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ON THE DEBT TO GDP RATIO IN MONOPOLISTIC COMPETITION

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Abstract

Research background: Many people in Japan and other countries believe that national finances will not stand still and will go bankrupt if deficits continue at present level. It is like a religion. In this paper, however, we prove that this religion is wrong by using a simple mathematical model.
Research methodology: This paper uses a macroeconomic model based on microeconomic foundations for consumers and firms with overlapping generations of consumers under monopolistic competition.
Results: Although fiscal failure or collapse is generally defined as the government debt to GDP ratio going beyond a finite value to an infinite value, i.e., diverging, the following analysis theoretically proves that such a failure does not occur under stable prices or under a constant inflation rate. It is often said that the government debt to GDP ratio will continue to increase if the interest rate on government bonds exceeds the economic growth rate, but we prove that such a fiscal failure does not occur even when the interest rate of government bonds exceeds the growth rate. This paper shows that the debt to GDP ratio converges to or remains at a finite value over time and that the budget deficit over a given period is equal to the difference between the savings of the younger generation of consumers and the savings of the older generation of consumers.
Novelty: There are few papers that analyze the fact that financial collapse does not occur using mathematical models, and it is worthwhile to do so.

Keywords: debt to GDP ratio, overlapping generations model, monopolistic competition, fiscal collapse

JEL classification: E24, E62

Introduction

There is a strong belief among many that government debt must eventually be repaid through taxes, and that new government spending through the issuance of government bonds will be a burden on future generations. However, the author believes that this may not be the case. Regardless of who purchases government bonds, government spending increases the financial assets of those who receive the spending, while taxation decreases the financial assets of people. The difference between the two is the budget deficit, and if the budget deficit continues, the financial assets held by people will continue to increase along with government debt. What kind of problems will these accumulated financial assets cause? People’s consumption is considered to depend on accumulated assets as well as income each period. Therefore, if people’s financial assets increase along with government debt, this will increase consumption, leading to higher prices under full employment. Although the ratio of government debt to GDP is often considered more problematic than the total amount of government debt, the ratio of government debt to GDP will decline as prices rise and nominal GDP increases. This paper will focus its analysis on that point.

Whenever a government implements any policy, financial resources are always an issue. Increasing taxes reduces people’s disposable income, which reduces consumption and worsens the economy. When the economy is booming or at full employment, new government spending becomes an inflationary factor and tax increases are necessary to control it, but even when the economy is not doing well, the question of financial resources becomes an issue. This is because of the assumption that government bonds must be repaid, which I wrote about at the beginning of this paper. Government spending increases demand, while taxes reduce demand by reducing disposable income. It is a government’s duty to balance the two and to implement necessary public policies. Even if the result of government policy is a budget deficit and the accumulation of government debt, there is no problem if full employment is maintained at stable prices, i.e., constant prices or mild inflation. Pensions and other social security benefits should be promoted, as well as the development of airports and the construction of new high-speed railroads, if technically feasible. The technology and production capacity of a country and its people are the constraints, not its financial resources.

Many people in Japan and other countries believe that national finances will not stand still and will go bankrupt if deficits continue at present levels. It is like a religion. In this paper, however, we prove that this religion is wrong by using a simple mathematical model. This paper uses a macroeconomic model based on microeconomic foundations for consumers and firms with overlapping generations of consumers under monopolistic competition. Although fiscal failure or collapse is generally defined as government debt to GDP ratio going beyond a finite value to an infinite value, i.e., diverging, the following analysis theoretically proves that such a failure does not occur under stable prices or under a constant inflation rate.

As Blanchard (2022, 2023) notes, many discussions of debt to GDP ratio use simple calculations based on comparisons of primary budget balances, the interest rate of government bonds, and the economic growth rate. But is the argument not so simple? Assuming a steady state of full employment, which may or may not include inflation, the size of the budget deficit to achieve this is naturally determined, and the larger the budget deficit is, the higher the inflation rate is. On the other hand, the higher (lower) the propensity to consume is, the smaller (larger) the budget deficit required to achieve full employment under constant price or inflation is.

It is often said that if the interest rate on government bonds exceeds the economic growth rate, the government debt-GDP ratio will continue to increase (Domar, 1944), Yoshino and Miyamoto (2020). However, this paper shows that no divergence in the government debt-GDP ratio occurs even when the interest rate on government bonds exceeds the real economic growth rate. When the interest rate on government bonds exceeds the real growth rate, a budget surplus excluding interest payments is necessary to achieve full employment at constant prices, but when the budget runs a deficit, inflation occurs and a situation arises where the interest rate on government bonds falls below the nominal growth rate. The inflation rate should be a little above the difference between the interest rate on government bonds and the real growth rate so that the interest rate is smaller than the nominal growth rate.

In another paper we examined the debt-GDP ratio using a simple macroeconomic model without microeconomic foundation. In this paper we consider the problem of debt to GDP ratio using an overlapping generations model in monopolistic competition according to Otaki (2007, 2009, 2015). In the next section, we present the microeconomic foundations of our model, utility maximization of consumers and profit maximization of firms. We suppose that the savings of consumers are done through interest-generating government bonds. In Section 3 we examine the debt to GDP ratio under full employment with constant prices, and prove the main results. In Section 4 we consider the case of inflation.

1. Microeconomic foundations for economic modelling

In this section we present the models of consumers, firms, and the government.

Consumers: A two-generation overlapping generations model

Following the model used by Otaki (2007, 2009, 2015), we use a two-period (generation) overlapping generations (OLG) model for consumers and a model of monopolistic competition for firms. Consumers live over two periods: the younger period and the older period. They work only in the younger period. They then consume goods in the older period, using savings carried over from the younger period. Savings are done through interest-generating government bonds. Let r be the nominal interest rate on government bonds. There is a continuum of goods indexed by $z \in [1, 1]$. Each good is produced monopolistically by firm z with labor as the only factor of production by a constant returns to scale technology. The number (or size) of firms is normalized as 1. Let $c_i(z)$ be the consumption of good z in Period i, $i = 1, 2$, of a consumer and $p_i(z)$ be the price of good z in Period i, $i = 1$ denotes the younger period for consumers, and $i = 2$ denotes their older period. Consumers choose their consumptions over two periods to maximize their utility subject to the budget constraint. Let L be the employment. L_f be the labor supply or the employment in the full employment state. It is constant.

Let

$$C_1 = \left(\int_0^1 c_1(z)^{1-\frac{1}{\eta}} dz\right)^{\frac{1}{1-\frac{1}{\eta}}}, \quad C_2 = \left(\int_0^1 c_2(z)^{1-\frac{1}{\eta}} dz\right)^{\frac{1}{1-\frac{1}{\eta}}}$$

be the consumption baskets in each period of a consumer and

$$P_1 = \left(\int_0^1 p_1(z)^{1-\eta} dz\right)^{\frac{1}{1-\eta}}, \quad P_2 = \left(\int_0^1 p_2(z)^{1-\eta} dz\right)^{\frac{1}{1-\eta}}$$

be the price indices in each period. η is the inverse of the degree of differentiation of the goods. $\eta > 1$ and in the limit when $\eta \to \infty$, the goods are homogeneous. The utility of consumption is represented by:

$$C_1^{\alpha} C_2^{1-\alpha}$$

$0 < \alpha < 1$. Denote the disposable income over two periods of an employed or an unemployed consumer by DI. The budget constraint for a consumer is

$$\int_0^1 p_1(z)c_1(z)dz + \frac{1}{1+r}\int_0^1 p_2(z)c_2(z)dz = DI$$

The Lagrange function is:

$$\mathcal{L} = C_1^{\alpha}C_2^{1-\alpha} - \lambda\left(\int_0^1 p_1(z)c_1(z)dz + \frac{1}{1+r}\int_0^1 p_2(z)c_1(z)dz - DI\right)$$

From them we obtain (please see Appendix)

$$c_1(z) = \frac{\alpha DI}{P_1}\left(\frac{P_1}{p_1(z)}\right)^{\eta}$$

and

$$c_2(z) = \frac{(1-\alpha)(1+r)DI}{P_2}\left(\frac{P_2}{p_2(z)}\right)^{\eta}$$

They are the demand functions for good z of a consumer in their younger and older periods. The demand of an older generation consumer in the same period is:

$$c_1^o(z) = \frac{(1-\alpha)(1+r)DI^o}{P_1}\left(\frac{P_1}{p_1(z)}\right)^{\eta}$$

DI^o is the disposable income of an older generation consumer. The total nominal spending (demand) is written as:

$$[\alpha DI + (1-\alpha)(1+r)DI^o]L_f + G$$

G is fiscal spending, which we will consider a little later. DI and DI^o are the average income of employed and unemployed younger generation consumers and that of the older generation consumers who were employed or unemployed in their younger period.

The government

The government determines the demand for good z, $g(z)$, given its expenditure G to maximize

$$\mathcal{G} = \left(\int_0^1 g(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{1}{1-\frac{1}{\eta}}}$$

subject to

$$\int_0^1 p_1(z)g(z)dz = G$$

The condition for maximization of G is:

$$\left(\int_0^1 g(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{1}{1-\frac{1}{\eta}}-1} g(z)^{-\frac{1}{\eta}} - \lambda_g p_1(z) = 0 \tag{1}$$

λ_g is the Lagrange multiplier. This means

$$\left(\int_0^1 g(z)^{1-\frac{1}{\eta}}dz\right)^{-1} \int_0^1 g(z)^{1-\frac{1}{\eta}}dz - \lambda_g^{1-\eta}\int_0^1 p_1(z)^{1-\eta}dz = 0$$

and

$$\left(\int_0^1 g(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{1}{1-\frac{1}{\eta}}-1} \int_0^1 g(z)^{1-\frac{1}{\eta}}dz - \lambda_g \int_0^1 p_1(z)g(z)dz = 0$$

Thus, we have

$$\left(\int_0^1 p_1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}} = P_1 = \frac{1}{\lambda_g}$$

and

$$\mathcal{G} = \lambda_g G = \frac{G}{P_1}$$

Again from (1)

$$\mathcal{G}^{\frac{1}{\eta}} g(z)^{-\frac{1}{\eta}} = \left(\frac{G}{P_1}\right)^{\frac{1}{\eta}} g(z)^{-\frac{1}{\eta}} = \frac{p_1(z)}{P_1}$$

This means

$$g(z) = \frac{G}{P_1}\left(\frac{P_1}{p_1(z)}\right)^{\eta}$$

This is the demand for good z of the government. In the equilibrium

$$g(z) = \frac{G}{P_1}$$

Firms: Monopolistic competition with production by labor

Now let us consider the profit maximization of a firm. The profit of the firm producing good z is

$$\pi(z)=\left(p_1(z)-\frac{w}{y}\right)\frac{\left[\alpha DI+(1+r)(1-\alpha)DI^o\right]L_f+G}{P_1}\left(\frac{P_1}{p_1(z)}\right)^{\eta}$$

w is the nominal wage rate and y is the labor productivity. The condition for profit maximization with respect to $p_1(z)$ given P_1 is:

$$\frac{\partial\pi(z)}{\partial p_1(z)}=\left\{\left[\alpha DI+(1+r)(1-\alpha)DI^o\right]L_f+G\right\}P_1^{\eta-1}\left[(1-\eta)p_1(z)^{-\eta}+\eta\frac{w}{y}p_1(z)^{-\eta-1}\right]=0$$

From this

$$p_1(z)=-\frac{\eta}{1-\eta}\frac{w}{y}=\frac{1}{1-\frac{1}{\eta}}\frac{w}{y}$$

All firms have the same cost function. In the equilibrium the prices of all goods are equal. Then,

$$P_1=p_1(z)=\frac{1}{1-\frac{1}{\eta}}\frac{w}{y}$$

and

$$c_1(z)=\frac{\alpha DI}{P_1},\quad c_1^o(z)=\frac{(1+r)(1-\alpha)DI^o}{P_1}$$

in the equilibrium. Hereafter we call P_1 and P_2 the prices in Periods 1 and 2.
The total demand for the goods is:

$$\frac{\left[\alpha DI+(1+r)(1-\alpha)DI^o\right]L_f+G}{P_1}$$

In the equilibrium the total demand equals the total supply which is yL. L is the employment. Under full employment $L=L_f$.

The economy grows by technological progress. The labor productivity increases at the rate $\gamma-1>0$ from a period to the next period. The nominal wage rate w also increases at the rate $\gamma-1$ under constant prices (without inflation). In a period, Period 1, it is y. If $w=y$,

$$P_1=\frac{1}{1-\frac{1}{\eta}}$$

Let Π_1 be the profit return of firms for a consumer in Period 1. Then,

$$\Pi_1 = \left(\frac{1}{1-\frac{1}{\eta}} - 1 \right) w \frac{L}{L_f} = \left(\frac{1}{\eta - 1} \right) w \frac{L}{L_f} > 0$$

For simplicity we assume that every younger generation consumer receives the profits including consumers who are unemployed. Of course, we can assume that only the consumers who are employed receive the profits. Essentially, the result is the same. In the next period (Period 2) the labor productivity increases at the rate of $\gamma - 1$. Thus, the profit return of firms for a consumer in that period is:

$$\Pi_2 = \left(\frac{1}{\eta - 1} \right) \gamma w \frac{L}{L_f}$$

Note that γw is the nominal wage rate (without inflation) in Period 2.

2. Economic growth, budget deficit and the debt to GDP ratio

2.1. Economic growth and budget deficit

We suppose full employment under constant prices, that is, $P_1 = P_2 = \frac{1}{1-\frac{1}{\eta}} \frac{w}{y}$. Thus,

$$\Pi_1 = \left(\frac{1}{\eta - 1} \right) w, \quad \text{and} \quad \Pi_2 = \left(\frac{1}{\eta - 1} \right) \gamma w$$

Let T be the (total) tax in Period 1. The younger generation consumers pay the taxes. Suppose $w = y$ in Period 1. The disposable income of younger generation consumers in total is:

$$yL_f + \Pi_1 L_f - T$$

The (total) savings of the younger generation consumers in Period 1 is:

$$(1 - \alpha)(yL_f + \Pi_1 L_f - T)$$

They purchase government bonds by their savings. Their consumption in the next period is the sum of their savings and the interest as follows,

$$(1 + r)(1 - \alpha)(yL_f + \Pi_1 L_f - T)$$

The fiscal spending and the tax in Period 2 are γG and γT, and the nominal wage rate is γy. The consumption of the younger generation consumers in Period 2 is:

$$\alpha(\gamma y L_f + \Pi_2 L_f - \gamma T) = \alpha\gamma(yL_f + \Pi_1 L_f - T)$$

The nominal total supply in Period 2 is:

$$\gamma P_1 yL_f$$

The nominal total demand is:

$$\alpha\gamma(yL_f + \Pi_1 L_f - T) + (1 + r)(1 - \alpha)(yL_f + \Pi_1 L_f - T) + \gamma G$$

From the equilibrium between the total supply and the total demand, we get

$$\gamma P_1 yL_f = \alpha\gamma(yL_f + \Pi_1 L_f - T) + (1 + r)(1 - \alpha)(yL_f + \Pi_1 L_f - T) + \gamma G$$

From this

$$\gamma yL_f = \alpha\gamma(yL_f + \Pi_1 L_f - T) + (1 + r)(1 - \alpha)(yL_f + \Pi_1 L_f - T) + \gamma G - \gamma(P_1 - 1)yL_f$$

Since $P_1 = \dfrac{1}{1-\dfrac{1}{\eta}}$ and $\Pi_1 = \dfrac{1}{\eta-1}y = \left(P_1 - 1\right)y$, we get

$$\gamma yL_f = \alpha\gamma(yL_f + \Pi_1 L_f - T) + (1 + r)(1 - \alpha)(yL_f + \Pi_1 L_f - T) + \gamma G - \gamma \Pi_1 L_f$$

This is rewritten as:

$$\gamma G - \gamma T + (1 + r)(1 - \alpha)(yL_f + \Pi_1 L_f - T) - \gamma\Pi_1 L_f = \gamma yL_f - \alpha\gamma(yL_f + \Pi_1 L_f - T) - \gamma T$$

Then,

$$\gamma(G - T) + r(1 - \alpha)(yL_f + \Pi_1 L_f - T) = \gamma(1 - \alpha)(yL_f + \Pi_1 L_f - T) - (1 - \alpha)(yL_f + \Pi_1 L_f - T) =$$
$$= (\gamma - 1)(1 - \alpha)(yL_f + \Pi_1 L_f - T) \tag{2}$$

(2) implies that if $\gamma > 1$, the budget deficit, including the interest payments to the government bonds, $\gamma(G - T) + r(1 - \alpha)(yL_f + \Pi_1 L_f - T)$, should be positive, that is, we need a budget deficit to maintain full employment under constant prices. We also find that the budget deficit equals the difference between the savings of the younger generation consumers and the savings of the older generation consumers. The savings of the younger generation consumers is

$$\gamma(1 - \alpha)(yL_f + \Pi_1 L_f - T)$$

and the savings of the older generation consumers is:

$$(1 - \alpha)(yL_f + \Pi_1 L_f - T)$$

Note that in (2) L_f and Π_1 are constant under full employment. Thus, (2) defines the budget deficit which is necessary and sufficient to realize full employment under constant prices given *T*. Summarizing the results, we obtain

Proposition 1. In order to achieve and maintain full employment under economic growth with constant prices, a budget deficit (*including* interest payments on government bonds) is necessary. The budget deficit equals the difference between the savings of the younger generation of consumers and the savings of the older generation of consumers.

A shortfall in the budget deficit (*including* interest payments) leads to a recession with involuntary unemployment. An excess budget deficit thus triggers inflation.

2.2. When the interest rate is higher than the growth rate under constant prices.

Let us consider a case where the interest rate is higher than the growth rate, that is, $r > \gamma - 1$ under constant prices. In (2) we find that if $r > \gamma - 1$, the budget deficit, excluding interest payments to government bonds, $\gamma(G - T)$, should be negative, that is, we need a budget surplus excluding interest payments to maintain full employment under constant prices. Under a balanced budget we have $r = \gamma - 1$, that is, the interest rate is equal to the growth rate. Thus, we have shown the following proposition.

Proposition 2. If the interest rate is greater than the growth rate, a budget surplus, excluding interest payments on government bonds, is needed to maintain full employment at constant prices. Under a balanced budget the interest rate is equal to the growth rate.

2.3. The debt to GDP ratio

Since we investigate the long run value of the debt to GDP ratio, we assume full employment. In this sub-section we consider a problem when the prices are constant. In Section 3.3 we examine a case of continuing inflation. First, consider the budget deficit in the first period of the world. If the world starts from this period, there is no older generation consumer. In this subsection we call the first period of the world, Period 1. Let $\hat{G}$ be the fiscal spending in Period 1. The total demand in Period 1 is:

$$\alpha\left(yL_f + \Pi_1 L_f - T\right) + \hat{G}$$

Π_1 is the profit in Period 1 which equals:

$$\Pi_1 = \left(\frac{1}{\eta - 1}\right)y = \left(P_1 - 1\right)y$$

The total supply is $P_1 yL_f$. Therefore, we have:

$$P_1 yL_f = \alpha\left(yL_f + \Pi_1 L_f - T\right) + \hat{G} = \Pi_1 L_f + yL_f$$

which means:

$$\hat{G} - T = (1-\alpha)\left(yL_f + \Pi_1 L_f - T\right)$$

Thus, the budget deficit in Period 1 equals the savings of the younger generation consumers.

From (2) the budget deficit, including interest payments to government bonds, in Period $n \geq 2$ is represented as:

$$\gamma^{n-1}(G - T) + r\gamma^{n-2}(1 - \alpha)(yL_f + \Pi_1 L_f - T) = \gamma^{n-2}(\gamma - 1)(1 - \alpha)(yL_f + \Pi_1 L_f - T)$$

Then, the accumulated budget deficit, or the government debt, from Period 1 to n is:

$$\sum_{k=2}^{n} \gamma^{k-2}(\gamma-1)(1-\alpha)\left(yL_f + \Pi_1 L_f - T\right) + (1-\alpha)\left(yL_f + \Pi_1 L_f - T\right) =$$

$$= \sum_{m=1}^{n-1} \gamma^{m-1}(\gamma-1)(1-\alpha)\left(yL_f + \Pi_2 L_f - T\right) + (1-\alpha)\left(yL_f + \Pi_2 L_f - T\right) =$$

$$= \frac{\left(\gamma^{n-1}-1\right)(\gamma-1)}{\gamma-1}(1-\alpha)\left(yL_f + \Pi_2 L_f - T\right) + (1-\alpha)\left(yL_f + \Pi_2 L_f - T\right) =$$

$$= \gamma^{n-1}(1-\alpha)\left(yL_f + \Pi_2 L_f - T\right)$$

Since the national income in Period n is $\gamma^{n-1} P_1 yL_f$, the debt to GDP ratio is

$$\frac{\gamma^{n-1}(1-\alpha)\left(yL_f + \Pi_2 L_f - T\right)}{\gamma^{n-1} P_1 yL_f} = \frac{(1-\alpha)\left(yL_f + \Pi_2 L_f - T\right)}{P_1 yL_f}$$

This is a finite number, and we obtain the following result.

Proposition 3. Under full employment with constant prices, the debt-GDP ratio converges to a finite value, or it stays a finite value through time.

The larger (smaller) the propensity to consume α is, the smaller (larger) the budget deficit required to achieve full employment under a constant rate of price increase is.

In this case, the budget deficit is kept at a level that does not cause inflation.

3. Continuing inflation by an excess budget deficit and the debt to GDP ratio

3.1. Continuing inflation by an excess budget deficit

In this section we examine inflation by the excess budget deficit and the debt to GDP ratio under continuing inflation. We assume full employment also in this section. In this case, the budget deficit is not kept at a level that does not cause inflation, and inflation is triggered by an excess budget deficit. However, if the real growth rate is replaced by the nominal growth rate, the argument in the previous section holds almost unchanged. Let $\xi P_1 = \frac{\xi}{1-\frac{1}{\eta}}$ be the price of the goods in Period 2, the nominal wage rate be $\gamma\xi y$, the fiscal spending be ζG. Assume that the excess budget deficit and inflation in Period 2 were predicted by the older generation consumers when they were young (in Period 1), and the younger generation consumers think that the price in Period 3 is ξ times the price in Period 2, and so on. Let $w = y$ in Period 1. The profit return for a consumer is

$$\xi\Pi_2 = \gamma\xi\Pi_1 = \left(\frac{1}{\eta-1}\right)\gamma\xi y$$

We assume that the nominal value of the tax is $\gamma\xi T$. The nominal total supply in Period 2 is:

$$\gamma\xi P_1 y L_f$$

The savings of the older generation consumers is:

$$(1-\alpha)(yL_f + \Pi_1 L_f - T)$$

Their consumption is:

$$(1+r)\,(1-\alpha)(yL_f + \Pi_1 L_f - T)$$

The consumption of the younger generation consumers is:

$$\alpha\gamma\xi(yL_f + \Pi_1 L_f - T)$$

$\gamma\xi - 1$ is the nominal growth rate. Their savings is:

$$(1-\alpha)\gamma\xi(yL_f + \Pi_1 L_f - T)$$

The nominal total demand is:

$$\alpha\gamma\xi(yL_f + \Pi_1 L_f - T) + (1+r)(1-\alpha)(yL_f + \Pi_1 L_f - T) + \zeta G$$

From the equilibrium between the total supply and the total demand, we obtain:

$$\gamma\xi P_1 yL_f = \alpha\gamma\xi(yL_f + \Pi_1 L_f - T) + (1 + r)(1 - \alpha)(yL_f + \Pi_1 L_f - T) + \zeta G$$

From this:

$$\gamma\xi yL_f = \alpha\gamma\xi(yL_f + \Pi_1 L_f - T) + (1 + r)(1 - \alpha)(yL_f + \Pi_1 L_f - T) + \zeta G - \xi(P_1 - 1)\,\gamma yL_f$$

Since $\Pi_1 = (P_1 - 1)y$, we get:

$$\gamma\xi yL_f = \alpha\gamma\xi(yL_f + \Pi_1 L_f - T) + (1 + r)(1 - \alpha)(yL_f + \Pi_1 L_f - T) + \zeta G - \gamma\xi\Pi_1 L_f$$

This is rewritten as:

$$\zeta G - \gamma\xi T + (1 + r)(1 - \alpha)(yL_f + \Pi_1 L_f - T) = \gamma\xi yL_f - \alpha\gamma\xi(yL_f + \Pi_1 L_f - T) + \gamma\xi\Pi_1 L_f - \gamma\xi T$$

Therefore,

$$\begin{aligned} &\zeta G - \gamma\xi T + r(1-\alpha)\left(yL_f + \Pi_1 L_f - T\right) = \\ &= (1-\alpha)\gamma\xi\left(yL_f + \Pi_1 L_f - T\right) - (1-\alpha)\left(yL_f + \Pi_1 L_f - T\right) = \\ &= (\gamma\xi - 1)(1-\alpha)\left(yL_f + \Pi_1 L_f - T\right) \end{aligned} \tag{3}$$

This is the budget deficit including interest payments to the government bonds. Comparing (3) with (2),

$$(\zeta G - \gamma\xi T) - \gamma(G - T) = \gamma(\xi - 1)(1 - \alpha)\,)(yL_f + \Pi_1 L_f - T)$$

When $(\zeta G - \gamma\xi T) - \gamma(G - T) > 0$, we have $\xi > 1$. This means inflation in Period 2. Summarizing the results in the following proposition.

Proposition 4. A budget deficit that exceeds the level necessary and sufficient to maintain full employment at constant prices will cause inflation[1].

(3) means that even in this case of inflation the budget deficit, including interest payments to the government bonds, equals the difference between the savings of the younger generation consumers and that of the older generation consumers. The savings of the younger generation consumers is

$$\gamma\xi(1 - \alpha)(yL_f + \Pi_1 L_f - T)$$

and the savings of the older generation consumers is:

$$(1 - \alpha)(yL_f + \Pi_1 L_f - T)$$

[1] We assume that the tax increases at the nominal growth rate. We can assume that the tax increases only at the rate of real growth rate, or the budget deficit increases due to tax cuts. However, in these cases, the effects may be complicated.

3.2. When the interest rate is higher than the nominal growth rate under inflation.

In (3) we find that if $r > \gamma\xi - 1$, the budget deficit, excluding interest payments to the government bonds, should be negative, that is, if the interest rate is higher than the nominal growth rate, we need a budget surplus excluding interest payments to maintain full employment under continuing inflation at the rate of $\xi - 1$. If the budget, excluding interest payments, is balanced, that is, $\zeta G - \gamma\xi T = 0$, for (3) to be satisfied we need

$$r = \gamma\xi - 1$$

Therefore, the interest rate must equal the nominal growth rate to maintain full employment under a balanced budget with continuing inflation at the rate of $\xi - 1$. We have shown the following proposition.

Proposition 5. When the interest rate is greater than the nominal growth rate, a budget surplus, excluding interest payments on government bonds, is necessary to maintain full employment while inflation continues. To maintain full employment under a balanced budget with continued inflation, the interest rate must equal the nominal growth rate.

In the case where the interest rate is higher than the real growth rate, a budget surplus (excluding interest payments) is necessary to achieve full employment under constant prices, but if the budget runs a deficit, inflation may occur, leading to a case where the interest rate is lower than the nominal growth rate. The inflation rate should be a little above the difference between the interest rate on government bonds and the real growth rate so that the interest rate is smaller than the nominal growth rate. Therefore, we have only mild inflation.

3.3. The debt to GDP ratio with continuing inflation

Like Section 3.3, the budget deficit in the first period of the world (Period 1) is:

$$\hat{G} - T = (1-\alpha)\left(yL_f + \Pi_1 L_f - T\right)$$

From (3) the budget deficit, including interest payments to the government bonds, in Period $n \geq 2$ is represented as:

$$(\gamma\xi)^{n-2}\left[\zeta G - \gamma\xi T + r(1-\alpha)\left(yL_f + \Pi_1 L_f - T\right)\right] = (\gamma\xi)^{n-2}(\gamma\xi - 1)(1-\alpha)\left(yL_f + \Pi_1 L_f - T\right)$$

Then, the accumulated budget deficit, or the government debt, from Period 1 to n is:

$$\sum_{k=2}^{n}(\gamma\xi)^{k-2}(\gamma\xi - 1)(1-\alpha)\left(yL_f + \Pi_1 L_f - T\right) + (1-\alpha)\left(yL_f + \Pi_1 L_f - T\right) =$$

$$= \sum_{m=1}^{n-1} (\gamma\xi)^{m-1} (\gamma\xi - 1)(1-\alpha)\left(yL_f + \Pi_2 L_f - T\right) + (1-\alpha)\left(yL_f + \Pi_2 L_f - T\right) =$$

$$= \frac{[\left(\gamma\xi\right)^{n-1} - 1\big](\gamma\xi - 1)}{\gamma\xi - 1}(1-\alpha)\left(yL_f + \Pi_2 L_f - T\right) + (1-\alpha)\left(yL_f + \Pi_2 L_f - T\right) =$$

$$= (\gamma\xi)^{n-1}(1-\alpha)\left(yL_f + \Pi_2 L_f - T\right)$$

Since the national income in Period n is $(\gamma\xi)^{n-1} P_1 L_f$, the debt to GDP ratio is

$$\frac{\left(\gamma\xi\right)^{n-1}(1-\alpha)\left(yL_f + \Pi_2 L_f - T\right)}{(\gamma\xi)^{n-1} P_1 y L_f} = \frac{(1-\alpha)\left(yL_f + \Pi_2 L_f - T\right)}{P_1 y L_f}$$

This is a finite number, and we find that we proved the following proposition.

Proposition 6. Under full employment followed by inflation, the debt-GDP ratio converges to a finite value, or it stays a finite value through time.

Proposition 6 is almost identical to Proposition 3. Therefore, fiscal instability and unsustainability do not arise whether inflation occurs or not. Even if a period consists of several years or even decades, the debt-GDP ratio is still a finite value.

Concluding Remarks

The objective of this paper was to theoretically prove that the increase in the government debt-GDP ratio is not a problem to be worried about under both stable prices and continuous inflation. Regardless of the relationship between government bond interest and the growth rate, the government debt-GDP ratio converges to a finite value, so no so-called fiscal collapse will occur. The main mechanism is that when interest on government debt is high, it increases consumption, so the budget deficit required to achieve full employment at constant prices becomes smaller. Alternatively, for the same budget deficit, the government debt-GDP ratio does not increase because of a higher rate of inflation. These conclusions seem to be consistent with the ideas of the Functional Finance Theory (Lerner, 1943, 1944) and MMT (Modern Monetary Theory, Kelton (2020), Mitchell, Wray, and Watts (2019), Wray (2015).[2]

When TV stations interview citizens about social security and public works projects such as railroads, they are met with responses such as, “I’m worried about the burden on my children and grandchildren”. That is not true. Social security is about income redistribution among those

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[2] Mochizuki (2020), Morinaga (2020), Park (2020) are references of MMT in Japan.

who are alive at that time, and the burden of public works projects is the inability to carry out other projects at that time, which basically has nothing to do with future generations. The burden on future generations is merely an accounting issue and is a meaningless argument from an economic standpoint.

This study focused exclusively on fiscal policy and did not address the issue of monetary policy. This may be one of the limitations of this paper. The model in this paper assumes that production is based only on labor, but even if a model including capital is used, monetary policy may be effective for a very short-term adjustment, but in the long run, monetary policy may not have an important impact on economic growth, and the role of fiscal policy is more important.

In future research, we would like to study the problem of government debt by using a discrete-time or continuous-time dynamic model that assumes people live indefinitely, rather than the two-generation overlapping model used in this paper for consumers, and an endogenous growth model for firms in which the economy grows through productivity gains from investment.

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Conflict of interest

The author states that the paper is completely original. The author also declares no conflict of interest.

Appendix

The first order conditions for utility maximization for consumers are:

$$\alpha C_1^{\alpha-1} C_2^{1-\alpha} \left(\int_0^1 c_1(z)^{1-\frac{1}{\eta}} dz \right)^{\frac{1}{1-\frac{1}{\eta}}-1} c_1(z)^{-\frac{1}{\eta}} - \lambda p_1(z) = 0 \tag{A1}$$

and

$$(1-\alpha) C_1^{\alpha} C_2^{-\alpha} \left(\int_0^1 c_2(z)^{1-\frac{1}{\eta}} dz \right)^{\frac{1}{1-\frac{1}{\eta}}-1} c_2(z)^{-\frac{1}{\eta}} - \frac{1}{1+r} \lambda p_2(z) = 0 \tag{A2}$$

From them, we have:

$$\alpha C_1^{\alpha-1} C_2^{1-\alpha} \left(\int_0^1 c_1(z)^{1-\frac{1}{\eta}} dz \right)^{\frac{1}{1-\frac{1}{\eta}}-1} c_1(z)^{1-\frac{1}{\eta}} - \lambda p_1(z) c_1(z) = 0$$

and

$$(1-\alpha) C_1^{\alpha} C_2^{-\alpha} \left(\int_0^1 c_2(z)^{1-\frac{1}{\eta}} dz \right)^{\frac{1}{1-\frac{1}{\eta}}-1} c_2(z)^{1-\frac{1}{\eta}} - \frac{1}{1+r} \lambda p_2(z) c_2(z) = 0$$

They mean:

$$\alpha C_1^{\alpha} C_2{}^{\alpha} - \lambda \int p_1(z) c_1(z) dz =$$

$$(1-\alpha) C_1^{\alpha} C_2^{1-\alpha} - \frac{1}{1+r} \lambda \int_0^1 p_2(z) c_2(z) dz = 0$$

Therefore, we have:

$$\int_0^1 p_1(z) c_1(z) dz = \frac{\alpha}{(1-\alpha)(1+r)} \int_0^1 p_2(z) c_2(z) dz$$

$$\int_0^1 p_1(z) c_1(z) dz = \frac{\alpha}{\lambda} C_1^{\alpha} C_2^{1-\alpha} = \alpha DI \tag{A3}$$

and

$$\int_0^1 p_2(z) c_2(z) dz = \frac{(1-\alpha)(1+r)}{\lambda} C_1^{\alpha} C_2^{1-\alpha} = (1-\alpha)(1+r) DI \tag{A4}$$

On the other hand, from (A1) and (A2)

$$\alpha^{1-\eta}\left(C_1^{\alpha-1}C_2^{1-\alpha}\right)^{1-\eta}\left(\int_0^1 c_1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c_1(z)^{1-\frac{1}{\eta}}-\lambda^{1-\eta}p_1(z)^{1-\eta}=0$$

$$(1-\alpha)^{1-\eta}\left(C_1^{\alpha}C_2^{-\alpha}\right)^{1-\eta}\left(\int_0^1 c_2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c_2(z)^{1-\frac{1}{\eta}}-\left(\frac{1}{1+r}\right)^{1-\eta}\lambda^{1-\eta}p_2(z)^{1-\eta}=0$$

Then,

$$\alpha^{1-\eta}\left(C_1^{\alpha-1}C_2^{1-\alpha}\right)^{1-\eta}\left(\int_0^1 c_1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c_1(z)^{1-\frac{1}{\eta}}dz-\lambda^{1-\eta}\int_0^1 p_1(z)^{1-\eta}dz=0$$

$$(1-\alpha)^{1-\eta}\left(C_1^{\alpha}C_2^{-\alpha}\right)^{1-\eta}\left(\int_0^1 c_2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c_2(z)^{1-\frac{1}{\eta}}dz-\left(\frac{1}{1+r}\right)^{1-\eta}\lambda^{1-\eta}\int_0^1 p_2(z)^{1-\eta}dz=0$$

They mean:

$$\alpha C_1^{\alpha-1}C_2^{1-\alpha}-\lambda\left(\int_0^1 p_1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}=0$$

$$\left(1-\alpha\right)C_1^{\alpha}C_2^{-\alpha}-\frac{1}{1+r}\lambda\left(\int_0^1 p_2(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}=0$$

Therefore, we get:

$$\alpha C_1^{\alpha}C_2^{1-\alpha}-\lambda P_1C_1=0,\ \left(1-\alpha\right)C_1^{\alpha}C_2^{1-\alpha}-\frac{1}{1+r}\lambda P_2C_2=0$$

By (A3) and (A4)

$$P_1C_1=\frac{\alpha}{\lambda}C_1^{\alpha}C_2^{1-\alpha}=\alpha DI,\ P_2C_2=\left(1-\alpha\right)\frac{1+r}{\lambda}C_1^{\alpha}C_2^{1-\alpha}=\left(1-\alpha\right)\left(1+r\right)DI$$

From them and (A1), (A2)

$$P_1C_1\left(\int_0^1 c_1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c_1(z)^{-\frac{1}{\eta}}=P_1C_1^{\frac{1}{\eta}}c_1(z)^{-\frac{1}{\eta}}=\left(P_1C_1\right)^{\frac{1}{\eta}}c_1(z)^{-\frac{1}{\eta}}P_1^{1-\frac{1}{\eta}}=p_1\left(z\right)$$

$$\frac{1}{1+r}P_2C_2\left(\int_0^1 c_2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c_2(z)^{-\frac{1}{\eta}}=\frac{1}{1+r}P_2C_2^{\frac{1}{\eta}}c_2(z)^{-\frac{1}{\eta}}=$$

$$=\frac{1}{1+r}\left(P_2C_2\right)^{\frac{1}{\eta}}c_2(z)^{-\frac{1}{\eta}}P_2^{1-\frac{1}{\eta}}=\frac{1}{1+r}p_2\left(z\right)$$

They mean:

$$c_1(z) = \frac{\alpha DI}{P_1}\left(\frac{P_1}{p_1(z)}\right)^{\eta}$$

and

$$c_2(z) = \frac{(1-\alpha)(1+r)DI}{P_2}\left(\frac{P_2}{p_2(z)}\right)^{\eta}$$

They are the demand functions for good z of a consumer in his younger and older periods.

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