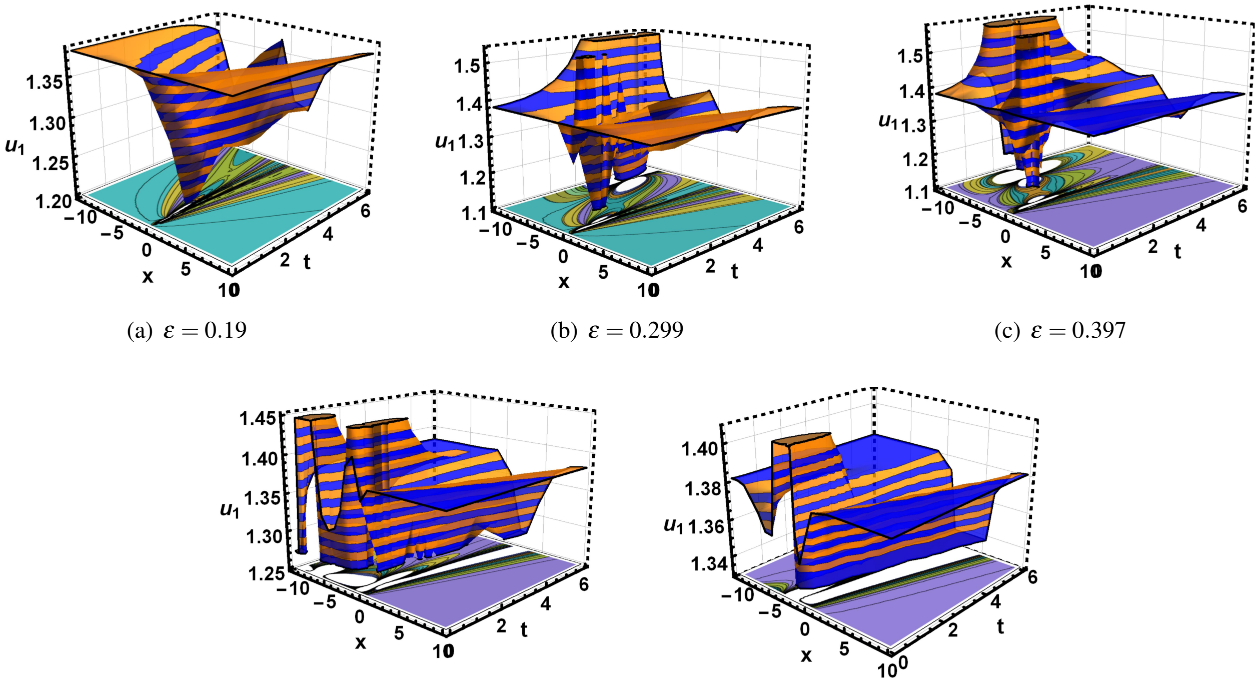
$$u_{15}(x,t)=\ln\left(-\frac{2c\delta_0\delta_2}{p}+\frac{c\Delta}{p\left(\cos\left(\sqrt{-\Delta}\xi\right)-1\right)}+\phi_0\right), \tag{23}$$

$$\begin{aligned}u_{16}(x,t)=\ln\Bigg(&\frac{1}{p\left(d+e\sin\left(\sqrt{-\Delta}\xi\right)\right)^2}\Bigg(-c\delta_1^2e\left(d\sin\left(\sqrt{-\Delta}\xi\right)+e\right)-c\sqrt{-\Delta}e\sqrt{\Delta(d-e)(d+e)}\cos\left(\sqrt{-\Delta}\xi\right)\\&+c\delta_0\delta_2\left(-2d^2+e^2\cosh\left(2\sqrt{\Delta}\xi\right)+3e^2\right)+\frac{1}{2}p\phi_0\left(2d^2+4de\sin\left(\sqrt{-\Delta}\xi\right)-e^2\cos\left(2\sqrt{-\Delta}\xi\right)+e^2\right)\Bigg)\Bigg),\end{aligned} \tag{24}$$

$$u_{17}(x,t) = \ln\left(-\frac{4c\delta_0\delta_1\delta_2}{\delta_1 p + \sqrt{-\Delta}p\tan\left(\frac{\sqrt{-\Delta}\xi}{2}\right)} + \frac{4c\delta_0^2\delta_2^2\left(\cos\left(\sqrt{-\Delta}\xi\right)+1\right)}{p\left(\delta_1\cos\left(\frac{\sqrt{-\Delta}\xi}{2}\right) + \sqrt{-\Delta}\sin\left(\frac{\sqrt{-\Delta}\xi}{2}\right)\right)^2} + \phi_0\right), \tag{25}$$

$$u_{18}(x,t) = \ln\left(-\frac{4c\delta_0\delta_1\delta_2}{\delta_1 p - \sqrt{-\Delta}p\cot\left(\frac{\sqrt{-\Delta}\xi}{2}\right)} - \frac{4c\delta_0^2\delta_2^2\left(\cos\left(\sqrt{-\Delta}\xi\right)-1\right)}{p\left(\delta_1\sin\left(\frac{\sqrt{-\Delta}\xi}{2}\right) - \sqrt{-\Delta}\cos\left(\frac{\sqrt{-\Delta}\xi}{2}\right)\right)^2} + \phi_0\right), \tag{26}$$

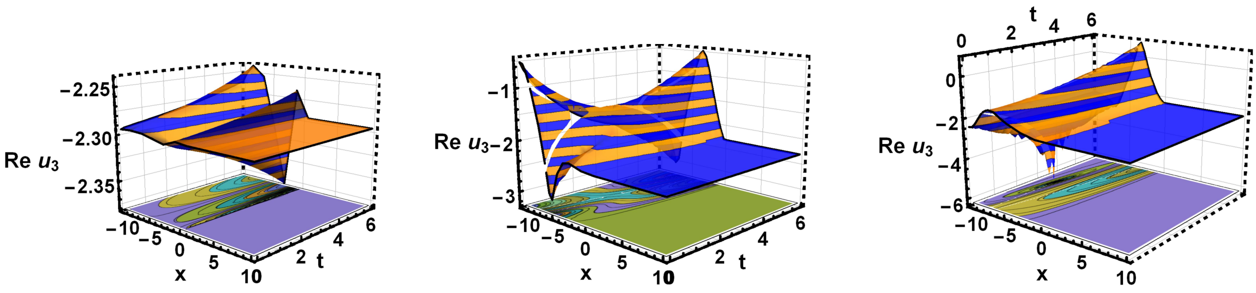
where ξ is expressed in Eq.(3). All the obtained solutions are written in the form of Eq.(5), such that $u = \ln v$. We simulate the wave behaviours of the solutions of the governing model.



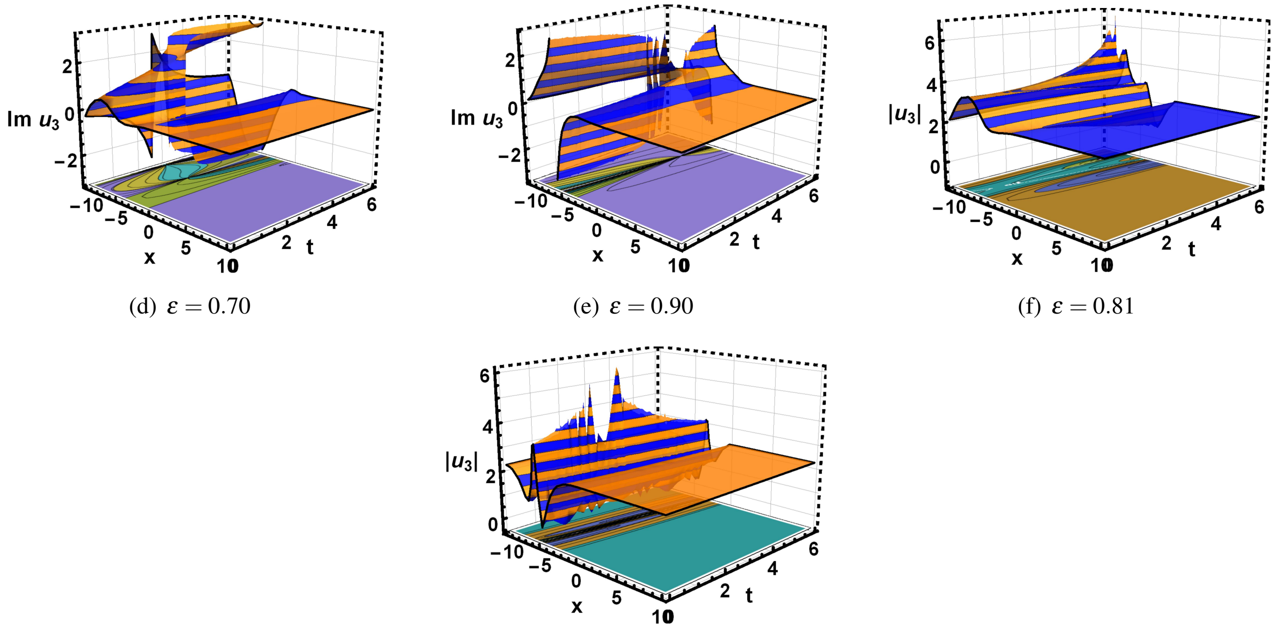
(a) $\varepsilon = 0.19$ (b) $\varepsilon = 0.299$ (c) $\varepsilon = 0.397$

(d) $\varepsilon = 0.578$ (e) $\varepsilon = 0.765$

Fig. 1 Sketches of the Eq. (9) with fractional parameter.



(a) $\varepsilon = 0.46$ (b) $\varepsilon = 0.787$ (c) $\varepsilon = 0.833$



(d) $\varepsilon = 0.70$ (e) $\varepsilon = 0.90$ (f) $\varepsilon = 0.81$

(g) $\varepsilon = 0.92$

Fig. 2 Sketches of the Eq.(11) with fractional parameter.

3.2 Application of the KMM

For $n = 2$, the general solution forms in KMM [38] can be described as

$$v(\xi) = \rho_0 + \rho_1 \Omega(\xi) + \rho_2 \left(\Omega(\xi)\right)^2, \tag{27}$$

with

$$\Omega^{'}(\xi) = \sqrt{\gamma_1 \Omega(\xi)^4 + \gamma_2 \Omega(\xi)^3 + \gamma_3 \Omega(\xi)^2 + \gamma_4 \Omega(\xi) + \gamma_5}. \tag{28}$$

The following solutions are obtained by employing (27) along with (28) in (6):

- **Case-1** For $\gamma_5 = \frac{(4\gamma_1\gamma_3-\gamma_2^2)^2}{64\gamma_1^3}$, $\gamma_4 = \frac{\gamma_2(4\gamma_1\gamma_3-\gamma_2^2)}{8\gamma_1^2}$ offers $\rho_2 = \frac{2\gamma_1\rho_1}{\gamma_2}$, $p = \frac{c\gamma_2}{\rho_1}$, $r = \frac{c(8\gamma_1\rho_0-\gamma_2\rho_1)(\gamma_2^2\rho_1+4\gamma_1\gamma_2\rho_0-4\gamma_1\gamma_3\rho_1)^2}{64\gamma_1^3\gamma_2\rho_1}$,

$q = -\frac{c(3\gamma_2\rho_0-\gamma_3\rho_1)(\gamma_2^2\rho_1+4\gamma_1\gamma_2\rho_0-4\gamma_1\gamma_3\rho_1)}{4\gamma_1\gamma_2\rho_1}$. As a result we have the following solutions:
When $\gamma_1 > 0$ and $8\gamma_1\gamma_3 - 3\gamma_2^2 < 0$, we get the solutions as follows:
The dark type soliton solution

$$u_1(x,t) = \ln\left(\frac{\frac{(3\gamma_2^2-8\gamma_1\gamma_3)\rho_1 \tanh^2\left(\frac{\sqrt{3\gamma_2^2-8\gamma_1\gamma_3}\,\Gamma(\beta+1)(x^{\varepsilon}-ct^{\varepsilon})}{4\sqrt{\gamma_1}\,\varepsilon}\right)}{\gamma_2} + 8\gamma_1\rho_0 - \gamma_2\rho_1}{8\gamma_1}\right). \tag{29}$$

The singular soliton solution

$$u_2 = \ln\left(-\frac{\gamma_3\rho_1 \coth^2\left(\frac{\sqrt{\gamma_1(3\gamma_2^2-8\gamma_1\gamma_3)}\,\Gamma(\beta+1)(x^{\varepsilon}-ct^{\varepsilon})}{4\gamma_1\varepsilon}\right)}{\gamma_2} + \frac{\gamma_2\rho_1\left(3\coth^2\left(\frac{\sqrt{\gamma_1(3\gamma_2^2-8\gamma_1\gamma_3)}\,\Gamma(\beta+1)(x^{\varepsilon}-ct^{\varepsilon})}{4\gamma_1\varepsilon}\right)-1\right)}{8\gamma_1} + \rho_0\right). \tag{30}$$

When $\gamma_1 > 0$ and $8\gamma_1\gamma_3 - 3\gamma_2^2 > 0$, we get the periodic solutions as follows:

$$u_3(x,t) = \ln\left(\frac{\frac{(8\gamma_1\gamma_3-3\gamma_2^2)\rho_1\tan^2\left(\frac{\sqrt{8\gamma_1\gamma_3-3\gamma_2^2}\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{4\sqrt{\gamma_1}\varepsilon}\right)}{\gamma_2}+8\gamma_1\rho_0-\gamma_2\rho_1}{8\gamma_1}\right), \tag{31}$$

$$u_4(x,t) = \ln\left(\frac{\gamma_3\rho_1\cot^2\left(\frac{\sqrt{\gamma_1(8\gamma_1\gamma_3-3\gamma_2^2)}\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{4\gamma_1\varepsilon}\right)}{\gamma_2}-\frac{\gamma_2\rho_1\left(3\cot^2\left(\frac{\sqrt{\gamma_1(8\gamma_1\gamma_3-3\gamma_2^2)}\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{4\gamma_1\varepsilon}\right)+1\right)}{8\gamma_1}+\rho_0\right) \tag{32}$$

- **Case-2** For $\gamma_5 = \frac{\gamma_2^2(16\gamma_1\gamma_3-5\gamma_2^2)}{256\gamma_1^3}$, $\gamma_4 = \frac{\gamma_2(4\gamma_1\gamma_3-\gamma_2^2)}{8\gamma_1^2}$ offers $\rho_2 = \frac{2\gamma_1\rho_1}{\gamma_2}$, $p = \frac{c\gamma_2}{\rho_1}$, $r = -\frac{c(8\gamma_1\rho_0-\gamma_2\rho_1)^2(-5\gamma_2^2\rho_1-8\gamma_1\gamma_2\rho_0+16\gamma_1\gamma_3\rho_1)}{256\gamma_1^3\rho_1}$, $q = \frac{c(8\gamma_1\rho_0-\gamma_2\rho_1)(-9\gamma_2^2\rho_1-24\gamma_1\gamma_2\rho_0+32\gamma_1\gamma_3\rho_1)}{64\gamma_1^2\rho_1}$. As a result we have the following solutions:

When $\gamma_1 < 0$ and $8\gamma_1\gamma_3 - 3\gamma_2^2 < 0$, we get: The bright soliton solution

$$u_5(x,t) = \ln\left(\frac{\frac{2(3\gamma_2^2-8\gamma_1\gamma_3)\rho_1\operatorname{sech}^2\left(\frac{\sqrt{8\gamma_1\gamma_3-3\gamma_2^2}\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{2\sqrt{2}\sqrt{\gamma_1}\varepsilon}\right)}{\gamma_2}+8\gamma_1\rho_0-\gamma_2\rho_1}{8\gamma_1}\right). \tag{33}$$

When $\gamma_1 > 0$ and $8\gamma_1\gamma_3 - 3\gamma_2^2 > 0$, we get the hyperbolic solution as follows:

$$u_6(x,t) = \ln\left(\frac{\frac{2(8\gamma_1\gamma_3-3\gamma_2^2)\rho_1\operatorname{csch}^2\left(\frac{\sqrt{8\gamma_1\gamma_3-3\gamma_2^2}\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{2\sqrt{2}\sqrt{\gamma_1}\varepsilon}\right)}{\gamma_2}+8\gamma_1\rho_0-\gamma_2\rho_1}{8\gamma_1}\right). \tag{34}$$

When $\gamma_1 > 0$ and $8\gamma_1\gamma_3 - 3\gamma_2^2 < 0$, we get the periodic solutions as follows:

$$u_7(x,t) = \ln\left(\frac{\frac{2(3\gamma_2^2-8\gamma_1\gamma_3)\rho_1\sec^2\left(\frac{\sqrt{3\gamma_2^2-8\gamma_1\gamma_3}\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{2\sqrt{2}\sqrt{\gamma_1}\varepsilon}\right)}{\gamma_2}+8\gamma_1\rho_0-\gamma_2\rho_1}{8\gamma_1}\right), \tag{35}$$

$$u_8(x,t) = \ln\left(\frac{\frac{2(3\gamma_2^2-8\gamma_1\gamma_3)\rho_1\csc^2\left(\frac{\sqrt{3\gamma_2^2-8\gamma_1\gamma_3}\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{2\sqrt{2}\sqrt{\gamma_1}\varepsilon}\right)}{\gamma_2}+8\gamma_1\rho_0-\gamma_2\rho_1}{8\gamma_1}\right). \tag{36}$$

- **Case-3** For $\gamma_2 = 0$, $\gamma_4 = 0$, $\gamma_5 = 0$ and $\gamma_3 > 0$, offers $\rho_1 = 0$, $c = \frac{p\rho_2}{2\gamma_1}$, $r = \frac{2(\gamma_1 p\rho_0^3-\gamma_3 p\rho_0^2\rho_2)}{\gamma_1}$, $q = \frac{2\gamma_3 p\rho_0\rho_2-3\gamma_1 p\rho_0^2}{\gamma_1}$. As a result we have the following exponential solution:

$$u_9(x,t) = \ln\left(\frac{16\gamma_3^2\rho^2\rho_2 e^{\frac{2\sqrt{\gamma_3}\Gamma(\beta+1)\left(x^\varepsilon - \frac{p\rho_2 t^\varepsilon}{2\gamma_1}\right)}{\varepsilon}}}{\left(\gamma_1\gamma_3 - 4\rho^2 e^{\frac{2\sqrt{\gamma_3}\Gamma(\beta+1)\left(x^\varepsilon - \frac{p\rho_2 t^\varepsilon}{2\gamma_1}\right)}{\varepsilon}}\right)^2} + \rho_0\right). \tag{37}$$

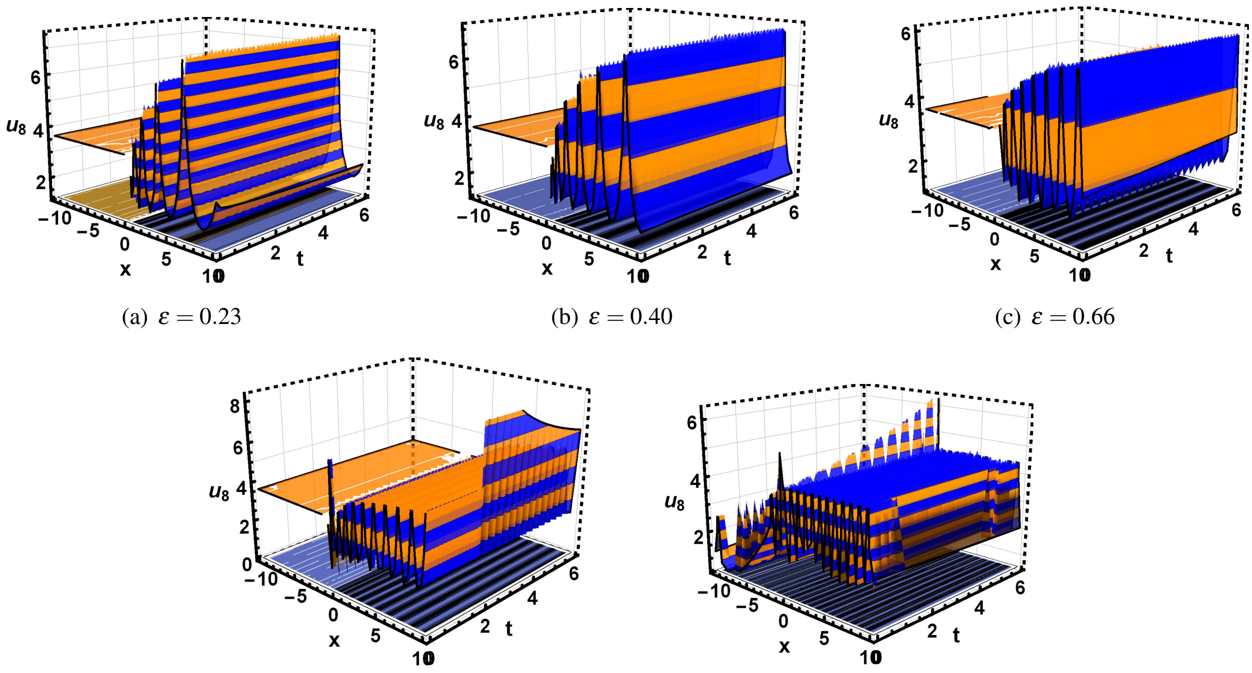
Taking $\gamma_1 = -\frac{4\rho^2}{\gamma_3}$ in Eq. (37), we have bright soliton solution

$$u_{10}(x,t) = \ln\left(\frac{\gamma_3^2\rho_2 \operatorname{sech}^2\left(\frac{\sqrt{\gamma_3}\Gamma(\beta+1)(x^\varepsilon - ct^\varepsilon)}{\varepsilon}\right)}{4\rho^2} + \rho_0\right). \tag{38}$$

Similarly, taking $\gamma_1 = \frac{4\rho^2}{\gamma_3}$ in Eq. (37), we have

$$u_{11}(x,t) = \ln\left(\frac{\gamma_3^2\rho_2 \operatorname{csch}^2\left(\frac{\sqrt{\gamma_3}\Gamma(\beta+1)(x^\varepsilon - ct^\varepsilon)}{\varepsilon}\right)}{4\rho^2} + \rho_0\right). \tag{39}$$

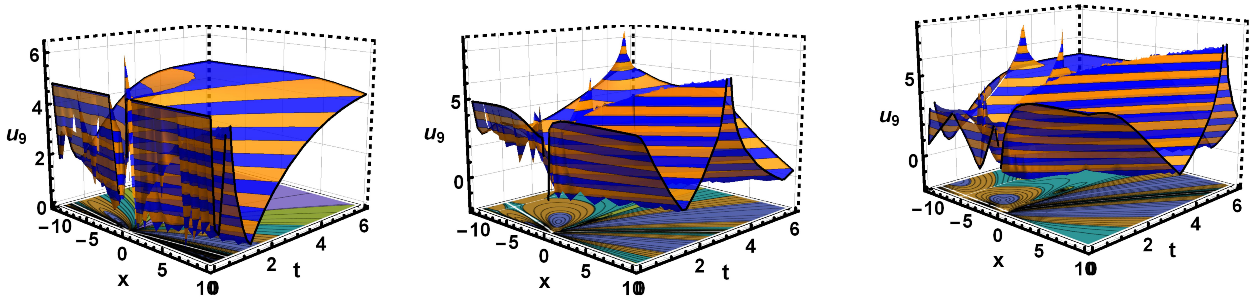
Remark: All the obtained solutions are written in the form of Eq. (5), such that $u = \ln v$.



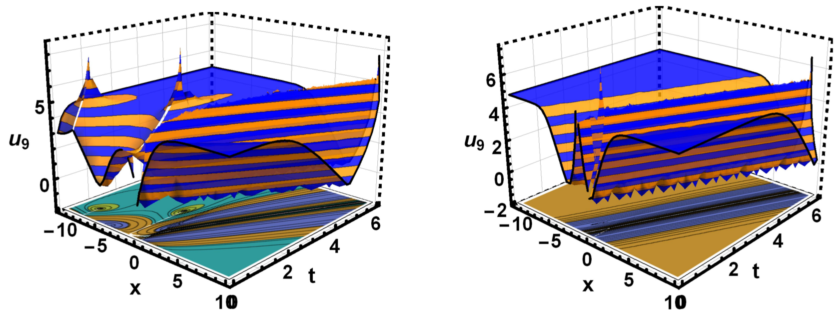
(a) $\varepsilon = 0.23$ (b) $\varepsilon = 0.40$ (c) $\varepsilon = 0.66$

(d) $\varepsilon = 0.76$ (e) $\varepsilon = 0.986$

Fig. 3 Sketches of the Eq. (36) with fractional parameter.

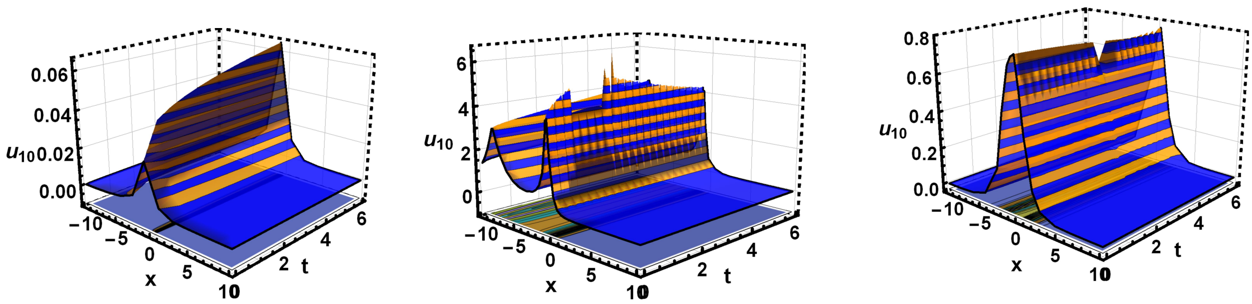


(a) $\varepsilon = 0.13$ (b) $\varepsilon = 0.28$ (c) $\varepsilon = 0.486$



(d) $\varepsilon = 0.686$ (e) $\varepsilon = 0.96$

Fig. 4 Sketches of the Eq. (37) with fractional parameter.



(a) $\varepsilon = 0.23$ (b) $\varepsilon = 0.46$ (c) $\varepsilon = 0.76$

Fig. 5 Sketches of the Eq. (38) with fractional parameter.

3.3 Application of the MGERIFM

The solution via MGERIFM [39] is described as:

$$v(\xi) = c_0 + \sum_{r=1}^{n} c_i \left(\underbrace{\iint \cdots \int}_{r} v(\xi) d\xi d\xi \cdots d\xi \right)^{r} + \sum_{r=1}^{n} d_r \left(\underbrace{\iint \cdots \int}_{r} v(\xi) d\xi d\xi \cdots d\xi \right)^{-r}. \tag{40}$$

The solution to equation (40) for $n = 2$ is as follows:

$$v(\xi) = c_0 + c_1 \int \phi(\xi) d\xi + d_1 \left(\int \phi(\xi) d\xi \right)^{-1} + c_2 \left(\int \phi(\xi) d\xi \right)^{2} + d_2 \left(\int \phi(\xi) d\xi \right)^{-2}, \tag{41}$$

where $v(\xi)$ is defined by

$$\phi(\xi)=\frac{\tau_1 e^{\xi s_1}+\tau_2 e^{\xi s_2}}{\tau_3 e^{\xi s_3}+\tau_4 e^{\xi s_4}}. \tag{42}$$

Moreover, we proceed as:
Case-1: Choosing $[\tau_1,\tau_2,\tau_3,\tau_4]=[-i,-i,-i,-i]$ and $[s_1,s_2,s_3,s_4]=[1,-1,0,0]$, Eq. (42) takes the following form

$$\phi(\xi)=\cosh(\xi). \tag{43}$$

Inserting Eq. (43) into Eq. (41), implies that

$$v(\xi)=c_0+c_1\sinh(\xi)+c_2\sinh^2(\xi)+\frac{d_1}{\sinh(\xi)}+\frac{d_2}{\sinh^2(\xi)}. \tag{44}$$

The solutions are as follows when Eq. (44) is inserted into Eq. (6):
For $d_1=0,\ c_1=0,\ c_2=0,\ c=\frac{d_2 p}{2},\ r=2c_0^2 p\,(c_0-d_2),\ q=c_0 p\,(2d_2-3c_0)$, we have:

$$u_1(x,t)=\ln\left(\frac{2c\operatorname{csch}^2\left(\frac{\Gamma(\beta+1)(x^{\varepsilon}-ct^{\varepsilon})}{\varepsilon}\right)}{p}+c_0\right). \tag{45}$$

Case-2: Let $[\tau_1,\tau_2,\tau_3,\tau_4]=[2i,-2i,4i,4i]$ and $[s_1,s_2,s_3,s_4]=\left[\frac{1}{2},-\frac{1}{2},0,0\right]$, Eq. (42) transforms to sine hyperbolic function

$$\phi(\xi)=\frac{1}{2}\sinh\left(\frac{\xi}{2}\right). \tag{46}$$

Plugging Eq. (46) into Eq. (41), offers

$$v(\xi)=c_0+c_1\cosh\left(\frac{\xi}{2}\right)+c_2\cosh^2\left(\frac{\xi}{2}\right)+\frac{d_1}{\cosh\left(\frac{\xi}{2}\right)}+\frac{d_2}{\cosh^2\left(\frac{\xi}{2}\right)}. \tag{47}$$

Applying Eq. (47) to Eq. (6), we get:
For $d_1=0,\ c_1=0,\ c_2=0,\ c=-2d_2 p,\ r=2c_0^2 p\,(c_0+d_2),\ q=c_0(-p)\,(3c_0+2d_2)$, we get the solution as follows:

$$u_2(x,t)=\ln\left(c_0-\frac{\operatorname{csch}^2\left(\frac{\Gamma(\beta+1)(x^{\varepsilon}-ct^{\varepsilon})}{2\varepsilon}\right)}{2p}\right). \tag{48}$$

Case-3: Taking $[\tau_1,\tau_2,\tau_3,\tau_4]=[1,-1,i,i]$ and $[s_1,s_2,s_3,s_4]=[i,-i,0,0]$, Eq. (42) converts to the periodic function

$$\phi(\xi)=\sin(\xi). \tag{49}$$

Putting Eq. (49) into Eq. (41), gives

$$v(\xi)=c_0+c_2(-\cos(\xi))^2+c_1(-\cos(\xi))+\frac{d_1}{-\cos(\xi)}+\frac{d_2}{(-\cos(\xi))^2}. \tag{50}$$

Incorporating Eq. (50) in Eq. (6), we have:
For $d_1=0,\ c_1=0,\ c_2=0,\ c=\frac{d_2 p}{2},\ r=2c_0^2 p\,(c_0+d_2),\ q=c_0(-p)\,(3c_0+2d_2)$, we obtain:

$$u_3(x,t)=\ln\left(\frac{2c\sec^2\left(\frac{\Gamma(\beta+1)(x^{\varepsilon}-ct^{\varepsilon})}{\varepsilon}\right)}{p}+c_0\right). \tag{51}$$

Case-4: Choosing the parameters $[\tau_1,\tau_2,\tau_3,\tau_4]=[1,1,1,1]$ and $[s_1,s_2,s_3,s_4]=[i,-i,0,0]$, Eq. (42) offers

$$\phi(\xi)=\cos(\xi). \tag{52}$$

Manipulating Eq. (52) and Eq. (41), we have

$$v(\xi)=c_0+c_1\sin(\xi)+c_2\sin^2(\xi)+\frac{d_1}{\sin(\xi)}+\frac{d_2}{\sin^2(\xi)}. \tag{53}$$

Plugging Eq. (53) into Eq. (6), give the following solutions:
When $d_1=0,\ c_1=0,\ c_2=0,\ c=\frac{d_2 p}{2},\ q=c_0(-p)(3c_0+2d_2),\ r=2c_0^2 p(c_0+d_2)$, we get:

$$u_4(x,t)=\ln\left(\frac{2c\csc^2\left(\frac{\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{\varepsilon}\right)}{p}+c_0\right). \tag{54}$$

Case-5: Taking the parameters $[\tau_1,\tau_2,\tau_3,\tau_4]=[2,2,2,2]$ and $[s_1,s_2,s_3,s_4]=\left[\frac{2}{5},\frac{2}{5},0,0\right]$, Eq. (42) gives

$$\phi(\xi)=e^{\frac{2\xi}{5}}. \tag{55}$$

Manipulating Eq. (55) and Eq. (41), we have

$$v(\xi)=c_0+\frac{1}{2}c_1\left(5e^{\frac{2\xi}{5}}\right)+c_2\left(\frac{5}{2}e^{\frac{2\xi}{5}}\right)^2+\frac{d_1}{\frac{5}{2}e^{\frac{2\xi}{5}}}+\frac{d_2}{\left(\frac{5}{2}e^{\frac{2\xi}{5}}\right)^2}. \tag{56}$$

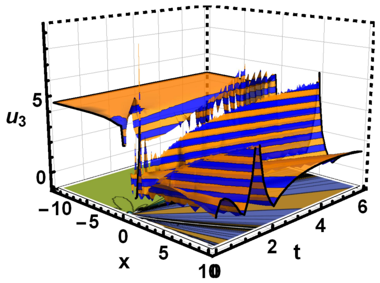
Inserting Eq. (56) into Eq. (6), give the following solution:
When $d_1=0,\ c_1=0,\ p=0,\ r=\frac{1}{25}(-16)c\left(c_0^2-4c_2d_2\right),\ q=\frac{16cc_0}{25}$, we get:

$$u_5(x,t)=\ln\left(\frac{4}{25}d_2e^{-\frac{4\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{5\varepsilon}}+\frac{25}{4}c_2e^{\frac{4\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{5\varepsilon}}+c_0\right). \tag{57}$$

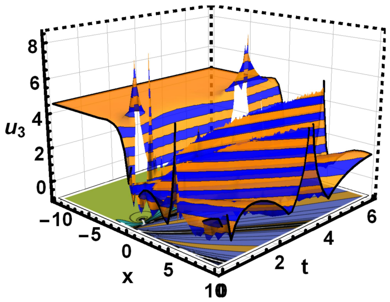
Next, the hyperbolic solution is written as

$$\begin{aligned}u_6(x,t)=\ln\Bigg(&\frac{25}{4}c_2\left(\cosh\left(\frac{4\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{5\varepsilon}\right)+\sinh\left(\frac{4\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{5\varepsilon}\right)\right)+c_0\\&+\frac{4}{25}d_2\left(\cosh\left(\frac{4\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{5\varepsilon}\right)-\sinh\left(\frac{4\Gamma(\beta+1)(x^\varepsilon-ct^\varepsilon)}{5\varepsilon}\right)\right)\Bigg).\end{aligned} \tag{58}$$

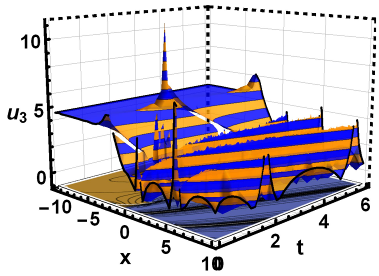
Remark: All the obtained solutions are written in the form of Eq. (5), such that $u=\ln v$.



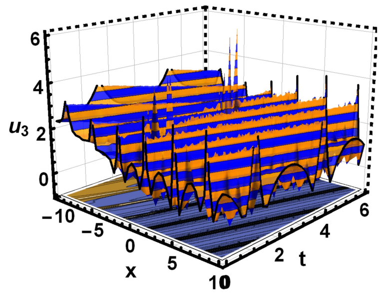
(a) $\varepsilon = 0.23$



(b) $\varepsilon = 0.43$

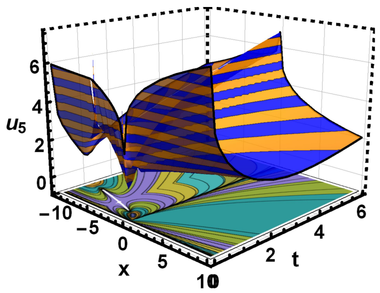


(c) $\varepsilon = 0.73$

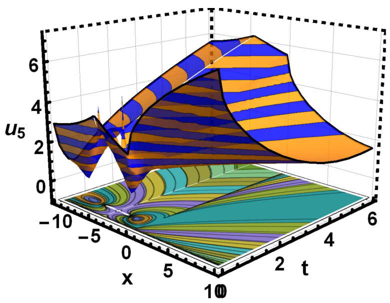


(d) $\varepsilon = 0.966$

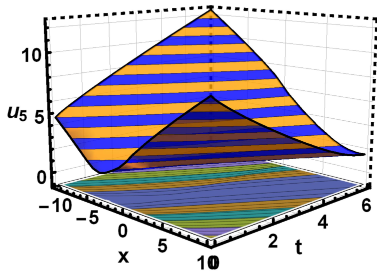
Fig. 6 Sketches of the Eq. (51) with fractional parameter.



(a) $\varepsilon = 0.29$



(b) $\varepsilon = 0.49$



(c) $\varepsilon = 0.86$

Fig. 7 Sketches of the Eq. (57) with fractional parameter.

4 Conclusions

In this paper, we have discussed the nonlinear truncated M-fractional ZSE and investigated the fractional parametric effect on the dynamics of wave movement. A variety of solutions have been secured through the implementation of the recently advanced mathematical methods. For the proposed model, our methods have effectively identified many types of propagating wave solutions, demonstrating their versatility and applicability in various scenarios. The physical characteristics of the developed solutions have been illustrated by a variety of Figures (1-7) and it has been observed that a slight change in the fractional parameter offers the different nature of the wave solutions. The obtained findings may be of interest for further investigation of frameworks that are designed to address nonlinear issues in the field of applied sciences. The results are fascinating from both a theoretical and practical perspective, especially in terms of the behavior of optical components. The presented techniques are the most practical, smoothly, and straightforward approaches to addressing different models in applied sciences, offering a wide range of exact solutions compared to previous methodologies. In future, these techniques can be extended for the solution of higher non-linear systems.

5 Declarations

5.1 Conflict of interest:

The authors declare that there is no conflict of interest regarding the publication of this paper.

5.2 Author's contributions:

J.M.-Methodology, Resources, Writing-Original Draft, Methodology, Investigation, Visualization. U.Y.-Validation, Conceptualization, Formal Analysis, Data Curation, Writing-Review and Editing. The paper has been submitted with the knowledge and consent of all authors.

5.3 Funding:

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5.5 Data availability statement:

All data that support the findings of this study are included within the article.

5.6 Using of AI tools

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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