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Original Contributions - Originalbeiträge

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Order and Super-summativity: Mathematics and Psychology

Introduction

The 20th century witnessed the development, growth, and revolution of two distinct fields of inquiry: mathematics and psychology. In mathematics[1], with the advent of Set Theory, the German mathematician Georg Cantor (1845–1918) shook the foundations of the discipline by introducing the international community to a new way of identifying and utilizing actual infinity. In psychology, it was Gestalt Theory that brought forth a comprehensive and revolutionary worldview, which on one hand adhered to the parameters of scientific research and study, and on the other addressed certain concepts and themes that had, for many centuries, been excluded from the realm of scientific inquiry — foremost among these being the direct experience that an individual perceives as unique and private.

The importance of these revolutionary "events" lies in the fact that, for the international community — whether scientific or mathematical — the theories developed over centuries of analysis and experimentation were no longer sufficient to formulate a general "unicum" that could encompass and encapsulate all the principles of a given discipline. Against the backdrop of the well-known crisis of foundations, Cantor's Set Theory and Gestalt Psychology converge on a unified path: a shared approach to the complex organization of reality. In both fields, the concepts of order in infinite totalities and super-summativity become essential tools for constructing a deeper and more nuanced understanding of reality — a new paradigm in which the whole, the totality, assumes a significance that transcends and rises above the mere sum of its individual parts.

This paradigm not only redefines the foundations of the respective disciplines but also paves the way toward a worldview where the concept of complexity becomes the central focus of scientific knowledge and philosophical inquiry.

I. The Power of the Infinite

In the summer of 1884, the philosopher Carl Friedrich Stumpf (1848–1936) was invited to Halle to take over Ulrici's position as a colleague of Haym and

[1] About twenty years after the formulation of set theory, logic witnessed a new challenge to the foundations of mathematics with Gödel's formulation of the two incompleteness theorems for mathematics.

J. E. Erdmann. In the same year, the mathematician Georg Cantor experienced two events that were both significant for his life: he suffered the first episode of what would later be identified as the mental illness that ultimately led to his isolation in a psychiatric hospital, where he died in 1918; and he completed the publication of a fundamental work, the collection of six essays titled *On Infinite Linear Manifolds of Points* (1879–1884) in Halle, where Cantor spent most of his career. Evidence of an encounter between Stumpf and Cantor can be found in a recent article, *Stumpf and Husserl on Phenomenology and Descriptive Psychology*. In the first paragraph, *Stumpf as Mentor, Advisor, and Paternal Friend*, the author states: «[…] Brentano recommends Husserl to study with Stumpf (1989, 83 f) in the hope that he would also benefit from the assistance of G. Cantor, who was also in Halle at that time for the mathematical aspect of his research» (Fisette 2009, p.176). As we will see in more detail in the conclusion of this article, Cantor was not solely focused on the strictly mathematical aspect of his theorems. In fact, he also conveyed and described his thoughts from a philosophical perspective. Although there is no evidence of Cantor explicitly approaching metaphysics or phenomenology, it is documented that in 1887, «Husserl presented a *Probevorlesung* on the debate surrounding the psychology of introspection to the members of a jury composed once again of Cantor and Stumpf» (Fisette 2009, p.177). Moreover, Carl Stumpf, the mentor of Gestalt Psychology, took an interest in analysing the concepts of set and multiplicity, thereby demonstrating a particular inclination toward an approach that extended beyond purely psychological research. In contrast, among the proponents of Gestalt theory, there is no direct connection to the ideas proposed by the German mathematician. Nevertheless, it is interesting to observe a redefinition of foundations. Indeed, as we shall see, Cantor defines cardinality not in an elementaristic sense but as a particular type of set; similarly, for the Gestalt tradition, the "cardinality" of the set of perceptual elements cannot be a single perceptual stimulus but is instead a figurative ensemble.

The theoretical foundation of the following work lies in the definition of key concepts from Cantor's set theory and Gestalt theory. Specifically, the concept of order will be analysed, as it plays a central role in Cantor's theory in determining both differences and hierarchies among infinite sets. Depending on the ordering of individual elements, various well-ordered infinite sequences of numbers can be formed. Another fundamental concept is that of structure: a totality, an infinite set, is not merely an aggregate of mathematical objects, but can be represented as an autonomous mathematical entity, an object with unique properties and a well-defined description. These concepts are also observable within Gestalt theory, particularly in the work of Wolfgang Köhler, Christian von Ehrenfels, and Max Wertheimer. The order of elements and the totality that emerges are fundamental in the analysis of the perception of reality, where what emerges is what is defined by the term Gestalt. The total experience that is perceived is not merely the sum of individual sensory elements. If we examine reality through the lens of the infinite, we may likewise conclude that a set can be represented by a number, which is why the number — mathematics' quintessential unit — is itself something complex. What underpins both mathematical and psychological analysis is the observable and perceivable reality, which is therefore also linked to thought.

I.1

The first problem to address in constructing a mathematical theory is the characterization of the formal object. In the case of set theory, this specifically involves defining what a set is. «In principle and in summary, we can affirm that originally, by 'set,' Cantor meant a multiplicity of objects, belonging to any conceptual domain, such that they can be comprehended by the intellect in a single act» (my translation, Ferrario 2021, p.7). The inclusion in a set depends, at first glance, on whether or not it leads to a contradiction: if forming a totality of elements results in an issue of consistency, making it impossible to speak of unity, then such a grouping must be defined as a plurality, not a set. From this definition, Cantor establishes two conditions: the existence of a collection of elements is not sufficient to generate a set; and a set is a collection of objects that is itself an object. Only for absolute infinity do these characterizations not apply, as one can then speak of a single, uncountable totality, such as "the class of all conceivable things". This means that, on the one hand, it is possible to speak of a countable infinity, such as an infinite set of numbers that can be enumerated one by one. On the other hand, there might exist something that is infinite yet also the greatest of all: the absolute infinity, which cannot be described in terms of sets or groupings. The concept of a set must then be considered under three fundamental aspects, which form a kind of hierarchy, from the most general to the most specific, for defining a set. First, a well-defined set (*wohldefinierte Menge*) is defined as a general set of mental objects, with the sole characteristic of being able to determine whether or not a specific object belongs to the set. More specifically, an ordered set (*geordnete Menge*) is defined as having the additional property (of order) such that, for any two elements, it is always possible to determine the predecessor or successor according to a law that establishes an order among the elements. Finally, given these definitions, a well-ordered set (*wohlgeordnete Menge*) is a collection where there is always a first element, and each element, except the last, has a successor.

Once the formal object is defined, the next step is to analyze the relationship between two sets or between a set and the elements belonging to it. One of the immediately observable properties between two sets is the concept of equivalence. Two finite sets, for example, A and B, are said to be equivalent if, when counting their respective elements, they are found to have the same number. For instance, if A={1,2,3,4,5,6} and B={1,4,9,16,25,36}, both sets have six elements. This number is generally defined as the cardinal number, which for finite sets corresponds to the result obtained by counting each element of the set "one by one".

The case of infinite sets is different, as enumeration is no longer sufficient since it is not possible to count an infinite number of elements one by one. For this reason, set theory introduced the concept of the power of a set. «We say that two aggregates M and N are equivalent [have the same power] […] if it is possible to put them […] in such a relation to one another that to every element of each one of them corresponds one and only one element of the other» (Cantor 1915, pp. 86-87). This relationship is defined as a one-to-one correspondence. When placing the set $\mathbb{N}$ of positive integers and one of its subsets, such as the set of positive even integers, in a one-to-one correspondence, one can observe that each element of the first is associated with one and only one element of the second. This results in a fundamental and unique property of infinite sets: any infinite set has a power equivalent to that of one of its infinite subsets. The result Cantor achieved is significant, both for its historical context and for the international debate it spurred throughout the 20th century. Cantor's discovery is that in the theory of determined infinite sets — distinct from the absolute infinite — the infinite part and the whole have the same cardinality. In general set theory, cardinality, which is representable by a number, is formally itself a set: that is, cardinality is an equivalence class of a set. In other words, given a generic set C, its equivalence class $[C]_{\approx}$ is equal to the set of sets D that are in one-to-one correspondence with C, i.e., $\{D|D\approx C\}$ (where ≈ represents the one-to-one correspondence between C e D).

Cantor designates the cardinality of countable infinity $\mathbb{N}$ with a new number, $\aleph_0$ (aleph-null), representing the first level of infinity. The cardinality of each set allows these sets to be arranged within a hierarchy where it is absolutely certain where it begins, namely at $\aleph_0$, and where it cannot go, namely the totality of the absolute infinite. In between[2], there likely exists an infinite number of infinite sets, each named for its own cardinality. While mathematics has not yet required progressing beyond $\aleph_1$, there is nothing to preclude the future need to introduce an infinite set of cardinality $\aleph_2$, corresponding to the result of $2^{\aleph_1}$, and so on.

The central aspect of Cantor's vision can be seen in the following quote: «Der Fall unendlicher Mengen ist anders, da das Zählen nicht mehr ausreicht, da es unmöglich ist, eine unendliche Anzahl von Elementen einzeln zu zählen. Aus diesem Grund wurde in der Mengenlehre das Konzept der Mächtigkeit einer

[2] One of the problems encountered by the theory after Cantor's death was determining whether something intermediate could exist between $\aleph_0$ and the next cardinal $\aleph_1$. Cantor's hypothesis, known as the *continuum hypothesis*, states that $\aleph_1$ corresponds exactly to two raised to the power of $\aleph_0$, where exponentiation of a transfinite cardinal is the only operation that allows one to "go beyond" and obtain a higher cardinality.

Menge eingeführt. «[…]so daß zweien Mengen dann, aber auch nur dann *gleiche* Mächtigkeit zugestanden wird, wenn sie nach irgendeinem Gesetze einander gegenseitig-eindeutig zugeordnet werden können»[3] (Cantor 1932, S.151).

I.2

To introduce transfinite ordinal numbers, which constitute a further innovation in Cantor's theory, it is necessary to start from the foundations of number construction. The sequence (I) of positive integers — a well-ordered set — 1, 2, 3, …, ν, …, originates from the repeated placement and grouping of initially given units. The first principle of production, according to Cantor, is precisely that the construction of integers follows from the addition of a unit to a given number. Since the sequence so defined is infinite, and there is no maximum number among them — meaning no number x exists such that its successor $x+1$ cannot be constructed — it is permissible to conceive of a new number, called ω, as the limit toward which the numbers ν tend, making ω the first integer greater than all preceding numbers (cf. Cantor 1915, p.56).

«The transfinite ordinal number ω, being the immediate successor of the infinite sequence of natural integers, is, so to speak, the first, simplest, and most fundamental transfinite number. Cantor invites us to think of this new number as the expression of the fact that the infinite set of natural integers is currently given in its enumerable sequence up to its limit ω» (my translation, Ferrario 2021, p.4).

By applying the first principle and adding further units after ω, a new sequence of numbers is obtained, similar to the previous one, formed by a progression of ω+1, ω+2, …, ω+ν, ….

The possibility of identifying a limit for an infinite sequence of numbers is provided by the second principle of production, which states that for any determined infinite sequence of integers, a new number can always be identified as the limit of the preceding ones. With the foundations of transfinite ordinal number construction established, it is interesting to mention some arithmetic operations involving transfinite numbers. If we add a unit at the beginning of the progression and define this operation as 1+ω, it is easy to note that the sequence will be 1,1,2,3,...ω, and therefore still of the type ω. If, however, a unit is added "at the end" of the progression (in this case written as ω+1), the resulting sequence will be 1,2,3,...ω,ω+1. This latter sequence represents a completely different enumeration from the first (cf. Ferrario 2021, p.5), as it is necessary for ω to be

[3] «The conception of number which, in the finite, has only the background of enumeration, splits, in a manner of speaking, when we raise ourselves to the infinite, into the two conceptions of power [...] and enumeration [...]; and, when I again descend to the finite, I see just as clearly and beautifully how these two conceptions again unite to form that of the finite integer» (Cantor 1915, p.52).

present as an internal element of the progression to perform the operation ω+1. Another discovery highlighted by Cantor in his theory is a peculiar characteristic observable only in the extension of transfinite arithmetic: in transfinite arithmetic, the commutative property of addition no longer applies. For example, while 3+5=5+3, generally $a+\omega \neq$ (not equal) $\omega+a$, where a represents any positive integer. It is important to refer to positive integers, or natural numbers, as ω\omegaω is understood as the limit toward which natural numbers converge. By applying these two principles, it is possible to define the ordinal number of a progression of infinite ω-progressions, i.e., ωω; or ωωω, thus constructing an ascending infinite scale of transfinite ordinal numbers. All these numbers belong to the second numerical class, where the numbers of the first class are the finite ordinal numbers. With the first principle — the ability to add a unit to an already given number — and the second principle of generation — the ability to define the limit number of a sequence of numbers — an infinite sequence of numbers, whether of natural numbers or transfinite numbers, is established. For every element in the sequence, one can always add a unit, just as it is always possible to define a new element, not belonging to the sequence, which can be defined as its limit. In the progression of $\omega^{\omega}, \omega^{\omega\omega}, \omega^{\omega\omega\omega}, \ldots$, the limit is the ordinal ε_0.

I.3

In conclusion, Cantor's set theory offers particular points for reflection, which will be analysed in the following paragraph. Specifically, Cantor analyses totality as a structured unity, endowed with its own identity and internal organization. A central role is attributed to the concept of order, which, as we have seen, makes it possible to create a hierarchy among infinite sets and to clearly distinguish between different forms of infinity. For example, one of Cantor's conclusions — both paradoxical and innovative — is the result that the set of positive integers $\mathbb{N}$ and the set of real numbers $\mathbb{R}$ have different cardinalities. Specifically, the set $\mathbb{R}$ has a power directly greater than that of the set $\mathbb{N}$, demonstrating that they are not merely aggregates of elements but autonomous entities with unique properties.

A fundamental aspect of Cantor's work is the emergence of new intrinsic properties of sets, which arise only when one goes beyond the simple and evident presence of elements and considers how the parts are organized within the whole. For instance, while it is trivial that a positive integer is a finite number, taken as a whole, this constitutes a determinate countable infinity. While classical notation generally assumes that the sequence of positive integers is 1,2,3,…1, 2, 3, …1,2,3,…, tending to infinity, using the symbol ∞ to indicate indeterminate or potential infinity, Cantor surpasses this tradition by introducing ω as a symbol representing the concreteness, the fact that «infinity is no longer the nebulous domain glimpsed beyond an unreachable finite. On the contrary, it is the finite that now appears, under the guise of infinity, in its most authentic and complete form» (my translation, Ferrario 2021, p.9).

Finally, Cantor defined the concept of hierarchy or stratification, which progresses from the countable to the uncountable and beyond, to comprehend the complexity of infinite sets. Each level of this hierarchy represents a new totality that cannot be reduced to the previous levels. For instance, the set $\mathbb{R}$, with its greater cardinality compared to $\mathbb{N}$, belongs to a higher level of infinity, highlighting the stratified complexity of mathematical totalities.

Thus, through Cantor's contributions, it is recognized that the whole is never a simple sum of its parts, offering us a broader perspective on how relationships and structures can define aggregates as autonomous and significant entities.

II. Unity and Totality

The Austrian philosopher Christian von Ehrenfels (1859–1932) published in 1890 — around the same time that Cantor published and presented his most complete version of set theory — an essay titled *On Gestalt Qualities* (*Über Gestaltqualitäten*) in the journal "Vierteljahrsschrift für wissenschaftliche Philosophie". The starting point of the essay is a reference to the 1886 work of E. Mach, *Empfindungen Contributions to the Analysis of Sensations* (*Beiträge zur Analyse der*). In particular, Mach claimed that each of us is capable of directly perceiving spatial forms and melodies. But what are these forms? What is a melody? «Eine bloße Zusammenfassung von Elementen, oder etwas diesem gegenüber Neues, welches zwar mit jener Zusammenfassung, aber doch unterscheidbar von ihr vorliegt?»[4] (Ehrenfels 1890, S. 2).

In the next section (II.1), Ehrenfels' theory will be analysed in more detail. From this preliminary discussion, which focuses on the concepts of super-summativity and transposability, the text will proceed (II.2-3) to a discussion on spontaneous grouping and sensory organization, the latter analysed by the psychologist Wolfgang Köhler (1887–1967).

II.1

«Ehrenfels seeks to be faithful to the reality (veridicality) of our perception of what is complex. There is something there, he insists, which we perceive through specific types of complex networks of acts of presentation (perception, memory, and imagination) of what is simple, whenever we perceive a melody, a rhythm, or any other Gestalt quality» (Smith 1988, p.130).

[4] «Is a melody a mere sum (*Zusammenfassung*) of elements, or something novel in relation to this sum, something that certainly goes hand in hand with, but is distinguishable from the sum of elements?» (Smith 1988, p.83).

The first issue clarified by Ehrenfels in his essay is the problem of terminology. Ehrenfels suggests that the term *Gestalt* should be generalized: a spatial form is perceived starting from a complex of sensations of individual elements, each with its own spatial determinations. Thus, a crucial point in Ehrenfels' perspective, as well as that of Gestalt psychology proponents, is the assertion that our total experience is something distinct from the experience of a mere sum of sensory elements (cf. Smith 1988, p.14). Ehrenfels' famous example is that of listening to a melody. Suppose a series of tones $t_1, t_2, \ldots, t_n$ is apprehended by a conscious subject as a tonal *Gestalt*. The question arises whether the consciousness S, in apprehending the melody, contributes something more to its presentation than the distinct individuals taken together. "Gestalt qualities" are characterized by two fundamental properties: super-summativity and transposability. The first, as mentioned above, defines that Gestalt qualities cannot be derived from the sum of individual sensations. Köhler emphasized the importance of mutual influence among the parts of the whole for Gestalten to emerge, meaning only if there is

Diagram 1.

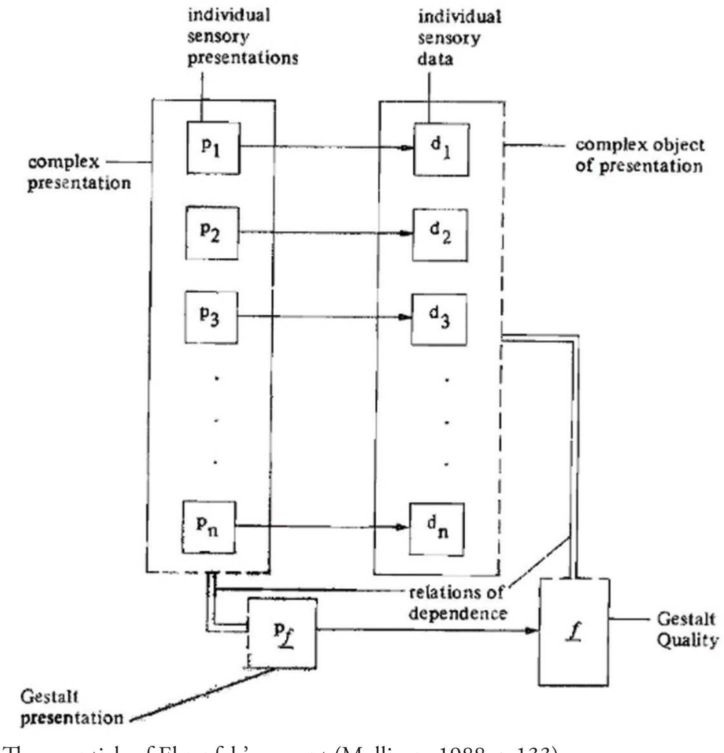

Fig. 1. The essentials of Ehrenfels' account (Mulligan, 1988, p.133).

proximity in space and time between the parts. Transposability is a sufficient criterion to affirm the independence of Gestalt qualities from their parts: the parts can be physically substituted (tones can be transposed to another key), but the melody remains the same (cf. Toccafondi 2000, p.72).

To synthesize the essential elements of Ehrenfels' text, the diagram depicted in figure 1 may be useful. On the left column, there is a succession of individual sensory presentations. On the right column, there is a progression of individual sensory data. Each perceived stimulus *p* is connected to each sensory datum *d* by a relationship of dependence. Similarly, there exists a dependency relationship between the Gestalt presentation and the Gestalt quality.

II.2

From Köhler's *Gestalt Psychology* (*Psychologische Probleme*), the following conceptual question can be extrapolated: How does the world appear to us? The study of perception addresses various fronts of interest: from the necessity of a scientific method of analysis to the importance of going beyond it, engaging in discussions on subjective, private experiences lived by each individual.

An example from daily life is as follows. In a room, one can clearly distinguish a window opposite the entrance, a bed on the left, a table on the right with several books placed against the wall, and, near the bed, a white wardrobe with a mirror on the left door. Each of these objects is recognizable, describable, and separable from the surrounding context; in other words, it appears as a distinct unit, separated from the background. All of this forms what is called the visual field. The visual field can be defined as the area perceivable by an individual's gaze at a given moment, within which objects and surfaces take on an organized structure. It is important to note that the division observed is concrete in the sense that it is presented directly before the observer's eyes (cf. Wertheimer 1938, p.71). The ability to divide objects within a field does not depend on recognizing them; it is entirely independent of practical knowledge or the meaning we may attribute to them. A very simple example can be drawn directly from a quote by Köhler.

«Man wird etwa sagen, das im Nebel unerkannt bleibende Ganze erscheine als etwas für sich, weil dieser Bereich dunkler sei als der graue Nebel ringsum. Freilich sei, wie unser Beispiel zeige, nicht der Einfluss unseres früher erworbenen Wissens gerade hinsichtlich dieses ganz bestimmten Empfindungskomplexes erforderlich, damit er als eine Einheit für sich aufgefasst werde»[5] (Köhler 1933, S. 96).

[5] «When I look into a dark corner, or when I walk through mist in the evening, I frequently find before me an unknown something which is detached from its environment as a particular object, while at the same time I am entirely unable to say what kind of thing it is. Only afterwards may I discover its nature in this sense» (Köhler 1980, p.140).

The possibility that a sensory world laden with meaning appears to an adult's eyes, according to Köhler, arises precisely from the original and natural factor of object division within a field. A telling example is provided by the author himself, who observes the case of adults with congenital blindness beginning to see after a surgical intervention.

«Gewöhnlich gibt der Patient keine befriedigende Antwort, wenn man ihn nach Form und Bedeutung eines Gegenstandes fragt, den er von seinem früheren Leben her durch Abtasten kennen muss, der ihm aber jetzt zum erstenmal optisch und unter Ausschluss des Tastsinnes dargeboten wird. […].Wird er gefragt, was "das da" vor ihm sei, so versteht er in der Regel die Frage und versucht, "das da" zu benennen oder irgendwie einzuordnen. Offenbar gibt es also auch für ihn von vornherein etwas wie eine aus gesonderte Einheit, auf die er die Frage bezieht und welche er zu beurteilen versucht. Wenigstens wenn der Gegenstand einfache kompakte Form hat, muss nicht erst gelernt werden, "welche Mannigfaltigkeit von Empfindungen als ein Ding zu behandeln ist" […]»[6] (Köhler 1933, S. 102).

II.3

When elements are segregated from their background, that is, from their surrounding environment, they can belong to a larger whole in which each part is intrinsically determined by the whole to which it belongs (cf. Wertheimer 1938, p.15). Being part of a whole, as one of its components, while simultaneously being an autonomous unit, are qualities identifiable in the case of sensory organization: when we observe any object, it is impossible to distinguish with the naked eye the potentially infinite sensory units that compose a single object in the visual field. What we see is a set of stimuli — a *complex object of presentation* as represented in figure 1 — organized in such a way as to construct the image of that object. This is rigorously referred to as *spontaneous grouping*.

The first author to recognize the importance of spontaneous grouping — which follows an order established by intrinsic laws — was Max Wertheimer (1880–1943), who described it in his work *Laws of Organization in Perceptual Forms* (*Untersuchungen zur Lehre von der Gestalt*)[7].

To begin with, if stimuli are considered as points, one can observe that we generally do not experience the "number of points" as the "number of individual objects".

[6] «When during the first post-operational tests the patient is shown an object which he knows by touch from previous life, he can seldom give a satisfactory response. With very few exceptions he does not recognize such forms when now given only in vision. […] When asked about "that thing" which he has before him, he understands the question. […] If the object has a simple and compact form, he need not learn what "aggregate of sensations" he must regard as one thing» (Köhler 1980, pp.149-150).

[7] In particular, two specific principles introduced by Wertheimer will be analysed, which can be highlighted using Cantor's theory.

Instead, larger sets are presented in the experience, which are both segregated and related to other sets (cf. Wertheimer 1938, p.72).

«In each of the [next] cases, that form of grouping is most natural which involves the smallest interval. They all show, that is to say, the predominant influence of what we may call The Factor of Proximity» (ibid.). In fact, in figure 2, we have an example.

However, the case of proximity does not always appear natural or spontaneous; at times, the result may seem confusing or unpleasant. «Sometimes a revolt against the originally dominant Factor of Proximity will occur and the shifted dots themselves thereupon constitute a new grouping whose common fate it has been to be shifted above the original row. The principle involved here may be designated *The Factor of Uniform Destiny* (or of "*Common Fate*")» (Wertheimer 1938, p.78). Wertheimer, in particular, is talking about the row that is represented in figure 3. Let us consider this string of dots as an example: a *Strukturgerecht* sequence might be d, e, f, as it involves an entire group of naturally correlated dots. If some dots are removed to form a sequence such as c, d, e, the result would be a *Strukturwidrig* succession, «because the common fate (i.e., the shift) to which these dots are subjected does not conform with their natural groupings» (Wertheimer 1938, p.78).

II.4

The two factors proposed above are fundamental to presenting two conceptually different theories, as they allow us to go beyond the boundaries of disciplines and enter, through the "doors" of mathematics and psychology, into the neutral

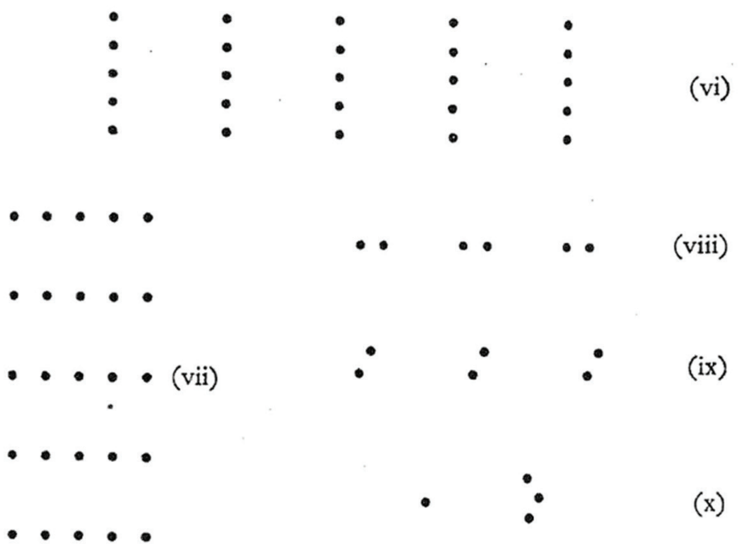

Fig. 2. An example of seeing what the objective arrangement dictates (Wertheimer, 1938, p.73).

territory of reality. Let us start with a question: can we speak of the reality or existence of whole numbers?

«Einmal dürfen wir die ganzen Zahlen insofern für wirklich ansehen, als sie auf Grund von Definitionen in unserm Verstande einen ganz bestimmten Platz einnehmen […]. Dann kann aber auch den Zahlen insofern Wirklichkeit zugeschrieben werden, als sie für einen Ausdruck oder ein Abbild von Vorgängen und Beziehungen in der dem Intellekt gegenüberstehenden Außenwelt gehalten werden müssen […]»[8] (Cantor 1932, S. 181).

In both reality and intellect, it is possible to conceive of a number that encapsulates the concept of something being infinitely large. In his search for foundations and ultimate principles, Cantor defied the late 19th- and 20th-century mathematical tendency to rebuild the discipline's basis from an exclusively finitist approach. The first step in defining infinity and incorporating it into the foundation of the discipline was to identify a "larger," infinite number representing an infinite set. Thus, a single element, a unit, can also be defined as a set, a totality, once the boundaries of finite mathematics are transcended.

Cantor's mathematical representation of reality aligns with the study of perception conducted by Gestalt psychologists. In particular, referring back to the previous discussion, two fundamental cases were studied based on Wertheimer's laws.

In the first case (fig. 2), the experiment concerns the extrapolation of the arrangement of certain points. For example, when observing a pattern, one might typically describe it as "I see five rows of five points each" rather than "I see five columns composed of five points each," or other configurations. The first description is spontaneously agreed upon. The arrangement defined by the principle of proximity is observable with the same clarity in both the perception of reality — where each sensory unit is ordered so that a specific figure is perceived — and in the reality of mathematical logic, where order relationships are codified in terms of a function: the relationship between elements defines the cardinality of a set.

In the second case (fig. 3), the experiment focuses on highlighting an intrinsic structure. A second, deeper type of ordering is defined here, which comes into play only when observing the totality of the presented figure. A simple example is given by observing figure 4.

[8] «We may regard the whole numbers as 'actual' in so far as they, on the ground of definitions, take a perfectly determined place in our understanding, are clearly distinguished from all other constituents of our thought, stand in definite relations to them, and thus modify, in a definite way, the substance of our mind. We may ascribe 'actuality' to them in so far as they must be held to be an expression or an image (Abbild) of processes and relations in the outer world, as distinguished from the intellect» (Cantor 1915, p.67).

Fig. 3. A group of naturally related dots (Wertheimer, 1938, p.77).

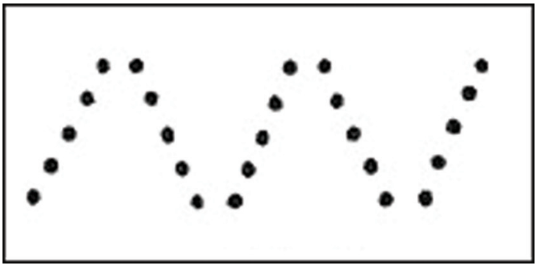

Fig. 4. The Factor of Direction (Wertheimer 1938, p.81).

When we asked to complete the image, one is generally inclined to continue the same pattern, even though no explicit instructions are given on how to proceed. In this example, the organization of the points reveals a type of structure that can spontaneously be reconstructed without relying on prior knowledge. Every complex structure then forms a totality, within which each structure becomes an element itself, and so on. The same reasoning connects to Cantor's analysis of infinite sequences of infinite numbers. Indeed, as we have seen, the number ω can be placed in an infinite sequence, applying the two principles of number construction. The sequence $\omega,\omega+1,\omega+2,\ldots$ has $\omega\omega$, also defined as 2ω, as its limit, which in turn can be the first number of the sequence $2\omega,2\omega+1$, etc., and so on. Each of these ω-sequences can be traced back to the totality defined as the second class, followed by a third class, a fourth class, and so on, extending to infinite numerical classes.

Conclusion

With the definitions of order, order relations, sets, and structures, an argument has been built around the possibility of reasoning from both perspectives that emerged in Germany at the end of the 19th century. The reality described in the pages of these authors is accessible to scholars, presented to the observer in an order that is necessary and foundational, first for the perception of spaces and objects, and subsequently for knowledge itself. Reality is nearly infinitely rich in stimuli, entities, and very small particles; each object is a collection of these elements. The reality of Cantorian mathematics, with its structures and principles of elementaristic and set-based organization, overlaps with the reality described by Gestalt psychology, with its perceptual images and organizational laws. There is no need to speak of mathematical reality, scientific reality, psychological reality, and so forth, because each of these fields of research describes a perspective on reality that can be observed in many diverse ways. One may speak of spiritual reality or objective reality: nothing changes the fact that everything observed always remains before our eyes, whether perceivable or not. Instead of the observer, there is always an intellect that judges, thinks, and imagines.

In *Productive Thinking*, Wertheimer analyses what thought ultimately consists of «[…] envisaging, realizing structural features and structural requirements […], which involves […] that there be operations of structural grouping and segregation, of centering, etc.; that operations be viewed and treated in their structural place, role, dynamic meaning, including realization of the changes which this involves; realizing structural transposability, structural hierarchy, and separating structurally peripheral from fundamental features […]; looking for structural rather than piecemeal truth» (Wertheimer 2019, pp. 190-191).

The nature of reality, as described, reflects the nature of human thought, where one can again speak of transposability, grouping, and segregation. Thus, thought shares the same objective as humanity's engagement with reality: to grasp increasingly complex and profound structures. In contrast to fragmented and incomplete truths, the need to comprehend complexity was emphasized by Cantor in his reformulation of the principles of number construction through the introduction of infinite numbers. With infinite numbers, the boundaries of mathematical horizons have expanded, allowing for the determination of infinity, which was historically referred to as the indefinite.

«Zeigt es sich aber, daß der Verstand auch in bestimmtem Sinne unendliche, d.i. *über-endliche* Zahlen definieren und voneinander unterscheiden kann, so muß entweder den Worten ,,endlicher Verstand" eine erweiterte Bedeutung gegeben werden, wonach alsdann jener Schluß aus ihnen nicht mehr gezogen werden kann; oder es muß auch dem menschlichen Verstand das Prädikat ,,unendlich" in gewissen Rücksichten zugestanden werden, was meines Erachtens das einzig Richtige ist»[9] (Cantor 1932, p. 176).

Not only in mathematics but also in human thought, the conception of a determined infinity allows for the introduction of the very concept of infinity

[9] «If it becomes evident that the intellect can, in a certain sense, define and distinguish infinite — that is, supra-finite — numbers, then either the term "finite intellect" must be given an expanded meaning, in which case the conclusion drawn from it can no longer hold; or the predicate "infinite" must also be attributed to the human intellect in certain respects, which, in my opinion, is the only correct conclusion» (Cantor 1932, p. 176) (Translation: E.O.).

as a real and defined object, with its own laws and specific analysis. However, a line of thought that extends to the boundaries of a conceivable infinity cannot be constrained in its concreteness and action. For this reason, according to Cantor the concepts of structure, order, and complexity, as evoked by Cantor, complement the freedom[10] of mathematical thought, namely, the freedom to experiment, explore, and justify one's own vision of reality.

Summary

The 20th century marked a paradigm shift in both mathematics and psychology. Georg Cantor's Set Theory redefined the concept of infinity, introducing a hierarchy of transfinite numbers and new approaches to understanding infinite totalities. Similarly, Gestalt Psychology revolutionized the study of perception by emphasizing super-summativity and transposability, which highlighting the autonomy of complex structures relative to their components. This article explores the conceptual intersections between these disciplines, focusing on the shared notions of order, structure, and complexity. Integrating these perspectives, it proposes a unified framework in which wholes transcend the sum of their parts.
Keywords: super-summativity, order, set, infinity, transfinite.

Zusammenfassung

Ordnung und Übersummativität: Mathematik und Psychologie

Das 20. Jahrhundert markierte einen Paradigmenwechsel sowohl in der Mathematik als auch in der Psychologie. Georg Cantors Mengenlehre definierte das Konzept der Unendlichkeit neu, indem sie eine Hierarchie über-endliche Zahlen und neue Ansätze zum Verständnis unendlicher Gesamtheiten einführte. Gleichzeitig revolutionierte die Gestaltpsychologie das Studium der Wahrnehmung, indem sie Prinzipien wie Über-Summativität und Transponierbarkeit betonte, die die Autonomie komplexer Strukturen gegenüber ihren individuellen Komponenten hervorheben. Dieser Artikel untersucht die konzeptionellen Überschneidungen zwischen diesen Disziplinen und konzentriert sich auf die gemeinsamen Begriffe Ordnung, Struktur und Komplexität. Durch die Integration dieser Perspektiven wird ein einheitlicher Rahmen vorgeschlagen, in dem das Ganze die bloße Summe seiner Teile übersteigt und eine gemeinsame epistemologische Grundlage in der Erforschung der Realität widerspiegelt.
Schlagwörter: Über-Summativität, Ordnung, Menge, Unendlichkeit, Transfinite.

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[10] With the sole rule of avoiding contradictions.

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