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Modelling the diameter distribution of near-natural even-aged European beech forests using forest inventory data – A comparison of methods

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Abstract

Tree diameter distribution is a key descriptive parameter of forest stands, yet studies on its modelling for major Central European forest types remain limited, particularly in near-natural forests. This study aimed to i) identify the best modelling method and ii) test the applicability of concentric fixed-area sample plots for modelling diameter distribution in even-aged, near-natural European beech stands. Diameter distribution was modelled using the two-parameter Weibull function based on data from 118 concentric fixed-area sample plots (200–400 m^2 each). Several modelling methods were tested, including the parameter prediction method, parameter recovery method and combinations of the two. Truncated samples were addressed using three approaches: applying expansion factors, incorporating truncation into the function formulation and ignoring truncation. Models were evaluated based on the ability to predict relative frequencies of trees per 5-cm diameter classes and key plot parameters (quadratic mean diameter, basal area and dominant diameter). Some of the 14 developed models performed similarly well; however, the parameter recovery model incorporating expansion factors was identified as the most suitable for forest management. The selected plot parameters were predicted with an acceptable bias, precision and accuracy, while prediction of relative frequencies within diameter classes was less reliable, particularly for thinner trees. The study also revealed considerable heterogeneity in tree diameters in near-natural beech forests. Overall, the findings demonstrate that concentric fixed-area sample plots are a feasible data source for modelling diameter distribution in near-natural beech forests. However, limitations related to heterogeneity, plot size and truncated samples must be considered in future applications.

Key words: *Fagus sylvatica* L.; Weibull function; concentric fixed-area sample plots; parameter prediction method; parameter recovery method; close-to-nature forests

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1. Introduction

Diameter distribution models are key components of frameworks commonly used for modelling growth and yield in forest stands (Burkhart & Tomé 2012). Diameter distribution, i.e. the frequency distribution of trees in a stand according to their diameter at breast height (dbh), is one of the most descriptive and important stand characteristics, providing insights into stand structure, stability, competitive situation, growth patterns and potential timber assortments. It also serves as the basis for determining necessary silvicultural measures and the type of harvesting, as well as for simulating a stand's development (Pretzsch 2010; Burkhart & Tomé 2012; Schütz & Rosset 2016). Several forest growth models operate at the stand level, where knowledge on stand diameter distribution is essential, e.g., SiWaWa (Schütz et al. 2018). Information about stand diameter distribution can also be used in individual tree growth models (e.g., Silva (Pretzsch et al. 2002), Sybila (Fabrika et al. 2018)) when individual tree data is unavailable.

The application of distribution models is currently the most common method of expressing the diameter distribution of forest stands (e.g., Rijal & Sharma 2023). A widely used approach to deriving information on stand diameter structure from stand and site parameters

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involves parametrizing one of the theoretical distributions (Kangas & Maltamo 2006). Among these, the Weibull distribution function stands out for its widespread use (Weibull 1951; Nord-Larsen & Cao 2006; Palahí et al. 2006, 2007; Sghaier et al. 2016; Schmidt et al. 2020). The Weibull distribution function is capable of describing a wide range of unimodal distributions, from reversed-J-shaped to right- and left-skewed, as well as exponential and normal distributions (Bailey & Dell 1973). The Weibull function can be configured as either a two- or three-parameter function. It includes the scale parameter *b* and the shape parameter *c*; in the three-parameter version, it includes an additional location parameter *a* (Bailey & Dell 1973). In forest management, it is important that the Weibull distribution parameters can easily be linked to (easily measurable) stand parameters (e.g., Schütz et al. 2018). A key feature of the Weibull function is that the 63rd percentile of the diameter distribution is equal to the scale parameter *b* or the sum of parameters *b* and *a* in the two- or three-parameter function, respectively (Bailey & Dell 1973).

In forest management, the (Weibull) diameter distribution function parameters *a*, *b* and *c* are commonly expressed as a function of different stand, site and climate variables (e.g., Cao et al. 2022; Guo et al. 2022). Two main methods are used for parameterization of the Weibull distribution function: i) the parameter prediction method (PPM) and ii) the parameter recovery method (PRM), although the two methods can also be combined (e.g., Poudel & Cao 2013; Mehtätalo & Lappi 2020; Schütz & Rosset 2020). In the PPM, the function parameters are predicted using regression models based on stand, site, climate and other variables (Kangas & Maltamo 2000). Usually, these models are established in two consecutive steps. First, the Weibull function is fitted to data using one of several methods, with maximum likelihood estimation being the most frequently used (e.g., Schmidt et al. 2020), although other methods, such as the modified moments method, have also been proposed (e.g., Akram & Hayat 2014; Bončina et al. 2022). In the second step, the calculated Weibull function parameters are expressed with regression models composed of various stand, site and climate predictors. These two steps can also be combined, and regression models for function parameters can be solved simultaneously in one step. For example, Cao (2004) proposed cumulative density function regression, while Mehtätalo & Lappi (2020, p. 273) suggested generalized linear modelling.

The PRM derives the parameters of the Weibull function from the moments or percentiles of the diameter distribution, which are themselves estimated from (measured or modelled) stand attributes (Newton et al. 2005). This method requires at least as many distribution-related explanatory variables as there are parameters in the Weibull function. These variables can be moments (e.g., mean, variance), derived from moments (e.g., quadratic mean) or percentiles (e.g., median) of the empirical distribution, allowing the PRM to be divided into two main sub-methods: moment-based (e.g., Rijal & Sharma 2023) and percentile-based (McEwen & Parresol 1991; Gorgoso et al. 2007). Both moments and percentiles can be used in recovery equations (e.g., Cao 2004; Siipilehto & Mehtätalo 2013). In mature even-aged stands, the PRM has been found to be more applicable than the PPM due to its connection with stand parameters (Kangas & Maltamo 2000; Siipilehto & Mehtätalo 2013; Schmidt et al. 2020).

Various data sources can be utilized for modelling diameter distributions of forest stands. Due to its high cost, full callipering is seldom performed at the stand level and even less frequently across larger forest areas. To rationalize forest inventory, inventory methods based on sampling were developed. The best-known examples are national forest inventories (NFI) performed in most countries across Europe (Tomppo et al. 2010), which are applied to obtain parameters on forest state and dynamics (Vidal et al. 2016). Forest inventory data are usually characterized by a large number of relatively small plots, typically less than 1,000 m^2 (Tomppo et al. 2010; Vidal et al. 2016). Across Europe, multiple circular concentric fixed-area plots are often used (McRoberts et al. 2010). These samples are truncated, meaning that not every tree has the same probability of being observed, as threshold (truncation) diameters vary with distance from the plot centre (Schmidt et al. 2020). To derive plot parameters for such truncated samples, the number of stems within respective circles are weighted with expansion factors (EF) to account for varying selection probabilities (Köhl 2001; Kangas & Maltamo 2006). A multi-truncated formulation that accounts for different threshold diameters has also been proposed for fitting and modelling the diameter distribution from such data (Nanos & Sjöstedt de Luna 2017; Mey et al. 2021). Another characteristic of field measurement is that trees are not usually measured from 0, but from a set measurement threshold. In European forest inventories, this measurement threshold ranges between 0 and 12.5 cm (Köhl 2001; Kangas & Maltamo 2006; Poljanec 2010; Tomppo et al. 2010; Vidal et al. 2016; Gschwantner et al. 2022). Not measuring all trees from 0 results in a left-truncated distribution. Some studies have addressed this by applying the left-truncated formulation of the Weibull function (Palahí et al. 2007; Gorgoso et al. 2012; Schmidt et al. 2020; Ciceu et al. 2021). Nevertheless, the complete, non-truncated formulation of the Weibull function has also been widely applied to describe such data (Kangas & Maltamo 2000; Cao 2004; Merganič & Sterba 2006; Nord-Larsen & Cao 2006; Gorgoso et al. 2007; Lei 2008; Poudel & Cao 2013; Siipilehto & Mehtätalo 2013; Podlaski & Roesch 2014; Sanquetta et. al. 2014; Sghaier et al. 2016; Schütz & Rosset 2020; Cao 2022; Guo et al. 2022).

Given that national forest inventories are common across Europe (Tomppo et al. 2010) and are continuously improving as a data source (Vidal et al. 2016), it is important to assess the suitability of concentric fixed-area sample plots for estimating the diameter distribution of forest stands (see also Mey et al. 2021, 2023). Studies using data from sample plots (e.g., < 500 m^2) have mostly been limited to even-aged stands, such as homogeneous pure stands of pine (Kangas & Maltamo 2000; Newton et al. 2005; Poudel & Cao 2013) or eucalyptus plantations (Schmidt et al. 2020) characterized by trees with small diameters (< 20 cm) (Newton et al., 2005). Few studies have focused on the stand diameter distribution of European beech (*Fagus sylvatica* L., hereafter beech) forests, despite being one of the most common tree species in Central Europe (Bohn et al. 2003; Köhl et al. 2020; Fuchs et al. 2024). Some beech studies have utilized data from relatively large experimental plots (Nord-Larsen & Cao 2006; Schütz et al. 2018), while there is a lack of studies focusing on near-natural beech forest by using data from sample plots (but see Mey et al. 2021, 2023). Beech is among the most common tree species in Slovenia (Slovenia Forest Service 2014; Bončina et al. 2021) and typically forms homogenous stands (Bončina 2012). Our study focused on homogeneous, even-aged beech forests, as the broader objective was to develop growth models and further improve the SiWaWa forest growth model, which is specifically designed for even-aged stands. A central aspect of this research involved modelling diameter distribution in even-aged beech forests using concentric fixed-area sample plots.

Therefore, the objective of our study was to compare the performance of diameter distribution models developed using various methods to estimate plot parameters of even-aged near-natural beech forests, based on data from concentric fixed-area sample plots. In addition, we aimed to assess the applicability of concentric fixed-area sample plots for modelling the diameter distribution of even-aged near-natural beech forests. We hypothesized that i) the PRM would outperform other methods in modelling diameter distribution, and ii) models considering truncated samples would produce less biased, more accurate and more precise diameter estimates.

2. Methods

2.1. Study area

Forests in Slovenia cover 1.2 million ha and grow under a wide range of geo-climatic conditions across the Eastern Alps, the Dinaric Mountains, the sub-Mediterranean region and the Pannonian Basin. Slovenian forests exhibit well-preserved structures and tree species compositions (Bončina et al. 2021), which are largely attributed to small-scale, close-to-nature management practices. These practices primarily employ irregular shelterwood and group and single-tree selection systems (Poljanec et al. 2010; Klopčič & Bončina 2011). Our study was conducted in the Alpine region of north-west Slovenia and comprised approximately 67,500 ha of forests (Fig. 1). The average growing stock is 285 m^3 per ha, and the mean annual increment is 7.1 m^3 per ha. The potential natural vegetation on almost 72% of the forest area consists of beech forest types, mainly mountainous beech forests on predominant carbonate bedrock and mixed beech-conifer forests, with silver fir (*Abies alba* Mill.) and Norway spruce (*Picea abies* [L.] H. Karst.) as the main conifers (Bončina et al. 2021). However, tree species composition has shifted because of human influence, with Norway spruce being the most dominant species (63.3% of growing stock), followed by beech (23.6%) and silver fir (4.1%) (Slovenia Forest Service 2012).

2.2. Data description

The data were obtained from permanent sample plots (hereafter PSP, n ≈ 8,983) of the Slovenia Forest Service, which are systematically distributed across the study area (SFS 2014). This inventory system is primarily used for forest management planning and operates in parallel with the NFI system; however, the plots used in both inventory methodologies are similar in size and shape, the tree measurement threshold are country-wise often the same (Kušar et al. 2010; Poljanec 2010; Gschwantner et al. 2022). Pure beech stands were defined as those where beech accounts for ≥ 80% of the plot basal area. Plots with a homogeneous (i.e. even-aged) structure were identified by a Gini index < 0.33 (Trifković et al. 2023). The Gini index is a measure of structural diversity on a scale from 0 to 1, with higher values indicating more heterogeneous structures. It is frequently used in forestry to assess stand structural homogeneity or heterogeneity (O'Hara et al. 2007; Duduman 2011; Klopčič & Bončina 2011). The Gini index was calculated at the plot level based on the basal area (BA) of trees (Dănescu et al. 2016). To classify forests into homogeneous and heterogenous categories, k-means cluster analysis was performed, resulting in two clusters (Trifković et al. 2023). The threshold value of 0.33 was adopted in accordance with the literature, as Duduman (2011) defined this threshold between homogenous and heterogenous forests at 0.35 and O'Hara et al. (2007) at 0.3.

Each plot in the study area is a circular fixed-area plot of 400 m^2 (radius = 11.28 m), and all trees with a diameter at breast height (dbh) ≥ 10 cm were recorded, including their distance and azimuth from the plot centre. These trees are remeasured every ten years. Plots containing fewer than 10 trees were excluded from the analysis due to insufficient sample size for modelling diameter distribution (sensu Palahí et al. 2006; Akram & Hayat 2014). The final PSP database comprised 118 plots (Fig. 1; Table 1).

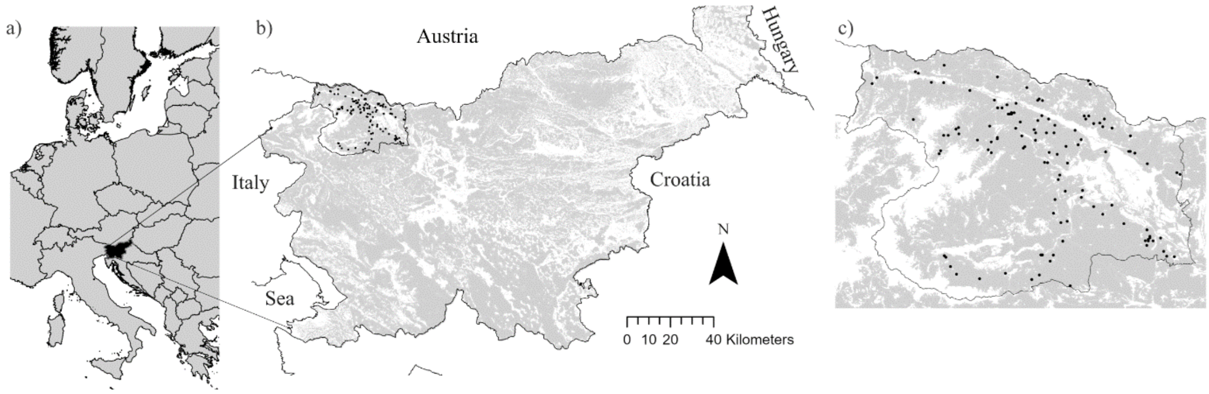

Fig. 1. Location of the analysed sample plots (black dots, n = 118) in the forests of Slovenia.

Table 1. Stand and site characteristics of plots (n = 118, 400 m^2 each).

	Mean	Median	SD	Min–Max
Quadratic mean diameter QMD (cm)	26.1	26.2	6.8	13.2–40.2
Basal area BA (m^2 per ha)	34.0	33.5	11.0	6.1–64.6
Number of trees per ha N (n per ha)	705	600	327	275–1800
Mean diameter D_{MEAN} (cm)	25.1	24.8	6.6	13.0–38.8
Dominant diameter D_{DOM} (cm)	36.1	36.6	7.7	17.5–51.3
Maximum diameter D_{MAX} (cm)	40.5	41	9.1	20–63
Minimum diameter D_{MIN} (cm)	12.7	11	3.7	10–26
Elevation (m)	932	904.5	258	477–1537
Slope (°)	23.8	26	9.3	0–46
Gini index	0.28	0.30	0.04	0.15–0.33
Proportion of beech (% in BA)	93	93	6	80–100

Measurements of all trees with dbh ≥ 10 cm enabled us to test whether the inventory concept of two concentric fixed-area subplots is suitable for estimating the diameter distribution of beech forests. This inventory approach is the most common in Slovenia (Poljanec et al. 2010) and is also widely used within NFI in many European countries (Vidal et al. 2016). For this purpose, we converted each 400 m^2 fixed-area plot into two concentric subplots of 200 m^2 and 400 m^2, with different measurement thresholds: trees with a dbh of 10–29.9 cm were recorded in the 200 m^2 plot (radius = 7.98 m), while trees with a dbh ≥ 30 cm were recorded in the 400 m^2 plot (radius = 11.28 m). In practice, this means that trees with dbh < 30 cm located more than 7.98 m from the plot centre were not included in the simulated inventory. Therefore, our simulated inventory data are truncated at two measurement thresholds, namely 10 cm and 30 cm, representing the first and second truncation points, respectively. To scale up the per-plot-parameters to their hectare values, trees within respective concentric plots were weighted with an expansion factor (EF) of 50 (200 m^2) or 25 (400 m^2). The use of EF is a common method to account for different selection probabilities when multiple concentric fixed-area subplots are used (e.g., Köhl 2001; Kangas & Maltamo 2006; McRoberts et al. 2010; Álvarez-González et al. 2014). They are defined as the relationship between the reference area (1 ha) and the subplot area, adjusting the values of the number of sampled trees to per-hectare values (Álvarez-González et al. 2014).

Data from the simulated inventory design were used for modelling the diameter distribution using various methods (Table 2).

2.3. Modelling procedure

The two-parameter Weibull distribution function (Weibull 1951) was used to model the diameter distribution of beech forests. Trees on plots were grouped into 5-cm diameter classes. The cumulative distribution function (CDF) of the two-parametric Weibull function is expressed in equation 1:

$$F(x) = 1 - e^{-\left(\frac{x}{b}\right)^{c}} \qquad [1]$$

The corresponding probability density function (PDF) is defined in equation 2:

$$f(x) = \frac{c}{b} \cdot \left(\frac{x}{b}\right)^{c-1} \cdot e^{-\left(\frac{x}{b}\right)^{c}} \qquad [2]$$

where b is the scale parameter, c is the shape parameter and x represents the independent variable, i.e. tree dbh.

To express the Weibull distribution function parameters b and c as functions of stand characteristics, four main modelling concepts were applied (Table 2): the two-step parameter prediction method (PPM) (Clutter & Bennett 1965; Poudel & Cao 2013; Ciceu et al. 2021), the one-step PPM (Cao 2004), the parameter recovery method (PRM) (McEwen & Parresol 1991; Gorgoso et al. 2007; Rijal & Sharma 2023) and a combined PPM and PRM (Mehtätalo & Lappi 2020).

A multi-truncated inventory design was considered in several ways. Some models ignored truncation entirely, some used expansion factors (EF) to address different selection probabilities within subplots, and some incorporated truncation directly into the function formulation (Table 2). Since ignoring truncation tends to produce rough error estimates (Nanos & Sjöstedt de Luna 2017) and has not been tested in the near-natural even-aged beech forests, we were interested in comparing the modelling performance of such models despite their expected inferiority (Nanos & Sjöstedt de Luna 2017).

In the two-step PPM modelling procedure, the first step involves fitting the Weibull function to empirical data. This was achieved using four different methods, each maximizing the likelihood of the sampling distribution via the *mle* function of the stats4 R package (R Core Team 2023). The first method resulted in the development of model Mod2PPM, which used the simulation data and ignored truncation. The second method produced model Mod2PPMEF, where the same procedure was applied as in Mod2PPM, but the data were multiplied by EF to account for different selection probabilities within two concentric fixed-area subplots. Truncated samples can also be addressed by incorporating the plot design directly into the Weibull function formulation. Some studies have considered multiple measurement thresholds within the Weibull function formulation (e.g., Nanos & Sjöstedt de Luna 2017; Mey et al. 2021). To achieve this, two additional methods were applied. Model ModWW was developed using the weighted Weibull method (Mehtätalo & Lappi 2020, p. 359), which uses sampling weights based on tree dbh thresholds and plot sizes (similar to EF). The original formulation was adapted to fit our inventory design (Supplementary Material SM1). Due to nonconvergence when fitting ModWW, two plots were excluded from the modelling procedure using this method. The second method applied is based on multiple truncated probability density functions (Nanos & Sjöstedt de Luna 2017) and resulted in the model ModNSL. The original version of the method was adapted to our inventory design with two concentric fixed-area plots (Supplementary Material SM2). Due to nonconvergence when fitting ModNSL, three plots were excluded from the modelling procedure.

In the second step of the PPM modelling, the Weibull function parameters *b* and *c* were modelled using linear regression, with plot parameters as independent variables (Mehtätalo & Lappi 2020, p. 372). Equations 3 and 4, adopted from Mehtätalo & Lappi (2020), were applied in all four models developed by the two-step PPM modelling:

$$\ln(b) = k_0 + k_1 \cdot \ln(QMD) \qquad [3]$$

$$\ln(c) = k_0 + k_1 \cdot QMD + k_2 \cdot BA \qquad [4]$$

where QMD is the quadratic mean diameter, BA is the basal area and k_0, k_1 and k_2 the regression coefficients.

In the one-step modelling method, the Weibull function parameters *b* and *c* were estimated simultaneously, as this method combines the two steps of the PPM into a single estimation process. Although this approach is similar to simultaneously fitting two regression equations for *b* and *c* (eq. 3 and 4), the objective is to minimize the sum of squared errors with respect to the cumulative density function rather than to *b* and *c* (Cao 2004). The method proposed by Cao (2004) was used in three models: ModCao used data without EF multiplication, ModCaoEF used data multiplied by EF to account for truncation, and model ModCao was upgraded to model ModCaoWW by applying sampling weights (Supplementary Material SM3).

We also developed three additional models using the generalized linear modelling proposed by Mehtätalo & Lappi (2020, p. 373) and Cao (2004). In the latter, the plot component (i.e. the weighting of the likelihoods) was removed. Model ModGLM was developed using raw simulation data, while Model ModGLMEF was developed using data multiplied by EF to account for truncation. In addition, model ModGLM was upgraded to model ModGLMWW by using sampling weights (Supplementary Material SM4).

Using the PRM approach, two additional models were developed: ModPRMEF1 and ModPRMEF2. The function parameters were derived from the mean diameter D_{MEAN} and QMD, while the per-hectare values of BA and number of trees N were also used in the calculation. The PRM models relied on different predictor variables

Table 2. The two-parametric Weibull diameter distribution models developed by combining modelling methods and considering truncated samples.

Modelling method	PPM		Combined PPM and PRM	PRM
Truncation treatment	Two-step modelling	One-step modelling		
Ignoring truncation	Mod2PPM	ModCao ModGLM	ModCOMB	
Truncation with EF	Mod2PPMEF	ModCaoEF ModGLMEF	ModCOMBEF	ModPRMEF1 ModPRMEF2
Truncation with NSL	ModNSL			
Truncation with WW	ModWW	ModCaoWW ModGLMWW		

than the PPM models. Data multiplied by EF were analysed using the *recweib* function from the *lmfor* R package (Mehtätalo & Kansanen 2015).

Finally, a combined PPM and PRM modelling approach was applied to develop two additional models: ModCOMB and ModCOMBEF. The scale parameter *b* was estimated according to the PPM (Mehtätalo & Lappi 2020, p. 373), while the shape parameter *c* was calculated using the PRM via the *scaleQMDMean* function in the *lmfor* R package (Mehtätalo & Kansanen 2015). Model ModCOMB ignored truncation, while ModCOMBEF incorporated it by multiplying the data by EF.

2.4. Verification of the simulated forest inventory design

Verification of the simulated forest inventory design was conducted to assess the reliability of the estimates obtained using two concentric fixed-area subplots (i.e. 200 m^2 and 400 m^2 measuring trees of dbh ≥ 10 cm and ≥ 30 cm, respectively). Specifically, we tested whether the key plot parameters (QMD, BA, D_{DOM} and N) estimated from the simulated inventory differed significantly from the empirical stand parameters derived from all trees in each sample plot (i.e. fixed area of 400 m^2 measuring trees of dbh ≥ 10 cm). A paired t-test for dependent samples was used to evaluate these differences (Zuur et al. 2007; Zar 2010).

2.5. Model performance evaluation

Model performance was assessed based on two main criteria. First, we assessed the models' ability to predict the diameter distribution, expressed as the relative and absolute frequencies of trees per 5-cm diameter classes. Second, we compared the empirical and modelled values of three key plot parameters (BA, QMD and D_{DOM}) obtained from the predicted diameter distribution. Differences between empirical and estimated plot parameters predicted by different models were evaluated with indicators for bias (the mean error, ME, calculated as a difference between the empirical value and the modelled value), precision (standard deviation of ME, SD) and accuracy (root mean square error, RMSE, and mean absolute percentage error, MAPE) at the plot level (e.g., Gorgoso 2007, 2012). To assess whether the estimated plot parameters QMD, BA and D_{DOM} differed significantly between models, an ANOVA for dependent samples was performed (Zuur et al. 2007). If significant differences were found, pairwise comparisons were conducted using the paired t-test with Bonferroni correction for multiple comparisons (Zar 2010). To assess over- and underestimation of the number of trees per diameter class, we compared absolute and relative frequencies per diameter class with ME and MAE. Relative frequencies per diameter class enabled the comparison of plots with different numbers of trees (after Gorgoso 2012). ME and MAE were compared within each diameter class and as average ME and MAE across all diameter classes. All statistical analyses were performed using R (R Core Team 2023).

3. Results

3.1. Verification of the simulated forest inventory design

On average, the estimated plot parameters obtained from the simulated inventory design with two concentric fixed-area subplots (200 m^2 and 400 m^2) differed only slightly from the empirical plot parameters derived from all measured trees in a sample plot (fixed area of 400 m^2); however, for some plots deviations were substantial. Estimates of QMD and D_{DOM} from the simulated inventory fell within the interval of ±10% of the empirical values in more than 96% of plots (Fig. 2). However, the estimates for BA and N were less accurate, with only 51% and 40% of plots, respectively, falling within this interval. No significant differences were found in the mean differences between empirical and simulated inventory design for the paired values of BA and N ($p > 0.05$). However, significant differences were observed between the paired values of QMD and D_{DOM} (p-values of 0.03 and 0.02, respectively).

3.2. Performance of the diameter distribution models

Estimation of frequencies within 5-cm diameter classes
The mean MAE between empirical and estimated relative frequencies within diameter classes ranged from 0.07 to 0.08 for the majority of models. On average, the models underestimated frequencies (all mean ME ≥ 0). The lowest mean MAE and RMSE values were observed for ModPRMEF1. In contrast, ModPRMEF2 performed worse than the others (Table 3).

Analyses per diameter classes showed that, in general, the accuracy of predicting relative frequencies was better in larger diameter classes, as MAE decreased with increasing tree diameter (Fig. 3a). The lowest MAE across most diameter classes was observed for ModPRMEF1. Bias was also highest in the smallest diameter class (10–15 cm). Most of the models underestimated the number of trees in the smallest (10–20 cm) and largest diameter classes (> 30 cm) (Fig. 3b), while overestimating the number of trees in the 20–30 cm diameter class. Some models also overestimated the number of

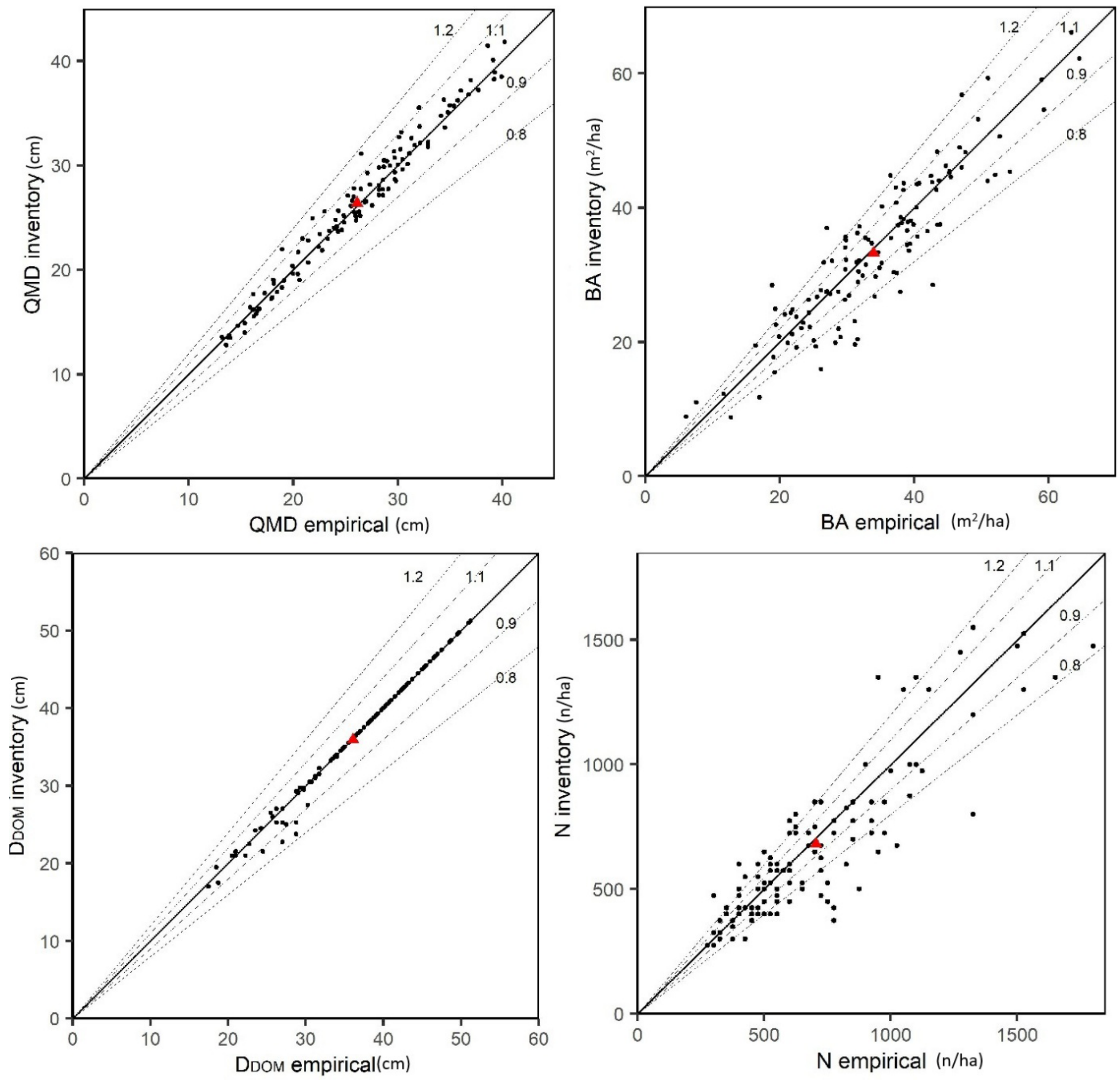

Fig. 2. Comparison of empirical plot parameters, calculated from fixed-area plots (400 m^2 each), with plot parameters derived from the simulated inventory design featuring two concentric fixed-area subplots (200 m^2 and 400 m^2 each) (n = 118). Red triangles indicate the mean value of each plot parameter.

Table 3. Mean values of mean error (ME), standard deviation of ME (SD), mean absolute error (MAE), root mean square error (RMSE) between empirical frequencies and the relative and absolute frequencies estimated by the diameter distribution models for each diameter class (n = 118).

Diameter distribution model	Absolute frequencies				Relative frequencies			
	ME	SD	MAE	RMSE	ME	SD	MAE	RMSE
Mod2PPM	14	33	59	86	0.011	0.014	0.08	2.79
Mod2PPMEF	14	33	53	93	0.011	0.013	0.08	2.56
ModWW	24	44	54	84	0.020	0.029	**0.07**	2.59
ModNSL	26	40	59	82	0.024	0.024	0.08	2.77
ModCao	16	33	60	97	0.014	0.016	0.08	2.82
ModCaoEF	15	33	53	86	0.013	0.015	**0.07**	2.58
ModCaoWW	24	43	55	95	0.021	0.029	**0.07**	2.68
ModGLM	15	33	58	94	0.013	0.015	0.08	2.78
ModGLMEF	14	33	53	84	0.012	0.013	0.07	2.56
ModGLMWW	25	44	55	93	0.022	0.029	**0.07**	2.65
ModCOMB	25	45	58	89	0.020	0.028	0.08	2.68
ModCOMBEF	21	41	55	82	0.017	0.024	0.08	2.58
ModPRMEF1	12	34	**46**	**66**	0.009	0.010	**0.07**	**2.22**
ModPRMEF2	3	32	139	217	0.001	0.018	0.19	7.22

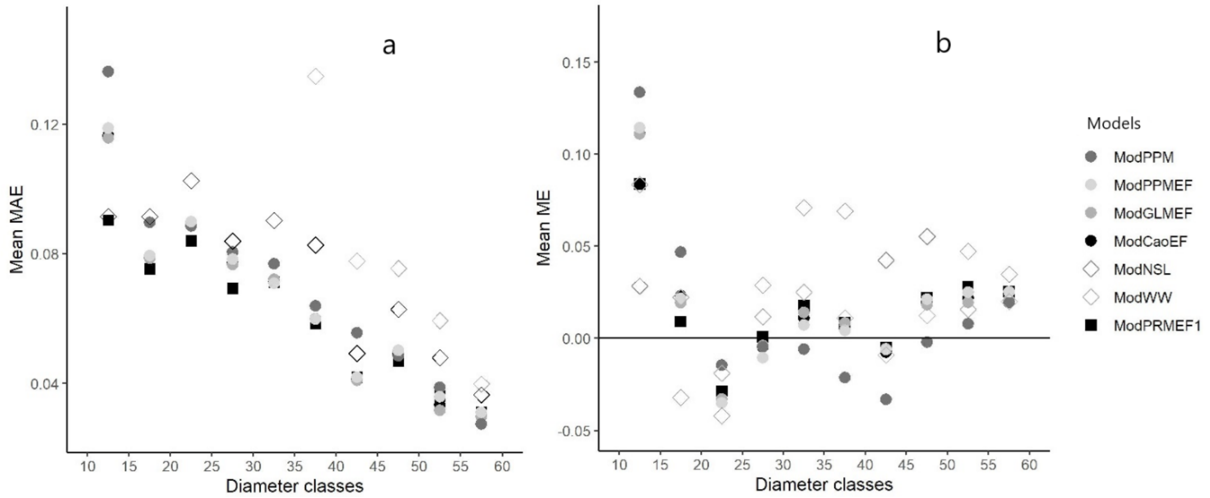

Fig. 3. The accuracy of selected diameter distribution models, measured by mean absolute error MAE (a), and bias, measured by mean error ME (b), in relative frequencies per 5-cm diameter class (models developed using the one- and two-step parameter prediction method (PPM) are marked with ○, models that considered truncated samples within their formulation are marked with ◊, and models developed using the parameter recovery method (PRM) are marked with □).

large trees (up to 50 cm in dbh). When considering ME, ModNSL performed best in the smallest diameter class (10–15 cm); however, its performance was much poorer in larger diameter classes, where numerous extreme deviations (outliers) were detected.

Our results demonstrated that the application of EF (e.g., ModPPMEF, ModCaoEF) resulted in higher accuracy compared to models that ignored truncated samples entirely (e.g., ModPPM) (Fig. 3). However, models that incorporated truncated samples directly within their formulation (e.g., ModNSL, ModWW) performed less effectively, except in the smallest diameter class. Models based on the PPM modelling, whether implemented in one step (e.g., ModCaoEF, ModGLMEF) or two steps (e.g, ModPPMEF), showed similar performance (Fig. 3). Nevertheless, ModPRMEF1 emerged as the best performing model for estimating frequencies across diameter classes.

3.3. Estimation of the selected plot parameters

We detected statistically significant differences in model performance (ANOVA, $p < 0.001$; Supplementary Material SM5). However, some models performed similarly well in estimating the selected plot parameters, as indicated by measures of bias (ME), precision (SD) and accuracy (RMSE and MAPE) (Fig. 4, Table 4).

Table 4. Performance of the developed models in estimating selected plot parameters: quadratic mean diameter (QMD), basal area (BA) and dominant diameter (D_{DOM}). Mean values of mean error (ME), SD (standard deviation) of ME, root mean square error (RMSE) and mean absolute percentage error (MAPE) between empirical and estimated plot parameters are shown (n = 118). Significant differences between empirical and estimated parameters are indicated in the mean error (ME) column (ns – no significant differences were found, *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$). The best-performing model within each performance indicator (ME, SD, RME, MAPE) is highlighted in bold.

	QMD				BA				D_{DOM}			
	ME	SD	RMSE	MAPE	ME	SD	RMSE	MAPE	ME	SD	RMSE	MAPE
Mod2PPM	2.25***	1.41	2.65	0.09	4.51***	5.50	7.10	0.19	1.42***	2.20	2.61	0.06
Mod2PPMEF	0.76***	**1.18**	1.40	**0.04**	**0.64** ns	4.93	4.95	0.13	**0.00** ns	2.15	2.14	0.05
ModWW	**–0.04****	1.43	1.72	0.06	–1.54 ns	5.07	5.38	0.14	–0.20 ns	2.32	2.37	0.05
ModNSL	–3.66***	2.00	4.02	0.14	–9.73***	6.13	11.07	0.28	–3.65***	2.35	4.29	0.10
ModCao	2.31***	1.31	2.65	0.09	4.73***	5.51	7.24	0.20	2.17***	2.14	3.05	0.08
ModCaoEF	0.85***	**1.18**	1.45	**0.04**	0.86 ns	4.95	5.00	0.13	0.56 ns	2.15	2.21	0.05
ModCaoWW	0.10 ns	1.42	1.41	0.05	–1.17 ns	4.81	4.93	0.12	0.63 ns	2.19	2.27	0.05
ModGLM	2.30***	1.39	2.69	0.09	4.68***	5.53	7.23	0.19	2.09***	2.20	3.03	0.08
ModGLMEF	0.85***	**1.18**	1.45	0.04	0.86 ns	4.95	5.00	0.13	0.59 ns	2.21	2.28	0.06
ModGLMWW	0.09 ns	1.56	1.56	0.05	–1.20 ns	4.97	5.09	0.13	0.60 ns	2.26	2.33	0.06
ModCOMB	–2.59***	1.19	2.84	0.11	–7.59***	**4.46**	8.79	0.23	–3.48***	2.16	4.09	0.10
ModCOMBEF	–2.50***	**1.18**	2.76	0.10	–7.33***	4.49	8.59	0.22	–3.56***	2.17	4.17	0.10
ModPRMEF1	0.26 ns	1.22	1.24	**0.04**	–0.69 ns	4.72	**4.75**	**0.12**	–0.58 ns	**1.73**	**1.82**	**0.04**
ModPRMEF2	0.13 ns	1.19	**1.20**	**0.04**	–1.00 ns	4.71	4.79	**0.12**	–8.49***	2.54	8.86	0.24

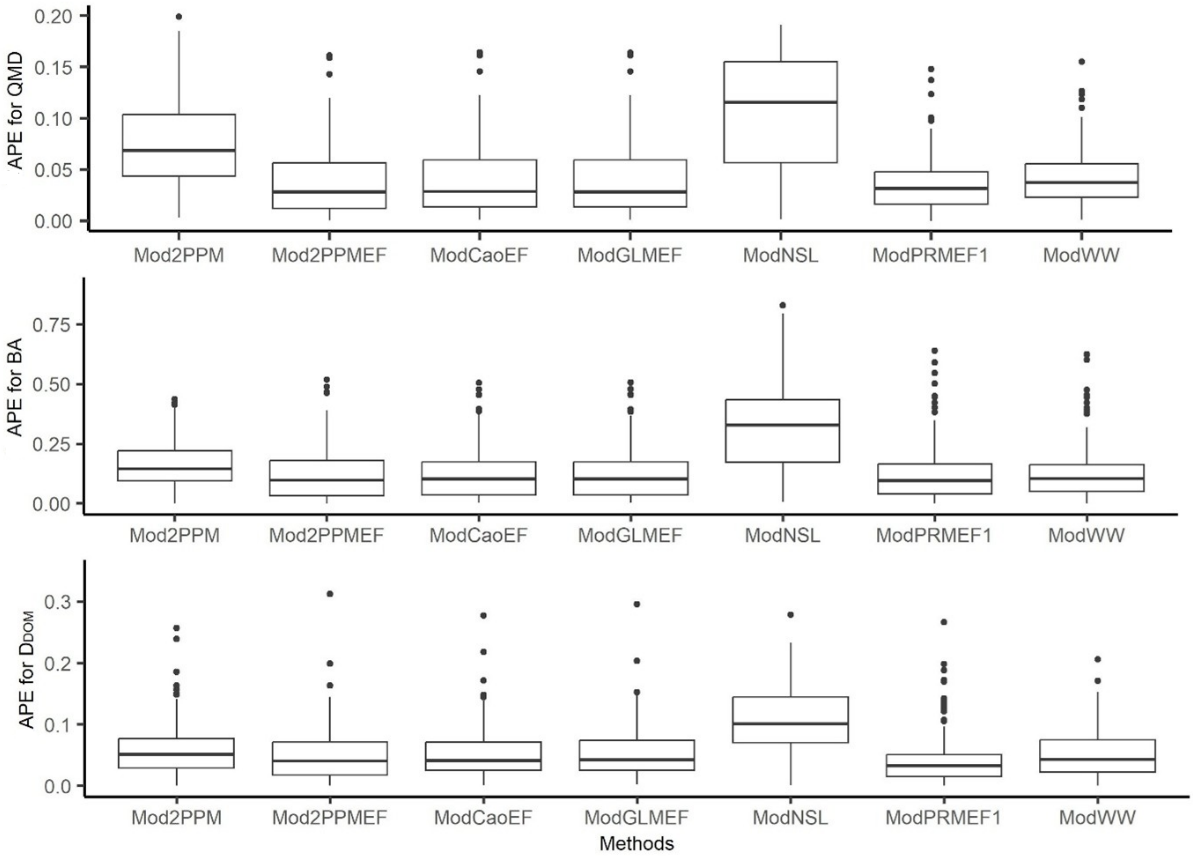

Fig. 4. Accuracy of selected models, measured by absolute percentage error (APE) in estimating plot parameters: quadratic mean diameter (QMD), basal area (BA) and dominant diameter (D_{DOM}) (some extreme outliers are not shown).

In general, models that accounted for the truncated sampling design outperformed those that ignored it. The smallest ME values between empirical and modelled parameters were –0.04 for QMD, 0.64 for BA and 0 for D_{DOM}, respectively. When considering bias, models based on the PPM outperformed those based on the PRM, although ModPRMEF1 also exhibited low bias. In terms of accuracy, the lowest MAPE and RMSE were achieved by ModPRMEF1. Paired t-tests revealed significant differences between the empirical and estimated pairs of stand parameters for most models (Table 3). With respect to detecting significant differences, ModCaoWW and ModGLMWW preformed best, followed by Mod2PPMEF, ModWW, ModCaoWW, ModGLMEF ModPRMEF1 and ModPRMEF2.

Stand age, indirectly expressed by D_{DOM} classes, influenced the performance of plot parameter estimation, especially for BA and D_{DOM}, while the prediction of QMD was less affected (Fig. 5). MAPE between empirical and estimated BA and D_{DOM} was lower in stands with higher D_{DOM}.

3.4. Model application

Estimates of plot parameters modelled with the best-performing model, ModPRMEF1, fell within the interval of ±10% of the empirical values for 96%, 57% and 89% of plots for QMD, BA and D_{DOM}, respectively (Fig. 6). For QMD and BA, these results are reasonably close to those obtained from the simulated inventory design (Fig. 2), while for D_{DOM}, the performance was 7% poorer. The estimated and empirical pairs of QMD and D_{DOM} significantly differed ($p < 0.05$ and $p < 0.001$, respectively), whereas no statistical differences were found between the pairs of estimated and empirical values of BA and N ($p > 0.05$).

4. Discussion

There is no consensus among researchers on the most appropriate method for estimating Weibull function parameters (e.g., Siipilehto & Mehtätalo 2013), and our study confirms this. We found that ModPRMEF1, the model developed using the PRM, performed best in predicting plot parameters and frequencies across diameter classes. When predicting plot parameters QMD, BA and D_{DOM}, ModPRMEF1 was among the most accurate mod-

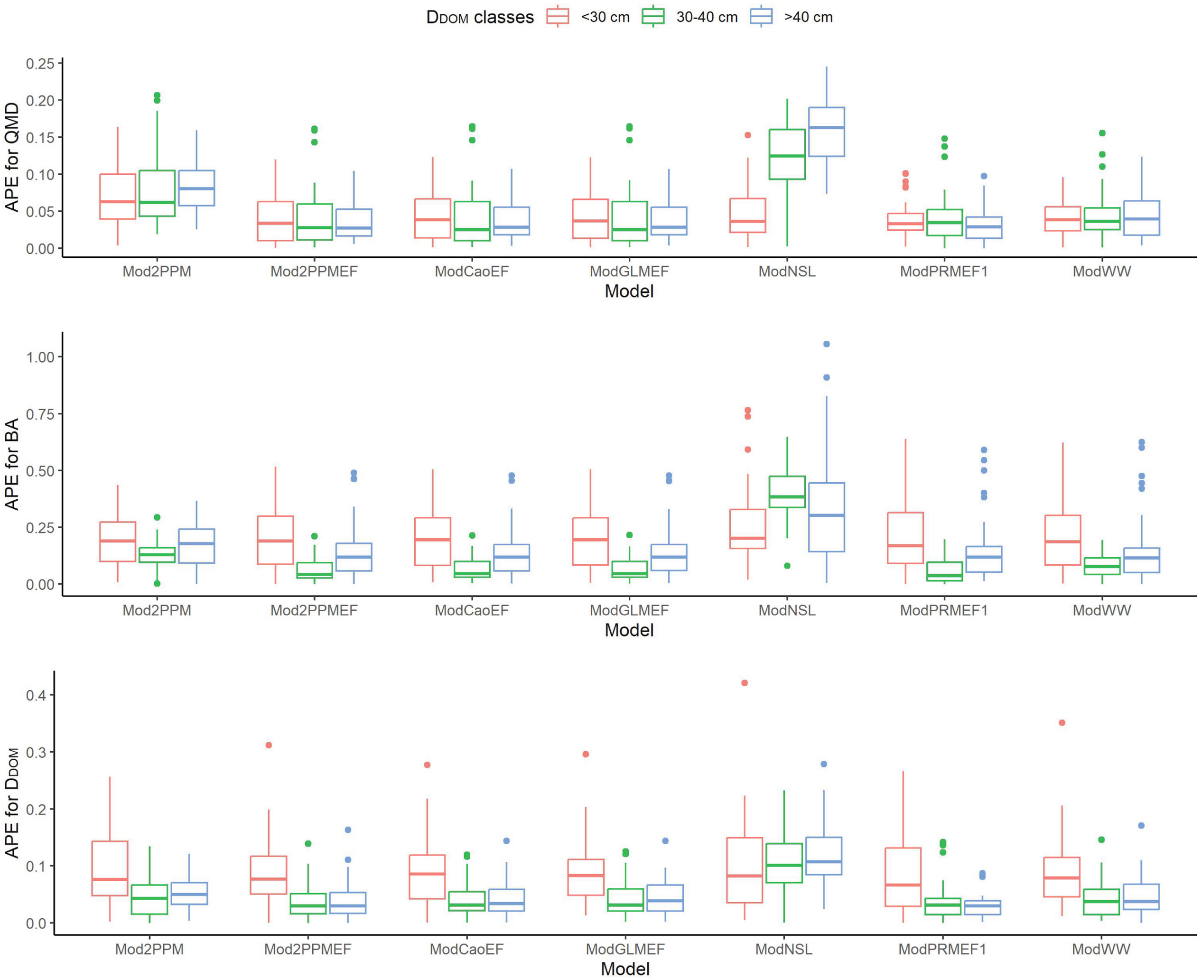

Fig. 5. Accuracy between empirical and estimated quadratic mean diameter (QMD), basal area (BA) and dominant diameter (D_{DOM}) based on empirical dominant diameter (D_{DOM}) classes (only selected models are shown).

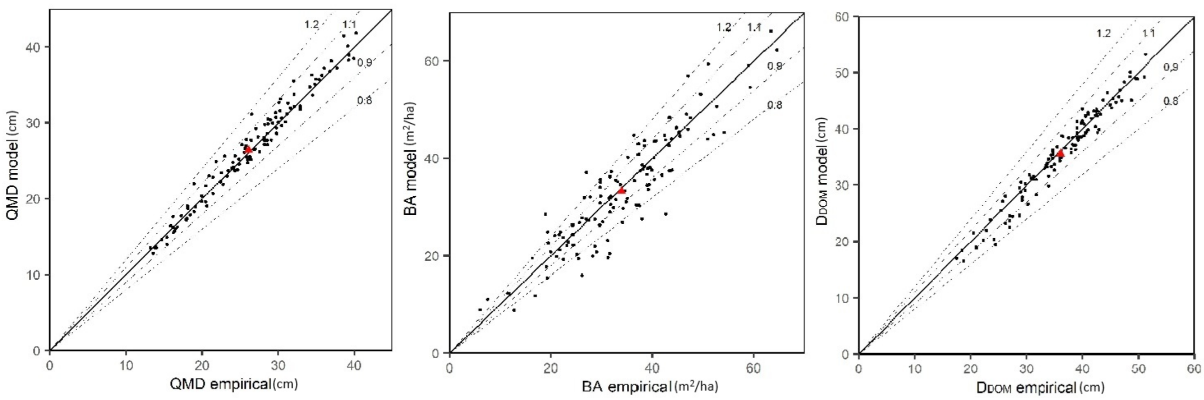

Fig. 6. Comparison of empirical (calculated from all trees on a plot) and estimated plot parameters (modelled with the best-performing model, ModPRMEF1) (n = 118); red triangles indicate mean values.

els, and no significant differences were found between modelled and empirical values for these variables. However, PPM models such as Mod2PPMEF, ModCaoEF, ModCaoWW, ModGLMWW and ModGLMEF performed almost equally well and could also be suitable for estimating the diameter distribution of even-aged near-natural beech forest stands. PPM models (i.e. Mod2PPMEF, ModWW) performed slightly better than PRM models in predicting stand parameters when considering the bias indicator (ME). While Cao (2004) advocated for one-step modelling over the two-step approach, our results showed comparable accuracy for models developed using both methods.

Because of their compatibility with certain stand variables (e.g., BA and QMD) (Newton et al. 2005), several studies have recommended the use of the PRM for older even-aged stands (e.g., Kangas & Maltamo 2000; Siipilehto & Mehtätalo 2013; Schmidt et al. 2020). In our study area, mature stands predominated, as indicated by mean D_{DOM} and QMD values of 36 cm and 26 cm, respectively. Despite this, it is important to acknowledge that the PRM has known shortcomings. In particular, it does not incorporate any information on dominant trees in the calculation procedure (Siipilehto & Mehtätalo 2013), making it inherently less suitable when predicting D_{DOM} is a primary objective. Interestingly, this limitation did not affect our results, as ModPRMEF1 was the most accurate model for estimating D_{DOM}. Notably, ModPRMEF1 was the only model containing D_{MEAN} as a predictor, while other models relied on QMD.

ModPRMEF1 showed the highest accuracy in modelling frequencies per diameter class, although differences between models were not substantial. On average, the MAE in relative frequencies was 0.07, which corresponds to an average of 46 trees per ha per 5-cm diameter class. This value was three times higher than that reported by Gorgoso et al. (2007), who studied even-aged birch stands using larger plots with more than 30 trees per plot. The highest MAE values (> 0.09) were observed in the smallest diameter classes (< 20 cm), indicating that the modelled diameter distributions were less accurate for thinner trees. For most models, we observed a tendency to underestimate frequencies in the 10–20 cm diameter class, slightly overestimate them in the 20–30 cm diameter class and then underestimate again in larger diameter classes (> 30 cm). There are likely several reasons for these patterns. First, the inconsistent convergence of the MLE estimator may play a role (Akram & Hayat 2014). Second, the (simulated) inventory design may have contributed, as trees with dbh < 30 cm were only sampled in the smaller 200 m^2 plot. Third, the categorization of even-aged and uneven-aged forests based on the Gini index may have influenced the outcome (Duduman 2011; Trifković et al. 2023). In our case, the Gini index was calculated using the basal area of each tree, which means that thinner trees contributed very little to the index despite being abundant within a plot. Although the analysed stands were classified as even-aged and were expected to exhibit a unimodal distribution, some plots were more accurately described by bi- or multimodal distributions (Merganič & Sterba 2006). This "less even-aged" structure of near-natural beech stands can be attributed to several factors. One explanation is the relatively high number of understorey trees with dbh between 10 and 20 cm, or even smaller, which are typical in near-natural forests (Von OHeimb et al. 2005). In stands of shade-tolerant species, inverse J-shaped diameter distributions are common. These can arise either from sustained ingrowth (Heiri et al. 2009) or from the retention of understorey and intermediate-layer trees, which is typical in close-to-nature forest management systems such as the irregular shelterwood system (Diaci et al. 2021). These trees are not removed during silvicultural interventions such as thinning from above (Bončina et al. 2007), but are often left to provide ecological benefits, such as shading the forest floor, protecting tree trunks or promoting self-pruning in the lower stems of crop trees (Diaci et al. 2021). These "less even-aged" structures may also result from unplanned development of understorey or intermediate layers due to improper or inconsistent silvicultural measures. Additionally, less intensive and/or irregular forest management can contribute to structural complexity (Pach & Podlaski 2015; Podlaski et al. 2019). In Slovenia, the lower intensity of forest management in some forest areas may be related to the high proportion of privately-owned forests (77%), which are often small and fragmented (Medved et al. 2010; Ficko 2019). Natural disturbances may also play a role in shaping these complex stand structures (Merganič & Sterba 2006). Excluding structurally complex stands may improve the quality of modelling results, and the proportion of excluded stands can also serve as an indicator of forest structural heterogeneity (e.g., Mey et al. 2021). To ensure that all investigated stands feature a unimodal diameter distribution, filtering could involve classifying stands by developmental phase, visually inspecting histograms using kernel density estimation with Scott's rule of thumb or examining the relationship between QMD and D_{DOM} (Mey et al. 2023). Some studies have excluded understorey trees altogether (Kangas & Maltamo 2000; Nord-Larsen & Cao 2006) or modelled diameter distribution separately by tree social class (Zasada & Cieszewski 2005). However, in line with our study objectives, we did not apply such filtering. Plots can also be excluded during the modelling procedure due to non-convergence of the fitting methods (e.g., Akram & Hayat 2014; Bončina et al. 2022). Furthermore, our criterion for defining even-aged stands was based on a Gini index threshold of 0.33 (Trifković et al. 2023), which is in accordance with other studies (e.g., O'Hara et al. 2007; Duduman 2011). However, our findings reaffirmed that the Gini index (and other structural diversity indices) does not unambiguously identify even-aged stands, and stands of very different structures can exhibit the same diversity index (Duduman 2011). In addition, our mean and median values of the Gini index were close to the threshold of 0.33, indicating less homogeneous stands.

One characteristic of forest inventory data is the presence of truncated samples (Vidal et al. 2016). In our study, models that accounted for truncation outperformed those that ignored it. Although the use of forest inventory data theoretically favours left-truncated (Palahí et al. 2007; Gorgoso et al. 2012; Ciceu et al. 2021) or multi-truncated formulations (e.g., Nanos & Sjöstedt de Luna 2017; Mey et al. 2021), the complete, non-truncated formulation of the Weibull distribution function has still frequently been applied (Cao 2004; Nord-Larsen & Cao 2006; Gorgoso et al. 2007; Lei 2008; Poudel & Cao 2013; Siipilehto & Mehtätalo 2013; Podlaski & Roesch 2014; Sanquetta et. al. 2014; Sghaier et al. 2016; Schütz et al. 2018; Rijal & Sharma 2019; Schütz & Rosset 2020; Cao 2022; Guo et al. 2022). For multi-truncated inventory designs, the use of expansion factors (EF), which account for different selection probabilities, proved to perform as well or better than methods that explicitly account for truncation within the function formulation (e.g., ModWW, ModNSL). However, when using EF to account for truncated samples, QMD estimation can still be biased if certain spatial tree patterns are present on a plot (e.g., inhomogeneous intensity Poisson pattern), although the estimates remain unbiased for random and clustered tree patterns (Nanos & Sjöstedt de Luna 2017).

ModNSL outperformed other models in predicting frequencies in the smallest diameter class (10–15 cm); however, its performance was much poorer in larger diameter classes. A notable shortcoming of ModWW and ModNSL was their extremely inaccurate prediction of plot parameters in 2% and 3% of plots, respectively. One potential reason for this poorer performance is convergence failure due to a low number of trees per plot. Furthermore, we observed that the starting values for parameters *b* and *c* had a strong influence on the high non-convergence rate in the ModWW modelling process (results not shown).

Although concentric fixed-area circular plots with truncated samples can be used for modelling diameter distributions (e.g., Mehtätalo & Lappi 2020; Mey et al. 2021), some limitations must be acknowledged. Individual plots may still show considerable discrepancies between empirical and simulated stand parameter values. Significant differences were detected between the empirical plot parameters and those derived from the simulated inventory design for D_{DOM} and QMD (but not for N and BA). Although no statistically significant differences were found, it is important to recognize that this test assesses the mean difference across all paired observations. Consequently, localized deviations may persist, potentially introducing bias in the prediction of stand parameters and in the estimation of frequencies across diameter classes. Additionally, while EF adjust for the different selection probabilities related to the second truncation point (measurement threshold of 30 cm), they do not address the first truncation point (measurement threshold at 10 cm). Applying the complete Weibull distribution to describe truncated data can introduce systematic errors in the parameters. This bias is more pronounced when QMD is close to the tree dbh measurement threshold (Zutter 1986), which represents the truncation point of the diameter distribution. In our study, the measurement threshold was 10 cm, and only 5% of plots had QMD below 15 cm, meaning only a small number were affected. The results for these young stands should thus be interpreted with caution. If the focus is on young stands or ingrowth, it may be more appropriate to use a left-truncated Weibull formulation, which explicitly accounts for the lower measurement limit, or to model diameter distributions using basal area (BA) rather than tree count (N), as BA is less sensitive to the omission of small-diameter trees.

Moreover, estimations of plot parameters BA and D_{DOM} were less reliable in younger stands and more accurate in older stands with higher D_{DOM}. However, besides age, D_{DOM} also reflects the thinning concept, particularly when thinning from above is applied (Bončina et al. 2007). This could be attributed to the more homogeneous structure of older stands, their greater distance from the first truncation point of the diameter distribution (10 cm) and the reduced or negligible influence of the second truncation point (30 cm) (Fig. 2 and 6). In contrast, QMD estimation was less affected by D_{DOM} classes, likely due to the strong relationship between QMD and scale parameter *b* (Schütz & Rosset 2020).

The sampling subplots used in our study were 200 m^2 and 400 m^2 in size, whereas the common inventory design in Slovenia includes subplots of 200 m^2 and 500 m^2 (Poljanec 2010). Larger plots (> 0.1 ha) are more commonly used for modelling diameter distributions (Sghaier et al. 2016; Schütz & Rosset 2020), while small-sized plots have been applied much less frequently (e.g., Ciceu et al. 2021; Mey 2021), at least for near-natural forests. The number of trees per plot appears to be a more important factor than plot size. Several studies have used a minimum number of trees per plot as a criterion for determining sample plot size (Gorgoso et al. 2012; Ciceu et al. 2021). For example, older stands with lower stand densities require larger plots to encompass a sufficient number of trees (e.g., Pretzsch 2010). Recommendations range from at least 10 trees per plot (e.g., Lei 2008; Poudel & Cao 2013; Akram & Hayat 2014) to 15–30 or more trees per plot (Gorgoso et al. 2012; Sghaier et al. 2016). By adopting the less strict criterion, our analysis excluded plots with fewer than 10 trees, which primarily resulted in the elimination of plots in mature stands with low tree density. However, Noel et al. (1998) argued that even plots exceeding 1 ha may still lack some information on tree diameters at larger spatial scales (i.e. the stand level). In our study area, stands are relatively small due to the application of close-to-nature forest management, and a plot can lie within two different stands. From this perspective, working with small sample plots is more challenging than working with larger research sample plots that have been selected to represent more or less homogenous stand conditions. Additionally, even even-aged stands exhibit considerable structural heterogeneity under close-to-nature silviculture principles. The horizontal structure of these stands is notably patchy, with some cohorts measuring 0.1–0.5 ha or even smaller. Consequently, a sample plot can encompass structural elements from two or more cohorts, and even larger sample plots (e.g., 1 ha) would likely capture similar heterogeneity.

The predictions of plot parameters had adequately low bias and were sufficiently precise and accurate to be applicable in forest management. The mean MAPE for the best-performing model, ModPRMEF1, was 4%, 12% and 4% for QMD, BA and D_{DOM}, respectively. The estimated plot parameters closely matched the empirical plot parameters (Fig. 6). Dominant trees are crucial for the management of even-aged forests (Bončina et al. 2007), and the model performed well in this regard, particularly in predicting D_{DOM} and the frequencies of thicker trees. Larger bias in the prediction of frequencies of thinner trees (and therefore also BA) is of lesser importance for forest management. Despite these observed biases, we can conclude that data from concentric fixed-area sample plots can be used as a reliable source for modelling diameter distributions when such data are needed for forest management practices.

5. Conclusion

In this study, we tested several modelling techniques for estimating the diameter distribution of even-aged near-natural European beech forests using the Weibull function. While the differences between the developed models were not substantial, ModPRMEF1 emerged as the most suitable model for this purpose. It is based on the parameter recovery method (PRM) and accounts for truncated sampling design by applying expansion factors (EF), which proved to be an effective method for dealing with multiple subsample plots and truncated samples. However, its effectiveness was limited to addressing the second truncation point, but it does not resolve issues related to the first truncation point (the minimal diameter threshold). Although models based on the parameter prediction method (PPM) also performed well, the selected PRM model performed slightly better overall.

The selected model provided adequately accurate estimations of plot parameters and exhibited one of the lowest biases. However, estimations of frequencies per diameter class were less reliable. The lower accuracy in predicting these frequencies is likely due to the heterogeneity of near-natural beech forests. These forests often have a high number of small-diameter trees (10–20 cm), likely due to beech ingrowth as a shade tolerant species, close-to-nature management practices with thinning from above, which retain understorey trees, or less intensive forest management. Estimations could likely be improved by applying stricter criteria for defining even-aged, homogeneous stands or by focusing exclusively on the upper tree-layer.

Our study confirms that data from inventory sample plots are a viable source for modelling diameter distributions, although some limitations should be acknowledged. Predictions of BA and D_{DOM} were more accurate in older stands, and frequency predictions were more reliable for thicker trees. Thus, this type of data may be more reliable in older, more homogeneous stands. Despite these limitations, the developed models appear adequate for practical application in forest management, including forest management planning and inclusion in forest development simulators where information on diameter distribution is required.

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Supplementary Material SM1

To develop model ModWW, we adapted the Weighted Weibull method (Mehtätalo & Lappi 2020, p. 359), which uses weights based on tree dbh thresholds when estimating diameter distribution models from samples of concentric inventory plots. The plot radius varies between 7.98 m and 11.28 m according to the tree diameter in the Slovene inventory design. We used the following equations in R to develop the ModWW model:

- CDF for ModWW:

```
cdfWW <- function (x, c, b, limit=0) {
F <- (pweibull(x, c, b) – pweibull (limit, c, b)) / (1 – pweibull (limit, c, b))
F[x < limit] <- 0
F
}
```

- PDF of a ModWW:

```
pdfWW<-function(x, c, b, limit=0){
f<-dweibull(x, c, b)/(1-pweibull(limit, c, b))
f[x<limit]<-0
w<-rep(1, length(x))
w[x<30]<-(7.98/11.28)^2
Iwf<-(1- cdf_ww (30, c, b, limit=limit))+
(7.98/11.28)^2*( cdfWW (30, c, b, limit=limit)- cdfWW (10, c, b, limit=limit))
w*f/Iwf
}
```

- Loglikelihood for ModWW:

```
nLLWW <-function (lc, lb){
c<-exp(lc)
b<-exp(lb)
 -sum (log(pdfWW ( x, c=c, b=b, limit=10.0)))
}
```

where *c* is the shape parameter, *b* is the scale parameter, *x* is a sample of trees in a plot, *pweibull* is the function used to compute CDF for the Weibull distribution, and *dweibulll* is the function used to compute PDF for the Weibull distribution.

Reference used:

Mehtätalo, L., Lappi, J., 2020: Biometry for Forestry and Environmental Data. With Examples in R. CRC Press, 426 p.

Supplementary Material SM2

To develop model ModNSL, we adapted the method proposed by Nanos and Sjöstedt de Luna (2017), which is based on multiple truncated probability density functions. Our adaptation of the original R script was made to suit our inventory design with two concentric fixed-area plots.

The eqaution for calculating the Loglikelihood for the model ModNSL:

$$\log L\,(c,b) = n[\log(c) - c\log(b)] + \sum_{s=1}^{2} n_s\left[\left(\frac{y_s}{b}\right)^c\right] + (c-1)\sum_{s=1}^{2}\sum_{i=1}^{n_s}\log x_{is} - \sum_{s=1}^{2}\sum_{i=1}^{n_s}\left(\frac{x_{is}}{b}\right)^c$$

where *c* is the shape parameter, *b* is the scale parameter, *n* is the number of trees per plot, n_s is the number of trees per subplot, *x* is a tree in a plot, *y* is the limit for truncation, and s is the subplot.

Reference used:

Nanos, N., Sjöstedt de Luna, S., 2017: Fitting diameter distribution models to data from forest inventories with concentric plot design. Forest Systems, 26:e01S.

Supplementary material SM3

To develop model ModCaoWW, sampling weights proposed in the Weigthed Weibull (after Mehtatalo et al. 2011, Mehtetalo & Lappi 2020, p. 359) were incorporated in function for developing model ModCao (Cao 2004). We used the following equations in R to develop the ModCaoWW model:

- CDF for ModCaoWW:

```
cdfCaoWW<- function (x, c, b, limit=0) {
F <- (pweibull(x, c, b) - pweibull(limit, c, b)) / (1 - pweibull(limit, c, b))
F[x < limit] <- 0
F
}
```

- PDF for ModCaoWW:

```
pdfCaoWW<-function (x, c, b, limit=0){
f<-dweibull(x, c, b)/(1-pweibull(limit, c, b))
f[x<limit]<-0
w<-rep(1, length(x))
w[x<30]<-(7.98/11.28)^2
Iwf<-(1- cdfCaoWW (30, c, b, limit=limit))+ (7.98/11.28)^2*( cdfCaoWW (30, c, b, limit=limit)- cdfCaoWW (10, c, b, limit=limit))
w*f/Iwf
}
```

- Loglikelihood for ModCaoWW:

```
nLLCaoWW<-function(x,sc1,sc2,sh1,sh2,sh3) {
b<-exp(sc1+sc2*log(QMD))
c<-exp(sh1+sh2*QMD+sh3*BA)
-sum(1/n*log(pdfCaoWW (x, c=c, b =b, limit=10)))
}
```

where *c* is the shape parameter, *b* is the scale parameter, *n* is the number of trees per plot, *x* is a sample of trees in a plot, *QMD* is the QMD of a stand on a plot, *BA* is the BA of a stand on a plot, *sc1*, *sc2*, *sh1*, *sh2*, *sh3* are coefficents in the equations for estimating *b* and *c*, *pweibull* is the function used to compute CDF for the Weibull distribution, and *dweibulll* is the function used to compute PDF for the Weibull distribution.

References used:

Cao, Q. V., 2004: Predicting Parameters of a Weibull Function for Modeling Diameter Distribution. Forest Science, 50:682–685.

Mehtätalo, L., Comas, C., Pukkala, T., Palahi, M., 2011: Combining a predicted diameter distribution with an estimate based on a small sample of diameters. Canadian Journal of Forest Research, 41:750–762.

Mehtätalo, L., Lappi, J., 2020: Biometry for Forestry and Environmental Data. With Examples in R. New York, Chapman and Hall/CRC, 426 p.

Supplementary Material SM4

To develop model ModGLMWW, sampling weights proposed in the Weigthed Weibull (after Mehtatalo et al. 2011; Mehtetalo and Lappi 2020, p. 359) were incorporated into the function for developing the ModGLM model (Mehtetalo & Lappi 2020, p. 373). We used the following equations in R for developing the ModGLMW model:

- CDF for ModGLMWW:

```
cdfModGLMWW <- function(x, c, b, limit=0) {
F <- (pweibull(x, c, b) - pweibull(limit, c, b)) / (1 - pweibull(limit, c, b))
F[x < limit] <- 0
F
}
```

- PDF for ModGLMWW:

```
pdfModGLMWW <-function(x, c, b, limit=0){
f<-dweibull(x, c, b)/(1-pweibull(limit, c, b))
f[x<limit]<-0
w<-rep(1, length(x))
w[x<30]<-(7.98/11.28)^2
Iwf<-(1- cdfModGLMWW (30, c, b, limit=limit))+ (7.98/11.28)^2*( cdfModGLMWW (30, c, b, limit=limit)- cdf-ModGLMWW (10, c, b, limit=limit))
w*f/Iwf
}
```

- Loglikelihood for ModGLMWW:

```
nLLGLMWW<-function(x,sc1,sc2,sh1,sh2,sh3) {
b<-exp(sc1+sc2*log(QMD))
c<-exp(sh1+sh2* QMD +sh3*BA)
 -sum(log(pdf_ModGLMWW (x, c=c, b =b, limit=10)))
}
```

where *c* is the shape parameter, *b* is the scale parameter, *x* is a sample of trees in a plot, *QMD* is the QMD of a stand on a plot, *BA* is the BA of a stand on a plot, *sc1*, *sc2*, *sh1*, *sh2*, *sh3* are coefficents in the equations for estimating *b* and *c*, *pweibull* is the function used to compute CDF for the Weibull distribution, and *dweibull* is the function used to compute PDF for the Weibull distribution.

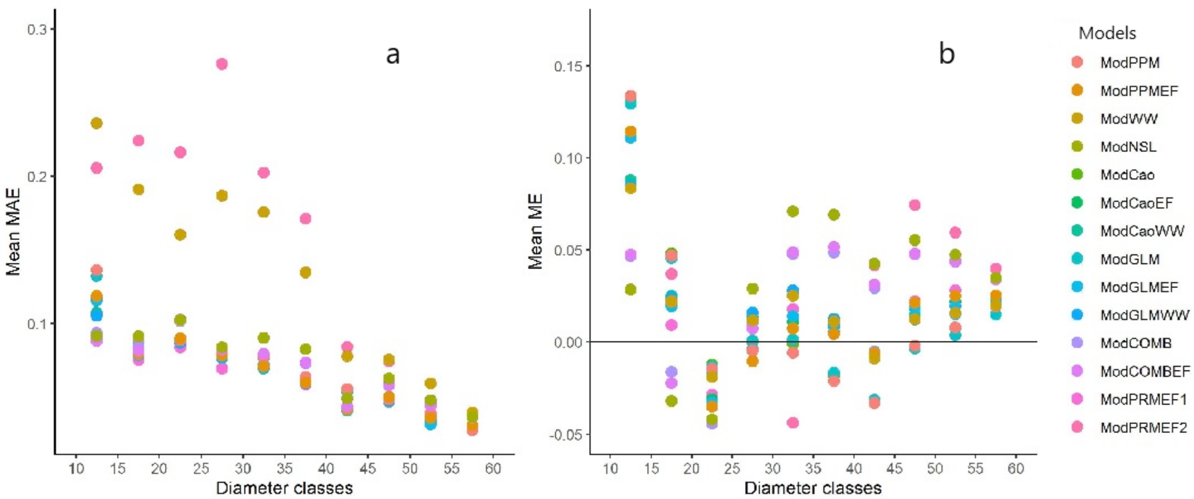
References used:

Mehtätalo, L., Comas, C., Pukkala, T., Palahi, M., 2011: Combining a predicted diameter distribution with an estimate based on a small sample of diameters. Canadian Journal of Forest Research, 41:750–762.

Mehtätalo, L., Lappi, J., 2020: Biometry for Forestry and Environmental Data. With Examples in R. New York, Chapman and Hall/CRC, 426 p.

Supplementary Material SM5

The accuracy, measured by mean absolute error MAE (a) and bias, measured by mean error ME (b) in relative frequencies per 5-cm diameter classes.



Supplementary Material SM6

Pairwise comparisons between the (empirical) plot parameters obtained from all measured trees at a sample plot (EMP) and plot parameters computationally derived from the developed diameter distribution models. P values from t-test with Bonferroni correction are shown (ns – no significant differences were found, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$). Comparisons where no significant differences were detected ($p > 0.05$) are highlighted in bold.

Plot quadratic mean diameter QMD:

	EMP	Mod2PPM	Mod2PPMEF	ModWW	ModNSL	ModCao	ModCaoEF	ModCaoWW	ModGLM	ModGLMEF	ModGLMWW	ModCOMB	ModCOMBEF	ModPRMEF1	ModPRMEF2
EMP	—														
Mod2PPM	0.000 ***	—													
Mod2PPMEF	0.000 ***	0.000 ***	—												
ModWW	0.005 **	0.000 ***	0.589	—											
ModNSL	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—										
ModCao	0.000 ***	**0.203** ns	0.000 ***	0.000 ***	0.000 ***	—									
ModCaoEF	0.000 ***	0.000 ***	0.000 ***	0.008 **	0.000 ***	0.000 ***	—								
ModCaoWW	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—							
ModGLM	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	—						
ModGLMEF	0.000 ***	0.000 ***	0.000 ***	0.010 **	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	—					
ModGLMWW	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	—				
ModCOMB	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—			
ModCOMBEF	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.002 **	—		
ModPRMEF1	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.025 *	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	—	
ModPRMEF2	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	—

Plot basal area BA:

	EMP	Mod2PPM	Mod2PPMEF	ModWW	ModNSL	ModCao	ModCaoEF	ModCaoWW	ModGLM	ModGLMEF	ModGLMWW	ModCOMB	ModCOMBEF	ModPRMEF1	ModPRMEF2
EMP	—														
Mod2PPM	0.000 ***	—													
Mod2PPMEF	**1.000** ns	0.000 ***	—												
ModWW	**1.000** ns	0.000 ***	**0.145** ns	—											
ModNSL	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—										
ModCao	0.000 ***	0.019 *	0.000 ***	0.000 ***	0.000 ***	—									
ModCaoEF	**1.000** ns	0.000 ***	0.000 ***	0.001 **	0.000 ***	0.000 ***	—								
ModCaoWW	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—							
ModGLM	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	—						
ModGLMEF	**1.000** ns	0.000 ***	0.000 ***	0.001 **	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	—					
ModGLMWW	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	—				
ModCOMB	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—			
ModCOMBEF	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—		
ModPRMEF1	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.003 **	0.000 ***	0.000 ***	**0.288** ns	0.000 ***	0.000 ***	—	
ModPRMEF2	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	—

Dominant diameter of a plot D_{DOM}:

	EMP	Mod2PPM	Mod2PPMEF	ModWW	ModNSL	ModCao	ModCaoEF	ModCaoWW	ModGLM	ModGLMEF	ModGLMWW	ModCOMB	ModCOMBEF	ModPRMEF1	ModPRMEF2
EMP	—														
Mod2PPM	0.000 ***	—													
Mod2PPMEF	**1.000** ns	0.000 ***	—												
ModWW	**1.000** ns	0.000 ***	**1.000** ns	—											
ModNSL	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—										
ModCao	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—									
ModCaoEF	**0.623** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—								
ModCaoWW	**0.251** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	—							
ModGLM	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	—						
ModGLMEF	**0.561** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	**1.000** ns	0.000 ***	—					
ModGLMWW	**0.574** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	**1.000** ns	0.000 ***	**1.000** ns	—				
ModCOMB	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—			
ModCOMBEF	0.000 ***	0.000 ***	0.000 ***	0.000 ***	**1.000** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 **	—		
ModPRMEF1	0.044 *	0.000 ***	**0.225** ns	**0.240** ns	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—	
ModPRMEF2	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	—

Supplementary Material SM 7

Examples of empirical diameter distributions for three selected plots, with corresponding modelled distributions derived from the selected models: Mod2PPMEF, ModNSL, ModWW, and ModPRMEF1.