

APPLICATION OF FUZZY AHP IN PRIORITY BASED SELECTION OF FINANCIAL INDICES: A PERSPECTIVE FOR INVESTORS

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ABSTRACT

By providing important indicators, financial indices help investors make educated judgements regarding their assets, much like vital sign monitors for the financial markets. The best way for investors to keep up with the market and make strategic adjustments is to keep an eye on these indexes. Researching the most important financial indexes for making educated investing decisions is, thus, quite relevant. Finding the most essential financial indices from an investing standpoint and assigning a weight to each of those indexes are the main goals of this research. A weighted score is derived by combining four financial indices in a Multi-Criteria Decision-Making (MCDM) technique. These objectives are then pursued. Triangular Fuzzy Numbers (TFNs) and the Fuzzy Analytic Hierarchy Process (F-AHP) are used to determine the weights of criteria in this technique. Using these methods together, the research hopes to provide a thorough analysis of the role that different financial indexes have in informing investment choices. This study emphasizes the paramount importance of considering the Price Earning to Growth (PEG) ratio when making investment decisions, followed by the Debt Equity Ratio. Price to Book Value and Dividend Yield, while relevant, carry comparatively less weightage in the overall assessment. Investors are advised to use these insights as a guideline in their financial analysis and decision-making processes.

Keywords: Financial Indices, Priority Selection, Multi-Criteria Decision-Making (MCDM), Triangular Fuzzy Number (TFN), Fuzzy Analytic Hierarchy Process (F-AHP)

1. INTRODUCTION

The stock market which is known to us as the share market is a platform where a person individually and any organization purchase and sell ownership shares of publicly traded companies. These shares represent a claim on the assets and earnings of the company. The market serves as a means for companies to raise capital by issuing shares, and for investors to potentially earn profits through price appreciation or dividends. It is essential to the global economy since it makes financial flows easier and allows businesses to develop and flourish. There are two major parts of buying stocks are trading and Investment. Buying and selling financial products, such as stocks, bonds, commodities, or currencies, with the intention of turning a profit is referred to as trading. Traders typically operate in relatively short time frames, sometimes within seconds, minutes, hours, or days. To make well-informed judgments, they employ a variety of tech-

niques, such as sentiment analysis, technical analysis, and fundamental analysis. Investment involves allocating money or resources into an asset or venture with the expectation of generating a return over an extended period. Unlike trading, investors typically have a longer-term perspective and are willing to withstand market fluctuations. Typical investment vehicles include mutual funds, equities, bonds, and real estate. The goal of investing is to grow wealth over time through appreciation, dividends, or interest.

1. 1. STRUCTURE OF STOCK MARKETS IN INDIA

The two main stock exchanges in India are the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE). The key indices include the BSE Sensex and the NSE Nifty. India's stock market was one of the largest in the world by market capitalization, with companies spanning various sectors. Foreign Institutional Investors (FIIs) wield considerable influence within the Indian stock market, and their financial contributions can exert a substantial influence on market fluctuations. Some Investment Instruments are:

- **Equities:** Equities or stocks symbolize ownership stakes in companies and are bought and sold on stock markets. They are a popular investment choice for many.
- **Bonds:** India has a well-developed bond market, with various types of bonds available, including government bonds, corporate bonds, and municipal bonds.
- **Mutual Funds:** Mutual funds combine funds from numerous investors to allocate them into a diversified array of stocks, bonds, or various securities.
- **Real Estate:** Real estate is a popular investment avenue in India, with residential and commercial properties being the primary sectors.
- **Gold:** India has a strong affinity for gold, and it is a common form of investment. It can be purchased in various forms, including jewelry, coins, and bars.

Government Policies and Regulations:

- **SEBI (Securities and Exchange Board of India):** SEBI serves as the regulatory body responsible for supervising the securities market in India. It regulates stock exchanges, brokers, and various market participants.
- **RBI (Reserve Bank of India):** RBI regulates the banking sector and formulates monetary policy, which indirectly affects investment decisions.

1. 2. PRESENT TRADING & INVESTMENT SCENARIO IN INDIA

Recent Trends and Developments:

- **Digitalization and Fintech:** India has seen a surge in digital trading platforms and fintech companies, making it easier for individuals to invest in various financial instruments.
- **Retail Participation:** The Indian stock market has witnessed increased participation from retail investors, partly due to easier access through online platforms.
- **Initial Public Offerings (IPOs):** There was a surge in IPO activity, with several companies going public to raise capital for expansion.
- **Foreign Direct Investment (FDI):** India remained an attractive destination for Foreign Direct Investment (FDI) in diverse sectors, encompassing technology, e-commerce, and manufacturing.
- **Policy Changes:** The government introduced a range of policy alterations and initiatives with the goal of enticing foreign investments and enhancing the business environment's ease of operation. Before the covid-19 pandemic, the stock market in India was primarily almost untouched. There was only a dedicated group of retail investors, along with

many foreign institutional investors (FII) and domestic institutional investors (DII). Mainstream media and public knowledge revolved around the risks associated with the stock market while staying oblivious to the opportunities to build long-term wealth. But in 2020, the number of retail investors achieved a sharp spike. The number of active demat account holders rose by 1.4 crores, a figure higher than the previous three years. At a time when businesses shut down and people's livelihoods were impacted, what could have been the possible reasons for an influx of investors in the stock market? Then explore the rise of Indian investors and look at the factors that led to a sudden shift from traditional investment avenues.

In January 2023, there were 110 million demat account holders in India, up significantly from 84 million in 2022. Despite these substantial figures, only around 3% of Indian families are said to be actively involved in the stock market. But as compared to other countries, India's performance is lacking in terms of the number of committed retail investors. Tragically, just 3% of Indians put their money into the stock market. In comparison, 13% of Chinese, 33% of British, and 55% of Americans do similar activities.

The recent developments in the industry have pointed to a positive development. More individual investors were selling their shares before 2019, suggesting they were cautious to acquire more. The net selling of equities was larger than the net buying. However, net purchases increased significantly at the turn of the 2020 calendar year, leading to a net inflow of 51,200 cores (INR). People are becoming more at ease with the stock market and are putting a large portion of their disposable money into shares, as this change in investor behaviour demonstrated.

1. 3. FUNDAMENTAL ANALYSIS

The purpose of fundamental analysis in stock market research is to determine a stock's true value by looking at a number of economic and financial metrics. The process includes looking at things like the company's financials, the condition of the industry, the leadership, the competition, and the macroeconomic data. Finding out if a stock is overpriced or under-priced will help investors make educated decisions about buying or selling. Focusing on a company's fundamental health and performance, rather than just market sentiment or technical chart pattern, is the main focus of this strategy. Financial ratios, indices, and indicators form the backbone of fundamental research.

1. 4. FINANCIAL INDICES

The performance and stability of a firm or the financial market as a whole may be evaluated using financial indices or ratios. By presenting financial data in a consistent and readily digestible manner, they facilitate analysis and comparison. Profitability Ratios gauge "a company's capacity to generate profits concerning its revenue, assets, or equity," and they are one of several common financial rankings or measurements. Some examples are the Net Profit Margin and Return on Investment (ROI). The liquidity ratio is a measure of a company's capacity to meet its short-term obligations using the funds on hand. Solvency Ratios, which measure a company's ability to meet its long-term financial commitments, and the "Current Ratio and Quick Ratio" are two examples. The "Debt to Equity Ratio" and the "Interest Coverage Ratio" serve as examples. "Efficiency Ratios" measure how well a business turns its assets into cash. "Market Ratios" and "Inventory Turnover and Asset Turnover" are two examples of similar tools that may help investors gauge a company's value. Two such metrics are earnings per share ("EPS") and the price-to-earnings (P/E) ratio. Conversely, indices are collective metrics that follow the value of a collection of assets or securities. They provide a snapshot of how a market, industry,

or class of assets is doing as a whole. Investors, analysts, and companies rely on financial indices for financial analysis, which in turn aids in decision-making about investments, operations, and strategy.

1. 5. CONTEXT

Financial indices are crucial for fundamental analysis because they provide a snapshot of the overall performance of a specific market, sector, or asset class. Here's why they're important:

- **Benchmarking:** Indices function as reference points for investors to measure the performance of their investments compared to the overall market or a particular industry. This helps investors assess if their investments are outperforming or underperforming relative to the market.
- **Market Trends:** They reflect the collective sentiment and trends in the market. Analysing indices can reveal if the market is bullish (rising) or bearish (falling), which can influence investment decisions.
- **Diversification:** Indices represent a diversified basket of assets. Through an examination of these indices, investors can obtain access to a diverse array of companies or assets without the need to individually invest in each one.
- **Risk Assessment:** They offer information about the degree of risk linked to a specific market or industry. For instance, a volatile index suggests higher market risk, while a stable one indicates lower risk.
- **Sectorial Analysis:** Indices are often categorized by sectors (e.g., technology, healthcare, energy). This allows investors to focus on specific industries or areas of interest.
- **Investment Strategy:** They help investors formulate or adjust their investment strategies. For instance, if an index indicates a strong performance in a particular sector, an investor might allocate more funds to that area.
- **Historical Performance:** Past performance of an index can provide valuable information about how it might behave in the future. Nonetheless, it's crucial to bear in mind that past performance does not serve as a reliable predictor of future outcomes.
- **Liquidity and Accessibility:** Many indices have associated exchange-traded funds (ETFs) or mutual funds that track their performance. This provides investors with a convenient way to gain exposure to a specific market or sector.

Financial indices are like vital sign monitors for the financial markets. They offer key metrics that investors can use to make informed decisions about their investments. By tracking indices, investors can stay attuned to market trends and adjust their strategies accordingly. In this context conducting a study on which financial indices are crucial for invest decision making is very pertinent.

1. 6. MULTI-CRITERIA DECISION MAKING

With the use of MCDM approaches, we may organise evaluations and our prioritisations of options according to how well they perform across different criteria. These strategies are incredibly helpful for decision-making and are used extensively in many different areas, including public policy, engineering, economics, environmental management, and business. There are many different MCDM strategies accessible, and each one has its own set of pros and cons. A few examples of popular methods are AHP, TOPSIS, PROMETHEE, and ELECTRE (Elimination and Choice Expressing Reality). Other popular methods include MARCOS, WASPAS, and ARAS. Researchers have looked at a variety of MCDM methods in the well-known fore-named research. Here, we have some of them like (Taletović, 2023) takes a look at a few

well-known MCDM strategies that are commonly used in warehouse management. Choosing logistics distribution channels for delivering the final product: utilizing the FUCOM-MARCOS Model introduced by [Stevic et al. \(2023\)](#). Later on, [Krstić et al. \(2023\)](#) employs an innovative hybrid Multiple Criteria Decision-Making (MCDM) model that combines the best-worst method (BWM) and comprehensive distance-based ranking (COBRA) within a grey analytical framework, enabling a nuanced assessment that accommodates uncertainty effectively. Further, [Petrovic et al. \(2023\)](#) proposed to study Serbia's passenger and freight road transport operational efficacy on an annual basis using Entropy and TOPSIS techniques. The comparing members of the European Union on the logistics performance index: making use of the Bonferroni operator in an integrated MCDM structure explained by [Hadžikadunić et al. \(2023\)](#). Recently, [Khan et al. \(2024\)](#) analyzed strategies for sustainable urban development through the Use of spherical CRITIC-WASPAS models. [Chen et al. \(2023\)](#) developed a method for assessing the carrying capacity of geological environments by employing GIS-AHP analysis. It focuses on 10 indicator layers, encompassing social, ecological, and post-earthquake geological data, as well as disaster distribution and societal evolution. How to use the TOPSIS method to evaluate and prioritise autonomous solutions for sustainable warehouse intra-logistics operations was focused by [Dabić-Miletić and Raković \(2023\)](#). Choosing the most appropriate "MCDM" method depends on the specific characteristics of the decision problem, the availability of data, the complexity" of criteria interactions, and the favours of the decision maker.

1. 7. JUSTIFICATION OF F-AHP

Fuzzy Analytic Hierarchy Process (Fuzzy AHP) ([Onay et al., 2016](#)) is a decision-making methodology that extend traditional AHP technique to manage unsteadiness and indeterminacy in decision-making problems. Some justifications for using this technique are Uncertainty and Fuzziness in Decision-making, Subjectivity and Multi-Criteria Decision Making, Inconsistent and Incomplete Data, Trade-offs and Ranking, Flexible Modelling, Complex Decision Scenarios, Project Evaluation and Selection, Risk Assessment, Sensitivity Analysis, Applicability in Various Fields. In shorts, the justification for using Fuzzy AHP lie in their ability to handle uncertainty, subjective preferences, multi-criteria decision-making, and complex decision scenarios, making them valuable tools for making informed and robust decisions in real-world situations.

1. 8. BENEFICIARIES

The potential beneficiaries could include for conduct research on the stock market:

- **Investors:** They can make more informed decisions about buying, selling, or holding stocks.
- **Traders:** They can use your research for short-term trading strategies.
- **Financial Analysts:** Your findings could aid in their market analyses and forecasts.
- **Academics:** Your research has the potential to add to the existing knowledge base in the fields of finance and economics.
- **Financial Advisors:** They can use your insights to provide better advice to their clients.
- **Regulators:** Your research may provide insights into market trends and behaviors, aiding in policy decisions.
- **Companies:** They may use your research to understand investor sentiment and market dynamics.
- **General Public:** Those interested in finance can gain a better understanding of the market through your research.

Though the impact and relevance of any research will depend on its quality, originality and applicability.

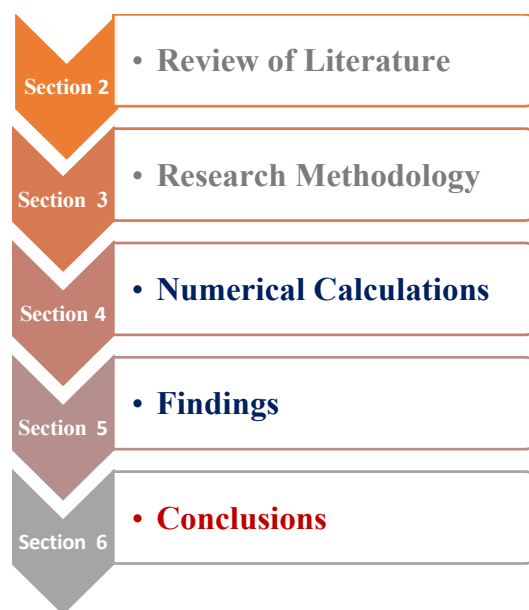
1. 9. NOVELTIES

The novelty in our research lies in applying Fuzzy AHP to evaluate and give precedence to financial indices. This approach introduces a quantitative and systematic method for determining the relative importance of different financial indicators, potentially offering more nuanced and accurate insights for decision-making in financial analysis as well as investment point of view.

1. 10. STRUCTURE OF THE STUDY

The rest portion of the paper is set for the following way as presented in the figure 1:

Figure 1. Structure of the study



Source: Own creation

2. REVIEW OF LITERATURE

After conducting an extensive literature review, this study has noted that numerous research endeavours have been undertaken across diverse fields of finance and other industries, employing a range of MCDM techniques to address multi-criteria decision-making challenges. Various MCDM approaches are available for conducting comparative analyses and establishing rankings. Researchers have adopted different combinations of these methods based on their specific study's requirements to determine the most suitable alternative.

Table 1. An extensive Review of Literature

Authors	Methodology Applied	Objectives
(Gallizo et al., 2002)	Bayesian hierarchical model based on the partial adjustment model	Analyse financial ratios through the Central Balance Sheet Office (CBSO) of The Bank of Spain.
(Laitinen, 2006)	Financial statement method	Evaluate Nokia's prospective growth from 1990 to 2000.

Authors	Methodology Applied	Objectives
(Johnson et al., 2006)	AHP	Choose the authentication level that best fits your needs and preferences.
(Öcal et al., 2007)	Factor analysis	Identifying financial metrics for assessing the financial trajectory of the Turkish construction sector over a five-year span is crucial.
(Shiue et al., 2008)	Empirical experimentation	To evaluate the financial well-being of 537 companies that are publicly traded on the Taiwan Stock Exchange Corporation.
(Wang et al., 2008)	Clustering method	Group financial ratios from various companies based on similar fluctuations.
(Sun & Li, 2009)	Financial distress prediction (FDP) method	Efforts are made to propose a serial combination system involving multiple classifiers, leveraging the class-specific expertise of diverse base classifiers.
(Xidonas et al., 2009)	ELECTRE Tri	In the process of choosing stocks through financial analysis, the selection of equities is conducted.
(Zangouinezhad & Moshabaki, 2011)	Fuzzy multiple attribute decision making (FMADM)	To assess the effectiveness of a university across the four knowledge-based dimensions of a balanced scorecard, one can evaluate its performance.
(Kung et al., 2011)	Fuzzy AHP & Fuzzy TOPSIS	Identify the most fitting company among the five options.
(Low & Hsueh Chen, 2012)	FAHP & Fuzzy Delphi method (FDM)	To assess the main metric obtained from 188 valid responses at an operational hospital in Taiwan.
(Seruffo et al., 2012)	fuzzy AHP & TOPSIS & ELECTRE III	In the context of the integrated services digital broadcasting terrestrial standard, when choosing the initial connection point for access, it is crucial to maintain a high standard of service quality. This is particularly important when employing a videoconferencing application for seamless communication.
(Božanić et al., 2015)	Modified AHP	Fuzzy numbers are utilised to establish the weights of the criterion and alternative values since they are great at expressing ambiguity and uncertainty. Following the AHP procedure, we were able to ascertain criteria function values for all of the options that were considered; these values were in line with the values dictated by the level of conviction.
(Božanić et al., 2016)	fuzzy AHP, MABAC	So that we may rank the options and get the weight coefficients of the criterion. When deciding where to set up laying-up positions, this hybrid approach might help with decision making.
(Ghosh & Jana, 2017)	VIKOR	To assess and contrast the financial success of nine information technology firms listed on the National Stock Exchange in India.
(Ghunaim & Dichter, 2019)	(FAHP) & KNIME data mining platform	To choose the optimal classifier for software development projects.
(Nazzal et al., 2019)	SPSS program	To identify the predominant histological subtypes of breast cancer in the West Bank region of Palestine, one would seek to determine the most frequently observed tissue patterns in cancerous breast cells.

Authors	Methodology Applied	Objectives
(Jana et al., 2019)	Data Envelopment Analysis (DEA)	To ascertain the suitable timing for adjusting the portfolio.
(Prajapati et al., 2020)	Fuzzy AHP and fuzzy TOPSIS	Enhance the current maritime model to facilitate seamless connections between industry professionals and logistics firms in the maritime sector.
(Bobar et al., 2020)	Fuzzy AHP-Z Number Model - Fuzzy MABAC	To find the Ranking and Assessment of the Efficiency of Social Media
(Bajec & Tuljak-Suban, 2019)	MCDM	In choosing a third-party logistics provider (3PLP), the process involves selecting a suitable external partner for managing and optimizing various aspects of logistics and supply chain activities.
(Jana, 2021)	MAGDM method -bipolar fuzzy MABAC	For the selection of renewable energy power generation project.
(Ghorui et al., 2021)	AHP, TOPSIS, WASPAS, SAW	Evaluation of performance for School Teacher Recruitment using
(Kumar & Nagarajan, 2021)	FAHP	To identify a suitable resolution for distributing spectrum among secondary users.
(Ghosh et al., 2021)	DEA & Structural Equation Modelling (SEM)	To assess the operational effectiveness of life insurance companies operating in India from 2010 to 2017.
(Jana & Basu, 2021)	TOPSIS	To assess the financial well-being of pharmaceutical companies operating in the Indian market.
(Ala et al., 2023)	Neutrosophic-Based Multi-Objective Grey Wolf Optimizer	For Ensuring the Security and Resilience of Sustainable Energy
(Bozanic et al., 2023)	Modified AHP with TFN	The goal of this presentation is to shed light on the challenge of determining a ship's navigational path through flooded regions by weighing the potential hazards associated with each potential route.
(Jana et al., 2023)	Linguistic q-rung orthopair fuzzy Choquet integral approach	In this study, the authors tackle the problem of how to choose the most environmentally friendly method of urban package distribution. Its goal is to facilitate the discovery of a solution that is ecologically sound, socially responsible, and economically feasible for postal and logistics corporations, regulatory bodies, and municipal authorities.
(Sharma et al., 2023)	AHP	To create a design for a forging die.
(Nandi et al., 2023)	HFS, GHFS, GHPFN, GHPFN TOPSIS	Evaluation of the treatment options for COVID-19 patients
(Shekar & Mathew, 2023)	Fuzzy AHP	Utilizing a combination of methodologies to evaluate groundwater potential zones (GWPZs) and pinpoint appropriate locations for artificial recharge sites.
(Jana et al., 2023)	Type-2 fuzzy VIKOR method	The goal of this paper is to present an improved extended version of interval type-2 fuzzy VIKOR method for relieving the drawbacks of usual IT2FVIKOR.
(Baskaran et al., 2023)	AHP and GIS technique	Evaluation of land appropriateness for agricultural purposes.
(Jana et al., 2024)	Hybrid MCDM method - bipolar fuzzy approach	For economic condition analysis.

Source: Own creation

The objectives of this study are

- (i) To find the most important financial indices for investment point of view.
- (ii) To find the weightage of importance of selected financial Indices.

3. RESEARCH METHODOLOGY

3.1. SELECTION OF CRITERIA

In this study, we initially compile a set of performance assessment metrics derived from financial statement indicators and ratios. Subsequently, we conduct a survey involving SEBI registered financial advisors and experts. The questionnaire is structured in accordance with the AHP format and encompasses the aforementioned financial statement ratios. The outcomes aim to offer insights and guidance to investors for shaping their future investment strategies. To identify the primary criteria, six SEBI registered experts were enlisted to participate in the survey.

After taking the experts opinion the four financial indices were considered for this study such as Price Earnings to Growth ratio (PEG), Price to Book Value (PB), Debt-Equity Ratio (DE) and Dividend Yield (DY) which are most important for fundamental analysis of a stock.

In Table 2, formulas for financial indices are defined.

Table 2. Formula for selected Financial Indices (Criteria)

Financial Indices	Formula
<i>Price Earning to Growth Rat</i>	<i>Price Earning to Growth Ratio</i> $= \frac{\text{Price to Earnings Ratio}}{\text{Earnings Growth Rate}}$
<i>Debt to Equity Ratio (PB)</i>	<i>Debt to Equity Ratio</i> = $\frac{\text{Total Debt}}{\text{Total Shareholders' Equity}}$
<i>Price to Book Value (DE)</i>	<i>Price to Book Value</i> = $\frac{\text{Market Price per Share}}{\text{Book Value per Share}}$
<i>Dividend Yield (DY)</i>	<i>Dividend Yield</i> = $\frac{\text{Dividend per Share}}{\text{Market Price per Share}}$

Source: Own creation

3.2. ARITHMETIC OPERATIONS OF TFN

Fuzzy set theory was introduced by Zadeh, (1965).

Let $E = (e_1, e_2, e_3)$ and $F = (f_1, f_2, f_3)$ be two different TFN.

(a) Addition:

$$(E + F) = (e_1 + f_1, e_2 + f_2, e_3 + f_3) \quad (1)$$

(b) Subtraction:

$$(E - F) = (e_1 - f_3, e_2 - f_2, e_3 - f_1) \quad (2)$$

(c) Multiplication:

$$(E \times F) = (e_1 f_1, e_2 f_2, e_3 f_3) \quad (3)$$

(d) Scalar Multiplication:

$$\theta E = (\theta e_1, \theta e_2, \theta e_3) \quad (4)$$

(e) Division:

$$\left(\frac{E}{F}\right) = \left(\frac{e_1}{f_3}, \frac{e_2}{f_2}, \frac{e_3}{f_1}\right) \quad (5)$$

(f) Inverse:

$$E^{-1} = \left(\frac{1}{e_3}, \frac{1}{e_2}, \frac{1}{e_1}\right) \quad (6)$$

3. 3. FUZZY AHP

The AHP, introduced by Saaty in 1980 offers a framework for evaluating the relative significance of various activities within a multi-criteria decision-making context (Saaty, 1980). It relies on pairwise comparisons for its assessments. In 1983, Van Laarhoven and Pedrycz, present a fuzzy method for choosing among a number of alternatives under conflicting decision criteria (Van Laarhoven et. al., 1983).

Two most popular Fuzzy AHP methods which are used to calculate the weights of the criteria are:

(i) Geometric Mean Method of Fuzzy AHP (GMM F-AHP):

It was introduced by Buckley, (1985).

(ii) Extent Analysis Method of Fuzzy AHP (EAM F-AHP):

It was introduced by Chang in 1996 Chang, (1996).

Nevertheless, the conventional AHP model encounters certain issues as identified by Yang and Chen in 2004. They noted that AHP is primarily suited for situations with well-defined data, leading to a potentially skewed and imprecise judgment scale (Yang & Chen, 2004). Furthermore, the subjective opinions and preferences of decision makers heavily influence the outcomes of AHP. To address these concerns, several researchers have integrated fuzzy theory with AHP in order to enhance its handling of uncertainty (Fattoruso et. el., 2022). Researchers are concentrated their presently in some complex intelligent decision-making problems related AHP (Fattoruso et. el., 2022) and TOPSIS model (Modibbo et. al., 2022).

3. 3. 1. STEPS OF GMM F-AHP Step 1: Develop Traditional Hierarchical Structure

The AHP involves decomposing a complex problem into smaller parts, creating a hierarchical structure. This structure consists of a top level representing the primary goal, a second level for criteria, and the lowest level for decision alternatives. The hierarchy can extend beyond three levels, with the top level signifying the overarching goal, the second level encompassing criteria, and the lowest level comprising alternatives. Additional levels may incorporate sub-criteria.

Step 2: Create Pair-wise Comparison Matrix with Scale of relative importance

Construct fuzzy pair-wise comparison matrices of order $n \times n$, if there are m criteria. Through expert questionnaires, each expert is asked to assign linguistic terms by TFN.

The elements a_{ij} can be placed as the degree of preferences of i th criterion over j th criterion.

Step 3: Construct Fuzzified Pairwise Comparison Matrix

In this step, to get fuzzified comparison matrix we have to transfer real elements of A into fuzzy numbers.

$$A = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & \tilde{a}_{nn} \end{bmatrix}$$

Step 4: Computation of Fuzzy Geometric Mean Value for each criterion

Geometric Mean Method, proposed by Buckley, (1985), is used to find geometric mean value of the fuzzy comparison values of criterion for each criterion by using the Equation (7)

$$\tilde{r}_i = \left((r_{11} * r_{21} * \dots * r_{n1})^{\frac{1}{n}}, (r_{12} * r_{22} * \dots * r_{n2})^{\frac{1}{n}}, \dots, (r_{1n} * r_{2n} * \dots * r_{nn})^{\frac{1}{n}} \right) = (r_{i1}, r_{i2}, \dots, r_{in}) \quad (7)$$

Step 5: Compute the fuzzy weights ($\tilde{w}_i$)

This step is dedicated for finding out the fuzzy weights for every criterion by using the formula

$$\tilde{w}_i = \tilde{r}_i \otimes \{\tilde{r}_1 \oplus \tilde{r}_2 \oplus \dots \oplus \tilde{r}_n\}^{-1} \quad (8)$$

Step 6: De-Fuzzification

This step has performed to De-Fuzzify the fuzzy weights by using the formula of “Centre of Area (COA)” and Best Non-fuzzy Performance (BNP) both.

$$Centre\ of\ Area\ (COA) = w_i = \left(\frac{l+m+u}{3} \right) \quad (9)$$

$$BNP_{Ci} = \frac{[(U_{Wi}-L_{Wi})+(M_{Wi}-L_{Wi})]}{3} + L_{Wi} \quad (10)$$

Step7: Normalization of Weights

If the total criteria weight is greater than 1, it is not acceptable. So the weights are generally normalized to get the total weights 1.

3. 3. 2. STEPS OF EAM F-AHP:

Step 1: Create Pairwise Comparison Matrix with Scale of relative importance

Construct fuzzy pair-wise comparison matrices of order $n \times n$, if there are m criteria. Through expert questionnaires, each expert is asked to assign linguistic terms by TFN.

The elements a_{ij} can be placed as the degree of preferences of i th criterion over j th criterion.

Step 2: Construct Fuzzified Pairwise Comparison Matrix

In this step, to get fuzzified comparison matrix we have to transfer real elements of A into fuzzy numbers.

$$A = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & \tilde{a}_{nn} \end{bmatrix} \quad (11)$$

Step 3: Calculate the Fuzzy Synthetic Extent

In this step we have to calculate fuzzy synthetic extent with respect to i^{th} alternative by the following formula:

$$S_i = \sum_{j=1}^n \tilde{a}_{ij} \left[\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij} \right]^{-1} \quad (12)$$

Step 4: Calculate the degree of possibility

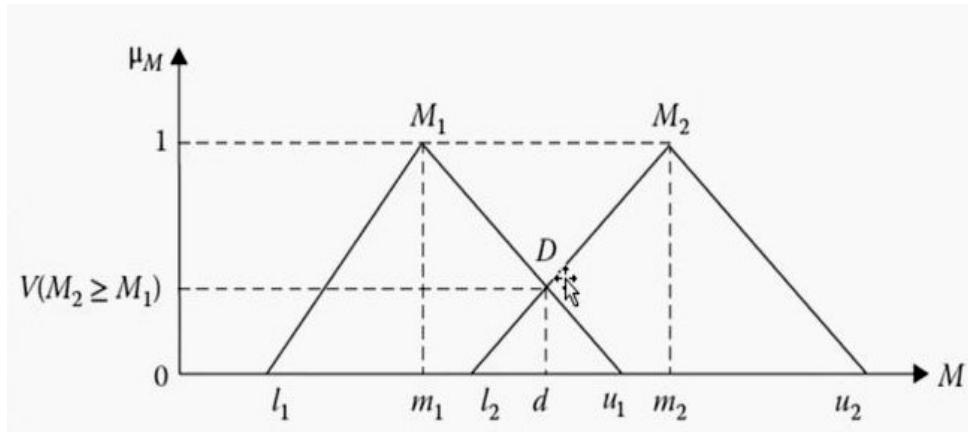
Two Fuzzy numbers are compared so as to get the degree of possibility of one fuzzy number greater than other fuzzy number. Suppose we have two Fuzzy numbers $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$. Then the degree of possibility of $M_2 \geq M_1$ is calculated using the following formula:

$$V(M_2 \geq M_1) = \sup [\min (\mu_{M_1(x)}, \mu_{M_2(y)})] = hgt(M_1 \cap M_2) = \mu_{M_2}(d)$$

$$= \begin{cases} 1; & \text{if } m_2 \geq m_1 \\ 0; & \text{if } l_1 \geq u_2 \\ \frac{(l_1 - u_2)}{(m_2 - u_2) - (m_1 - l_1)}; & \text{otherwise} \end{cases} \quad (13)$$

Where $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$

Figure 2. Degree of Possibility in Diagram



Source: (Hangshenas et. al., 2017)

In the Figure 2, d is the ordinate of the highest intersection point between M_1 and M_2 . We can visualize D is the highest intersection point. In the figure l_1, m_1 and u_1 are the ordinates of the membership function M_1 . Also l_2, m_2 and u_2 are the ordinates of the membership function M_2 .

Step 5: Calculate the degree of possibility for a convex fuzzy number to be greater than k convex fuzzy number

In this step we have to calculate the value of V . These values will be calculated by using the following formula:

$$V(S \geq S_1, S_2, \dots, S_k) = \min V(S \geq S_i), \text{ where } i = 1, 2, \dots, k; \quad (14)$$

Step 6: Calculate the weight vector and normalize the non-fuzzy weight vector

Now we use the following formula:

$$d'(A_i) = \min V(S_i \geq S_k), i, k = 1, 2, \dots, n; k \neq i; \quad (15)$$

$$W' = \{d'(A_1), d'(A_2), \dots, d'(A_n)\}^T; \quad (16)$$

3.3.3. CONSISTENCY CHECK

In this step we have to check the whether the calculated values of weights are usable or not. For this we have to take the same pair-wise comparison matrix which is not normalized. First we have to calculate Consistency Index (C.I) by the following formula:

$$\text{Consistency Index (C.I)} = \frac{\lambda_{\max} - n}{n-1} \quad (17)$$

$$\text{Where } \lambda_{\max} = \frac{\sum_{i=1}^n \frac{WSV_i}{CW_i}}{n} \quad (18)$$

and n is the number of compared elements. Here *WSV* & *CW* stands for Weighted Sum Value and Criteria weights respectively.

Now we have to calculate the Consistency Ratio (C.R) by the following formula:

$$\text{Consistency Ratio} = \frac{\text{Consistency Index (C.I)}}{\text{Random Index (R.I)}} \quad (19)$$

We will take the value of RI from provided table of RI.

If we find the C.R value less than 0.10, pairwise matrix is reasonably consistent. Here 0.10 is the standard value as AHP allows up to 10% inconsistency. If the inconsistency is more than 10%, then matrix needs to be reconsidered. If the check of consistency is ok, the criteria weights can be used by the decision maker for their further calculations. So the pairwise comparison matrix is reasonably consistent. Otherwise we have to reconstruct the pairwise comparison matrix.

4. NUMERICAL CALCULATIONS

Firstly we have to define a Scale of Relative Importance according to degree of importance. In Table 3 it is defined.

Table 3. Scale of Relative Importance

Degree of importance	Definition
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2, 4, 6, 8	Intermediate values
$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$	Values for inverse comparison

Source: (Saaty, 1980)

Now Pair-wise comparison matrix is created with the help of “Scale of Relative Importance”. The Pair-wise comparison matrix gives relative importance of different criteria with respect to goal. Here we have 4×4 matrix as we have 4 criteria.

To calculate the value in each cell:

- Now what will be the value in (PEG, PB) cell?

To the experts PEG is “strong importance” than PB (It may vary person to person). If PB is x , then PEG is $5x$.

$$\text{Value (PEG, PB)} = \frac{\text{Row element}}{\text{Coloumn element}} = \frac{5x}{x} = 5$$

$$\text{Value (PB, PEG)} = \frac{1}{5}$$

- Now what will be the value in (PEG, DE) cell?

To the experts PEG is “moderate to strong importance” than DE (It may vary person to person). If DE is x , then PEG is $4x$.

$$\text{Value (PEG, DE)} = \frac{\text{Row element}}{\text{Coloumn element}} = \frac{4x}{x} = 4$$

$$\text{Value (DE, PEG)} = \frac{1}{4}$$

- Now what will be the value in (DE, PB) cell?

To the experts DE is “equal to moderate importance” than PB (It may vary person to person). If PB is x , then DE is $2x$.

$$\text{Value (DE, PB)} = \frac{\text{Row element}}{\text{Coloumn element}} = \frac{2x}{x} = 2$$

$$\text{Value (PB, DE)} = \frac{1}{2}$$

Similarly it can be assigned the values to each cell. In Table 4 the complete pair-wise comparison matrix is presented.

Table 4. Pair-wise Comparison Matrix

Criteria	PEG	PB	DE	DY
PEG	1	5	4	7
PB	$\frac{1}{5}$	1	$\frac{1}{2}$	3
DE	$\frac{1}{4}$	2	1	3
DY	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{3}$	1

Source: Own calculation

In the context of relative importance scales, crisp numeric values such as 1, 3, 5, 7, and 9 are commonly observed. In Fuzzy Logic, these precise numerical values undergo a transformation into Fuzzy numbers. This involves the conversion of linguistic terms into membership functions, where the triangular segment within the given figure is recognized as the triangular membership function. This particular membership function assumes a triangular shape, hence its name.

It is important to note that Triangular Membership Function is just one type among various others, including Trapezoidal membership function, Bell-shaped membership function, etc. The Fuzzy value is generally represented as $\mu_A(x) = \tilde{A} = (1, 2, 3)$, here 1, 2, 3 together known as Fuzzy number which is associated with the membership function. This approach addresses the limita-

tion of assigning a single numeric value to a term. For instance, designating “Moderate” with the value 3 may not adequately capture the nuances between 2.5 and 3.5. The introduction of Fuzzy numbers resolves this issue by accommodating a range of values, allowing for a more nuanced and flexible representation of linguistic terms in the Fuzzy Logic framework. In the Table 5 Fuzzy Scale of Relative Importance is defined. Here the crisp numbers like 1, 3, 5, 7 and 9 is replaced with the Fuzzy number.

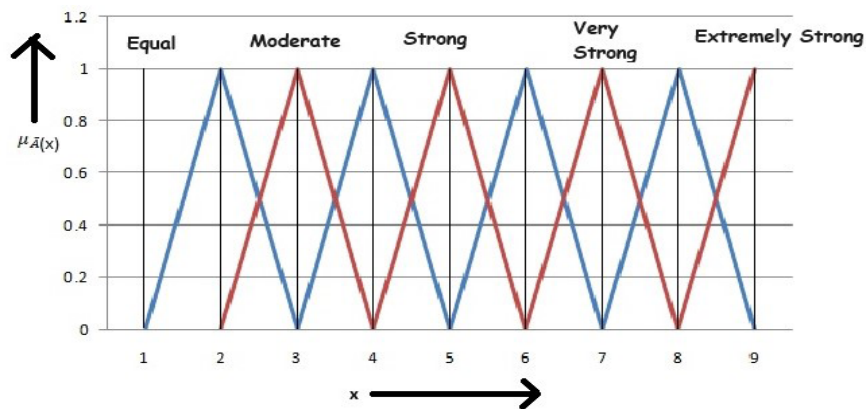
Table 5. Fuzzy Scale of Relative Importance

Linguistic Terms	Fuzzy Numbers
<i>Equal</i>	<i>(1, 1, 1)</i>
<i>Moderate</i>	<i>(2, 3, 4)</i>
<i>Strong</i>	<i>(4, 5, 6)</i>
<i>Very Strong</i>	<i>(6, 7, 8)</i>
<i>Extremely Strong</i>	<i>(9, 9, 9)</i>
<i>Intermediate Values</i>	<i>(1, 2, 3)</i>
	<i>(3, 4, 5)</i>
	<i>(5, 6, 7)</i>
	<i>(7, 8, 9)</i>

Source: [Zavadskas et al. \(2020\)](#)

In the Figure 3 diagram of Fuzzy Scale of Relative Importance sketched.

Figure 3. Diagram of Fuzzy Scale of Relative Importance



Source: [\(Hsieh et al. 2015\)](#)

In the Table 6 Crisp & Fuzzy Scale of Relative Importance represented together.

Table 6. Crisp & Fuzzy Scale of Relative Importance together

Linguistic Terms	Crisp Numbers	Fuzzy Numbers
<i>Equal</i>	1	(1, 1, 1)
<i>Moderate</i>	3	(2, 3, 4)
<i>Strong</i>	5	(4, 5, 6)
<i>Very Strong</i>	7	(6, 7, 8)
<i>Extremely Strong</i>	9	(9, 9, 9)
<i>Intermediate Values</i>	2	(1, 2, 3)
	4	(3, 4, 5)
	6	(5, 6, 7)
	8	(7, 8, 9)

Source: Zavadskas et al. (2020)

Table 7 replacing the crisp numeric value to the Fuzzy numbers in the Pair-wise comparison matrix to get the Fuzzified Pair-Wise comparison matrix.

Table 7. Fuzzified Pair-Wise comparison matrix

Criteria	PEG	PB	DE	DY
PEG	(1, 1, 1)	(4, 5, 6)	(3, 4, 5)	(6, 7, 8)
PB	$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	(1, 1, 1)	$(\frac{1}{3}, \frac{1}{2}, \frac{1}{1})$	(2, 3, 4)
DE	$(\frac{1}{5}, \frac{1}{4}, \frac{1}{3})$	(1, 2, 3)	(1, 1, 1)	(2, 3, 4)
DY	$(\frac{1}{8}, \frac{1}{7}, \frac{1}{6})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	(1, 1, 1)

Source: Own calculation

4. 1. CALCULATIONS WITH GMM F-AHP

Geometric Mean Method of Fuzzy AHP (GMM F-AHP) is used to calculate the weights of the criteria.

$\tilde{P} \otimes \tilde{Q} = (l_1 \otimes l_2, m_1 \otimes m_2, u_1 \otimes u_2)$ where $l_1 \geq 0, l_2 \geq 0$. This is equation used to multiply two fuzzy numbers. Table 8 calculates the Fuzzy Geometric Mean Value using Equation (7).

Table 8. Calculation of Fuzzy Geometric Mean Value

Criteria	PEG	PB	DE	DY	Fuzzy Geometric Mean Value ($\tilde{r}_i$)
PEG	(1, 1, 1)	(4, 5, 6)	(3, 4, 5)	(6, 7, 8)	(2.91, 3.44, 3.94)
PB	$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	(1, 1, 1)	$(\frac{1}{3}, \frac{1}{2}, \frac{1}{1})$	(2, 3, 4)	(0.58, 0.74, 1)
DE	$(\frac{1}{5}, \frac{1}{4}, \frac{1}{3})$	(1, 2, 3)	(1, 1, 1)	(2, 3, 4)	(0.80, 1.11, 1.41)
DY	$(\frac{1}{8}, \frac{1}{7}, \frac{1}{6})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	(1, 1, 1)	(0.30, 0.35, 0.45)

Source: Own calculation

$$\tilde{r}_1 = \left((1 * 4 * 3 * 6)^{\frac{1}{4}}, (1 * 5 * 4 * 7)^{\frac{1}{4}}, (1 * 6 * 5 * 8)^{\frac{1}{4}} \right) = (2.91, 3.44, 3.94) \text{ and so on.}$$

Next the Fuzzy weights for every criterion are calculated in Table 9 using the Equation (8) i.e.

$$\tilde{w}_i = \tilde{r}_i \otimes \{\tilde{r}_1 \oplus \tilde{r}_2 \oplus \dots \oplus \tilde{r}_n\}^{-1}$$

Table 9. Calculation of Fuzzy weights for every criterion

Criteria	Fuzzy Geometric Mean Value ($\tilde{r}_i$)	Fuzzy Weights ($\tilde{w}_i$)
PEG	(2.91, 3.44, 3.94)	$(2.91, 3.44, 3.94) \otimes \left(\frac{1}{6.80}, \frac{1}{5.64}, \frac{1}{4.58}\right) = (0.43, 0.61, 0.86)$
PB	(0.58, 0.74, 1)	$(0.58, 0.74, 1) \otimes \left(\frac{1}{6.80}, \frac{1}{5.64}, \frac{1}{4.58}\right) = (0.08, 0.13, 0.22)$
DE	(0.80, 1.11, 1.41)	$(0.80, 1.11, 1.41) \otimes \left(\frac{1}{6.80}, \frac{1}{5.64}, \frac{1}{4.58}\right) = (0.12, 0.20, 0.31)$
DY	(0.30, 0.35, 0.45)	$(0.30, 0.35, 0.45) \otimes \left(\frac{1}{6.80}, \frac{1}{5.64}, \frac{1}{4.58}\right) = (0.04, 0.06, 0.09)$

Source: Own calculation

$$(\tilde{r}_1 \oplus \tilde{r}_2 \oplus \tilde{r}_3 \oplus \tilde{r}_4)^{-1} = \left(\frac{1}{6.80}, \frac{1}{5.64}, \frac{1}{4.58}\right)$$

$$\tilde{A}_1 \otimes \tilde{A}_2 = (l_1, m_1, u_1) \otimes (l_2, m_2, u_2) = (l_1 * l_2, m_1 * m_2, u_1 * u_2)$$

Now we have to De-Fuzzify these four Fuzzy Numbers to get crisp numerical values. Table 10 shows the value of crisp weight after doing De-Fuzzification by using Equation (10) i.e.

Centre of Area (COA) = $w_i = \left(\frac{l+m+u}{3}\right)$ and Equation (10) i.e. both simultaneously. Then took mean of COA & BNP.

Table 10. Crisp Weight after De-Fuzzification

Criteria	Fuzzy Weights ($\tilde{w}_i$)	Crisp Weight (w_i) (COA)	Crisp Weight (w_i) (BNP)	Mean $w_i = \frac{COA+BNP}{2}$
PEG	(0.43, 0.61, 0.86)	0.633	0.633	0.633
PB	(0.08, 0.13, 0.22)	0.145	0.143	0.144
DE	(0.12, 0.20, 0.31)	0.207	0.210	0.208
DY	(0.04, 0.06, 0.09)	0.068	0.063	0.066
				<i>Total</i> = 1.058

Source: Own calculation

$$COA_{PEG} = \frac{0.43 + 0.61 + 0.86}{3} = 0.633 \text{ and } BNP_{PEG} = \frac{[(0.86 - 0.43) + (0.61 - 0.43)]}{3} + 0.43 = 0.633$$

Now in the Table 10 we can see that the total criteria weight is 1.058 which is not acceptable as it is greater than 1. So the weights are generally normalized to get the total weights 1.

Table 11. Normalized Weight

Criteria	Crisp Weight (w_i)	Normalized Weight
PEG	0.633	$\frac{0.633}{1.058} = \mathbf{0.640}$
PB	0.144	$\frac{0.144}{1.058} = \mathbf{0.162}$
DE	0.208	$\frac{0.208}{1.058} = \mathbf{0.170}$
DY	0.066	$\frac{0.066}{1.058} = \mathbf{0.028}$
	<i>Total</i> = 1.058	<i>Total</i> = 1

Source: Own calculation

In the Table 11 the weights are normalized by dividing each weight by the total weight i.e. 1.058. Now the sum of the normalized weights is 1.

4. 2. CALCULATIONS WITH EAM F-AHP

First we have to choose the pairwise comparison matrix as represented in Table 7.

Firstly we have to calculate the value of $\sum_{j=1}^n \tilde{a}_{ij}$ as shows in Table 12.

Table 12. Calculation of $\sum_{j=1}^n \tilde{a}_{ij}$

Criteria	PEG	PB	DE	DY	$\sum_{j=1}^n \tilde{a}_{ij}$
PEG	(1, 1, 1)	(4, 5, 6)	(3, 4, 5)	(6, 7, 8)	(14, 17, 20)
PB	$(\frac{1}{6}, \frac{1}{5}, \frac{1}{4})$	(1, 1, 1)	$(\frac{1}{3}, \frac{1}{2}, \frac{1}{1})$	(2, 3, 4)	(3.50, 4.70, 6.25)
DE	$(\frac{1}{5}, \frac{1}{4}, \frac{1}{3})$	(1, 2, 3)	(1, 1, 1)	(2, 3, 4)	(4.20, 6.25, 8.33)
DY	$(\frac{1}{8}, \frac{1}{7}, \frac{1}{6})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	$(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$	(1, 1, 1)	(1.62, 1.81, 2.17)
					$\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij} = (23.32, 29.76, 36.75)$
					$\left[\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij} \right]^{-1} = \left(\frac{1}{36.75}, \frac{1}{29.76}, \frac{1}{23.32} \right)$

Source: Own calculation

Now we have to calculate the fuzzy synthetic extent as shown in Table 13 with respect to alternative by using Equation (12).

Table 13. Calculation of Fuzzy Synthetic Extent

Criteria	$\sum_{j=1}^n \tilde{a}_{ij}$	$\sum_{j=1}^n \tilde{a}_{ij} \left[\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij} \right]^{-1}$	S_i (Fuzzy Synthetic Extent)
PEG	(14, 17, 20)	$(14, 17, 20) \otimes \left(\frac{1}{36.75}, \frac{1}{29.76}, \frac{1}{23.32} \right)$	$S_1 = (0.38, 0.57, 0.86)$
PB	(3.50, 4.70, 6.25)	$(3.50, 4.70, 6.25) \otimes \left(\frac{1}{36.75}, \frac{1}{29.76}, \frac{1}{23.32} \right)$	$S_2 = (0.09, 0.16, 0.27)$
DE	(4.20, 6.25, 8.33)	$(4.20, 6.25, 8.33) \otimes \left(\frac{1}{36.75}, \frac{1}{29.76}, \frac{1}{23.32} \right)$	$S_3 = (0.11, 0.21, 0.36)$
DY	(1.62, 1.81, 2.17)	$(1.62, 1.81, 2.17) \otimes \left(\frac{1}{36.75}, \frac{1}{29.76}, \frac{1}{23.32} \right)$	$S_4 = (0.04, 0.06, 0.09)$

Source: Own calculation

Therefore $l_1 = 0.38, m_1 = 0.57, u_1 = 0.86, l_2 = 0.09, m_2 = 0.16, u_2 = 0.27,$
 $l_3 = 0.11, m_3 = 0.21, u_3 = 0.36, l_4 = 0.04, m_4 = 0.06, u_4 = 0.09$

Now we are going to calculate the degree of possibility using equation (13). Here we need to calculate the twelve degree of possibility as we have four criteria. They are

$$V(S_1 \geq S_2) = 1; \text{ Since } m_1 > m_2$$

$$V(S_1 \geq S_3) = 1; \text{ Since } m_1 > m_3$$

$$V(S_1 \geq S_4) = 1; \text{ Since } m_1 > m_4$$

$$V(S_2 \geq S_1) = 1; \text{ Since } l_1 > u_2$$

$$V(S_2 \geq S_3) = 0.197$$

$$V(S_2 \geq S_4) = 1; \text{ Since } m_2 > m_4$$

$$V(S_3 \geq S_1) = 0.401$$

$$V(S_3 \geq S_2) = 1; \text{ Since } m_3 > m_2$$

$$V(S_3 \geq S_4) = 1; \text{ Since } m_3 > m_4$$

$$V(S_4 \geq S_1) = 1; \text{ Since } l_1 > u_4$$

$$V(S_4 \geq S_2) = 1; \text{ Since } l_2 > u_4$$

$$V(S_4 \geq S_3) = 0.181$$

Now we have to calculate the value of $V(S_1 \geq S_2, S_3, S_4), V(S_2 \geq S_1, S_3, S_4), V(S_3 \geq S_1, S_2, S_4),$
 $V(S_4 \geq S_1, S_2, S_3)$ by using equation (14)

$$V(S_1 \geq S_2, S_3, S_4) = 1; \text{ Since the minimum value of } V(S_1 \geq S_2) = 1, V(S_1 \geq S_3) = 1 \text{ and } V(S_1 \geq S_4) = 1 \text{ is } 1$$

$$V(S_2 \geq S_1, S_3, S_4) = 0.197; \text{ Since the minimum value of } V(S_2 \geq S_1) = 1, V(S_2 \geq S_3) = 0.197 \text{ and } V(S_2 \geq S_4) = 1 \text{ is } 0.197$$

$$V(S_3 \geq S_1, S_2, S_4) = 0.401; \text{ Since the minimum value of } V(S_3 \geq S_1) = 0.401, V(S_3 \geq S_2) = 1 \text{ and } V(S_3 \geq S_4) = 1 \text{ is } 0.401$$

$$V(S_4 \geq S_1, S_2, S_3) = 0.181; \text{ Since the minimum value of } V(S_4 \geq S_1) = 1, V(S_4 \geq S_2) = 1 \text{ and } V(S_4 \geq S_3) = 0.181$$

Now by applying Equation (15) and Equation (16) we have to calculate the weight vector and normalize the non-fuzzy weight vector.

$$\text{Therefore } W' = \left(\frac{1}{1.779}, \frac{0.197}{1.779}, \frac{0.401}{1.779}, \frac{0.181}{1.779} \right)^T = (0.562, 0.111, 0.225, 0.102)^T$$

Table 14. Non-Fuzzified Criteria Weights

Criteria	Non Fuzzified Weight (w_i)
PEG	0.562
PB	0.111
DE	0.225
DY	0.102

Source: Own calculation

4. 3. CONSISTENCY CHECK

In this step we are able to check the whether the calculated values are usable or not. For this we have to take the same pair-wise comparison matrix which is not normalized as shown in Table 15.

Table 15. Pair-wise comparison matrix in decimal form

Criteria	PEG	PB	DE	DY
PEG	1		4	7
PB	0.2	1	0.5	3
DE	0.25	2	1	3
DY	0.14	0.33	0.33	1

Source: Own calculation

Mean Weights can be obtained by taking simple average of the criteria weights of GMM F-AHP and EAM F-AHP value as shown in the Table 16.

Table 16. Mean Criteria Weights

Criteria	Criteria Weights by GMM F-AHP	Criteria Weights by EAM F-AHP	Mean Weights
PEG	0.640	0.562	0.601
PB	0.162	0.111	0.138
DE	0.169	0.225	0.197
DY	0.028	0.102	0.065

Source: Own calculation

Next step is to the multiplication of weights in each respective column as shown in Table 17.

Table 17. Pair-wise comparison matrix after multiplied by weights

Criteria Weights	0.601	0.138	0.197	0.065
Criteria	PEG	PB	DE	DY
PEG	1*0.601	*0.138	4*0.197	7*0.065
PB	0.2*0.601	1*0.138	0.5*0.197	3*0.065
DE	0.25*0.601	2*0.138	1*0.197	3*0.065
DY	0.14*0.601	0.33*0.138	0.33*0.197	1*0.065

Source: Own calculation

In this step of Table 18 we have to calculate the ratio of Weighted Sum Values (WSV) and Criteria Weights (CW).

Table 18. Calculation of the ratio of WSW & CW

Criteria	PEG	PB	DE	DY	Weighted Sum Values (WSV)	Criteria Weights (CW)	$\frac{WSV}{CW}$
PEG	0.601	0.682	0.783	0.447	2.517	0.601	$\frac{2.517}{0.601} = 4.168$
PB	0.121	0.138	0.098	0.192	0.547	0.138	$\frac{0.547}{0.138} = 4.007$
DE	0.151	0.273	0.197	0.192	0.812	0.197	$\frac{0.812}{0.197} = 4.144$
DY	0.084	0.045	0.065	0.065	0.258	0.065	$\frac{0.258}{0.065} = 4.038$

Source: Own calculation

$0.601+0.682+0.783+0.447=2.517$ & so on

Using Equation (18),

Using Equation (19), Consistency Index (C.I) = , where n is the number of compared elements. In this case n = 4 as we have 4 criteria.

Using Equation (19), Consistency Ratio =

We took the value of RI is equal to 0.90 for 4 criteria as shown in Table 19.

Here 0.10 is the standard value as AHP allows up to 10% inconsistency. If the inconsistency is more than 10%, then matrix needs to be reconsidered.

Table 19. Random Index Table up to 10 Criteria

Number of Criteria	1	2	3	4	5	6	7	8	9	10
Value of RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Source: (Saaty, 1980)

We can assume that the matrix is reasonably consistent. So we make continue with the process of decision making using AHP.

Table 20. Percentage of importance of Criteria Weights

Criteria	Criteria Weights	% of Importance (Rounded off)
PEG (Price Earning to Growth)	0.601	60%
PB (Price to Book Value)	0.138	14%
DE (Debt Equity Ratio)	0.197	20%
DY (Dividend Yield)	0.065	6%

Source: Own calculation

This percentage of criteria weights according to the importance as shown in the Table 20 can be used by the decision maker for their further investment decision making in the future.

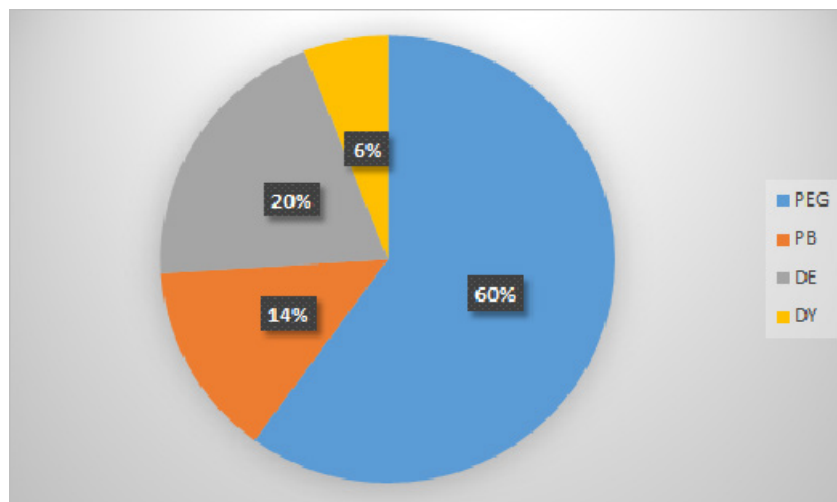
5. FINDINGS

The result shows that

- (i) The Price Earning to Growth (PEG) which carries maximum 60% weightage is the most important financial indices for investment point of view.
- (ii) The second most important financial Index is Debt Equity Ratio with 20% weightage.
- (iii) Price to Book Value and Dividend Yield has 14% and 6% of weightage respectively.

Priority percentage of criteria are represented in Figure 4.

Figure 4. Priority percentage of criteria



Source: Own creation

6. CONCLUSIONS

Based on our comprehensive study, it is evident that the Price Earning to Growth (PEG) ratio, allocated with the highest weightage of 60%, stands out as the most crucial financial index from an investment perspective. This indicates that investors should place significant emphasis for earnings. Following is the Debt Equity Ratio, holding a substantial 20% weightage. This signifies the importance of assessing a company's financial leverage and its ability to meet its financial obligations. A lower Debt Equity Ratio is generally considered more favourable as it suggests a lower level of financial risk. Price to Book Value, with a weightage of 14%, is the third most influential financial index in our analysis. This ratio is indicative of the market's valuation of a company in relation to its book value, providing insights into the underlying assets and liabilities. Finally, Dividend Yield, though not as heavily weighted, still holds significance with a 6% weightage. This metric provides valuable information to income-seeking investors, highlighting the return on investment through dividend pay-outs.

This study emphasizes the paramount importance of considering the Price Earning to Growth (PEG) ratio when making investment decisions, followed by the Debt Equity Ratio. Price to Book Value and Dividend Yield, while relevant, carry comparatively less weightage in the overall assessment. Investors are advised to use these insights as a guideline in their financial analysis and decision-making processes.

6. 1. LIMITATIONS

This study is

- Limited to four financial indices or ratios only.
- Bounded with six financial advisors only.
- Restricted to Triangular Fuzzy Numbers (TFN) only.
- Used a specific MCDM technique i.e. AHP technique.

6. 2. FUTURE SCOPES

The Study

- Has considered four financial indices only, other indices can be taken for analysis in future.
- Is bounded with six financial advisors only. The number financial advisors can be increased to get more precise result in future.
- Can be carried out using pentagonal fuzzy numbers, Hexagonal fuzzy numbers, Pythagorean fuzzy numbers etc.
- Has used one of the methods of MCDM technique for weightage calculations i.e. AHP, further work can be done using DEMATEL.

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