

Solution, Simulation and Optimization of the Equations Governing the Water Hammer Problem in the Case of a Valve Closing

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Abstract. This study provides a rigorous numerical investigation into transient hydrodynamic regimes (water hammer) induced by slow valve closure configurations within a Reservoir-Pipe-Valve Supply System (RPVSS). Utilizing the Method of Characteristics (MOC) to solve the fundamental hyperbolic Saint-Venant equations, this research evaluates and optimizes the transient response of four widely implemented piping materials: steel, ductile cast iron, glass-reinforced plastic (GRP), and high-density polyethylene (HDPE), under both standardized and optimized non-linear boundary conditions. The multi-parametric optimization framework reveals that each material exhibits an intrinsic, periodic distribution of the optimal closure shape exponent (m), systematically confined between converging upper ($m_{\max}$) and lower ($m_{\min}$) envelope curves. Cross-analysis of the transient dynamics demonstrates that while rigid metallic conduits (cast iron and steel) suffer severe initial overpressures up to 28.5 bar, they achieve rapid wave attenuation; conversely, visco-elastic HDPE inherently cushions the initial pressure shock to 20.5 bar but demands a broader stabilization window (up to 105 s) due to its lower acoustic wave celerity. Furthermore, the overpressure profiles exhibit distinct non-monotonic localized bumps resulting from constructive acoustic wave interference, initiating at a threshold of $1.4t_4$ and fading beyond $4t_4$ (where t_4 represents the transient reflection period). The resulting parametric predictive models demonstrate that optimal closure profiles systematically converge toward a convex configuration ($1.20 \leq m \leq 1.30$). These analytical envelopes provide a powerful, high-fidelity engineering tool, enabling operators to determine optimal valve closing schedules that simultaneously mitigate extreme overpressures below the material's nominal pressure capacity (PN) and prevent transient depressions from dropping beneath the liquid's vaporization threshold, thereby effectively eliminating cavitation risks.

Key words: Transient flow – water hammer, Optimal closure law – Pipe materials, Exponent envelope

1. Introduction

The water hammer phenomenon constitutes a critical hydraulic transient induced by an abrupt alteration in the momentum of a fluid within a closed conduit, typically triggered by valve closures or pump trips, wherein the propagation of the acoustic shock wave is governed by the fluid's bulk modulus and the pipe's structural elasticity. Although the theoretical foundations of wave propagation and the Method of Characteristics (MOC) trace back to the pioneering works of Lagrange (1788) and Monge (1789), with Monge's role in the development of the Method of Characteristics highlighted by Wood (1970), they were subsequently enriched by the seminal works of Joukowski (1904), Bergeron (1931), and Gariel's analyses of head losses and wave transmission (Remenieras 1961). More recently, advances in modern computing have considerably improved the numerical simulation of transient hydraulic flows. While extensive research has focused on finite volume (Christen et al 2010), finite element (Chen and Lee 2003), and finite difference methods (Liang et al 2006), the combination of the MOC and mixed schemes (Streeter and Benjamin 1983) has received less attention. Recent research highlights the crucial role of fluid-structure interactions (FSI) and the elastic properties of materials, demonstrating that conduits with a low modulus of elasticity (such as GRP) or composite and hybrid configurations (steel combined with HDPE/LDPE or rubber) optimize the attenuation of the maximum pressure surge. To mitigate transient pressures in water distribution networks, the strategic integration of polymeric and viscoelastic materials as alternatives to traditional metallic pipelines has been widely investigated. Gong et al (2018) demonstrated the feasibility of suppressing pressure surges by substituting a steel pipe with a plastic section, specifically along the branch connecting the transient source to the water main. This concept of infrastructure retrofitting was extended by Garg and Kumar (2020), who reported that deploying viscoelastic piping, either standalone or in hybrid configurations, provides an effective mechanism for water hammer control. Similarly, Triki (2020) validated a localized protection technique involving the replacement of highly vulnerable pipeline segments with polymeric conduits. However, the introduction of material heterogeneities alters the system's boundary dynamics. Ferrante (2021) established that the reflection coefficient at the junction of two distinct polymer pipes is strictly governed by their viscoelastic parameters and the transient time-history. Furthermore, under critical operational scenarios such as pump failure, Bettaieb et al (2020) confirmed that wave reflections originating from metallic-polymeric interfaces exert a significant influence on the overall pressure wave propagation across the distribution system. Kubrak et al (2021, 2022) in order to mitigate high pressures presents a new approach to include additional polymeric pipe in the steel pipeline system.

Concurrently, sensitivity analyses indicate that while geometric parameters (diameter, wall thickness) and flow characteristics exhibit strong correlations with overpressure (Jinhao et al 2021), the ultimate magnitude of the water hammer inherently

depends on the operational characteristics of the valves. Consequently, contemporary research has intensified around the development of Optimal Valve Closure Laws (OVCL) (Bazargan-Lari et al 2013, Wan and Li, 2016), leveraging non-linear kinetics (fast then slow phases) (Zheng et al 2022), broken or staggered closures (Herasymov et al 2019, Wan et al 2019a), or convex profiles modeled by exponential and linear approaches (Toumi and Sekiou 2024), in order to eradicate the risks of structural rupture and cavitation while maximizing the longevity and operational reliability of adduction networks.

Despite these insights, a gap remains in systematically coupling optimal valve closure laws with the specific viscoelastic and elastic properties of diverse pipeline materials under transient conditions. This research addresses this gap by investigating the transient response of a gravity-driven water supply system experiencing a downstream valve closure.

The original contributions of this study to the literature are threefold:

- **Multi-Material Comparative Analysis:** We evaluate the temporal evolution of total head, pressure, mean velocity, and transient flow rate across all discretization nodes for four distinct, commercially prominent buried pipeline materials: steel, cast iron, glass-fiber reinforced polyester (GRP), and high-density polyethylene (HDPE).
- **Exposant-Driven Optimization:** Utilizing the MOC to transform the governing Saint-Venant partial differential equations into numerically manageable total differential equations, we implement a mixed diagram approach to precisely integrate pipeline friction.
- **Material-Specific Optimal Law Synthesis:** We establish a rigorous optimization framework to identify the optimal valve closing laws tailored to each of the four materials. This optimization accounts for localized pressure drops, systemic characteristics, and varying closure times, contrasting our optimal profiles directly against standard industrial closure practices to demonstrate surge suppression efficiency.

For this purpose, four different network materials will be considered which are: steel, cast iron, fiber reinforced polyester and high density polyethylene (H.D.PE) and this for the case of buried pipes. This will be within our reach by using the method of characteristics allowing, on the one hand, to transform the two partial differential equations of Saint Venant into four integrable total difference equations in order to attend algebraic equations that are easy to manipulate numerically, and on the other hand, it allows the transition from a permanent flow to a transient flow. However, the calculation of the indicated above parameters requires first of all the search for the optimal valve closing laws for the four chosen materials while taking into account the pressure drop, all the characteristics of the system (RPVSS), the closing time and using the mixed diagram for the integration of the resistance part of the pipeline, then go and look for the corresponding values of overpressures and depressions as well as for the laws imposed the frequently applied ones. To do this, it is necessary to start

with, all the equations and relations governing transient flows in pressurized water supply and the existing laws of closing a valve.

2. Mathematical Model

The equations which can faithfully describe the phenomena of transient flow in hydraulic adduction pipeline are the basic hyperbolic equations demonstrated successively from the continuity equations and dynamics (Streeter and Benjamin 1983, Toumi and Remini 2021):

$$\begin{cases} \rho \frac{\partial u}{\partial x} + \frac{1}{a^2} \left(\frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} \right) = 0, \\ \rho \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} = -g (\sin(\alpha) + j), \end{cases} \quad (1)$$

where:

- P – pressure (Pa),
- ρ – density of the liquid, for water $\rho = 10^3$ (kg/m³),
- u – the flow velocity (m/s),
- g – acceleration due to gravity (m/s²),
- α – pipe inclination angle (degrees),
- j – head loss per unit length (m/m),
- a – wave velocity (m/s).

3. Materials and methods

3.1. Resolution Method and Integration of Differential System

The Method Of Characteristics (MOC) is the most powerful method for the numerical resolution of equations characterizing the transient flow under pressure; it transforms the two partial differential equations of the transient flow into four total differential equations (Ouragh 1971, Piskonov 1995, Toumi and Remini 2021).

Figure 1 illustrates the four equations obtained when using the characteristics method and the algebraic relations.

The system of total differential equations derived through the application of the Method of Characteristics to equation system (1) is elegantly expressed as follows:

$$\begin{cases} \frac{d}{dx} = \pm \alpha, \\ \frac{d}{dt} \left(\frac{\alpha Q}{gS} \pm H \right) + \alpha j = 0, \end{cases} \quad (2)$$

where Q is the flow rate (m³/s) and H is piezometric head.

The integration of the total differential equation system leads to a compact and solvable set of algebraic equations, formulated as follows (Toumi and Remini 2021):

$$\begin{cases} CP = H(i) + RQ(i) - T_1((Q(i+1) \cdot |Q(i)|)), \\ CM = H(i+2) - RQ(i+2) + T_1((Q(i+1)|Q(i+2)|)), \end{cases} \quad (3)$$

$$\begin{cases} H(i+1) = \frac{CP + CM}{2}, \\ Q(i+1) = \frac{CP - H(i+1)}{R}, \end{cases} \quad (4)$$

where:

$H(i)$ and $Q(i)$ denote the piezometric head and the flow rate under steady-state conditions, respectively; $H(i+1)$ and $Q(i+1)$ are successively the piezometric head and flow rate in transient flow; CP and CM are successively the positive and negative characteristics (Toumi and Remini 2021).

$$T_1 = B(x(i+2) - x(i+1)) = B\Delta x, \quad (5)$$

where T_1 – frictional head loss term per spatial step in s^2/m^4 ,

$$B = \frac{\lambda}{2gdS^2}, \quad (6)$$

where:

- B – head loss coefficient due to wall friction in s^2/m^5 ,
- d – internal diameter of the pipe in m,
- λ – coefficient of friction,
- Δx – space step in m,
- α – pipe inclination angle (degrees),
- S – pipe cross section in (m^2).

$$R = \frac{\alpha}{gS}, \quad (7)$$

where: R is the head-to-discharge conversion factor in (s/m^2).

j is expressed by the following relation:

$$j = \frac{\lambda}{2gdS^2} Q|Q|. \quad (8)$$

For the case of a buried pipe, wave celerity is done by the following expression (Ait Sliman 2018).

$$\alpha = \left[\frac{\rho}{K_1} \left(1 + K_1 (1 - v_m^2) \frac{2r}{E_m e_m} \right) [1 + F(P - P_a)] \right]^{-1/2}, \quad (9)$$

where:

$$F = (1 - v_m^2) \frac{r^2}{E_m e_m^2} (1 - 2v_m), \quad (10)$$

where:

- F – coefficient of variation of compressibility with respect to pressure (Pa^{-1}),
- E_m – Young's modulus of pipe walls material (Pa),
- v_m – Poisson's coefficient of the metal,
- e_m – pipe wall thickness (m),
- K_1 – bulk modulus of elasticity of water (Pa),
- r – pipe radius (m),
- P – hydrostatic pressure (Pa),
- P_a – backfill soil load (Pa).

$$P = (H_i - \Delta H) \rho g, \quad (11)$$

where:

- H_i – hydrostatic head at the considered node,
- ΔH – pressure drop between the tank and the considered node.

The fill earth load is calculated by the formula given by Marston written as follows:

$$P_a = C_d \gamma B_d H_1, \quad (12)$$

where:

- γ – specific weight of soil (N/m^3),
- B_d – trench width (m),
- H_1 – height of backfill soil above pipe (m),
- C_d – load coefficient.

$$C_d = \frac{\left(1 - e^{-2K_2\mu' \left(\frac{H_1}{B_d}\right)}\right)}{2K_2\mu'} \frac{B_d}{H_1}, \quad (13)$$

where: K_2 stands for Rankine's coefficient, and μ' is the parameter that describes vertical friction at the backfill and trench wall contact (Moser 2001).

The two last expressions (4) allow the calculation of the values of piezometric head and flow rate, in transient flow, at the interior nodes. However, the computation of the values of these parameters at boundary nodes requires other additional expressions depending on the conditions of the case to be studied.

3.2. Stability Condition of the Method Used

The stability of characteristics method (CM) is ensured when the stability criterion S_c is less than or equal to one.

$$S_c = \frac{a\Delta t}{\Delta x} = \frac{aN\Delta t}{L} \leq 1, \quad (14)$$

where:

- Δt – the time step,
- N – number of discretization nodes,
- L – length of the pipe.

In general, the MOC gives exact numerical results when $S_c = 1.0$ (John 2018, Goldberg and Wylie 1983).

3.3. Statement of the Problem

The mathematical models developed in this work are validated using a benchmarking hydraulic configuration, designed to reflect realistic operational conditions. The investigated system consists of a gravity-fed pipeline $L = 1000$ m supplied by a major upstream reservoir R featuring a 200 m overflow crest. This conduit discharges a constant steady-state flow of $0.355 \text{ m}^3/\text{s}$ into a motorized control valve positioned at a downstream elevation of 50 m. To simulate transient hydrodynamic phenomena, the initial steady flow is perturbed by manipulating the valve closure dynamics. Both rapid and gradual closure scenarios, whether operational or accidental, are considered to evaluate the system's transient response. The complete topology of this hydraulic network is schematized in Figure 1.

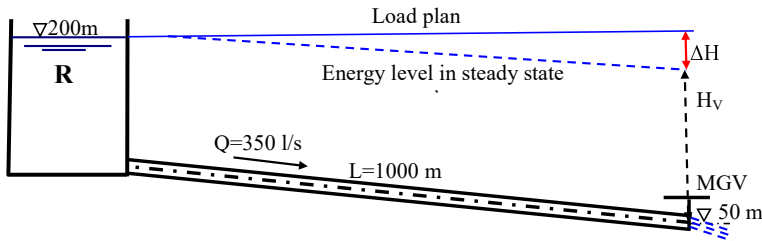


Fig. 1. Diagram of a motorized valve supplied with water from a reservoir

To evaluate the influence of structural elasticity on transient behavior, the pipeline is sequentially modeled using four standard piping materials: cast iron, steel, fiber-reinforced polyester, and high-density polyethylene (HDPE). Characterizing these widely adopted materials allows for the simulation and subsequent optimization of

material-specific valve closure laws. The following sections provide the comprehensive dataset and constitutive equations required to implement the proposed mathematical framework. Specifically, the formulation incorporates two distinct boundary conditions at the terminal nodes, which govern the transient flow dynamics and establish the operational system of equations presented below:

– **at the outlet of the tank (node number 1)**

$$H(1) = H_R, \quad (15)$$

$$Q(1) = \frac{H_R - CM}{R}, \quad (16)$$

$$CM = H(2) - Q(2)(R - T_1|Q(2)|), \quad (17)$$

$H(2)$ and $Q(2)$ are the piezometric head and discharge, respectively, computed at time t at node number 2, while H_R denotes the piezometric head at the reservoir.

– **at valves placed at the end of a water supply network (last node)**

The calculation of the pressure and the flow at this node is carried out according to the positive characteristic of the previous node, whose equations of this characteristic of the node are written as follows:

$$CP = H(N - 1) + Q(N - 1)(R - T_1|Q(N - 1)|), \quad (18)$$

$$Q(N) = \frac{CP - H(N)}{R}, \quad (19)$$

$$H(N) + RQ(N) = CP. \quad (20)$$

However, in the case where closing is controlled, the flow at the valve follows, over time, a function depending on the chosen closing law and in this case, the total head at the valve takes the following relationship:

$$H(N) = CP - RQ(N). \quad (21)$$

The flow rate at node number N (at the valve) is closely related to the flow rate through the valve. This last can take concave, linear or convex laws, as it is below (Provenzano et al 2011, Subani and Norsarahaida 2015):

$$Q_N^{k+1} = \left(1 - \left(\frac{k\Delta t}{C_t}\right)^m\right) Q_N^k, \quad (22)$$

where:

- m – mathematical exponent determining the valve closing law,
- C_t – the total valve closing time,
- Q^k, Q^{k+1} – successively the flow rates at times t and $t + \Delta t$,
- k – time increment index.

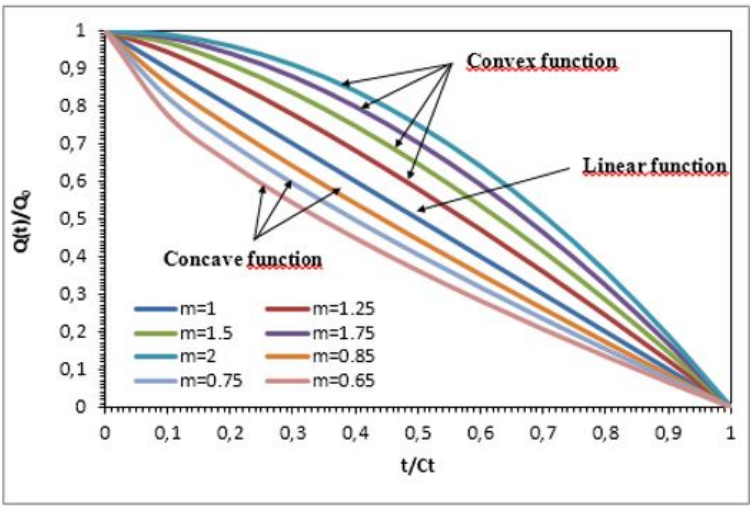


Fig. 2. Closing functions corresponding to different values of m

Table 1. Characteristics of the materials used

Materials kind	Steel	Cast Iron	HDPE	FRP
Elasticity modulus in Pascal	$E_A = 2.10^{11}$	$E_F = 1.03 \cdot 10^{11}$	$E_{PEHD} = 1.2 \cdot 10^9$	$E_{FRP} = 2.10^{10}$
Exterior diameter in (mm)	610	635	630	632
Interior diameter in (mm)	600	600	515.6	600
Pipe thickness e_1 in (mm)	5	17.5	57.2	16
Pipe roughness ε in (mm)	0.04	0.03	0.02	0.029
Length in (m)	1000	1000	1000	1000
Pipe nominal pressure in (bar)	42	42	16	16 to 20
Poisson ratio	0.30	0.26	0.45	0.29

Table 2. Reservoir, flow and fluid characteristics

Reservoir overflow level in (m)	200	Fluid elasticity modulus	$K = 2.0710^9$ Pa
Flow leaving the reservoir in (l/s)	350	Fluid temperature	15°C

Figure 2 shows the envelope curves of the possible laws for closing a valve in a hydraulic system (Reservoir-Pipe-Valve).

The exponent m within the nodal discharge equation introduces a high degree of functional variability. Consequently, identifying its optimal value constitutes a critical prerequisite for achieving efficient system management. To validate the algebraic formulations derived from the mathematical model, the framework is applied to the benchmark configuration illustrated in Figure 2. Initiating the transient state simulations requires a comprehensive dataset defining the physical properties of the fluid, the reservoir geometry, the hydraulic network, and the pipe materials. These parameters are systematically detailed in Tables 1 and 2.

4. Results and Discussion

To monitor the spatiotemporal evolution of hydraulic parameters – specifically manometric head, pressure head, velocity, and volumetric flow rate – a dedicated computational algorithm was developed. The operational logic of this numerical program is governed by the sequential flowchart presented below.

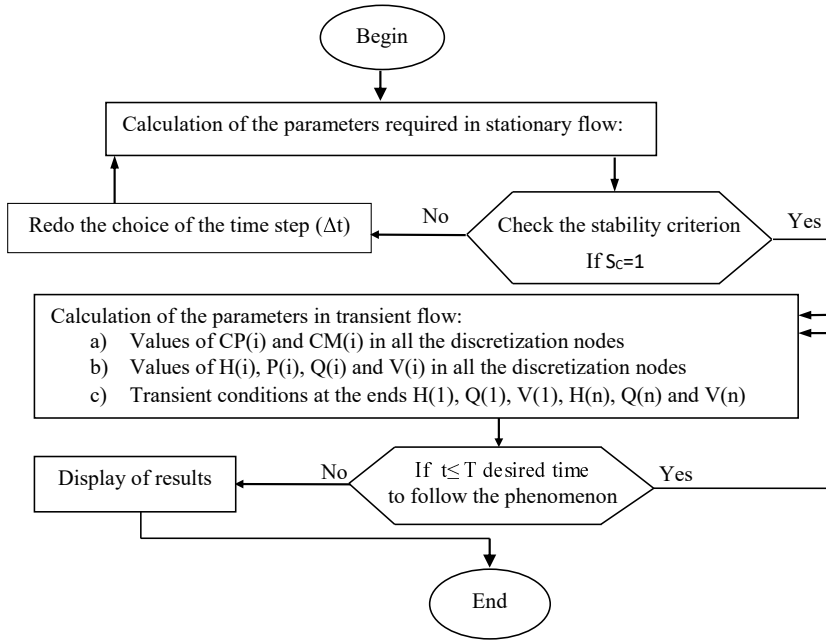


Fig. 3. Flowchart for calculating hydraulic parameters in a pressurized network during unsteady state for several laws of closing of a motorized valve

The computational procedure for determining the transient flow parameters is systematically outlined in the flowchart presented in Figure 3.

4.1. The Search for Optimal Valve Closing Laws

Steel, cast iron, fiber-reinforced polyester, and HDPE are widely utilized in water distribution networks, their structural response to transient hydrodynamic regimes requires deeper investigation. To elucidate the material-specific behavior under such dynamic stresses, this study analyzes a downstream control valve undergoing a gradual closure sequence. This operational scenario serves as a benchmark to characterize and compare the damping and propagation characteristics inherent to each piping material during transient events.

According to the criterion established by Dupont (1979), a valve closure is classified as slow when the total closure duration t exceeds the wave travel time required

for a complete round trip (back and forth) along the pipeline, defined by the condition $t > 2L/a$. Consequently, this study investigates the transient hydrodynamic response across pipe length, tracking the evolution of key hydraulic parameters from the initiation of the maneuver $t = 0$ through the valve slow closure.

Addressing this problem constitutes the primary objective of the present study. Prior to evaluating the transient parameter profiles, it is necessary to determine the optimal valve closure laws for the considered hydraulic configuration. This optimization process involves highly non-linear, iterative, and computationally intensive operations, which demand strict numerical stability and precise data interpretation. Because these closure laws are highly sensitive to a multi-parametric space – including network topology, closure duration, spatiotemporal discretization steps (Δt , Δx), and the transient wave period – this work establishes a generalized methodological framework. Given that the optimal closure dynamics are intrinsically system-specific, providing a unified operational procedure ensures the scalability and reproducibility of the model for diverse hydraulic systems.

This methodological approach primarily involves evaluating various closure law profiles – specifically concave, convex, and linear configurations – to identify the optimal operational trajectory. This comparative analysis is achieved through a generalized valve closure function governed by the shape exponent m , which systematically controls the curvature of the law. Furthermore, to capture the temporal evolution of the transient state, the system response is evaluated across multiple iterations of the transient reflection period t_4 . Within this framework, the characteristic time equation is expressed as follows:

$$t = k_3 t_4, \quad (23)$$

where k_3 takes positive values greater than 0.5.

The closing times taken start from $0.51 t_4$ to $10 t_4$ with a time step equal to 0.01, ie 950 points.

The results obtained of the optimal values of the exponent m , for the four materials are mentioned in figures 4(a), 4(b), 4(c) and 4(d).

The parametric curves of the optimal closure laws, plotted against the total valve closure duration for the four selected materials, demonstrate that the optimal control law is highly sensitive to both material properties and closure time scales. At slow closure durations, a significant divergence in the optimal law profiles is observed. However, as the closure time increases, these profiles converge toward a highly restricted envelope. Furthermore, the results indicate that higher material elastic moduli correlate with a narrower final convergence interval. The optimal shape exponent m exhibits a damped periodic behavior, where each material is characterized by a distinct transient reflection period t_4 and a unique response curve. Notably, when the closure duration exceeds $2t_4$, the interval of optimal laws restricts and exhibits a distinct clustering or thickening effect. This phenomenon is primarily attributed to the constructive interference and superposition of acoustic shock waves generated

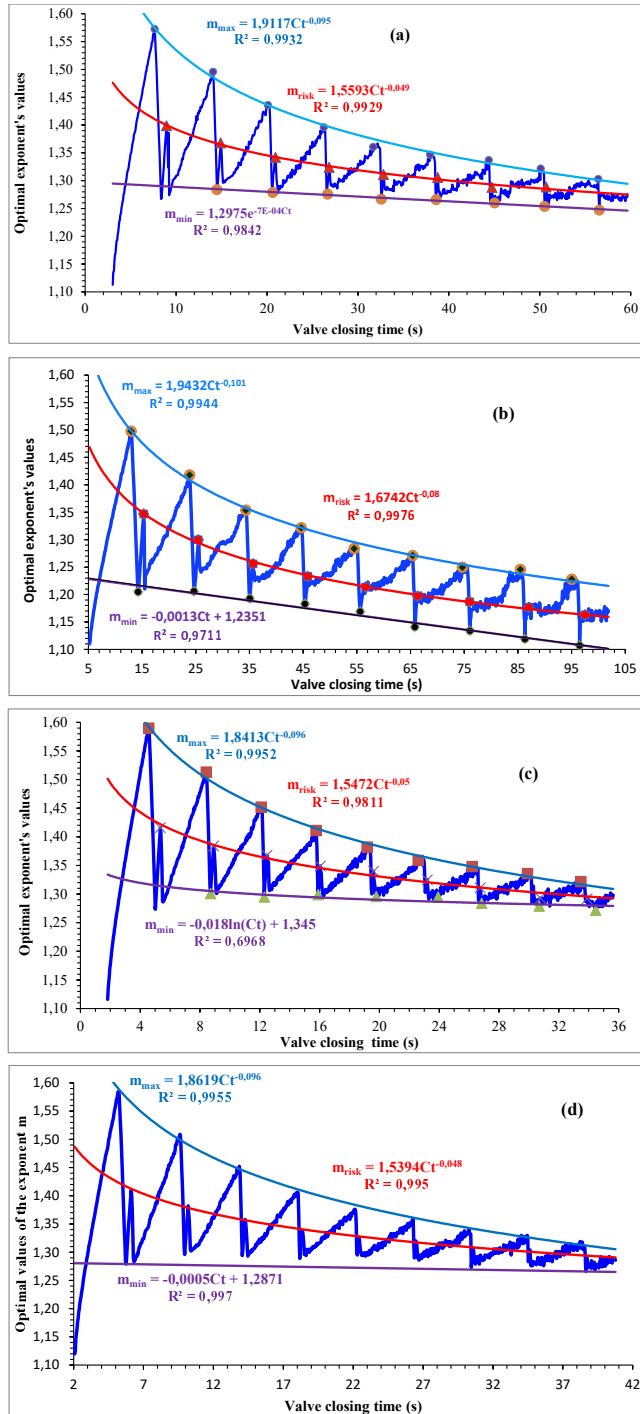


Fig. 4. Optimal exponent's values versus time plot, at the valve and the envelope curves PRV (a), HDPE (b), Cast Iron (c) and Steel (d)

during preceding transient cycles. To analytically characterize this behavior without requiring computationally intensive iterations, material-specific regression boundaries were established for each individual conduit type. For FRP (a), the damped oscillations converge smoothly toward an asymptote of $m = 1.27$ within a 60 s window, displaying a distinct exponential decay boundary for its minimum envelope $m_{\min} = 1.2975e^{-7.10-4Ct}$. Conversely, HDPE (b) requires a significantly prolonged tracking timeline up to 105 s due to its low wave celerity, where its lower boundary uniquely follows a steadily decreasing linear function ($m_{\min} = -0.0013Ct + 1.2351$), capturing the material's long-term viscoelastic memory. For the highly rigid metallic conduits, distinct structural signatures emerge: cast iron (c) exhibits rapid stabilization within 36 s but displays localized geometric perturbations along its minimum envelope governed by a logarithmic trend ($m_{\min} = -0.018 \ln(Ct) + 1.345$), whereas steel (d) demonstrates a flawless deterministic fit ($R^2 = 0.997$) along a virtually flat, linear lower boundary ($m_{\min} = -0.0005Ct + 1.2871$) over a 42 s window. While these lower boundaries ($m_{\min}$) prove strictly sensitive to the structural nature of the conduit, the risk-based middle envelopes (m_{risk}) maintain a mathematically universal power-law structure ($R^2 = 0.98$) across all four configurations, offering a standardized, predictive engineering framework to directly program optimal non-linear valve scheduling.

4.2. Overpressure and Optimal Laws Curves Variation Versus Time Plot at the Valve

To elucidate the physical mechanisms underlying the localized fluctuations – observed as small bumps within the optimal law profiles – the temporal evolution of the maximum overpressure was cross-analyzed with the corresponding optimal closure trajectories. Figures 5(a), 5(b), 5(c), and 5(d) illustrate these coupled transient responses over time for fiber-reinforced polyester (FRP), HDPE, cast iron, and steel, respectively.

The analysis of the transient hydrodynamic regimes induced by the slow closure of a downstream control valve reveals that the structural response and water hammer attenuation inherently depend on the mechanical and viscoelastic properties of the piping materials (FRP, HDPE, ductile cast iron, and steel). In accordance with Joukowski's theory, rigid materials such as cast iron and steel generate the highest initial overpressures (reaching 28 bar and 27 bar, respectively) but allow for a rapid stabilization of the pressure wave (in less than 40 s) due to a high wave celerity. Conversely, HDPE effectively dampens the initial pressure shock (maximum of 20.5 bar) at the expense of a significantly broader stabilization time window (up to 105 s). Nevertheless, these four representations clearly demonstrate that an increase in valve closing time does not systematically lead to a monotonic decrease in pressure values. Due to the periodic nature of the generated phenomenon, the overpressure curves are not smooth and exhibit localized bumps or secondary peaks. The first major bump initiates at a valve closing time equal to 1.4 times the wave reflection period (t_4). For

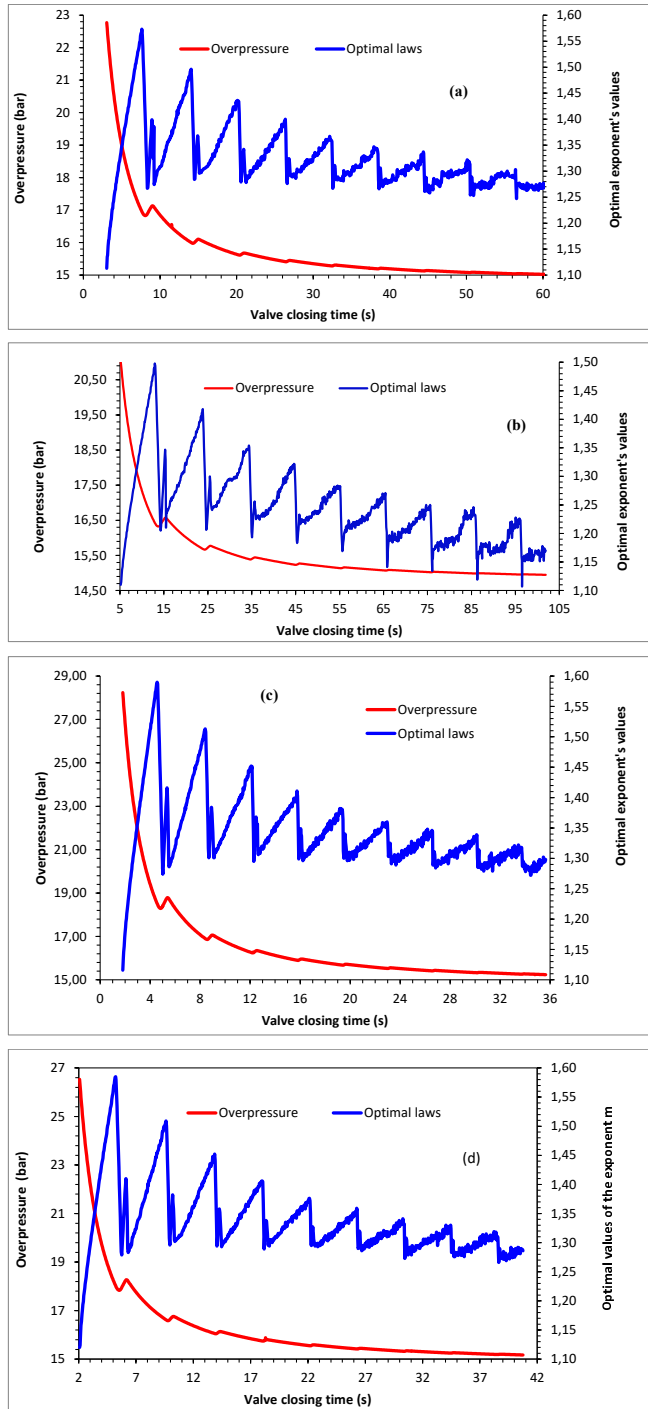


Fig. 5. Minimum Overpressure and exponent's values versus time plot, at the valve for PRV (a), HDPE (b), Cast Iron (c), and Steel (d)

closing times exceeding 4 times this reflection period ($4t_4$), these small bumps disappear, rendering subsequent overpressure variations very weak. Consequently, selecting an optimal valve closing time between these two threshold values ($1.4t_4 < t < 4t_4$) yields reasonable overpressures. Finally, the optimization of the valve maneuver law shows that across all materials, the optimal closure profile converges toward a convex configuration governed by a shape exponent m stabilized between 1.20 and 1.30 as the maneuver duration increases, confirming that a progressive, non-linear closure strategy remains essential to minimize the impact of transient overpressure waves in water distribution networks.

The figures highlight a sawtooth evolution of the optimal exponent m as a function of the valve closing time, punctuated by main peaks whose periodicity corresponds to the round-trip time of the pressure wave in the pipe. As the operating time approaches these characteristic values, a partial hydraulic resonance phenomenon occurs through constructive interaction between the valve movement and successive reflections of the shock waves. This locally amplifies the overpressure – visible as a slight rebound or a slowing down of the decay on the red curve – and requires a sharp increase in m to minimize the water hammer effect. Immediately following these peaks, the appearance of intermittent secondary peaks of lower amplitude reflects zones of imperfect yet still disruptive interference due to wave phase shifts, coupled with strong instability in transient sensitivity that forces the optimization algorithm to finely adjust the exponent. Finally, as the closing time increases (for example, beyond 55 seconds), the combined action of fluid friction, viscous dissipation, and the specific mechanical properties of each material damps the wave energy, which attenuates the resonance, progressively smoothes out the fluctuations, and stabilizes the system toward a final stable configuration.

4.3. Overpressure and Depression Variation Versus Time Plot, at the Valve

In the analysis of transient pipeline hydrodynamics, structural failure risks are governed not only by peak overpressures but equally by the magnitude of subsequent underpressures (depressions). Figures 6(a)–6(d) elucidate the spatiotemporal evolution of these dual transient envelopes across GRP, HDPE, ductile cast iron, and steel networks under linear ($m = 1$), quadratic ($m = 2$), and optimized valve closure laws. Critically, the quadratic law ($m = 2$) systematically establishes the worst-case outer boundaries – yielding the absolute maximum overpressures and the deepest depressions across all materials. Conversely, the optimized closure laws generate protective inner envelopes, dampening the violent pressure oscillations characterized by the linear law. From an industrial safety perspective, rigid conduits (cast iron and steel) exhibit extreme vulnerability during rapid maneuvers, where overpressures reach up to 28.5 bar and depressions plunge dangerously close to 1 bar, introducing a severe risk of fluid vaporization and subsequent cavitation. In contrast, the viscoelastic nature and lower wave celerity inherent to HDPE and GRP naturally buffer these extremes,

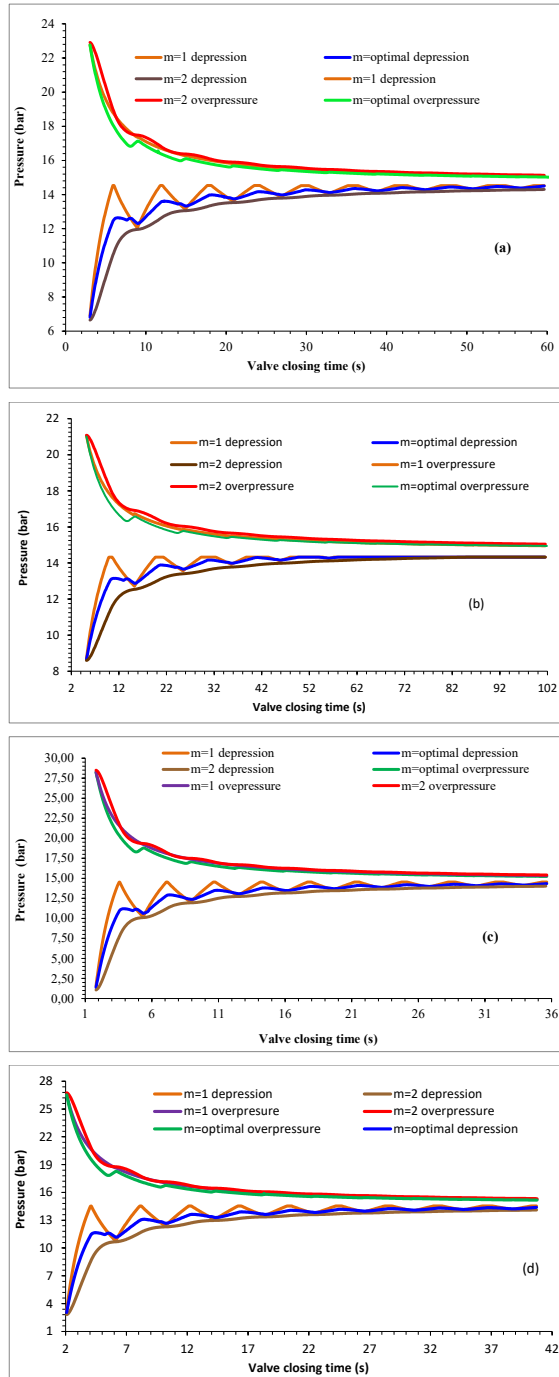


Fig. 6. Variation of overpressure and underpressure for optimal, linear and quadratic closing laws as a function of time closing valve for the, PRV (a), HDPE (b), cast iron (c) and steel (d)

maintaining safer depression floors (8.5 bar and 6.5 bar, respectively). These findings demonstrate that adopting an optimized, non-linear inner envelope closure profile is functionally vital to simultaneously prevent exceeding the pipeline's nominal pressure capacity and falling below the liquid's vaporization threshold.

5. Conclusions

The search for the optimal scheduling law governing the slow closure of a downstream control valve within an RPVSS configuration requires a systematic spatiotemporal assessment of transient overpressure and depression envelopes. This investigation proves that the optimal shape exponent m possesses an intrinsic periodic and damped oscillatory nature, strictly bounded by material-specific upper and lower mathematical regression curves. These envelope boundaries start with a wide divergence at rapid closure scales and systematically tighten over time, dramatically reducing the flutter margin of the exponent m . A key hydrodynamic finding of this work indicates that extending the valve closing time does not induce a monotonic reduction in transient stresses; rather, due to acoustic wave superposition, localized secondary pressure peaks are generated. The critical operational window to achieve reasonable and structurally safe transient profiles is explicitly restricted to a duration between $1.4t_4$ and $4t_4$, beyond which the secondary fluctuations fade into negligible variations.

Furthermore, the comparative material analysis underscores the necessity of matching the closure law to the pipeline's mechanical properties. While rigid steel and cast iron conduits exhibit severe vulnerabilities to structural bursting and deep vacuum pressures (plunging near 1 bar) that promote catastrophic cavitation, the flexible rheological structures of FRP and HDPE naturally buffer these extremes, securing safer depression floors (6.5 bar and 8.5 bar, respectively). Across all materials, as the closure duration expands, the optimal control laws universally converge toward a highly restricted convex envelope governed by a shape exponent m stabilized between 1.20 and 1.30. In conclusion, the predictive power-law and material-sensitive regression formulas developed in this study eliminate the necessity for computationally intensive non-linear iterations in pipeline design. They provide a standardized, robust engineering framework for directly programming non-linear valve maneuvers, ensuring long-term structural integrity against both overpressure bursts and sub-atmospheric cavitation implosions in water distribution networks

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