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The Potential of Prospect Theory under Inductive Reasoning

Alexandre Bevilacqua Leoneti[1], Luiz Flávio Autran Monteiro Gomes[2]

Abstract. The TODIM method was the first Multiple Criteria Decision Making (MCDM) approach to incorporate the advances of judgment and decision theory based on the findings of prospect theory. Since its advent, several studies have proposed new methods for the same purpose, such as Behavioral TOPSIS. Additionally, new versions of the original TODIM method have been developed to enhance its adherence to prospect theory. Among these approaches, the ExpTODIM version demonstrated high accuracy in predictions under deductive reasoning scenarios. In this research, an inductive reasoning analysis of the potential of ExpTODIM and Behavioral TOPSIS is conducted. The results are then compared with a native inductive reasoning method, namely multinomial regression analysis. It was found that ExpTODIM demonstrated strong potential for applications under an inductive reasoning perspective, which supports the application of prospect theory in tasks such as autonomous classification, a key area in artificial intelligence studies.

Keywords. TODIM, ExpTODIM, prospect theory, deduction, induction

1. Introduction

Analytical methods for supporting decision-making under multiple conflicting criteria have been successfully used in different fields of application for about five decades now. Such methods form the essential body of a branch of the Decision Sciences named Multiple Criteria Decision Making (MCDM). Within the MCDM field, specific methods have also emerged that incorporate evidence from the judgment and decision sciences [21].

Examples of MCDM methods that incorporate these evidences are those founded on prospect theory. This theory, developed by Daniel Kahneman and Amos Tversky [12], describes the phenomena of loss aversion, and has changed the interpretation of classical utility theory [7]. Loss aversion is the accentuated perception of loss relatively to the same magnitude of gains. Under this perspective, it is possible to affirm that people tend to care more to not lose than to gain. Among those MCDM methods, Behavioral TOPSIS (Technique

[1] University of São Paulo, School of Economics, Business Administration and Accounting, Research Group in Decision Sciences, Ribeirão Preto, Brazil, Email: ableoneti@usp.br
[2] Federal University of ABC, Research Program in Economics, São Paulo, Brazil, Email: luiz.autran@ufabc.edu.br

for Order Preference by Similarity to Ideal Solution) [27] and the TODIM (Interactive and Multicriteria Decision Making or TOmada de Decisão Interativa e Multicritério, in Portuguese) [9, 10] must be cited for their importance.

The Behavioral TOPSIS method is derived from one of the most known and applied MCDM methods, namely TOPSIS [11]. TOPSIS was originally designed to construe an index that merges two indicators, both based on the measures of distance in a Euclidian space. Being impossible to avoid arbitrariness on the merge of these different dimensions, TOPSIS has gained different refinements for improving its index [15], mainly for circumventing its associated ranking reversal problems [8]. Among these methods, the Behavioral TOPSIS approach proposes a straight-line equation in which the loss aversion parameter serves as the slope coefficient.

Notably, the TODIM method was the first MCDM approach to incorporate insights from judgment and decision theory, dating back to the 1990s. The TODIM method incorporates the findings of prospect theory into a pairwise comparison procedure through a mathematical function called the Phi function, which is used to measure the perceived values of gains and losses by the decision maker in accordance with the theory. Those quantities are then aggregated, and a score is provided for each alternative considering that people usually tend to avoid losses than attaining gains. With regards to the Phi function of TODIM, it should be added that it is based on a power function, which was arbitrarily defined. In this sense, several studies have proposed alternative mathematical functions as ways to improve the original TODIM's version. The offered justification is usually the aim of making the mathematical function more adherent to the principles of prospect theory.

Leoneti & Gomes [17] had investigated and concluded that all the previously modified versions of the TODIM method were also associated with variations of a power function. As an alternative, the authors proposed two new versions of TODIM method by replacing such functions with similar-shape ones, which were the exponential and logarithmical mathematical functions. Due to their respective foundational basis of mathematical functions, the two new methods have been named ExpTODIM and LogTODIM.

Accordingly, Leoneti & Gomes [17] proposed an experiment for investigating the adherence of different adaptations of the Phi function from TODIM methods to the prospect theory. The experiment performed included the evaluation of three different cases and had the participation of 100 volunteers from the University of São Paulo, totaling 300 observations. The volunteers had their preferences regarding the criteria elicited prior to the evaluation of each case. Then, the elicited preferences were transformed into weight vectors. The volunteers also provided a final ranking of the alternatives that were compared to the rankings generated by the different version of TODIM methods by using each respective weight vector. In this sense, it can be said that the experiment followed a deductive reasoning approach, aiming to mathematically describe the relationship between elicited preferences regarding the criteria and decision outcomes (rankings), by applying different versions of the TODIM method across multiple scenarios.

As a result, Leoneti & Gomes [17] compared the matches among the rankings generated by the different TODIM methods with the final ranking provided by the volunteers and found that the ExpTODIM and LogTODIM overcome the other versions of TODIM method, which were based on the power function. The ExpTODIM also presented higher performance in comparison with TOPSIS and Behavioral TOPSIS and presented lower incidence of ranking reversal when compared to LogTODIM. Finally, ExpTODIM does not incur any paradox related to the original TODIM method as reported in the literature, due to the bounded structure of the exponential function, and it is the least affected by data standardization. Hence, Leoneti & Gomes [17] demonstrated the potential applicability of prospect theory through the use of ExpTODIM, supported by a deductive reasoning approach.

While the ExpTODIM model demonstrated high potential for the applicability of prospect theory through a deductive reasoning approach, other methodological traditions in decision-making research have emerged from a contrasting perspective. Notably, inductive reasoning, which builds generalizable insights from empirical observations, has played a foundational role in the development of statistical modeling techniques.

Galileo's emphasis on empirical observation as the foundation of knowledge, advocating for systematic experimentation and the recognition of observable patterns, laid the groundwork for scientific methods that prioritize learning from data, a core tenet of inductive reasoning analysis. In the 19th century, statisticians such as Francis Galton and Karl Pearson formalized these principles, paving the way for regression techniques to model complex relationships between a dependent variable and a set of independent variables.

Among the regression analysis models, multinomial regression analysis stands out as a tool for deriving knowledge from categorical data with multiple outcomes, reflecting the philosophical lineage rooted in the early history of statistics. Multinomial regression extends this tradition by enabling researchers to analyze and predict outcomes across more than two discrete categories, based on empirical input. It allows for the modeling of probabilities associated with each possible outcome, offering nuanced insights into how multiple predictors influence categorical decisions or classifications.

The present research aims to evaluate the MCDM methods that apply the principles of prospect theory by conducting an inductive reasoning analysis grounded in a numerical perspective. It is hypothesized that those methods, which are built upon a robust framework of human judgment and choice theory, may yield as accurate predictions as the ones from a native inductive reasoning technique, i.e, the multinomial regression analysis. Within this context, the performance of ExpTODIM and Behavioral TOPSIS are also evaluated under identical conditions in which the weights assigned to variables are unknown and derived from empirical data. To achieve this, a unique set of weights is inferred from a large number of observations generated through a naïve stochastic search procedure with heuristic updating, based on a Monte Carlo approach. This procedure aligns with inductive reasoning, commonly employed in statistical methods.

Considering the substantial number of applications on car evaluation among MCDM papers [1, 5, 24, 25], this research choses to analyze the ExpTODIM and Behavioral TOPSIS methods under the same context. For achieving that, a database of car evaluation from Bohanec [2] was considered, which is available in the UC Irvine data repository. Subsequently, the research proposes a C++ coding for performing the calculations of ExpTODIM and Behavioral TOPSIS methods by using a Monte Carlo approach, which generates the data for the comparisons intended. Finally, the statistical software JASP was used for obtaining the outcomes from the multinomial regression analysis.

2. Background

In this section, the ExpTODIM and Behavioral TOPSIS methods are introduced. Then, the adherence of those methods with the additive formulation of the regression analysis models is emphasized. As notation for the next subsections, it should be considered that $\{x_{ij}\}$ is the decision matrix, where there are $i = 1, \dots, m$ alternatives and $j = 1, \dots, n$ criteria for their analysis.

2.1. The ExpTODIM method

The ExpTODIM is a discrete multicriteria method inspired by the exponential version of Kahneman & Tversky's [12] prospect theory that was proposed as an extension of the TODIM method introduced by Gomes & Lima [9, 10]. To make the prospect theory paradigm applicable to value judgments, the ExpTODIM method uses an additive difference function (Phi function). This is achieved by projecting the differences between the values of any two alternatives, perceived in relation to each criterion. This pairwise comparison procedure constitutes the core of the method's framework, since it allows the distinction between gains and losses and, particularly, it increases the perception of losses, as presumed by the prospect theory. The steps of the ExpTODIM method can be summarized as follows.

Firstly, it is necessary to standardize the criteria using a standardization technique, since the criteria can be from different magnitudes and types, such as benefit or cost criteria. The linear standardization technique is therefore applied, as Equations 1 and 2.

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}, i = 1, \dots, m; j = 1, \dots, n \text{ if benefit criteria} \tag{1}$$

$$r_{ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{m} \frac{1}{x_{ij}}}, i = 1, \dots, m; j = 1, \dots, n \text{ if cost criteria} \tag{2}$$

Then, it is necessary to calculate the Phi function for each criterion $j = 1, \dots, n$ in the pairwise comparisons between the alternatives $i = 1, \dots, m$ and $k = 1, \dots, m$, which is depicted by the Equation 3.

$$\varphi_j(r_i, r_k) = \begin{cases} w_j\left(1 - 10^{-\rho|r_{ij} - r_{kj}|}\right) & if\left(r_{ij} - r_{kj}\right) > 0 \\ 0 & if\left(r_{ij} - r_{kj}\right) = 0 \\ -w_j\lambda\left(1 - 10^{-\rho|r_{ij} - r_{kj}|}\right) & if\left(r_{ij} - r_{kj}\right) < 0 \end{cases} \tag{3}$$

for $\forall(i, k)$ where, $r_i = (r_{i1}, \dots, r_{ij}, \dots, r_{in})$, $r_k = (r_{k1}, \dots, r_{kj}, \dots, r_{kn})$, w_j is the weighting of criterion $j = 1, \dots, n$, λ is the amplification parameter commonly used to adjust the different responses regarding gains and losses, generally 2.25 as much for losses according to the findings of Tversky & Kahneman [26], and $\rho \in \mathbb{N}^*$ indicates how significant the decision is according to the perception of the agent, usually set as 3, a neutral point as preconized by Leoneti & Gomes [17].

As a last necessary procedure, it is necessary to determine the dominance ratio for alternatives $i = 1, \dots, m$, which can be performed by Equation 4.

$$\delta_i(r_i, r_k) = \sum_{j=1}^{n} \varphi_j(r_i, r_k) \tag{4}$$

where $\delta_i(r_i, r_k)$ is the dominance ratio that indicates the overall performance of alternative i under the pairwise procedure over all the $j = 1, \ldots, n$ criteria.

The calculation of the performance of each alternative $i = 1, \ldots, m$ based on the sum of the dominance ratio can be standardized to values between 0 and 1 for better interpretation of the results by using the Equation 5.

$$\xi_i = \frac{\sum_{k=1}^{m} \delta_i(r_i, r_k) - \min \sum_{k=1}^{m} \delta_i(r_i, r_k)}{\max \sum_{k=1}^{m} \delta_i(r_i, r_k) - \min \sum_{k=1}^{m} \delta_i(r_i, r_k)} \tag{5}$$

Since it is a linear standardization technique, it does not alter the original order of the alternatives. Alternatively, it is possible to calculate the performance of each alternative $i = 1, \ldots, m$ based solely on the sum of the dominance ratio by using Equation 6.

$$\xi_i = \sum_{k=1}^{m} \delta_i(r_i, r_k) \tag{6}$$

With the calculation of the performance of each alternative it is possible to order the alternatives according to the ξ_i values.

2.2. The Behavioral TOPSIS method

The TOPSIS method, proposed by Hwang and Yoon [11], is based on the pairwise comparison of alternatives within Euclidean space relative to the ideal positive and ideal negative solutions. It defines an index of similarity (or relative proximity) to the ideal positive solution and an index of dissimilarity (or relative distance) from the ideal negative solution. Behavioral TOPSIS, in turn, integrates the classical TOPSIS framework with the assumptions of prospect theory by incorporating these indices with the amplification parameter introduced by Tversky and Kahneman [26]. The steps of the Behavioral TOPSIS method are detailed below.

Firstly, it is necessary to standardize the criteria using a standardization technique, which is the vector standardization technique, as suggested by [11], shown in Equation 7.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1, \ldots, m; j = 1, \ldots, n \ \text{ if benefit criteria} \tag{7}$$

Next, it is necessary to weigh the standardized values by using $v_{ij} = w_j r_{ij}$, where w_j is the weighting value of the j-th criterion in $j = 1, \ldots, n$ and $i = 1, \ldots, m$. Then, that procedure allows identifying the ideal positive and negative alternatives. For this, the criteria are divided into two groups: (i) benefit criteria (C_1), where the higher the numerical value, the better; (ii) cost criteria (C_2), where the lower the numerical value, the better. For each criterion it is identified the ideal positive solution (A^+) and the ideal negative solution (A^-) by applying Equations 8 and 9, respectively.

$$A^+ = \{\upsilon_1^+, \upsilon_2^+, \dots, \upsilon_j^+, \dots, \upsilon_n^+\} = \{(\max_i \upsilon_{ij} | j \in C_1), (\min_i \upsilon_{ij} | j \in C_2) \,|\, i = 1,2,\dots,m\} \quad (8)$$

$$A^- = \{\upsilon_1^-, \upsilon_2^-, \dots, \upsilon_j^-, \dots, \upsilon_n^-\} = \{(min\, \upsilon_{ij} | j \in C_1), (\max_i \upsilon_{ij} | j \in C_2) \,|\, i = 1,2,\dots,m\} \quad (9)$$

where C_1 is the set of benefit criteria, with the preferable value being the highest, and C_2 is the set of cost criteria, with the preferable value being the lowest.

In the sequence, it is necessary to calculate the distance measurements for ideal positive and negative alternatives, which are respectively performed with Equations 10 and 11.

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}, i = 1, \dots, m \quad (10)$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}, i = 1, \dots, m \quad (11)$$

In its final step, it is possible to adopt the calculation of similarities by applying the amplificatory factor as proposed by Yoon & Kim [4], in the Equation 12.

$$V_i = D_i^- - \lambda D_i^+, i = 1, \dots, m \quad (12)$$

where λ is a loss aversion factor for which Yoon & Kim [4] argued that the value attributed to the loss aversion ratio (λ) may vary depending on the decision-maker behavior related to risk, where it can be defined as loss aversion ($\lambda > 1$), neutral ($\lambda = 1$), and risk-seeking ($\lambda < 1$).

Finally, in its original version of Behavioral TOPSIS, it is possible to order the alternatives according to the indexes V_i on the descending ordering from the index generated with similarity measurements based on the distances between the ideal positive and negative alternatives. However, the same linear standardization technique used in ExpTODIM (Equation 5) is also necessary, for allowing the comparison between these two methods. Therefore, with the calculation of the performance of each V_i it was also possible to order the alternatives according to the ξ_i values.

2.3. The regression analysis model

The regression analysis model is a classical statistical method designed to estimate the relationship between a dependent variable and several independent variables simultaneously. The method is based on the Ordinary Least Squares (OLS), which, under the Gauss-Markov assumptions, yields the Best Linear Unbiased Estimator (BLUE). These assumptions include a linear relationship between variables, random sampling of observations, no perfect multicollinearity among the independent variables, and homoscedasticity, meaning constant variance of the error terms across observations. The steps of the multiple regression model can be summarized as follows.

Firstly, it is necessary to specify the dependent variable y and the set of independent variables $x_1, \dots, x_n$. The general form of the model is represented in Equation 13.

$$y_i = \beta_0 + \sum_{j=1}^{n} \beta_j x_{ij} + \varepsilon_i \tag{13}$$

where β_0 is the intercept, β_j $(j = 1, \dots, n)$ are the coefficients that measure the marginal effect of each independent variable x_{ij} (*ceteris paribus* effect), and ε_i is the error term.

To estimate the coefficients, the ordinary least squares (OLS) procedure is applied, which minimizes the sum of squared residuals, given in Equation 14.

$$\min_{\beta} \sum_{i=1}^{m} \left(y_i - \beta_0 - \sum_{j=1}^{n} \beta_j x_{ij} \right)^2 \tag{14}$$

The closed-form solution for the estimated coefficients can be expressed in matrix notation, as presented in Equation 15.

$$\hat{\beta} = (X^T X)^{-1} X^T Y \tag{15}$$

where X is the $m \times (n+1)$ design matrix including the intercept, and Y is the $m \times 1$ response vector. Once the coefficients are estimated, it is possible to compute the fitted values and the residuals. The goodness of fit of the model can be evaluated using the coefficient of determination (R^2), which is defined in Equation 16.

$$R^2 = 1 - \frac{\sum_{i=1}^{m} (y_i - \hat{y_i})^2}{\sum_{i=1}^{m} (y_i - \bar{y})^2} \tag{16}$$

where $\hat{y_i}$ denotes the predicted values and $\bar{y}$ the sample mean of the dependent variable.

In turn, the multinomial regression analysis model is designed to estimate the relationship between a categorical dependent variable with more than two unordered outcomes and a set of independent variables. Unlike multiple linear regression, which predicts continuous outcomes, multinomial regression models the probabilities of each possible category of the dependent variable. This is achieved by comparing each category to a reference category using a logit link function within the framework of generalized linear models. The method is particularly useful in classification problems where the outcome variable represents nominal categories, such as consumer choices, medical diagnoses, or voting preferences.

The multinomial regression analysis model assumes a category as reference, and the model estimates the log-odds of each non-reference category relative to this baseline. For each observation, the log-odds of choosing category k over the reference category K is modeled as a linear combination of independent variables. This relationship is expressed by Equation 17.

$$log\left(\frac{P(y_i = k)}{P(y_i = K)}\right) = \beta_{0k} + \sum_{j=1}^{n} \beta_{jk} x_{ij} \ , k = 1, \ldots, K-1 \tag{17}$$

where β_{0k} is the intercept for category k, β_{jk} are the coefficients for each independent variable x_{ij}, and $\frac{P(y_i = k)}{P(y_i = K)}$ represents the odds of observation i falling into category k relative to the reference category K. For identification purposes, the coefficients associated with the reference category are normalized to zero.

To estimate the coefficients, the maximum likelihood estimation (MLE) procedure is applied. This involves maximizing the likelihood function that represents the probability of observing the given outcomes across all observations. Unlike ordinary least squares, there is no closed-form solution for the coefficients. Instead, iterative numerical methods such as Newton-Raphson are used to find the optimal values.

Once the coefficients are estimated, the predicted probabilities for each category are computed using the softmax function, as shown in Equation 18.

$$P(y_i = k) = \frac{exp(Z_{ki})}{\sum_{h=1}^{T} exp(Z_{hi})} \tag{18}$$

where $Z_{ik} = \beta_{0k} + \sum_{j=1}^{n} \beta_{jk} x_{ij}$ is the linear predictor for category k. These probabilities enable both categorical classification and a probabilistic interpretation of the model's predictions. With the estimation of the model parameters, it is possible to interpret the influence of each explanatory variable on the likelihood of each outcome category. The multinomial regression model thus serves as a powerful tool for prediction, decision-making, and scientific inference in contexts involving categorical outcomes.

2.4. The additive model and the statistical assumptions

It is noteworthy that ExpTODIM, Behavioral TOPSIS, and the regression analysis models share a fundamental structural resemblance through their additive model frameworks, where outcomes are derived by aggregating weighted contributions of multiple variables. In multiple regression, the predicted value of the dependent variable is computed as a linear combination of independent variables and their respective coefficients. In multinomial regression analysis, the predicted probabilities of the categorical outcomes are derived from a set of linear equations, where each equation represents a linear combination of independent variables and their corresponding coefficients for each category relative to a reference group.

Similarly, Behavioral TOPSIS evaluates alternatives by calculating distances to the ideal solutions based on weighted normalized criteria, and ExpTODIM extends this logic by incorporating psychological parameters from prospect theory, such as loss aversion and reference dependence. Hence, despite their differing theoretical foundations those models shared additive structures enable their comparison in terms of how they synthesize multidimensional data into a single evaluative output.

The research hypothesis is that MCDM methods supported by a sound decision and judgement theory, could be used as inferential methods. Once the hypothesis is confirmed, one key advantage of using ExpTODIM or Behavioral TOPSIS in place of traditional regression analysis lies in their freedom from strict statistical assumptions.

The regression analysis models rely heavily on conditions such as normally distributed errors, homoscedasticity, and linearity, which, if violated, can compromise the validity of its inferences. In contrast, ExpTODIM and Behavioral TOPSIS operate within a decision-theoretical framework, allowing for robust modeling of preferences and judgments without requiring distributional assumptions. This makes them particularly valuable in contexts where inferring human decision-making, uncertainty, or non-linear preferences are central, such as behavioral economics, psychology, or multi-criteria decision analysis.

By grounding their logic in cognitive theory rather than parametric optimization, these models would offer a flexible and psychologically informed alternative to inductive reasoning models, especially when empirical data is noisy, sparse, or not well-behaved.

3. Methodology

In this section the materials and methods used to evaluate the potential of prospect theory for multicriteria judgement and decision under inductive reasoning are presented. Firstly, a database was selected as the basis for the numerical evaluation of the potential of each method for its use in inductive reasoning. Then, the ExpTODIM and Behavioral TOPSIS methods were programming into a C++ language altogether with their respective pseudo-code for a Monte Carlo procedure. Finally, statistics for the performance evaluation were used for the comparative analysis.

3.1. The database for the numerical evaluation

The database selected to support the comparison evaluation was first introduced by Bohanec & Rajkovič [3] for evaluating a decision-making model according to the concept structure presented in Table 1.

Table 1 – Structure of the database

Variable	Description	Range
CAR	car acceptability	Unacceptable, acceptable, good, very
. PRICE		good
. . buying	buying price	
. . maint	price of the maintenance	Very high, high, medium, low
. TECH		Very high, high, medium, low
. . COMFORT	number of doors	
. . . doors	capacity in terms of persons to	2, 3, 4, 5 or more
. . . persons	carry	2, 4, more
. . . lug_boot	the size of luggage boot	Small, medium, big
. . safety	estimated safety of the car	Low, medium, high

Source: adapted from [3]

In the database, there are 1,728 car's evaluations based on six variables. For some variables, a transformation from a string into an integer was necessary, since the database would be used as the decision matrix for the methods' evaluation.

Consequently, the variables that were categorical were transformed into discrete variables, by the following procedure. For the cost variable buying (the higher the worse), which has the categories very high, high, median, low, the values of the variable were set as 1 to very high up to 4 to low, incremented by a unity at each level. For the cost variable maintenance (the higher the worse), which also has the categories very high, high, median, low, the values of the variable were similarly settled as 1 to very high up to 4 to low. For the benefit variable doors (the more the better), which has the categories as 2, 3, 4, 5 or more, the values of the variable were settled as 1 to a car with 2 doors up to 4 to a car with 5 or more doors, incremented by a unity each added door. For the benefit variable number of people (the more the better), which has the categories 2, 4, more, the values of the variable were set as 1 to a car that accommodates up to 2 persons, up to 3 to the car that accommodates more than 4 persons. For the benefit variable luggage boot (the larger the better), which has the categories small, medium, big, the values of the variable were set as 1 to a car that has a small luggage boot up to 3 to the car that has a big luggage boot. Finally, for the benefit variable of safety (the more the better), which has the categories low, medium, high, the values of the variable were settled as 1 to a car that has low estimated safety up to 3 to the car that has a high estimated safety.

Based on the database with the transformed variables, the cars should be classified as unacceptable, acceptable, good, and very good.

3.2. The pseudo-code

For evaluating ExpTODIM and Behavioral TOPSIS as inductive reasoning methods, a cross-selection procedure was applied. The original database was divided into two databases by means of a procedure of random selection. That procedure selects, in a random manner, a proportion of 80% of the original database to be used as the training environment. Then, the data from the training database was inserted into ExpTODIM and Behavioral TOPSIS as decision matrixes. The decision matrixes contained the variables buying price, price of the maintenance, number of doors, capacity in terms of persons to carry, the size of luggage boot, and estimated safety of the car. Notice that, although ExpTODIM and Behavioral TOPSIS include a prior step of data standardization, no standardization procedure was applied. This was intended for an adequate way for comparison with the multinomial regression model.

Following, a weighting vector was designed to be inserted as a necessary input of ExpTODIM and Behavioral TOPSIS methods. Here, the weighting vector was initially set as a vector of equal weights for all criteria. The arbitrary decision of starting by equal weights is justified by the intention to allow the MCDM methods to departure from a not biased scenario, in order to verify the chances of the Monte Carlo procedure to obtain significative weighting vectors. Considering that all the criteria were set as benefit criteria, given the transformations performed to the variables of the original data, a verification of the type of criteria was not necessary.

Subsequently, a naïve stochastic search procedure with heuristic updating, based on a Monte Carlo approach, was carried out to obtain the final values for the weighting vector. For that, a categorization procedure for transforming the standardized outputs from the methods into categorical values was necessary. The procedure uses the standardized outputs of ExpTODIM and Behavior TOPSIS for allocating them into categories that are defined based on thresholds, which starts by using arbitrary values. Recalling that the standardized outputs vary between 0 and 1, the categories were defined by using the thresholds: (i) very good, if the standardized score is higher than 0.75, (ii) good, if is lower than 0.75 and higher than 0.6, (iii) acceptable, if is lower than 0.6 and higher than 0.5, and (iv) not acceptable, if the score is lower than 0.5. Although arbitrary, the definition of the thresholds was based on a visual evaluation of the score distributions for each car's category.

Then, in the Monte Carlo approach, the weights and the thresholds were set as variables, with their initial values being set as mentioned previously[3]. The initial value of each variable was considered as the average of a normal distribution with a standard deviation of 0.15 and a simulation was performed by calculating the standardized scores of ExpTODIM and Behavioral TOPSIS, considering their respective transformation into a categorical variable for the purpose of classification. Finally, the classifications obtained through the application of ExpTODIM and Behavioral TOPSIS were compared to those from the original dataset using the selected testing portion of the sample, from which the performance metrics were derived.

Five metrics were chosen for evaluating the performance of the methods, namely: (i) micro-precision (overall precision); (ii) macro-precision (simple class average); (iii) weighted precision; (iv) macro-accuracy; and (v) relative matches (the number of matches for any position divided by the total of the sample). The micro-precision measures the overall precision of a classification model by aggregating all classes and treating every prediction equally, regardless of which class it belongs to. It gives a global view of model performance across all classes and is especially useful when caring more about individual predictions than class-specific performance. The metric for micro-precision can be seen in Equation 19.

$$mP = \frac{\sum_{k=1}^{T} TP_k}{\sum_{k=1}^{T}(TP_k + FP_k)} \tag{19}$$

where mP is the micro-precision metric, TP_k is the counting of true positives within the k-th class, FP_k is the counting of false positives within the k-th class, and T is the total of classes.

In its turn, the macro-precision metric is usually employed for measuring how well a model can correctly produce true positive predictions for a class compared to all instances that were predicted as that class (both true positives and false positives). That metric can be extended to a multi-class scenario by obtaining an average measure of the respective precision across all classes. The extended metric for macro-precision can be seen in Equation 20.

$$MP = \frac{1}{T}\sum_{k=1}^{T} Pr_k \tag{20}$$

[3] A limit to the weights that could vary between 0.05 up to 0.35 was also included, while for the thresholds a checker to ensure that the order of the thresholds follows the logic that goes from the very good to the unacceptable

where MP is the macro-precision metric, $Pr_k = \frac{TP_k}{TP_k+FP_k}$, is the precision for the k-th class, TP_k is the counting of true positives within the k-th class, and FP_k is the counting of false positives within the k-th class.

As the previous precision metrics can be sensitive to class imbalance, the weighted precision is a type of average precision that accounts for the relative size (support) of each class. Instead of treating all classes equally (as macro-precision does), weighted precision gives more influence on classes with more samples. The formula of weighted precision can be seen in Equation 21.

$$wP = \frac{\sum_{k=1}^{T} Pr_k . S_k}{\sum_{k=1}^{T} S_k} \tag{21}$$

were wP is the weighted precision metric, Pr_k is the precision for the k-th class, and S_k is the support, the number of true instances of k-th class.

Following, the macro-accuracy metric is the average of the per-class accuracies, which can be directly obtained within a multi-class scenario by Equation 22.

$$A = \frac{1}{T}\sum_{k=1}^{T} Acc_k \tag{22}$$

where A is the macro-accuracy metric, $Acc_k = \frac{TP_k + TN_k}{TP_k+FP_k+FN_k+TN_k}$, is the accuracy for the k-th class, TP_k is the counting of true positives within the k-th class, FP_k is the counting of false positives within the k-th class, FN_k is the counting of false negatives within the k-th class, and TN_k is the counting of true negatives within the k-th class.

Lastly, the number of matches between the predicted class and the original one were counted, that amount was divided by the number of elements in the sample, which created the relative number of matches. Then, those values were stored altogether with the other metrics for every simulation performed.

Finally, from 10 different executions of the procedure, it was possible to obtain the average values of performance from each metric, besides its variation and other measures such as minimum and maximum values, which made possible the comparison with a native inductive reasoning tool, the multinomial regression analysis. Figure 1 presents the summary of the pseudo-code, which was programmed in Microsoft Visual Studio 2019 by using the C++ language.

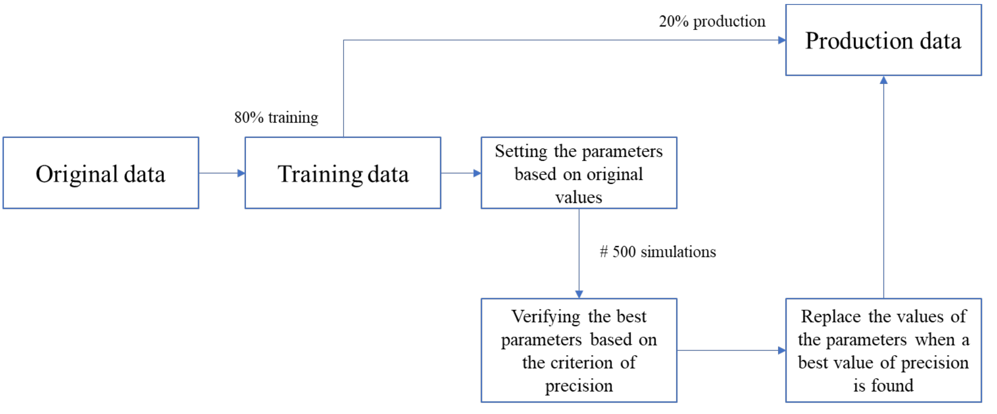

Figure 1 – Diagram of the naïve stochastic search procedure with heuristic updating, based on a Monte Carlo approach

Summarizing, the details of the procedure for obtaining the final values to the weighting vector and the thresholds from the training data are as follows. In 500 simulations, the stored values were arranged in descending order with basis on the macro-precision metric. The number of stored values was set as 10 for each variable. A trigger was set to each 10 simulations to do a verification. Then, when a new simulation obtained a value for the macro-precision metric higher than the lowest value from the descending ordered macro-precision values, the weight vector and the thresholds from the new simulation replaced the values respectively to the lowest average of macro-precision metric. The macro-precision metric was chosen because it is a more conservative metric, since it doesn't let strong performance on dominant classes overshadow weak performance on rare ones. That mechanism was included to serve as a component of memory. Therefore, the number of stored values was set up to the 10 best weights and thresholds, which the average was later used for generating the input values of ExpTODIM and Behavioral TOPSIS methods for the testing data.

In parallel, the multinomial regression analysis was run 10 times with the outputs from the software JASP, version 0.95.2, stored for the comparison analysis with the MCDM methods. MLE was used, and the training data was also set to 80% of the original data.

3.3. The results analysis

To assess the effectiveness of the ExpTODIM and Behavioral TOPSIS decision-making methods, the 10 most optimal configurations, comprising the best-performing weighting vectors and threshold values identified during the initial phase, were applied to the remaining 20% of the dataset, which had been reserved for testing purposes. This evaluation aimed to test the generalizability and robustness of both methods under previously unseen conditions.

For comparative analysis, descriptive statistical measures such as mean and range were computed for the performance scores generated by each method. These metrics provided insights into the consistency, sensitivity, and discriminative power of ExpTODIM and Behavioral TOPSIS across different simulations. Beyond numerical comparison, the conceptual foundations of each method were critically examined to evaluate their alignment with inductive reasoning processes.

This dual-layered evaluation enabled a comprehensive understanding of each method's strengths and limitations, particularly in terms of their applicability to real-world decision-making environments that rely on inductive reasoning.

4. Results and Discussions

Table 2 shows the proportion of cars in the sample.

Table 2 – Original data and class frequencies

	Original data	
Class	#	# (%)
Unacceptable	1,210	70.0
Acceptable	384	22.2
Good	69	3.9
Very good	62	3.8
Total	1,728	100.0

Each of the 10 times running the 500 simulations with the training data took about 47 seconds for Behavioral TOPSIS and 7 minutes and 56 seconds for ExpTODIM in a Windows PC using Intel Core i7 from 4th generation. The longer runtime observed for ExpTODIM can be attributed to its computational structure, which involves an extensive phase of pairwise comparisons among all alternatives. This phase is inherently demanding, as it evaluates the dominance relationships between each pair of alternatives, considering both gains and losses under prospect theory principles. Additionally, ExpTODIM includes a second aggregation phase that synthesizes these pairwise evaluations into final scores, further contributing to its computational load. In contrast, Behavioral TOPSIS operates more efficiently by comparing each alternative only against the ideal positive and negative alternatives. This streamlined approach significantly reduces the number of required computations, resulting in faster execution. The simplicity and directness of Behavioral TOPSIS in handling alternatives make it a clear advantage in scenarios where computational efficiency is critical. The time required for JASP to run each multinomial regression analysis was short, taking less than 5 seconds.

Subsequently, the average from the 10 best values for the weighting vector and the thresholds obtained from the Monte Carlo procedure were inserted into the ExpTODIM and Behavioral TOPSIS for evaluating the method with regards its potential for autonomous classification by using the remaining 20% of data. Figures 2, 3, 4, 5 and 6 present the measure of micro-precision, macro-precision, weighted precision, macro-accuracy, and relative matches, respectively. The blue dots represent, from left to right, respectively the minimum, average, and maximum values for each metric and method, while the blue lines indicate the range of values for each method within the corresponding metric.

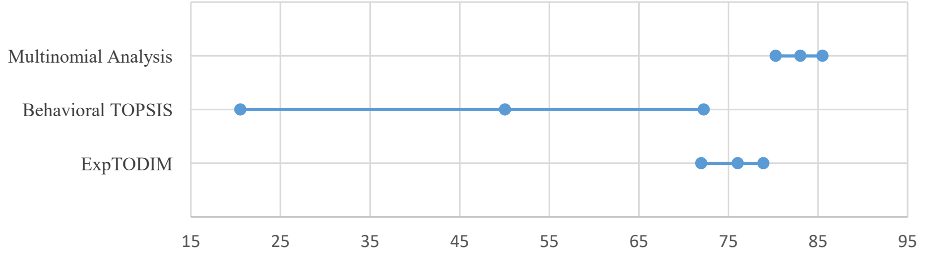

Figure 2 – The micro-precision metric

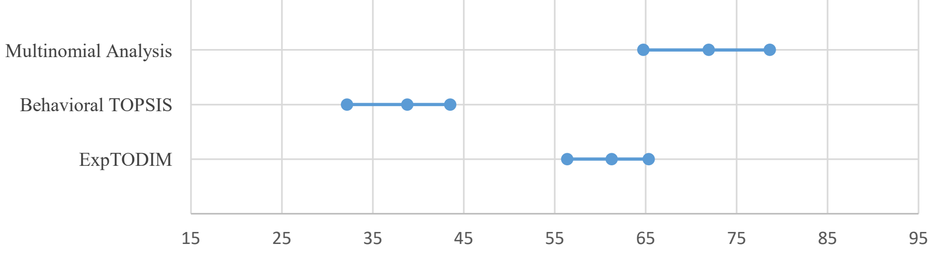

Figure 3 – The macro-precision metric

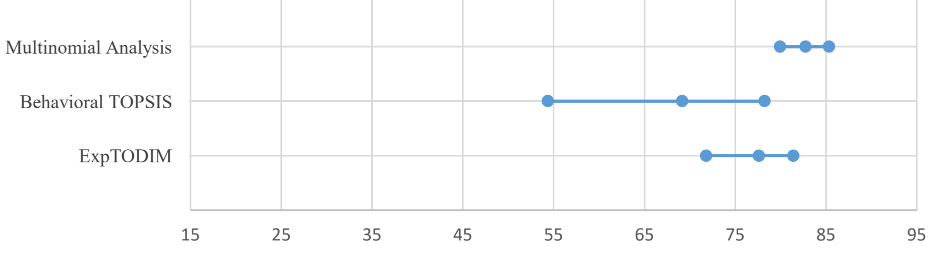

Figure 4 – The weighted precision metric

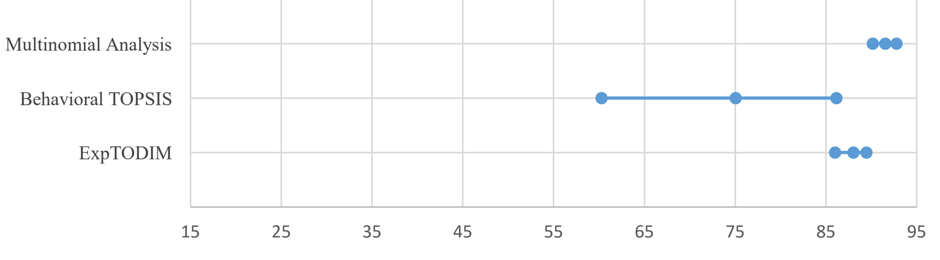

Figure 5 – The macro-accuracy metric

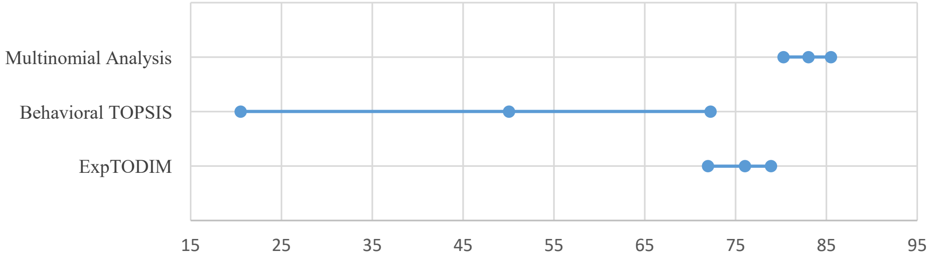

Figure 6 – The relative matches metric

It should be noticed that, although it reached good values for accuracy, the Behavioral TOPSIS reached lowest values for the measures of precision. Additionally, the Behavioral TOPSIS presented a high range for all the metrics, with the exception for the macro-precision metric. On the other hand, it should be emphasized that ExpTODIM achieved high values for all the metrics, very closely to the native statistic technique. It should be stressed out that such values were reached by means of a naïve procedure with not standardized data and, rather than produced to validate the procedure, the results mean discussing the potential use of Behavioral TOPSIS and ExpTODIM as methods for inductive reasoning.

The set of values and thresholds that leads to the best performance of ExpTODIM through the testing data, were found in the 7th simulation, which were 0.203 to the buying's criteria, and: 0.164 to maint, 0.093 to doors, 0.159 to persons, 0.145 to lug_boot, and 0.233 to safety. In its turn, the ranges of the thresholds for car evaluation obtained from the Monte Carlo procedure were: (i) very good if the score was higher than 0.912, (ii) good if the score was higher than 0.742, (iii) acceptable if the score was higher than 0.617, and (iv) not acceptable if the score is lower than 0.617. Under these parameters the maximum macro-accuracy achieved to ExpTODIM method was 89.45%, with weighted precision of 81.38%, as can be seen in the confusion matrix of Table 3.

Table 3 – Confusion matrix for the best performance achieved by ExpTODIM

	Unacceptable	Acceptable	Good	Very good
Unacceptable	225	17	6	0
Acceptable	26	34	10	0
Good	0	4	8	1
Very good	0	0	9	6

As can be seen in Table 3, the acceptable cars class shows the least discrimination compared to the other classes. The low performance in this class significantly diminished the overall performance of the ExpTODIM method.

For Behavioral TOPSIS, the set of values and thresholds that leads to the best performance of the method through the testing data, were found in the 9th simulation, which were 0.192 to the buying's criteria, and: 0.134 to maint, 0.098 to doors, 0.186 to persons, 0.139 to lug_boot, and 0.249 to safety. The ranges of the thresholds for car evaluation obtained from the Monte Carlo procedure were: (i) very good if the score was higher than 0.922, (ii) good if the score was higher than 0.875, (iii) acceptable if the score was higher than 0.745, and (iv) not acceptable if the score is lower than 0.745. Under these parameters the maximum macro-accuracy achieved by Behavioral TOPSIS was 86.12%, with 57.66% for weighted precision, as can be seen in the confusion matrix of Table 4.

Table 4 – Confusion matrix for the best performance achieved by Behavioral TOPSIS

	Unacceptable	Acceptable	Good	Very good
Unacceptable	249	0	0	0
Acceptable	71	0	0	0
Good	11	0	0	0
Very good	5	8	1	1

The class-wise distribution observed in the confusion matrix resulting from the Behavioral TOPSIS outcomes exhibited atypical patterns and irregular inter-class behavior. This anomaly underscores the inherent variability in task performance when approached through a non-statistical decision-making framework, highlighting the methodological divergence from conventional statistical models.

Finally, the best performance of multinomial regression analysis through the testing data was found in the 6th simulation. In this simulation the maximum macro-accuracy achieved by multinomial regression analysis was 92.75%, with 85.33% for weighted precision, as shown in the confusion matrix of Table 5.

Table 5 – Confusion matrix for the best performance achieved by multinomial regression analysis

	Unacceptable	Acceptable	Good	Very good
Unacceptable	228	14	4	0
Acceptable	19	46	1	0
Good	2	7	4	0
Very good	0	3	0	17

Based on the evaluation metrics applied, the similarity in performance between the ExpTODIM method and the multinomial regression analysis is particularly noteworthy. This finding represents a significant achievement, especially considering that ExpTODIM, grounded in prospect theory, reached comparable results through a relatively simplistic implementation, based on a naïve stochastic search procedure with heuristic updating. Despite the multinomial regression showing a slight edge in terms of precision and accuracy, the proximity of the metrics suggests that ExpTODIM holds substantial promise as a viable alternative for modeling inductive reasoning.

Importantly, this outcome supports the central hypothesis of the study, that prospect theory, traditionally applied in behavioral economics and decision-making under risk, can be effectively adapted to inductive reasoning tasks through the ExpTODIM framework. The method's ability to approximate the performance of a well-established statistical model[4], even considering the numerical procedure in its early form, highlights its potential for further refinement and optimization in future research.

In contrast, the Behavioral TOPSIS method, while demonstrating computational performance close to that of multinomial regression, did not yield metric results that substantiate the application of prospect theory in inductive reasoning. Part of the explanation could eventually be attributed to the new aggregation phase introduced to enable comparison among the evaluated methods. However, it should be noticed that, since this phase relied on a linear standardization technique, which preserves the original order of the alternatives, the observed performance limitations cannot be attributed particularly to that output-standardization process. In this sense, the finding is consistent with the structural characteristics of Behavioral TOPSIS, which relies heavily on distance-based pairwise comparisons. Such an approach can be particularly sensitive to unnormalized data, a common challenge in inductive reasoning contexts where input features may vary widely in scale and distribution.

This sensitivity to data normalization was also observed in deductive reasoning tasks [17], where Behavioral TOPSIS underperformed when applied to raw, unstandardized datasets. These findings underscore the importance of preprocessing and methodological alignment when applying distance-based techniques to cognitive modeling. This is particularly relevant given that Behavioral TOPSIS applies the loss amplification factor solely during the final amalgamation phase, limiting its influence on the aggregation of alternatives. In contrast, ExpTODIM incorporates the loss amplification throughout all pairwise comparison, allowing the psychological impact of losses to reverberate consistently across the entire decision-making process.

In fact, the ExpTODIM method belongs to the class of universal rationality MCDM methods, according to the typology proposed by [18]. This class assumes that rationality is universal, that is, the pairwise comparisons within methods of this class are grounded in a specific theory, implemented through a mathematical function that reflects the expected rationality. The results demonstrate that, as an MCDM method capable of incorporating the principles of prospect theory, the ExpTODIM method shows strong potential as an inductive reasoning model, achieving outcomes comparable to those generated by a native inductive reasoning model. Consequently, ExpTODIM had previously reached a prediction level as high as that of traditional MCDM methods, while also producing results strongly aligned with a native statistical method based on inductive reasoning in this research. These findings corroborate recent studies suggesting that MCDM methods can effectively perform tasks traditionally carried out by statistical methods [22].

Particularly, the use of the principles of prospect theory within a classification task, when applied from the perspective of the decision-making subjects, as in MCDM methods like ExpTODIM, offers advantages over its sole application within statistical models that operate directly on the data. For example, by embedding the amplifying factor into the subjective evaluation process, MCDM methods are able to capture and model the cognitive biases, preferences, and perceptions of individuals, rather than merely inferring patterns from data. This subject-centered approach aligns more closely with real-world decision contexts where

[4] That baseline performance considers some of the most applied autonomous methods for classification [2]. For instance, the Xgboost classification method, support vector classification, random forest classification, neural network classification, and logistic regression.

human judgment plays a central role, allowing for a more nuanced and psychologically grounded classification. Moreover, it allows the elicitation of preference structures that reflect loss aversion and risk sensitivity, core tenets of prospect theory, often overlooked or oversimplified in purely algorithmic models. In doing so, it strengthens human agency and control as systemic requirements for developing comprehensive frameworks for trustworthy artificial intelligence applications [23].

As a result, classifications derived from MCDM frameworks incorporating the principles of prospect theory not only maintain high predictive performance but also enhance interpretability and alignment with human decision behavior, bridging the gap between normative statistical inference and descriptive behavioral modeling.

5. Conclusions

The ExpTODIM method had previously demonstrated strong performance among multicriteria decision-making (MCDM) approaches when applied from a deductive reasoning perspective. The present study extends this understanding by revealing the method's promising potential in modeling inductive reasoning tasks.

Notably, the comparative analysis with multinomial regression, a well-established statistical model, showed that ExpTODIM achieved comparable performance metrics, despite relying on a relatively naïve stochastic search procedure with heuristic updating. This outcome represents a significant achievement, particularly given ExpTODIM's foundation in prospect theory, traditionally associated with behavioral economics and decision-making under risk [19].

While multinomial regression exhibited marginally superior precision and accuracy, the proximity of results reinforces the hypothesis that prospect theory can be effectively adapted to inductive reasoning through the ExpTODIM framework. This finding is especially relevant for artificial intelligence applications such as autonomous classification, where inductive reasoning plays a central role. Moreover, ExpTODIM's structural advantages, such as the absence of TODIM's paradox and reduced dependency on data normalization, further support its suitability for such tasks.

In contrast, Behavioral TOPSIS, although computationally efficient, did not demonstrate metric outcomes that substantiate its use as an inductive reasoning model grounded in prospect theory. Its reliance on distance-based pairwise comparisons and the restrict application of the loss amplification factor, only during the final aggregation phase, make it more sensitive to unnormalized data and less aligned with inductive reasoning modeling.

Although the current study does not intend to establish a formal methodology for applying ExpTODIM as an inductive reasoning model, the results highlight its potential and suggest that further methodological refinement is warranted. Future research could explore more sophisticated implementations beyond the naïve Monte Carlo-based procedure employed in this research.

The findings supplement earlier results and reveal new avenues for applying ExpTODIM in inductive reasoning contexts. The method's ability to approximate the performance of a native statistical model underscores its versatility and opens possibilities for broader applications in artificial intelligence and cognitive modeling. To support such endeavors, this study also introduces a spreadsheet tool for automating ExpTODIM's calculations, which is made available for academic use [16].

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