

MEASURE THE EFFICIENCY OF COMMERCIAL BANKERS – A NEW APPROACH

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Abstract: Evaluating the performance of banks is important to governments and bankers, but it is also important to investors. Usually, retail investors judge the strength of a bank by its return on capital ratio. However, that ratio does not fully assess how efficiently the inputs are used to produce the outputs. In this paper, we examine different methods to evaluate the efficiency of Vietnam's commercial banks. Three problems A, B, and C are proposed to measure efficiency. A Decision-Making Unit that is A-efficient is not necessarily B-efficient; similarly, a Decision-Making Unit that is B-efficient may not necessarily be C-efficient. Conversely, if a Decision-Making Unit is C-efficient, then it is certainly B-efficient and A-efficient. We have proven the existence of solutions to the problems and run experiments for 26 commercial banks in Vietnam. The obtained results prove the theory's correctness and measure banks' relative efficiency.

Keywords: Efficiency, Linear Programming, Data Envelopment Analysis (DEA), Commercial Banks, Performance.

JEL Classification: C61, C63, C67.

1 Introduction

The banking system is the lifeblood of the economy. That is enough to show the important role of banks for a country clearly. The economy is only strong when there is an efficient and transparent banking system. More than 30 years since Vietnam implemented economic reforms, the banking system has contributed significantly to that development and innovation. To get the current remarkable results, banks must overcome many difficulties to catch up with the general development trend of banking activities in the world and face challenges.

To assess how effectively Vietnam's commercial banking system operates, how it is affected when the environment changes, and what it actually contributes to developing a market economy become an urgent problem to be solved. In the past three decades, although there has been a remarkable development in both quality and quantity, according to the assessment of regional and international organizations such as ADB, WB, or IMF and famous credit rating organizations, for example, Fitch Rating or Moody's, the performance of Vietnamese banks is still weak. In recent years, the operation of banks has always faced many difficulties.

Some banks with too many bad debts have been arranged to merge with other banks under the direction of the State Bank. Compared with the time before 2007, the operating environment of banks is more complicated. Along with the rapid development in the past time in both the scale of operation and the ownership structure, accordingly, the participation of shareholders who are foreign financial institutions is increasingly expanding.

Metrics such as return on assets (ROA) or return on equity (ROE) are often used to assess bank efficiency. However, these financial ratios provide only a unidirectional and incomplete picture, and they can yield conflicting results depending on context and application (Berger and Humphrey, 1997; Avkiran, 1999).

Comparative financial ratios are frequently studied as part of efficiency or productivity studies in the banking industry. There is a collection of coefficients that are often utilized, and each one of them deals with a certain area of banking. Due to the fact that the banking sector employs several inputs to generate a variety of outputs, suitable pooling has been studied (Kim, 1986). The average practice cost functions have been estimated in several research. These methods were

successful in identifying average performance improvements but failed to take best practice banks' productivity into consideration. Other ways that incorporate numerous inputs/outputs and take into consideration the relative performance of banks have been developed as a result of the issues with the "traditional" approach to productivity. Frontier analysis is used in a number of studies to distinguish between successful and unsuccessful banking units using a predetermined set of criteria. These studies concentrate on how the efficiency frontier shifts and how banks function close to it. The first approach comprises locating effective production units and creating a linear efficiency frontier utilizing these effective production units. Charnes, Cooper, and Rhodes (1978) utilized linear programming techniques to find the efficiency units and came up with the moniker "envelope analysis" as a result of the first use of this technique (DEA).

Numerous studies have been conducted in industrialized nations, such as the Berg, et al. (1991) research for Norwegian banks and research by Resti (1997) for Italian banks. For Japanese banks, Altunbus, et al. (1999) and Drake and Hall (2000) conducted research, Rebelo and Mendes (2000) on behalf of banks in Portugal, Gilbert and Wilson (1998) investigated banks in Korea, Leightner and Lovell (1998) conducted research for Thai banks, and Leong (2002) for banks in Singapore and emerging nations, and research by Hung (2008) and Minh, et al. for Vietnamese banks (2013). One study that uses data on Singapore banks from the years 1993 to 1999 to create efficiency scores and ratings for Singapore banks is Leong, et al (2002). They next put these ratings and scores to the test using five robustness criteria created by Bauer, et al. (1997). Their method enables academics to test out several models and choose the best model for policy-making. The authors of the paper "Banking Efficiency in the Nordic Countries: A Four-Country Malmquist Index Analysis" (1995) are interested in how competitive variables affect the success of businesses. Commercial banks and other institutions explore this topic in Nordic banks. Using the non-parametric DEA method, the research produced the following results: deposits from organizations that finance, lending to financial institutions, the number of branches, and customer guarantees. Input variables included the value of machinery and equipment, labor (calculated by the number of

working hours), and operating costs. The findings indicate that the biggest banks in Sweden and Denmark are the most efficient and are thus most likely to branch out into markets beyond the Nordic area. The average total factor productivity in Australian banks was 1.013, according to Sathy (2001), who used the Malmquist index to analyze the change in productivity in these institutions between 1995 and 1999.

David Grigorian and Vlad Manole (2002) estimated commercial bank performance indicators by applying a version of Data Envelope Analysis (DEA) to bank-level data from a range of countries. In addition to emphasizing the importance of a number of bank-specific variables, the censored Tobit analysis shows that (1) foreign ownership with controlling power and corporate restructuring improve efficiency of commercial banks, (2) the effect of prudential tightening on the performance of banks varies among different prudential norms, and (3) consolidation is likely to improve the efficiency of banking operations. Overall, the study results confirm the usefulness of DEA when used to evaluate banking systems in developing countries and shed light on the question of optimal architecture of the banking system. The positive effects of market capitalization and concentration on the DEA indexes suggest that banking sectors with a few large, well-capitalized banks are likely to generate better performance and higher ratios and higher market penetration. Using an integrated technique of data envelope modeling (DEA) and random edge analysis, Nakhun and Necmi K. Avkiran (2009) investigate the link between post-crisis banking restructuring, country-specific circumstances, and bank performance in Asian nations from 1997 to 2001. The article focuses on bank ownership-related restructuring procedures. The findings demonstrate that while domestic mergers produce banks that are more effective overall, restructuring does not result in more effective banking systems. The high interest rates, market concentration, and economic development that are country-specific factors are primarily to blame for the banking system's inefficiencies. Fethi and Pasiouras (2010) show in their survey that the majority of empirical studies on banking efficiency using the DEA framework focus on banking technical efficiency and to some extent, cost efficiency, and there is a research gap in the studies examining the profit/revenue effect with DEA. The reason behind

these is the lack of good quality of the output price. It can be clearly seen that the studies on evaluating the efficiency of the commercial banking system's operation are quite diverse, most of these studies use the non-parametric technical model DEA combined with some other methods for the most comprehensive assessment.

The main contribution of this paper is the development of new standards for evaluating the efficiency of commercial banks. Specifically, we propose three criteria for measuring efficiency, referred to as A-efficiency, B-efficiency, and C-efficiency. Each criterion is constructed through a mathematical optimization model, which will be detailed in the following section.

Explanation of Models A, B, and C: In this paper, we introduce three mathematical optimization models—Model A, Model B, and Model C—to evaluate the efficiency of commercial banks from different perspectives. These models extend the classical input-oriented Data Envelopment Analysis (DEA) model originally proposed by Charnes, Cooper, and Rhodes (1978). Each subsequent model introduces increasingly stringent constraints, offering a more refined classification of efficient decision-making units (DMUs):

- **Model A (A-efficiency):** This model is a modified version of the traditional DEA linear programming formulation. A DMU is considered A-efficient if it uses the minimum possible input to produce a given level of output, with no slack or inefficiency present. The objective is to minimize the input-output ratio, subject to standard DEA constraints.
- **Model B (B-efficiency):** This model imposes stricter conditions than Model A. In addition to the DEA constraints, it includes normalization conditions that restrict the weights assigned to inputs and outputs, thereby enabling sharper discrimination among DMUs. A B-efficient DMU must also satisfy the conditions for A-efficiency.
- **Model C (C-efficiency):** Model C adds another layer of constraints, representing the most rigorous efficiency condition among the three. It examines the stability and consistency of a DMU's efficiency over more complex feasible regions. A DMU that is C-efficient must necessarily be both B-efficient and A-efficient.

All models are formulated as linear programming problems, and the existence of optimal solutions is mathematically ensured. The progression from Model A to Model C allows for a more nuanced and hierarchical assessment of bank performance under varying assumptions and conditions.

2 Methodology

In this paper, we propose three mathematical optimization models—named A, B, and C—based on linear programming and an extension of DEA. These models define different criteria of efficiency, each capturing specific aspects of performance. A DMU that is efficient in model C is necessarily efficient in models A and B, but not vice versa.

2.1 Production efficiency

Measuring the level of absolute efficiency is not feasible due to the lack of a common production function to determine the maximum output for all banks in the same industry. Farrell in 1957 first introduced a solution to measure the relative efficiency of production with enterprises with similar characteristics in an industry. This method aims to determine the difference in production efficiency between banks compared with (several) best efficient banks in (same) industry. Farrell also divides production efficiency into two parts: technical efficiency and allocative efficiency. Technical efficiency looks at whether a bank can produce maximum output with existing technology. Allocative efficiency considers whether a bank that is at maximum technical efficiency can optimize input prices when their prices are given (Afriat, 1972; Banker and Maindiratta, 1988). Figure 1 shows a simple depiction of technical and allocative efficiency under the assumption of production banks with constant economies of scale.

In Figure 1, IAI' is the isoquant curve of a bank, with two inputs x_1 and x_2 . The line SS' represents the toll line. The author further hypothesizes that, in the normal case, banks have a concave production function for the two inputs x_1 and x_2 .

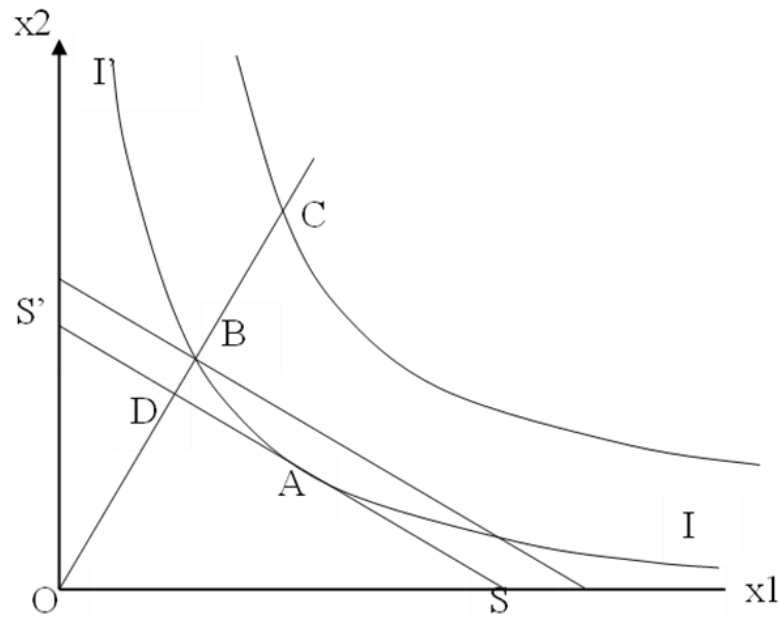


Figure 1. Technical and allocative efficiency
(Source: Afriat, 1972; Banker and Maindiratta, 1988)

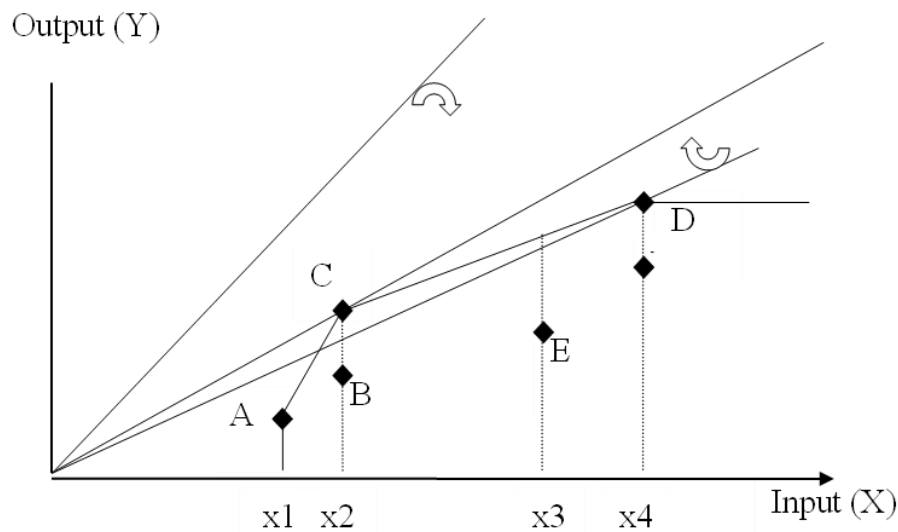


Figure 2. Technical efficiency
(Source: Authors' own research)

Micro theory using optimal producer behavior assumes that the bank will have a cost minimization objective and will be optimal at point A, where the rate of input substitution equals the relative price between two inputs. At point B, a bank can use the least amount of inputs to produce the same amount of output on the isoquant IAI'. However, if the bank has to use a combination of inputs at point C to get the same level of output as at point B, then it is inefficiently productive and needs to use more OC/OB times the number of

inputs relative to the bank at B. Other way, the producing bank at point B with the budget line SS' has not yet reached the optimal level of total input costs when input prices are known. The bank can produce at the same level of output at combination A with a cost at this point (equal to the cost at point D) and less than the cost at point B. Thus, allocative efficiency is measured by the ratio OD/OB .

A bank's level of productive efficiency is calculated as the product of its technical efficiency and its allocative

efficiency. In the case under consideration, production efficiency is expressed as the ratio:

$$\frac{OD}{OC} = \frac{OB}{OC} \cdot \frac{OD}{OB}$$

If a bank's production efficiency is equal to 1, it means that it has made the best use of production technology and combined inputs for optimal costs.

2.2 Technical Efficiency

Data Envelopment Analysis (DEA) is a non-parametric method used to assess the relative efficiency of decision-making units (DMUs) by comparing input-output ratios. Unlike regression-based approaches, DEA constructs an empirical efficiency frontier using linear programming, allowing for the evaluation of multiple inputs and outputs without requiring a specific functional form.

After giving an overview of production efficiency, the author focuses on technical efficiency. If a bank has a technical efficiency of one, it means that it is at its optimal (best) level of production relative to other banks in the industry. If a bank has a level of technical efficiency less than one, it means that the bank has not reached its maximum output level with current technology and given output. Technical efficiency is divided into net technical efficiency and economies of scale.

Figure 2 illustrates six points representing six banks with each combination of outputs and inputs at points A, B, C, D, E, and F. In this case, the production frontier is the curve ACD. Points A, C, and D lie on the production frontier, while points B, E, and F lie below this frontier. The line passing through the origin is used to represent the constant efficiency of the banks when the banks lie on this line (then increasing rates of input will lead to an increase in output at the same rate). The line passing through the origin is tangent to the production boundary at point C. The production bank at point C achieves maximum technical efficiency, which also means that it achieves both maximum net technical efficiency and a constant efficiency of scale. In general, points that are both on the production boundary and on the line passing through the origin will ensure maximum technical efficiency. However, a bank that does not achieve maximum technical efficiency may

achieve maximum net technical efficiency or maximum scale efficiency. Figure 2 shows that the points A and D are not on the line through the origin but on the production frontier, which means that the banks represented by points A and D are not at the scale of efficiency but achieve the maximum level of net technical efficiency. Points B and F guarantee maximum efficiency of scale because their inputs at x_2 and x_4 coincide with the inputs of banks C and D, which guarantee maximum net technical efficiency.

A score of E represents a bank that is not both economically efficient in terms of scale nor in terms of net technical efficiency because it is below the production frontier and has no inputs that coincide with the inputs of any bank that is efficient. maximum net technical result.

Subsequent studies in the aspect of relative efficiency, inheriting Farrell's thought, mainly focus on the production function estimation method (in some cases also known as production technology) and search forms of technology (Moorsteen, 1961; Afriat, 1972; Aigner, et al., 1977; Meeusen and Broeck, 1977; Greene, 1980; Kumbhakar, 1987). The production frontier is still widely used today, and a bank's production distance from this frontier is understood as the bank's own production inefficiency. The efficiency frontier can be fixed or it can be random, and the estimation method can be either parametric or non-parametric.

This relevant DEA (linear programming) paradigm will be briefly explained. For each DMU, we suppose that each bank has K inputs and M outputs. The inputs and outputs for the each DMU are represented by the vectors. We seek to determine the ratio of all outputs to all inputs for each bank (DMU), like $\frac{\sum_{k=1}^K u_{ik}y_{ik}}{\sum_{m=1}^M v_{im}x_{im}}$,

where u_i and v_i are weight vectors. To choose the optimal weights, the following problem is proposed:

$$\frac{\sum_{k=1}^K u_{ik}y_{ik}}{\sum_{m=1}^M v_{im}x_{im}}$$

with constraints

$$\begin{aligned} \frac{\sum_{k=1}^K u_{ik}y_{ik}}{\sum_{m=1}^M v_{im}x_{im}} &\leq 1 \\ u_{ik}, v_{im} &\geq 0 \\ i &= 1, 2, \dots, N \end{aligned}$$

There are infinitely many solutions with this model's representation, as is well known. You can prevent this by adding a constraint $\sum_{m=1}^M v_{im}x_{im} = 1$, and obtaining the multiplier form of the linear programming problem:

$$\sum_{k=1}^K u_{ik}y_{ik} \rightarrow \min$$

with constraints

$$\begin{aligned} \sum_{m=1}^M v_{im}x_{im} &= 1 \\ \sum_{k=1}^K u_{ik}y_{ik} - \sum_{m=1}^M v_{im}x_{im} &\leq 0 \\ u_{ik}, v_{im} &\geq 0 \\ i &= 1, 2, \dots, N \end{aligned}$$

Charnes, Cooper and Rhodes (1978) derive an equivalent envelope form from the dual property of this linear programming problem:

$$\min_{\theta, \lambda} \theta_i$$

with constraints

$$\begin{aligned} \sum_{j=1}^N \lambda_j y_{kj} &\geq y_{ki}, k = 1, 2, \dots, K \\ \sum_{j=1}^N \lambda_j x_{mj} &\leq \theta_i x_{mi}, m = 1, 2, \dots, M \\ \lambda_j &\geq 0 \\ i &= 1, 2, \dots, N \end{aligned}$$

2.3 Technical Efficiency Approach

The problems we consider here are quite similar to the problems DEA is considering, but we add additional variables so that the problem has a canonical form. In addition, we also consider different constraints to consider different levels of efficiency of banks. With different degrees of efficiency, a DMU may be effective in one measure but not at another.

We consider K decision-making units (DMUs) with an input matrix $X = (x_{ij}) \in R^{M \times K}$ (with M inputs) and an output matrix $Y = (y_{rj}) \in R^{N \times K}$ (with N outputs). Assume that the data matrices are positive, that is, $X > 0$ and $Y > 0$. To estimate the efficiency of the

$DMU(x_o, y_o)$, we consider the following model (called Model A), which is an improved model of the original DAE model.

Model A

$$\begin{aligned} r_A^* = \min_{l, s^-, s^+} r_A &= 1 - \frac{1}{M} \sum_{i=1}^M \frac{s_i^-}{x_{io}} \\ \text{subject to } \sum_{j=1}^K x_{ij} l_j + s_i^- &= x_{io} \\ \sum_{j=1}^K y_{rj} l_j - s_r^+ &= y_{ro} \\ l_j &\geq 0, s_i^- \geq 0, s_r^+ \geq 0 \end{aligned}$$

Here s^-, s^+ is the sub variable to give the problem (1) about the main form. From the objective function of the problem we see that $r_A^* \leq 1$. The following definition indicates when a DMU is called an effective DMU.

Definition 1. A $DMU(x_o, y_o)$ is an A-efficient if $r_A^* = 1$ và $s^- = s^+ = 0$.

Proof:

The constraint set can be rewritten as follows:

$$\begin{aligned} \sum_{j=1}^K \frac{l_j}{\frac{x_{io}}{x_{ij}}} + \frac{s_i^-}{x_{io}} &= 1 \\ \sum_{j=1}^K \frac{l_j}{\frac{y_{ro}}{y_{rj}}} - \frac{s_r^+}{x_{io}} &= 1 \\ l_j &\geq 0, s_i^- \geq 0, s_r^+ \geq 0 \end{aligned}$$

These are the polygons in space R^{K+M+N} with non-negative variables, so the set is bounded and the objective function is linear, so the problem always has a solution.

To measure the efficiency of DMUs, we consider the following model:

Model B

$$\begin{aligned} r_B^* = \min_{l, s^-, s^+} r_B &= q - \frac{1}{M} \sum_{i=1}^M \frac{s_i^-}{x_{io}} \\ \text{subject to } \sum_{j=1}^K x_{ij} l_j + s_i^- &= qx_{io} \\ \sum_{j=1}^K y_{rj} l_j - s_r^+ &= y_{ro} \\ l_j &\geq 0, s_i^- \geq 0, s_r^+ \geq 0 \end{aligned}$$

Definition 2. A $DMU(x_o, y_o)$ is B-efficient only if $r_B^* = q^* = 1$ and $s^- = s^+ = 0$.

Theorem 2. If $DMU(x_o, y_o)$ is B-efficient, then it is A-efficient.

Proof: If $DMU(x_o, y_o)$ is B-efficient it follows that

$$r_B^* = 1 \text{ and } s^- = s^+ = 0, \text{ then } r_A = 1 - \frac{1}{M} \sum_{i=1}^M \frac{s_i^-}{x_{io}} \text{ and}$$

$s^- = s^+ = 0$, and we have $r_A = r_A^* = 1$; therefore, $DMU(x_o, y_o)$ is A-efficient.

Theorem 3. $r_A^* \leq r_B^*$.

Proof: Let r_A^* is the minimum value of the objective function and the optimal solution is (s^-, s^+) it means that:

$$\begin{aligned} r_A^* &= 1 - \frac{1}{M} \sum_{i=1}^M \frac{s_i^-}{x_{io}} \\ \sum_{j=1}^K x_{ij} l_j + s_i^- &= x_{io} \\ \sum_{j=1}^K y_{rj} l_j - s_r^+ &= y_{ro} \\ l_j &\geq 0, s_i^- \geq 0, s_r^+ \geq 0 \end{aligned}$$

So $(q, s^-, s^+) = (1, s^-, s^+)$ is a solution of problem B

that $r_B^* = \min r_B = q - \frac{1}{M} \sum_{i=1}^M \frac{s_i^-}{x_{io}} \leq r_A^*$ bởi because r_A^* is a value of the objective function in problem B.

We now extend the constraints of problem B to form the following problem C:

Model C

$$\begin{aligned} r_C^* &= \min r_C = q - \frac{1}{M} \sum_{i=1}^M \frac{s_i^-}{x_{io}} \\ \text{subject to } \sum_{j=1, j \neq o}^K x_{ij} l_j + s_i^- &\leq q x_{io} \\ \sum_{j=1, j \neq o}^K y_{rj} l_j - s_r^+ &\leq y_{ro} \\ 0 &\leq q, l_j \leq 0, s_i^- \leq 0, s_r^+ \leq 0 \end{aligned} \quad (4)$$

Theorem 4. $r_B^* \leq r_C^*$.

Proof. Let r_B^* is the minimum value of the objective function and the optimal solution is (q^*, s^-, s^+) , that is, it satisfies the following constraints:

$$\begin{aligned} r_B^* &= q^* - \frac{1}{M} \sum_{i=1}^M \frac{s_i^-}{x_{io}} \\ \sum_{j=1}^K x_{ij} l_j + s_i^- &= q^* x_{io} \\ \sum_{j=1}^K y_{rj} l_j - s_r^+ &= y_{ro} \\ l_j &\geq 0, s_i^- \geq 0, s_r^+ \geq 0, q^* \geq 0 \end{aligned}$$

So $(q, s^-, s^+) = (q^*, s^-, s^+)$ is a solution of problem C

that $r_C^* = \min r_C = q - \frac{1}{M} \sum_{i=1}^M \frac{s_i^-}{x_{io}} \leq r_B^*$ because

r_A^* is a value of the objective function in problem C.

Comment: Through the above theorems, we see:

$1 \geq \rho_A^* \geq \rho_B^* \geq \rho_C^*$, so if a DMU is C-efficient, it will be A-efficient and B-efficient. With three models created, it is possible to evaluate different levels of effectiveness of DMUs

3 Experimental Study

We take the data of 26 joint-stock commercial banks in Vietnam over the period 2008–2019 with the input variables x1: customer deposits, x2: operating expenses, x3: Total assets, and x4: number of employees and output variables y1: profit after tax and y2: loans to customers. The results of running problems A, B, C for 26 Vietnamese commercial banks from 2008 to 2019 are given in the following tables.

Table 1. Results of running efficiency A, B, C for 26 banks from 2008 to 2019
(Source: Authors' own research)

-	-	2008			2009			2010		
Index	Bank name	A	B	C	A	B	C	A	B	C
1	Joint Stock Commercial Bank for Industry and Trade of Vietnam	0.98	0.96	0.79	0.99	0.99	0.93	0.99	0.97	0.92
2	Joint Stock Commercial Bank for Foreign Trade of Vietnam	1.00	1.00	0.97	0.97	0.90	0.84	1.00	1.00	0.86
3	Vietnam Technological and Commercial Joint Stock Bank	1.00	1.00	1.00	0.96	0.96	0.84	0.96	0.99	0.63
4	Joint Stock Commercial Bank for Investment and Development of Vietnam	1.00	1.00	1.00	0.99	0.99	0.88	1.00	1.00	1.00
5	Bank for Agriculture and Rural Development of Vietnam	1.00	1.00	1.00	0.99	0.99	0.92	1.00	1.00	1.00
6	Military Joint Stock Commercial Bank	1.00	1.00	0.99	1.00	1.00	1.00	1.00	0.99	0.74
7	Vietnam Prosperity Joint Stock Commercial Bank	0.87	0.85	0.64	0.78	0.60	0.60	0.99	0.99	0.78
8	Saigon Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	Saigon Thuong Tin Commercial Joint Stock Bank	0.92	0.77	0.67	0.94	0.83	0.82	0.95	0.86	0.86
10	Asia Commercial Joint Stock Bank	0.98	0.75	0.63	0.97	0.82	0.76	0.95	0.76	0.76
11	Vietnam Export-Import Joint Stock Commercial Bank	0.83	0.69	0.69	0.96	0.96	0.88	1.00	1.00	1.00
12	Saigon - Hanoi Commercial Joint Stock Bank	0.56	0.56	0.56	0.73	0.72	0.70	0.85	0.71	0.71
13	Vietnam Maritime Commercial Joint Stock Bank	0.95	0.95	0.70	1.00	1.00	0.99	0.94	0.97	0.77
14	Ho Chi Minh City Development Joint Stock Commercial Bank	1.00	1.00	1.00	0.75	0.75	0.75	0.90	0.72	0.56
15	Tien Phong Commercial Joint Stock Bank	0.71	0.00	0.00	0.83	0.46	0.34	0.99	0.96	0.42
16	Vietnam International Commercial Joint Stock Bank	0.83	0.61	0.61	0.88	0.59	0.59	0.91	0.72	0.72
17	Joint Stock Commercial Bank for Southeast Asia	0.99	0.99	0.70	0.99	0.99	0.91	0.99	0.80	0.79
18	Lien Viet Post Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.49
19	Orient Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	An Binh Joint Stock Commercial Bank	0.57	0.50	0.44	0.72	0.71	0.67	0.82	0.76	0.71
21	Viet A Commercial Joint Stock Bank	0.77	0.77	0.72	1.00	1.00	1.00	1.00	1.00	1.00
22	South Asia Commercial Joint Stock Bank	0.51	0.51	0.51	0.80	0.72	0.61	0.78	0.71	0.49
23	Saigon Industrial and Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	National Commercial Joint Stock Bank	0.52	0.52	0.46	0.77	0.77	0.77	0.70	0.70	0.70
25	Petrolimex Petroleum Joint Stock Commercial Bank	0.68	0.68	0.49	0.88	0.88	0.79	0.96	0.96	0.88
26	Viet Capital Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	0.78	0.76	0.60
-	Mean	0.87	0.81	0.75	0.92	0.87	0.83	0.94	0.90	0.78

-	-	2011			2012			2013		
Index	Bank name	A	B	C	A	B	C	A	B	C
1	Joint Stock Commercial Bank for Industry and Trade of Vietnam	0.99	0.90	0.90	1.00	1.00	1.00	1.00	1.00	1.00
2	Joint Stock Commercial Bank for Foreign Trade of Vietnam	1.00	1.00	0.93	1.00	1.00	1.00	1.00	1.00	1.00
3	Vietnam Technological and Commercial Joint Stock Bank	1.00	1.00	0.96	0.97	0.52	0.52	0.97	0.60	0.58
4	Joint Stock Commercial Bank for Investment and Development of Vietnam	0.99	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
5	Bank for Agriculture and Rural Development of Vietnam	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6	Military Joint Stock Commercial Bank	1.00	1.00	1.00	0.98	1.00	1.00	0.99	0.99	0.75
7	Vietnam Prosperity Joint Stock Commercial Bank	0.92	0.61	0.61	0.94	0.53	0.53	0.96	0.65	0.65
8	Saigon Commercial Joint Stock Bank	1.00	1.00	1.00	0.99	0.99	0.93	1.00	1.00	0.94
9	Saigon Thuong Tin Commercial Joint Stock Bank	0.97	0.86	0.81	0.97	0.83	0.83	1.00	1.00	1.00
10	Asia Commercial Joint Stock Bank	1.00	1.00	0.81	0.98	0.83	0.78	0.99	0.99	0.85
11	Vietnam Export-Import Joint Stock Commercial Bank	1.00	1.00	1.00	0.98	1.00	1.00	0.98	1.00	0.92
12	Saigon - Hanoi Commercial Joint Stock Bank	0.91	0.69	0.66	1.00	1.00	1.00	0.96	0.93	0.89
13	Vietnam Maritime Commercial Joint Stock Bank	1.00	1.00	0.72	0.99	0.38	0.37	0.99	0.90	0.90
14	Ho Chi Minh City Development Joint Stock Commercial Bank	0.82	0.61	0.57	0.97	0.65	0.64	0.94	0.90	0.87
15	Tien Phong Commercial Joint Stock Bank	0.97	0.93	0.29	0.68	0.47	0.46	1.00	1.00	1.00
16	Vietnam International Commercial Joint Stock Bank	0.94	0.67	0.67	0.94	0.72	0.70	0.93	0.72	0.64
17	Joint Stock Commercial Bank for Southeast Asia	1.00	1.00	0.76	1.00	0.98	0.36	1.00	1.00	0.77
18	Lien Viet Post Commercial Joint Stock Bank	0.99	0.99	0.72	0.99	0.99	0.82	0.99	0.98	0.68
19	Orient Commercial Joint Stock Bank	0.88	0.88	0.88	0.85	0.85	0.85	0.99	0.99	0.83
20	An Binh Joint Stock Commercial Bank	0.88	0.62	0.62	0.90	0.51	0.51	0.90	0.49	0.48
21	Viet A Commercial Joint Stock Bank	1.00	1.00	1.00	0.99	0.99	0.94	0.89	0.84	0.83
22	South Asia Commercial Joint Stock Bank	0.70	0.64	0.62	0.67	0.61	0.61	0.78	0.73	0.63
23	Saigon Industrial and Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
24	National Commercial Joint Stock Bank	0.98	0.98	0.76	0.83	0.80	0.74	0.83	0.49	0.48
25	Petrolimex Petroleum Joint Stock Commercial Bank	1.00	1.00	1.00	1.00	1.00	1.00	0.79	0.72	0.68
26	Viet Capital Commercial Joint Stock Bank	0.98	0.98	0.79	0.77	0.63	0.62	0.74	0.60	0.55
-	Mean	0.96	0.90	0.81	0.94	0.82	0.78	0.95	0.87	0.81

-	-	2014			2015			2016		
In- dex	Bank name	A	B	C	A	B	C	A	B	C
1	Joint Stock Commercial Bank for Industry and Trade of Vietnam	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	Joint Stock Commercial Bank for Foreign Trade of Vietnam	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	Vietnam Technological and Commercial Joint Stock Bank	0.97	0.62	0.62	0.97	0.88	0.84	1.00	1.00	1.00
4	Joint Stock Commercial Bank for Investment and Development of Vietnam	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	Bank for Agriculture and Rural Development of Vietnam	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6	Military Joint Stock Commercial Bank	1.00	1.00	0.86	1.00	1.00	1.00	1.00	1.00	0.96
7	Vietnam Prosperity Joint Stock Commercial Bank	0.97	0.92	0.71	1.00	1.00	1.00	1.00	1.00	1.00
8	Saigon Commercial Joint Stock Bank	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	Saigon Thuong Tin Commercial Joint Stock Bank	0.99	1.00	1.00	1.00	1.00	0.87	1.00	1.00	0.81
10	Asia Commercial Joint Stock Bank	0.99	0.96	0.88	0.99	0.99	0.91	1.00	1.00	0.95
11	Vietnam Export-Import Joint Stock Commercial Bank	0.95	0.86	0.81	0.95	0.95	0.91	0.95	0.90	0.88
12	Saigon - Hanoi Commercial Joint Stock Bank	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	Vietnam Maritime Commercial Joint Stock Bank	0.99	0.33	0.31	0.99	0.34	0.33	0.94	0.60	0.54
14	Ho Chi Minh City Development Joint Stock Commercial Bank	0.94	0.59	0.57	0.96	0.76	0.72	0.97	0.77	0.75
15	Tien Phong Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.77
16	Vietnam International Commercial Joint Stock Bank	0.94	0.77	0.68	0.94	0.81	0.75	0.95	0.94	0.89
17	Joint Stock Commercial Bank for Southeast Asia	1.00	0.89	0.67	0.99	0.79	0.70	0.91	0.82	0.78
18	Lien Viet Post Commercial Joint Stock Bank	0.99	0.64	0.59	0.93	0.69	0.68	0.99	0.91	0.84
19	Orient Commercial Joint Stock Bank	0.84	0.73	0.71	0.87	0.70	0.69	0.90	0.78	0.78
20	An Binh Joint Stock Commercial Bank	0.91	0.47	0.47	0.91	0.60	0.59	0.91	0.69	0.67
21	Viet A Commercial Joint Stock Bank	0.99	0.99	0.68	1.00	1.00	1.00	1.00	1.00	0.93
22	South Asia Commercial Joint Stock Bank	0.85	0.84	0.69	0.82	0.67	0.66	0.87	0.64	0.64
23	Saigon Industrial and Commercial Joint Stock Bank	1.00	1.00	1.00	0.72	0.66	0.66	0.75	0.71	0.69
24	National Commercial Joint Stock Bank	0.83	0.53	0.52	0.99	0.53	0.50	0.99	0.90	0.86
25	Petrolimex Petroleum Joint Stock Commercial Bank	0.78	0.63	0.63	0.78	0.72	0.70	0.75	0.75	0.75
26	Viet Capital Commercial Joint Stock Bank	0.79	0.69	0.65	0.76	0.60	0.60	0.83	0.76	0.73
-	Mean	0.95	0.83	0.77	0.95	0.83	0.81	0.95	0.89	0.85

-	-	2017			2018			2019		
Index	Bank name	A	B	C	A	B	C	A	B	C
1	Joint Stock Commercial Bank for Industry and Trade of Vietnam	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	Joint Stock Commercial Bank for Foreign Trade of Vietnam	1.00	1.00	0.85	1.00	1.00	1.00	1.00	1.00	1.00
3	Vietnam Technological and Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
4	Joint Stock Commercial Bank for Investment and Development of Vietnam	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	Bank for Agriculture and Rural Development of Vietnam	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6	Military Joint Stock Commercial Bank	0.98	0.84	0.84	0.99	0.87	0.87	0.99	0.91	0.90
7	Vietnam Prosperity Joint Stock Commercial Bank	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
8	Saigon Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
9	Saigon Thuong Tin Commercial Joint Stock Bank	0.99	0.80	0.79	0.99	0.98	0.80	1.00	1.00	0.86
10	Asia Commercial Joint Stock Bank	1.00	1.00	0.94	1.00	1.00	1.00	1.00	1.00	0.98
11	Vietnam Export-Import Joint Stock Commercial Bank	0.95	0.92	0.89	0.96	0.87	0.86	1.00	1.00	0.87
12	Saigon - Hanoi Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13	Vietnam Maritime Commercial Joint Stock Bank	0.99	0.99	0.73	1.00	1.00	0.60	1.00	1.00	0.67
14	Ho Chi Minh City Development Joint Stock Commercial Bank	0.97	0.77	0.77	0.98	0.87	0.84	1.00	1.00	1.00
15	Tien Phong Commercial Joint Stock Bank	0.95	0.80	0.78	0.97	0.84	0.82	1.00	1.00	0.93
16	Vietnam International Commercial Joint Stock Bank	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	Joint Stock Commercial Bank for Southeast Asia	0.92	0.83	0.81	1.00	1.00	0.89	1.00	1.00	0.92
18	Lien Viet Post Commercial Joint Stock Bank	0.96	0.83	0.83	0.97	0.89	0.88	0.97	0.97	0.92
19	Orient Commercial Joint Stock Bank	0.93	0.78	0.78	0.94	0.92	0.89	0.96	0.93	0.93
20	An Binh Joint Stock Commercial Bank	0.92	0.71	0.71	0.94	0.73	0.73	0.94	0.71	0.71
21	Viet A Commercial Joint Stock Bank	1.00	1.00	0.89	1.00	1.00	0.98	1.00	1.00	1.00
22	South Asia Commercial Joint Stock Bank	0.88	0.79	0.79	0.91	0.84	0.84	0.93	0.89	0.89
23	Saigon Industrial and Commercial Joint Stock Bank	0.74	0.66	0.66	0.77	0.68	0.68	0.78	0.66	0.66
24	National Commercial Joint Stock Bank	0.89	0.61	0.57	0.98	0.70	0.62	0.99	0.63	0.62
25	Petrolimex Petroleum Joint Stock Commercial Bank	0.87	0.87	0.86	0.80	0.78	0.78	0.98	0.98	0.82
26	Viet Capital Commercial Joint Stock Bank	0.84	0.78	0.73	0.85	0.71	0.71	0.88	0.85	0.79
-	Mean	0.95	0.88	0.85	0.96	0.91	0.88	0.98	0.94	0.90

According to the results, we see that Joint Stock Commercial Bank for Industry and Trade of Vietnam started to be effective from 2012 to the end of the research period in 2019 on all three measures A, B, and C. Meanwhile, Joint Stock Commercial Bank Vietnam's Foreign Trade section achieved A-efficiency

and B-efficiency right in 2008, and 2009 was not effective. 2010 and 2011 achieved A-efficiency and B-efficiency but not C-efficiency. From 2012 to 2019, VCB achieved efficiency on all three measures A, B, and C, except in 2017 when it did not achieve C-efficiency.

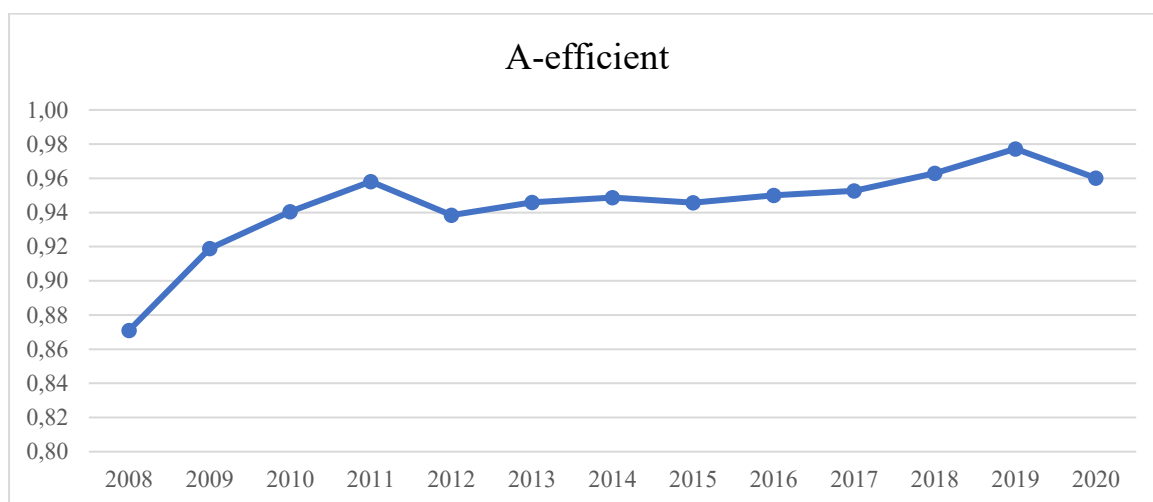


Figure 3. Average of A-efficiency of banks by year
(Source: Authors' own research)

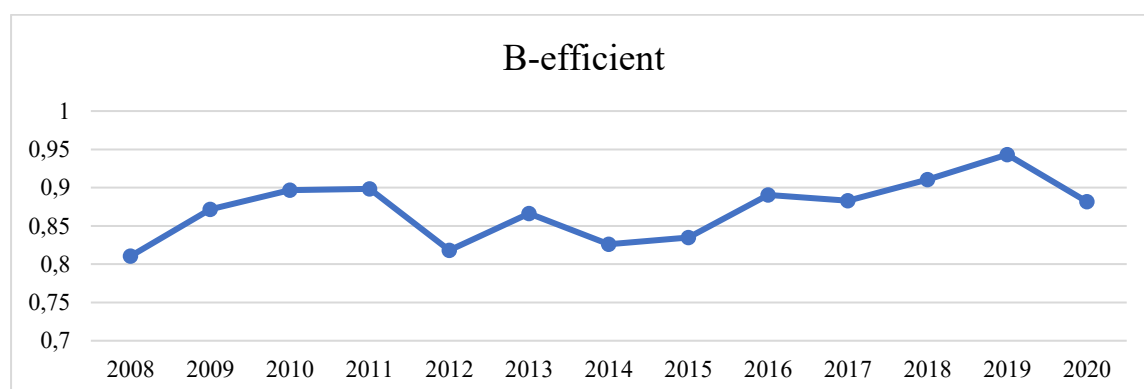


Figure 4. Average of B-efficiency of banks by year
(Source: Authors' own research)

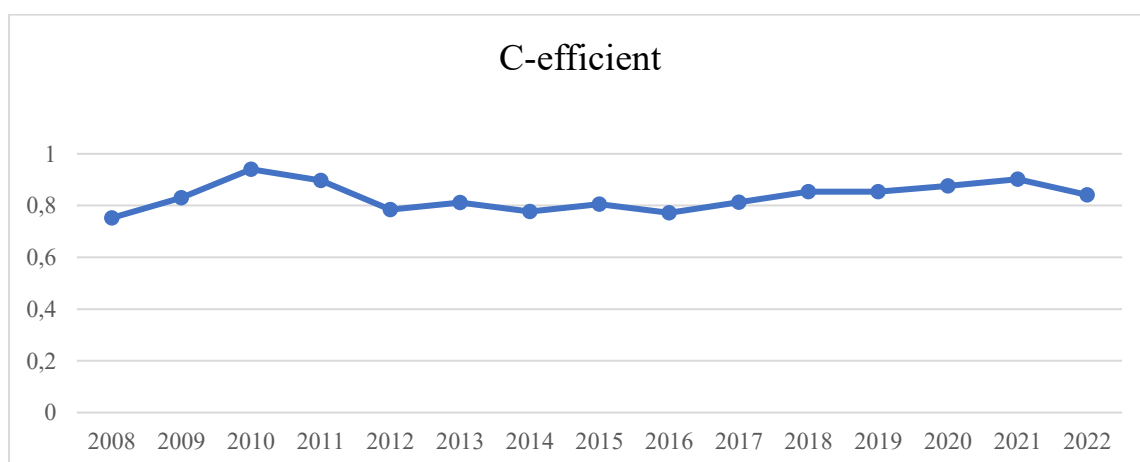


Figure 5. Average of C-efficiency of banks by year
(Source: Authors' own research)

Table 2. Rank commercial banks according to A-efficiency
(Source: Authors' own research)

Ranking A	Bank name	A	B	C
1	Joint Stock Commercial Bank for Investment and Development of Vietnam	0.998277	0.999337	0.988649
2	Bank for Agriculture and Rural Development of Vietnam	0.998029	0.999415	0.943496
3	Saigon Commercial Joint Stock Bank	0.996244	0.992549	0.983324
4	Joint Stock Commercial Bank for Foreign Trade of Vietnam	0.995866	0.991917	0.958274
5	Joint Stock Commercial Bank for Industry and Trade of Vietnam	0.995015	0.983591	0.962093
6	Military Joint Stock Commercial Bank	0.993642	0.968635	0.915633
7	Asia Commercial Joint Stock Bank	0.985293	0.915244	0.849693
8	Lien Viet Post Commercial Joint Stock Bank	0.983911	0.914495	0.803736
9	Vietnam Technological and Commercial Joint Stock Bank	0.982932	0.892264	0.843981
10	Joint Stock Commercial Bank for Southeast Asia	0.982697	0.9284	0.757848
11	Vietnam Maritime Commercial Joint Stock Bank	0.982044	0.786939	0.662337
12	Saigon Thuong Tin Commercial Joint Stock Bank	0.978398	0.916557	0.854431
13	Vietnam Export-Import Joint Stock Commercial Bank	0.962785	0.933981	0.895878
14	Vietnam Prosperity Joint Stock Commercial Bank	0.956497	0.858366	0.802339
15	Viet A Commercial Joint Stock Bank	0.954873	0.941508	0.895254
16	Vietnam International Commercial Joint Stock Bank	0.939183	0.808432	0.774638
17	Ho Chi Minh City Development Joint Stock Commercial Bank	0.937394	0.785937	0.758806
18	Orient Commercial Joint Stock Bank	0.930951	0.867593	0.850888
19	Tien Phong Commercial Joint Stock Bank	0.929272	0.803569	0.677132
20	Saigon - Hanoi Commercial Joint Stock Bank	0.918851	0.864685	0.857835
21	Saigon Industrial and Commercial Joint Stock Bank	0.892647	0.858865	0.857727
22	Petrolimex Petroleum Joint Stock Commercial Bank	0.866986	0.843114	0.797765
23	An Binh Joint Stock Commercial Bank	0.866659	0.640838	0.627537
24	National Commercial Joint Stock Bank	0.860827	0.674545	0.631195
25	Viet Capital Commercial Joint Stock Bank	0.853675	0.779614	0.733485
26	South Asia Commercial Joint Stock Bank	0.797864	0.712068	0.664535
	MEDIUM	0.943877	0.871633	0.821097

Table 3. Rank commercial banks according to B-efficiency
(Source: Authors' own research)

Ranking B	-	A	B	C
1	Bank for Agriculture and Rural Development of Vietnam	0.998029	0.999415	0.943496
2	Joint Stock Commercial Bank for Investment and Development of Vietnam	0.998277	0.999337	0.988649
3	Saigon Commercial Joint Stock Bank	0.996244	0.992549	0.983324
4	Joint Stock Commercial Bank for Foreign Trade of Vietnam	0.995866	0.991917	0.958274
5	Joint Stock Commercial Bank for Industry and Trade of Vietnam	0.995015	0.983591	0.962093
6	Military Joint Stock Commercial Bank	0.993642	0.968635	0.915633
7	Viet A Commercial Joint Stock Bank	0.954873	0.941508	0.895254
8	Vietnam Export-Import Joint Stock Commercial Bank	0.962785	0.933981	0.895878
9	Joint Stock Commercial Bank for Southeast Asia	0.982697	0.9284	0.757848
10	Saigon Thuong Tin Commercial Joint Stock Bank	0.978398	0.916557	0.854431
11	Asia Commercial Joint Stock Bank	0.985293	0.915244	0.849693
12	Lien Viet Post Commercial Joint Stock Bank	0.983911	0.914495	0.803736
13	Vietnam Technological and Commercial Joint Stock Bank	0.982932	0.892264	0.843981
14	Orient Commercial Joint Stock Bank	0.930951	0.867593	0.850888
15	Saigon - Hanoi Commercial Joint Stock Bank	0.918851	0.864685	0.857835
16	Saigon Industrial and Commercial Joint Stock Bank	0.892647	0.858865	0.857727
17	Vietnam Prosperity Joint Stock Commercial Bank	0.956497	0.858366	0.802339
18	Petrolimex Petroleum Joint Stock Commercial Bank	0.866986	0.843114	0.797765
19	Vietnam International Commercial Joint Stock Bank	0.939183	0.808432	0.774638
20	Tien Phong Commercial Joint Stock Bank	0.929272	0.803569	0.677132
21	Vietnam Maritime Commercial Joint Stock Bank	0.982044	0.786939	0.662337
22	Ho Chi Minh City Development Joint Stock Commercial Bank	0.937394	0.785937	0.758806
23	Viet Capital Commercial Joint Stock Bank	0.853675	0.779614	0.733485
24	South Asia Commercial Joint Stock Bank	0.797864	0.712068	0.664535
25	National Commercial Joint Stock Bank	0.860827	0.674545	0.631195
26	An Binh Joint Stock Commercial Bank	0.866659	0.640838	0.627537

Table 4. Rank commercial banks according to B-efficiency
(Source: Authors' own research)

Ranking C	-	A	B	C
1	Joint Stock Commercial Bank for Investment and Development of Vietnam	0.998277	0.999337	0.988649
2	Saigon Commercial Joint Stock Bank	0.996244	0.992549	0.983324
3	Joint Stock Commercial Bank for Industry and Trade of Vietnam	0.995015	0.983591	0.962093
4	Joint Stock Commercial Bank for Foreign Trade of Vietnam	0.995866	0.991917	0.958274
5	Bank for Agriculture and Rural Development of Vietnam	0.998029	0.999415	0.943496
6	Military Joint Stock Commercial Bank	0.993642	0.968635	0.915633
7	Vietnam Export-Import Joint Stock Commercial Bank	0.962785	0.933981	0.895878
8	Viet A Commercial Joint Stock Bank	0.954873	0.941508	0.895254
9	Saigon - Hanoi Commercial Joint Stock Bank	0.918851	0.864685	0.857835
10	Saigon Industrial and Commercial Joint Stock Bank	0.892647	0.858865	0.857727
11	Saigon Thuong Tin Commercial Joint Stock Bank	0.978398	0.916557	0.854431
12	Orient Commercial Joint Stock Bank	0.930951	0.867593	0.850888
13	Asia Commercial Joint Stock Bank	0.985293	0.915244	0.849693
14	Vietnam Technological and Commercial Joint Stock Bank	0.982932	0.892264	0.843981
15	Lien Viet Post Commercial Joint Stock Bank	0.983911	0.914495	0.803736
16	Vietnam Prosperity Joint Stock Commercial Bank	0.956497	0.858366	0.802339
17	Petrolimex Petroleum Joint Stock Commercial Bank	0.866986	0.843114	0.797765
18	Vietnam International Commercial Joint Stock Bank	0.939183	0.808432	0.774638
19	Ho Chi Minh City Development Joint Stock Commercial Bank	0.937394	0.785937	0.758806
20	Joint Stock Commercial Bank for Southeast Asia	0.982697	0.9284	0.757848
21	Viet Capital Commercial Joint Stock Bank	0.853675	0.779614	0.733485
22	Tien Phong Commercial Joint Stock Bank	0.929272	0.803569	0.677132
23	South Asia Commercial Joint Stock Bank	0.797864	0.712068	0.664535
24	Vietnam Maritime Commercial Joint Stock Bank	0.982044	0.786939	0.662337
25	National Commercial Joint Stock Bank	0.860827	0.674545	0.631195
26	An Binh Joint Stock Commercial Bank	0.866659	0.640838	0.627537

Vietnam Technological and Commercial Joint Stock Bank achieved efficiency at all three levels of A, B, and C in 2008, but the following years from 2009 to 2015, it was almost ineffective. However, starting from 2016 to the end of the research period, it is effective on all three measurements A, B, and C.

The Joint Stock Commercial Bank for Investment and Development of Vietnam (BIDV) and the Bank for Agriculture and Rural Development of Vietnam were effective in most of the study years except 2009.

Military Commercial Joint Stock Bank (MB) has been effective for many years, but for the last 3 years of 2017–2019, it has not been effective in most measures.

Saigon Commercial Joint Stock Bank for Industry and Trade was effective in the years 2008–2014, but in the period 2015–2019, it was not.

Vietnam Prosperity Joint Stock Commercial Bank has been effective in the years 2015–2019.

Saigon Commercial Joint Stock Bank is the only bank that achieved efficiency in all years. However, in 2012 and 2013, it did not achieve C-efficiency but achieved A-efficiency and B-efficiency.

Lien Viet Post Commercial Joint Stock Bank was effective in the first two years; Orient Commercial Joint Stock Bank was effective in the first three years, but there were no effective years in the following years.

An Binh Commercial Joint Stock Bank and National Commercial Joint Stock Bank did not have any effective years during the study period. On average, efficiency is lowest in 2008 and highest in 2019.

According to Figure 3, A-efficiency increased from 2008 to 2011 and then decreased in 2012 due to a series of banks with bad debts. It then increased slightly in the following years, reaching a peak average A-efficiency of 0.98 in 2019, and then decreased in 2020 due to the impact of the Covid-19 epidemic

B-efficiency is always lower than A-efficiency, and we see a year-to-year difference compared to A-efficiency.

C-efficiency (Fig. 5) peaked in 2010 and declined until 2016 began to recover. This result is quite consistent with the recovery of the Vietnamese economy. The recession started in 2010 and recovered from 2016. The year 2020 was effectively reduced due to the impact of the pandemic.

In Tables 2-4 there is a ranking of the banks' performance after averaged over the study period. Accordingly, Joint Stock Commercial Bank for Investment and Development of Vietnam ranked first in efficiency measures A and C and Bank for Agriculture and Rural Development of Vietnam ranked first in A- efficiency.

Nam A Commercial Joint Stock Bank, An Binh Commercial Joint Stock Bank, and Quoc Dan Joint Stock Commercial Bank were ranked last.

To compare with the ratings obtained from Model A, Model B, and Model C as above, we rerun the model of Charnes, Cooper, and Rhodes (1978) for the data of 23 commercial banks in 2008, 2009, 2019, and 2020. The result shows that the classification ability of the models proposed in this paper is much better than that of Charnes, Cooper, and Rhodes.

4 Discussion

The findings indicate that Vietnamese banks vary significantly in terms of efficiency across the three proposed metrics (A, B, and C). This highlights the complexity of banking performance, where conventional financial ratios such as ROA or ROE may fail to capture multidimensional inefficiencies. Compared to traditional DEA models (e.g., CCR model), our proposed models offer better classification and interpretation, confirming their robustness.

From a theoretical standpoint, the findings are consistent with the efficient frontier model, where banks closer to the frontier are deemed efficient. Furthermore, the notion of A-, B-, and C-efficiency enriches the understanding of production frontiers by introducing layered benchmarks. This contributes to financial intermediation theory by providing insight into how input–output allocations affect performance. Managerial implications include the following:

1. Banks with low efficiency scores should evaluate operational structures, particularly in areas like labor and asset utilization.
2. Regulators can use the models to monitor systemic risk or determine which banks require intervention.
3. Foreign investors may use efficiency scores to assess performance beyond profitability.

The study addresses the original question of how environmental and structural factors (e.g., reforms,

technology) impact bank efficiency by demonstrating that many banks improve performance over time, aligning with post-reform developments and digitalization.

5 Conclusion

This paper considers new metrics to determine the effectiveness of decision-making units. We have proved that the programming problems have solutions and compare the values of the solutions of the problems. We apply theoretical results and efficiency estimates to 26 Vietnamese commercial banks and compare their relative efficiency. The Joint Stock Commercial Bank for Investment and Development of Vietnam, the Bank for Agriculture and Rural Development of Vietnam, the Joint Stock Commercial Bank for Industry and Trade of Vietnam, the Joint Stock Commercial Bank for Foreign Trade of Vietnam. Nam and Saigon Commercial Joint Stock Bank have high rankings in the ranking of three efficiency measures. Nam A Commercial Joint Stock Bank, An Binh Commercial Joint Stock Bank, and National Commercial Joint Stock Bank have low rankings. These studies help managers, bankers, and stock investors gain an additional criterion for evaluating banks.

6 Acknowledgment

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8 Appendix

Appendix 1. Efficiency of 23 banks in 2008 was estimated by the model of Charnes, Cooper, and Rhodes (1978)

Bank Name	crste	Vrste	scale
(VPB) Joint Stock Commercial Bank for Foreign Trade of Vietnam	1	1	1
(BIDV). Joint Stock Commercial Bank for Investment and Development of Vietnam	1	1	1
(HDB). Ho Chi Minh City Development Joint Stock Commercial Bank	1	1	1
(TPB). Tien Phong Commercial Joint Stock Bank	1	1	1
(SEAB). Joint Stock Commercial Bank for Southeast Asia	1	1	1
(OCB). Orient Commercial Joint Stock Bank	1	1	1
(NAB). South Asia Commercial Joint Stock B	1	1	1
KLB). Kien Long Joint Stock Commercial Bank	1	1	1
(NVB). National Commercial Joint Stock Bank	1	1	1
(PGP). Petrolimex Petroleum Joint Stock Commercial Bank	1	1	1
(VIB). International Joint Stock Commercial Bank	0.97	0.97	1
(CTG). Joint Stock Commercial Bank for Industry and Trade of Vietnam	0.95	1	0.95
(SCB). Saigon Commercial Joint Stock Bank	0.84	1	0.84
(SGB). Commercial Joint Stock Bank Saigon Cong Thuong	0.84	1	0.84
(ABB). An Binh Joint Stock Commercial Bank	0.82	0.93	0.88
(MSB). Vietnam Maritime Commercial Joint Stock Bank	0.8	0.8	1
(VBP). Vietnam Prosperity Joint Stock Commercial Bank	0.69	1	0.69
(ACB). Asia Commercial Joint Stock Bank	0.65	1	0.65
(STB). Saigon Thuong Tin Commercial Joint Stock Bank	0.58	0.96	0.6
(EIB). Vietnam Export-Import Joint Stock Commercial Bank	0.58	0.71	0.82

(TCB). Vietnam Technological and Commercial Joint Stock Bank	0.54	0.84	0.64
(MBB). Military Joint Stock Commercial Bank	0.51	0.62	0.82
(SHB). Saigon – Hanoi Commercial Joint Stock Bank	0.5	0.62	0.82
Mean	0.84	0.93	0.89

Source: The author estimates from the data collected from the annual reports of banks.

Appendix 2. Efficiency of 23 banks in 2009 was estimated by the model of Charnes, Cooper, and Rhodes (1978)

Bank Name	crste	Vrste	scale
(BIDV). Joint Stock Commercial Bank for Investment and Development of Vietnam	1	1	1
(MBB). Military Joint Stock Commercial Bank	1	1	1
(MSB). Vietnam Maritime Commercial Joint Stock Bank	1	1	1
(VIB). International Joint Stock Commercial Bank	1	1	1
(OCB). Orient Commercial Joint Stock Bank	1	1	1
(NAB). South Asia Commercial Joint Stock Bank	1	1	1
(KLB). Kien Long Joint Stock Commercial Bank	1	1	1
(NVB). National Commercial Joint Stock Bank	1	1	1
(SGB). Commercial Joint Stock Bank Saigon Cong Thuong	0.97	1	0.97
(SEAB). Joint Stock Commercial Bank for Southeast Asia	0.94	1	0.94
(VBP). Vietnam Prosperity Joint Stock Commercial Bank	0.92	0.99	0.93
(VPB) Joint Stock Commercial Bank for Foreign Trade of Vietnam	0.9	1	0.9
(TPB). Tien Phong Commercial Joint Stock Bank	0.87	1	0.87
(TCB). Vietnam Technological and Commercial Joint Stock Bank	0.85	0.93	0.91
(PGP). Petrolimex Petroleum Joint Stock Commercial Bank	0.84	1	0.84
(SCB). Saigon Commercial Joint Stock Bank	0.82	0.88	0.93
(ACB). Asia Commercial Joint Stock Bank	0.81	1	0.81
(CTG). Joint Stock Commercial Bank for Industry and Trade of Vietnam	0.79	0.99	0.8
(EIB). Vietnam Export-Import Joint Stock Commercial Bank	0.79	0.82	0.96
(STB). Saigon Thuong Tin Commercial Joint Stock Bank	0.66	0.99	0.66
(HDB). Ho Chi Minh City Development Joint Stock Commercial Bank	0.6	0.67	0.89
(ABB). An Binh Joint Stock Commercial Bank	0.59	0.86	0.69
(SHB). Saigon – Hanoi Commercial Joint Stock Bank	0.5	0.66	0.77
Mean	0.86	0.95	0.91

Source: The author estimates from the data collected from the annual reports of banks

Appendix 3. Efficiency of 23 banks in 2019 estimated by the model of Charnes, Cooper, and Rhodes (1978)

Bank Name	crste	vrste	scale
(CTG). Joint Stock Commercial Bank for Industry and Trade of Vietnam	1	1	1
(BIDV). Joint Stock Commercial Bank for Investment and Development of Vietnam	1	1	1
(VBP). Vietnam Prosperity Joint Stock Commercial Bank	1	1	1
(SHB). Saigon – Hanoi Commercial Joint Stock Bank	1	1	1
(HDB). Ho Chi Minh City Development Joint Stock Commercial Bank	1	1	1
(VIB). International Joint Stock Commercial Bank	1	1	1
(NAB). South Asia Commercial Joint Stock Bank	1	1	1
(KLB). Kien Long Joint Stock Commercial Bank	1	1	1

(SGB). Commercial Joint Stock Bank Saigon Cong Thuong	1	1	1
(PGP). Petrolimex Petroleum Joint Stock Commercial Bank	1	1	1
(SEAB). Joint Stock Commercial Bank for Southeast Asia	0.94	0.94	0.999
(TPB). Tien Phong Commercial Joint Stock Bank	0.93	0.94	0.996
(OCB). Orient Commercial Joint Stock Bank	0.92	0.92	0.998
(SCB). Saigon Commercial Joint Stock Bank	0.89	0.91	0.97
(TCB). Vietnam Technological and Commercial Joint Stock Bank	0.85	0.85	0.996
(MSB). Vietnam Maritime Commercial Joint Stock Bank	0.84	0.88	0.954
(ACB). Asia Commercial Joint Stock Bank	0.8	0.81	0.989
(EIB). Vietnam Export-Import Joint Stock Commercial Bank	0.78	0.79	0.993
(MBB). Military Joint Stock Commercial Bank	0.77	0.8	0.961
(VPB) Joint Stock Commercial Bank for Foreign Trade of Vietnam	0.75	0.99	0.755
(ABB). An Binh Joint Stock Commercial Bank	0.74	0.84	0.883
(STB). Saigon Thuong Tin Commercial Joint Stock Bank	0.66	0.7	0.945
(NVB). National Commercial Joint Stock Bank	0.63	0.98	0.637
Mean	0.89	0.93	0.96

Source: The author estimates from the data collected from the annual reports of banks.

Appendix 4. Efficiency of 23 banks in 2020 was estimated by the model of Charnes, Cooper, and Rhodes (1978)

Bank Name	crste	vrste	scale
(BIDV). Joint Stock Commercial Bank for Investment and Development of Vietnam	1	1	1
(MBB). Military Joint Stock Commercial Bank	1	1	1
(VBP). Vietnam Prosperity Joint Stock Commercial Bank	1	1	1
(SHB). Saigon – Hanoi Commercial Joint Stock Bank	1	1	1
(MSB). Vietnam Maritime Commercial Joint Stock Bank	1	1	1
(VIB). International Joint Stock Commercial Bank	1	1	1
(SGB). Commercial Joint Stock Bank Saigon Cong Thuong	1	1	1
(PGP). Petrolimex Petroleum Joint Stock Commercial Bank	1	1	1
(CTG). Joint Stock Commercial Bank for Industry and Trade of Vietnam	0.99	1	0.987
(KLB). Kien Long Joint Stock Commercial Bank	0.91	1	0.911
(OCB). Orient Commercial Joint Stock Bank	0.86	1	0.86
(TPB). Tien Phong Commercial Joint Stock Bank	0.84	1	0.841
(HDB). Ho Chi Minh City Development Joint Stock Commercial Bank	0.77	1	0.77
(ACB). Asia Commercial Joint Stock Bank	0.76	1	0.759
(VPB) Joint Stock Commercial Bank for Foreign Trade of Vietnam	0.75	1	0.753
(SEAB). Joint Stock Commercial Bank for Southeast Asia	0.75	1	0.751
(NAB). South Asia Commercial Joint Stock Bank	0.75	1	0.751
(SCB). Saigon Commercial Joint Stock Bank	0.74	1	0.744
(EIB). Vietnam Export-Import Joint Stock Commercial Bank	0.71	1	0.706
(ABB). An Binh Joint Stock Commercial Bank	0.67	1	0.666
(TCB). Vietnam Technological and Commercial Joint Stock Bank	0.65	1	0.651
(STB). Saigon Thuong Tin Commercial Joint Stock Bank	0.63	1	0.633
(NVB). National Commercial Joint Stock Bank	0.6	1	0.595
Mean	0.84	1	0.843

Source: The author estimates from the data collected from the annual reports of banks.