

Aristotle's Notion of Deduction

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Abstract

Aristotle's notion of deduction (syllogism) differs from the conception of logical consequence in classical logic in two essential features, which are required by Aristotle's definition of syllogism and are incorporated into his formalisation of deduction: in addition to the standard necessary truth-preservation, Aristotle requires relevance of premises for the conclusion and non-repetition of premises in the conclusion. These requirements, together with Aristotle's conception of simple propositions, lead to the result that valid deductive steps (syllogisms) must have very specific forms, namely the well-known syllogistic shape. All other kinds of deduction lacking this shape, such as "syllogisms based on a hypothesis", can be considered "syllogisms" only in a relative sense: they are based on an assumption of the existence of genuine syllogistic deductions in the syllogistic shape. Aristotle's demands should cover all kinds of deduction: all valid deduction must be relevant and non-repetitive. This brings Aristotle's definition much closer to the intuition associated with the notion of logical consequence.

Keywords

Aristotle's deduction; logical consequence; relevance; syllogism; syllogism from a hypothesis

1 Introduction

Aristotle's conception of deduction differs quite significantly from that of classical logic. Among other things, I want to demonstrate two important points: (1) Aristotle requires that the premises in a deductive step be relevant to the conclusion and that the formal structure of valid inference steps reflect this requirement. All deductive steps should

thus have a specific syllogistic structure. Therefore, (2) all other apparent deductive steps lacking proper syllogistic structure are acceptable only as an abbreviated formulation of an argument composed of proper syllogisms. What is important here is that Aristotle considers conditional statements to express entailment rather than some combination of the truth values of the antecedent and the consequent.

Although “[t]he literature on Aristotle’s Syllogistic of the past 50 years has frequently suggested that Aristotle’s logic bears a similarity to modern relevance logic (...), it is surprising to see that there is hardly any attempt to give an argument for it”, as Philipp Steingkuger [2015: 1414] writes in the introduction to his very thorough analysis. However, even he does not want to “try to give an answer to the interesting question whether Aristotle had the intention or motivation to incorporate relevance into his syllogistic”, but only to try to evaluate “the thesis that Aristotle’s logic shows similarity to modern relevance logic”. In contrast to these analyses, I do not intend to look in detail for similarities between modern relevance logics and Aristotle’s conception, my aim is to emphasize that the request of relevance is an essential feature of Aristotle’s notion of deduction, a crucial part of his definition of deduction, and is incorporated into the formal structure of valid inference steps. Aristotle tries to show that only syllogistically formulated deductive steps formally satisfy the requirement of relevance of premises. Awareness of this fundamental requirement of relevance makes Aristotle’s conception much more plausible and consistent, and also more comprehensible to contemporary scholars brought up on classical logic.

Point (2) is a variant of the same issue: for Aristotle (as for many others), a conditional expresses *conditionality*, a connection between what is stated by the antecedent and what is stated by the consequent. It expresses consequence rather than a truth-functional connection as it is understood in classical logic. Although some scholars have pointed to this understanding of conditionals, little emphasis is placed on the fact that it makes the idea of the further analysing of conditionals and hypothetical arguments by means of categorical syllogisms perfectly consistent, and thus makes also consistent Aristotle’s claim that all deduction proceeds in syllogistic form. Although Aristotle does not state this further analysability of hypothetical arguments as explicitly as the requirement of relevance, the fact that it makes Aristotle’s conception much more plausible and elegant argues in favour of this interpretation of his somewhat sketchy comments.

2 Logical consequence

The notion of logical consequence is a crucial notion of logic and there have been various attempts to define it. An apt definition should capture the intuitive notion of logical consequence as accurately as possible: the definition should be met, ideally, by all and only those arguments that are intuitively considered valid. The accuracy of the definition is also highly important because another crucial notion of logic, namely the notion of logical derivability, is based on the notion of logical consequence. A derivability system which does not capture the validity of as many natural language arguments as possible is very limited.

In classical logic, logical consequence is standardly defined as “necessary truth preservation in virtue of form” (in the sense that it is, in virtue of form, “impossible for the premises to be true and the conclusion to be untrue” Beal & Restall [2019: 1; 3.1])—let us call this definition the ‘C-definition’. However, this definition has known, counterintuitive implications: every conclusion which is necessarily true in virtue of its form follows, according to the C-definition, from any premises; and premise(s) that are necessarily false in virtue of their form imply any conclusion (“paradoxes of strict implication”: *necessarium ad quodlibet sequitur; ex impossibili quodlibet sequitur*).¹ These implications of the definition do not correspond to common understanding: the proposition ‘It rains or it does not rain’ is standardly not considered to follow from ‘Two is an odd number’, nor the proposition ‘Every elephant can fly’ from ‘Venus is not Venus’; it seems to go against basic intuitions connected with the notion of consequence.

The C-definition thus seems to ignore an important intuition: besides the necessity in virtue of form (it cannot be the case that the premises are true and the conclusion false, whatever their content is) required by the C-definition, what is expected in case of valid inference in ordinary discourse is a certain connection between the content of the premises and the content of the conclusion—the conclusion is considered true *on the basis that* premises are true, or *because* premises are true. This is sometimes expressed by means of the notion of *relevance*: in a valid argument, the premises should be relevant (pertinent) to the conclusion. The requirements that the necessary truth preservation should be only in virtue of *form*, and that there should be, at the same time, some connection between the

¹ Cf. Read [2012: 19–20]: “It is well known that such a theory of validity has some unintuitive consequences. For example, it follows that all arguments with a necessarily true conclusion, and all those with inconsistent premises, are valid”.

content of the premises and of the conclusion may seem incompatible. However, this demand much better captures the intuition connected with the notion of logical consequence.

3 Aristotle's definition of syllogism

Let us now look at Aristotle's definition of deduction and compare it with the above C-definition of logical consequence:

A deduction [*syllogismos*] is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so. I mean by the last phrase that it follows because of them, and by this, that no further term is required from without in order to make the consequence necessary. [An. Pr. 24b19–22]

Let us call it the 'A-definition'.

The A-definition works with the notion of necessity ("follows of necessity"), but there is nothing about (logical) form. What is expressed in the A-definition additionally to the C-definition is the request of relevance: the conclusion necessarily should be true not only whenever the premises are true, but its truth should be, so to speak, the *result* of the truth of the premises ("follows of necessity *from their being so*", "it follows *because of* them [premises]").² The content of the conclusion thus cannot be independent of the content of premises. And since premises should be relevant to the conclusion, irrelevant premises should not be part of a valid argument.³

The A-definition also contains another requirement that has not yet been mentioned: the requirement that the conclusion should be "something other than what is stated". It may sound a bit odd: from the point of view of classical logic, arguments of the type 'if

² This 'because of' does not have to express some kind of causality (which Aristotle requires in the case of *demonstration*), it probably means that there is an "ontological" connection between the facts stated in the premises and the fact stated in the consequence, that "the validity of an argument was thought to be due to a real relation that holds between the facts referred to in the argument, if the premises are true. An argument, on this view, would be valid because, given that what is asserted by the premises is in fact the case, what is asserted by the conclusion cannot but be the case, too" [Frede 1987: 104–5].

³ Cf. Top. 161b29–30, see also Frede [1987: 116]. Note that although the idea of monotonicity of entailment seems quite natural to modern logic, it is in fact not very intuitive in the case of deduction: what sense does it make to claim that a conclusion is *deduced* from premises that have nothing to do with it?

p , then p' (p/p) or ‘if p and q , then p' ’ ($(p, q)/p$)⁴ are obviously valid. But it is important to notice that Aristotle talks about *deduction*. Does it make sense to say that p can be *deduced* from p ? Could we even call it *deduction* if Sherlock Holmes concluded from the fact that the victim has a wet coat that the victim has a wet coat? There is thus something about this requirement of Aristotle: it corresponds to common intuition about deduction.⁵ Deduction is the inference of a conclusion from premises, not a repetition of the premise(s).

Some interpreters find another requirement “hidden” in the A-definition where the plural is used: “certain *things* being stated...”, namely the requirement of multiple premises. Deduction seems to be possible, on this interpretation of the A-definition, only from more than one premise.⁶

Before stating the A-definition, Aristotle explains what a *proposition* is and which form it can have:

A proposition, then, is a statement affirming or denying something of something; and this is either universal or particular or indefinite. By universal I mean a statement that something belongs to all or none of something; by particular that it belongs to some or not to some or not to all; by indefinite that it does or does not belong, without any mark of being universal or particular. [An. Pr. 24a16–b16]

“Indefinite” propositions are those that lack an explicit quantifier: we must decide whether they are intended to be universal or particular, they are “hidden” universal or particular propositions. There are thus four kinds of simple propositions, namely universal affirmative, universal negative, particular affirmative and particular negative. Whether a proposition is particular or universal is standardly called its quantity, whether it is negative or positive is called its quality.

According to this conception of Aristotle, as every simple proposition affirms or

⁴ I will use $(p_1, p_2)/c$ as an economical notation for the argument:

p_1

p_2

—

c .

⁵ Cf. Kulicki [2020: 4], where he calls this feature ‘non-tautologicality’.

⁶ Cf. e.g. Frede [1987: 114], or Smith [2022: 3.2].

denies “something of something”, it contains two terms,⁷ which are traditionally called ‘subject’ and ‘predicate’. The situation is different in the logical formalisation of classical logic. Although some atomic formulas also contain two “terms”, *i.e.* a name of a monadic predicate (function) and a singular term (as an argument of the function), there are also atomic formulas expressing some relation and containing therefore more “terms”. And quantified propositions, which Aristotle considers simple, are not regarded as atomic in the formalisation of classical logic, although they are not complex in the same sense as other complex propositions (conjunctions, disjunctions, etc.), which can be parsed in more simple propositions.⁸ In any case, in the Aristotelian conception, all simple (including quantified) propositions contain two terms. (It is possible that the subject term and the predicate term of a proposition are one and the same term—*cf.* An. Pr. 63b39–64a6; yet the proposition still contains two distinct positions of the term.)

After the A-definition of deduction, Aristotle adds important rules for “conversions” of propositions:

It is necessary then that in universal attribution the terms of the negative proposition should be convertible, *e.g.* if no pleasure is good, then no good will be pleasure; the terms of the affirmative must be convertible, not however universally, but in part, *e.g.* if every pleasure is good, some good must be pleasure; the particular affirmative must convert in part (for if some pleasure is good, then some good will be pleasure)... [An. Pr. 25a1–13]

It seems that Aristotle considers these conversions to be a kind of variation (or weakening) of *one and the same* proposition, and therefore does not regard the process of converting to be *deduction*; in the inference ‘if no A is B, then no B is A’, the “conclusion” is, more or less,

⁷ It should be emphasized that the Aristotelian “term” (*horos*) is primarily not a language expression, but a part of a proposition; some terms can be expressed by a simple word, some need to be expressed by a complex expression, *cf.* An. Pr. 48a29–39; the same terms can be also denoted by different names, *cf.* Top. 103a6–38.

⁸ In classical logic, a complex sentence *e.g.* ‘ $P(a) \wedge Q(a)$ ’ contains two complete sentences, namely $P(a)$ and $Q(a)$. Quantified sentences, though not atomic, cannot be parsed into more simple sentences. The reason is that, since Tarski, the quantification is usually considered to be objectual, not substitutional. Therefore, a quantified sentence $\forall x P(x)$ cannot be understood as a (very long) conjunction of more simple sentences $P(a) \wedge P(b) \wedge P(c) \wedge \dots$ because it is necessary to take into account all individuals, not only the “named” ones.

the same as the “premise”,⁹ but in a deduction, according to A-definition, the conclusion should be “something other”.¹⁰ In the case of conversion per accidents (*i.e.* ‘if every A is B, then some B is A’) or in the case of simple weakening (‘if every A is B, then some A is B’), the “conclusion” is obviously “less the same” as the “premise”, because it is weaker, but it is still just “a way of restating part of what has been said” [Frede 1987: 114]. (Nevertheless, even if we want to consider weakening as deducing a *new* conclusion, the A-definition also makes the requirement of more than one premise, which is not satisfied in the case of conversion or weakening of a proposition, therefore these cases do not fall under the definition of deduction).

If this, together with the assumption regarding the form of simple propositions, is combined with the A-definition of deduction, it leads to some technical results concerning the form of a valid argument. Conclusion of a deduction has the subject-predicate structure, *i.e.* contains two terms, say A and B. In a valid argument (syllogism), the conclusion should be *different* from the premises; this means, as has just been said, that the conclusion cannot even be a variant (weakening) of any premise. Since, according to the A-definition, a conclusion cannot be properly deduced from only one premise, and each premise should be relevant to the conclusion, no premise can contain both terms A and B.¹¹ However, at the same time, as the premises should be relevant to the conclusion, they should say something about A or about B. There is thus only one possibility: there should be exactly two premises, one premise should contain the term A, the other premise the term B. And as the two premises should be somehow “interconnected” in order to reach the conclusion, there has to be a “middle term” contained in both premises, which enables both premises

⁹ Aristotle even says in one place that “...the proposition that B belongs to no A is identical with the proposition that A belongs to no B.” [An. Pr. 58a27–9]. However, this statement should be understood with some caution and in context, see Frede’s comments on it [1987: 114–5].

¹⁰ Cf. An. Pr. 40b30–41a20; see also, *e.g.*, Frede [1987: 114].

¹¹ Let us consider what the different options are: The conclusion may be a variation or weakening of the premise containing the same terms A and B as the conclusion; then the result is a single-premise argument that is not a syllogism according to the A-definition. Or the conclusion and the premise containing the same terms A and B can be contrary or contradictory; then it obviously cannot be a valid inference. Or the premise itself is not sufficient to ensure the truth of the conclusion, and a second premise is needed (in accordance with the A-definition); the second premise, to be relevant, must also contain both terms A and B; however, if we look at the square of opposition, there is no variant of two (or more) subject-predicate propositions containing the terms A and B that merely jointly ensure the truth of another proposition containing A and B.

to be “combined” into the conclusion.

Aristotle explains the structure of syllogisms and why they should contain three terms¹² in the following way:

If then one wants to deduce that A belongs or does not belong to B, one must assume something of something. If now A should be assumed of B, the proposition originally in question will have been assumed. But if A should be assumed of C, but C should not be assumed of anything, nor anything of it, nor anything else of A, no deduction will be possible. For nothing necessarily follows from the assumption of some one thing concerning some one thing. Thus we must take another proposition as well. If then A be assumed of something else, or something else of A, or something different of C, nothing prevents a deduction being formed, but it will not be in relation to B through the propositions taken. Nor when C belongs to something else, and that to something else and so on, no connexion however being made with B, will a deduction be possible in relation to B. For in general we stated that no deduction can establish the attribution of one thing to another, unless some middle term is taken, which is somehow related to each by way of predication. (...) So we must take a middle term relating to both, which will connect the predications, if we are to have a deduction relating this to that. [An. Pr. 40b30–41a20]

The fact that every syllogism has exactly two premises is therefore not accidental or arbitrary, it is a result of the A-definition of deduction and Aristotle's conception of the structure of propositions. A particular syllogism should be considered much more as a simple “deductive step” than as a kind of complete argument. By every deductive step, a “new” conclusion should be deduced, which is possible only by having exactly two premises of certain forms as input: one premise must contain the subject term of the conclusion, the other premise must contain the predicate term of the conclusion, and both premises must share one middle term.¹³ (Where it should be emphasized, I will call this pattern of deductive steps a ‘proper syllogism’.)

If there is a “syllogism” with more premises than two (the so called *sorites*), it is in

¹² There is the already mentioned possibility that the terms are in fact the same: see An. Pr. 63b39–64a6, where the proposition ‘No science is a science’ is taken as the conclusion of a syllogism. However, even here Aristotle consistently distinguishes the three terms A, B and C (and says: “let B and C stand for science”) – it seems that what matters is the position and the role of the terms, and one and the same term can play multiple roles.

¹³ Cf., for example, Lear [1980: 11–3].

fact, according to Aristotle, not one syllogism, but more syllogisms combined together [An. Pr. 41b36–42a15]—in other words, there are more deductive steps in the inference. Just as it is possible to recognize particular steps in an ordinary proof in classical logic, it is also possible to recognize particular deductive steps (syllogisms) in a process of inference.¹⁴ To give a simple example: consider the premises (P₁) ‘All primes are natural numbers’, (P₂) ‘Every natural number is an integer’ and (P₃) ‘No integer is an irrational number’. The conclusion: (C) ‘No prime is an irrational number’ can be inferred from the premises, but the inference consists of two syllogisms (deductive steps), namely (P₁, P₂)/ (C₁) ‘Every prime is an integer’ (*Barbara*) and (P₃, C₁)/C (*Celarent*).¹⁵

It is therefore needed to have exactly two premises built from exactly three terms to reach a deductive step according to the A-definition. However, only some forms of premises allow a conclusion to be drawn. It is the form (*i.e.* quality and quantity of propositions together with the position of terms) which assures the possibility of deduction, *i.e.* that a conclusion follows necessarily.¹⁶ If the validity is given by the structure of the argument, then it does not depend on the actual state of affairs and is thus necessary; if the validity is not given by the structure, but by the actual state of affairs, then this state could be different and therefore the conclusion does not follow necessarily.¹⁷

Therefore, in the exposition of his syllogistic system, Aristotle does not speak about individual arguments, but only about their structure: it is important whether the premises

¹⁴ For a more technical description see, *e.g.*, Smiley [1973].

¹⁵ *Barbara, Celarent, Darii, ...* are specific mnemonic words used by medieval scholars to encode valid syllogistic schemes.

¹⁶ However, Aristotle does not explicitly speak of ‘forms’ of propositions, and his systematisation of various kinds (forms) of propositions concern the said rather than the exact linguistic-syntactic structure (for example, the form of a proposition is determined by whether it says that “something belongs to all or none of something” [An. Pr. 24a16–b16], which can be expressed linguistically in various ways, not by whether the corresponding sentence contains any linguistic logical constants such as ‘every’ or ‘no’); *cf.*: “Aristotle did not formalize ordinary Greek sentences into some regimented language. He generalizes about arguments by describing the relationship between what their premisses and conclusion say, rather than by describing the syntactic form that the propositions would have to have” [Morison 2011: 182].

¹⁷ *Cf.* “Aristotle does not locate the explanation of the validity of this kind of argument in the meaning of some logical constant (in this case, the word ‘all’, or the symbol ‘a’ in some reconstructions of the system of syllogistic). It is, rather, that he locates the explanation of the validity of such arguments in what the premisses say: if things are to be as the premisses say they are, then such-and-such must be the case” [Morison 2011: 187].

are universal or particular, negative or positive, and whether the middle term appears in the place of the predicate in both premises, or in the place of the subject in both premises, or in the place of the predicate in one premise and the subject in the other premise (Aristotle distinguishes, according to the positions of terms, these three possible combinations—figures).¹⁸ Some combinations lead to a syllogism, *i.e.* a deductive step, and others do not, no matter what specific content all three terms have. What is decisive for the validity is thus only the mutual position of all three terms and general character (form) of all three propositions. The required relevance, *i.e.* that the conclusion is true because premises are true, is assured not by the specific content of the premises and the conclusion, but by their structure (including the fact that some terms have to be shared by a couple of propositions).

Aristotle goes through all possible combinations of the position of the middle term and of the general character of both premises and a conclusion and selects those which represent a valid deductive step (syllogism). Their validity is either evident (the syllogism is *perfect*) or can be shown by their *reduction* to a perfect syllogism.¹⁹ Invalidity of the remaining combinations is proven by showing counterexamples.²⁰ Because a deductive step must have a structure which represents one of the possible combinations, the combinations selected by Aristotle represent *all* possible (valid) deductive steps, and the system is complete.

It is worth repeating that the current demand on logical consequence, namely that the necessary truth preservation is *in virtue of the logical form* of propositions, is not explicitly

¹⁸ The fourth figure, as is well known, was distinguished as separate later.

¹⁹ See, for example, Patzig [1968: 43–87] for a detailed analysis.

²⁰ If two premises of a given structure do not lead to any conclusion, the counterexample consists of two triads of terms which make premises of the same form true and, at the same time, one triad makes false the corresponding universal affirmative conclusion, the other the corresponding universal negative conclusion. If the conclusion of both premises of a given form is sometimes positive, sometimes negative, none of the possible conclusions follow from the premises *of necessity*; therefore, premises with the given form cannot create any syllogism. More precisely: “His [Aristotle’s] method has been called proof by contrasted instances. It would perhaps be better called rejection by contrasted instances. For it consists in the production of two triads of terms such that in each the relations of the extremes to the middle are of the kinds under investigation, though in one the major belongs to all of the minor and in the other to none of the minor. Obviously the first triad is a counter-example to show that the specified relations do not allow for inference to a negative conclusion, whether universal or particular, while the second triad is a counterexample to show that the relations do not allow for inference to an affirmative conclusion, whether universal or particular. Together they suffice to show that the specified relations allow no conclusion at all.” [Kneale & Kneale 1984: 75].

required by Aristotle—this characteristic is simply a consequence of the A-definition. To meet the requirements of relevance and non-repetition, the premises need to have a certain structure, namely one premise needs to contain one term from the consequence and the middle term, the other premise needs to contain the second term from the consequence and the middle term; and to meet the requirement of necessary truth preservation, both premises can have only certain combinations of quantity and quality, as the other combinations enable the premises to be true and the conclusion false (there are counterexamples). So only propositions having a certain specific form can meet all three requirements of the A-definition. The validity of a deduction is therefore given “in virtue of form”, but this fact is not a part of Aristotle’s definition of deduction, it is a *result* of the definition.²¹

Whereas C-definition demands “necessary truth preservation”, the A-definition demands something stronger, namely that if the premises are true, the conclusion must be true *due to* the truth of premises. If the truth of the conclusion has nothing to do with the truth of premises, there is no deduction, even if necessary truth preservation in the sense of classical logic holds. The (potential) truth of premises must be *relevant* to the truth of the conclusion. However, the relevance is assured, in Aristotle’s conception, only by the formal shape of syllogisms. It is sometimes thought that relevance relates to content, and that it is therefore something that cannot be treated in the logic for which the only decisive aspect is the form (of sentences, arguments, etc.), not content.²² But this is not completely true: there is a feature which is associated with content, and yet is captured in formal logic, namely identity of denotation, which is expressed by identity of symbols (constants, variables, predicate symbols, etc.). The formula ‘ $p \rightarrow p$ ’ is considered always true, although its tautologicality depends not only on the structure of the formula, but also on the *content* of the antecedent and the consequent: this content is considered to be the

²¹ Compare Frede [1987: 103]: “And it seems that neither the Stoics nor the Peripatetics ever say that an argument is valid because of its logical form, which would be strange if they actually had thought that the validity had to be explained as being due to the form. And even when it is said that a certain form of argument is valid for every matter (*i.e.*, for every suitable substitution of the letters), this does not seem to be the same as saying that the validity is due to the form.”

²² Cf. “Relevant logicians point out that what is wrong with some of the paradoxes (and fallacies) is that the antecedents and consequents (or premises and conclusions) are on completely different topics. The notion of a topic, however, would seem not to be something that a logician should be interested in—it has to do with the content, not the form, of a sentence or inference. But there is a formal principle that relevant logicians apply to force theorems and inferences to ‘stay on topic’. This is the *variable sharing principle*.” [Mares 2020].

same because the symbol ' p ' is the same. Similarly, the formula ' $a = a$ ' is considered always true. The sameness of content is expressible even in fully formalised logic, because there is a fixed presupposition that one symbol must always have only one and the same *denotatum* (this presupposition is, in fact, not true in natural languages, there are a lot of equivocal or non-denoting expressions).

The A-definition therefore results in the criterion of relevance in a formal shape. Based on its requirement of “novelty” (the conclusion must be different from premises) and relevance, Aristotle can specify some necessary features of the required structure, the logical forms of the premises and the conclusion: two terms have to be shared by a premise and the conclusion, and one term has to be shared by both premises. Aristotle thus connects relevance with formal “sharing non-logical symbols” long before all modern attempts to build relevance logic. And the structure of a proper syllogism also assures that both premises are “used to derive the conclusion”, therefore also this demand of modern relevance logics is incorporated already in Aristotle's logic.²³

Although the A-definition can be understood as primarily defining logical consequence (logically valid arguments), the whole syllogistic system based on this definition should rather be, due to its formality, understood as a system of deduction or logical inference (inference rules).

4 Necessity wider than syllogism

Are proper syllogisms really the only possible pattern of deduction in accordance with A-definition? It seems that in some cases a “new” conclusion can necessarily be true because the premises are true (*i.e.* it seems to meet the A-definition) without having the Aristotelian syllogistic shape. One of Aristotle's own examples of these kinds is the following: (Ex) “if it is necessary that animal should exist, if man does, and that substance should exist, if animal does, it is necessary that substance should exist if man does”. However, according to Aristotle, those arguments are not deduction (“yet the conclusion has not been deduced”), because the propositions “are not in the shape we described” and “this has not however been deduced from the assumptions, but propositions are wanting” [An. Pr. 47a10–39]. Aristotle comments on his examples in the following way:

²³ See a very precise analysis in Steinkrüger [2015].

(N) We are deceived in such cases because something necessary results from what is assumed, since deduction [syllogismos] also is necessary. But that which is necessary is wider than deduction; for every deduction is necessary, but not everything which is necessary is a deduction. [An. Pr. 47a32–35]

Based on this claim of Aristotle, it is sometimes assumed that Aristotle recognizes two kinds of logical inference, namely in proper syllogistic shape and in “non-syllogistic” shape.²⁴ This would mean that the A-definition either allows also non-syllogistic shape of deduction or covers only one kind of inference. However, this interpretation is questionable. Though Aristotle’s formulation “that which is necessary is wider than deduction” as well as some other comments (e.g. An. Pr. 50a17–b4, 45b13–20) may give the impression that he has in mind a notion of inference broader than the syllogistic one, he repeatedly emphasizes that all deduction can be expressed in syllogistic form [An. Pr. 41b1–6, 45b36–46a3].²⁵

It is useful to take a closer look at the whole passage in which Aristotle speaks about necessity “wider than deduction”. At the beginning of it, he talks about the need to clearly express both required premises also in practical argumentation:

First then we must attempt to select the two propositions of the deduction (...) and if both have not been stated, we must ourselves assume the one which is missing. For sometimes men put forward the universal, but do not posit the proposition which is contained in it, either in writing or in discussion: or men put these forward, but omit those through which they are inferred, and invite the concession of others to no purpose. We must inquire then whether anything unnecessary has been assumed, or anything necessary has been omitted, and we must posit the one and take away the other, until we have reached the two propositions; for unless we have these, we cannot reduce arguments put forward in the way described. [An. Pr. 47a10–31]

Aristotle then states the examples, one of them is the above example (Ex). They should thus probably be examples of ill-formulated deduction where something is missing or not in an appropriate “shape”, as it is analysed in the quotation just given. And now Aristotle adds the claim (N) that “we are deceived in such cases because something necessary results from what is assumed”. Therefore, it seems that the main message is not that there are

²⁴ See, for example, Frede [1987: 115].

²⁵ See, for example, Patzig [1968: 132–3].

special kinds of non-syllogistic inference, but that there are arguments which may seem to be deductions (syllogisms), but in fact, some additions or reformulations are needed to become real formulations of deductive steps.

Consider now the example (Ex). Let 'A' mean animal, 'M' man, 'S' substance and 'E' to exist ("be an ens"). Then the (Ex) could be formalized in the following way:

(ExF) If some M is E, then some A is E
 If some A is E, then some S is E

If some M is E, then some S is E.²⁶

Now, it is more apparent that the argument seems to be logically valid (in an intuitive sense; and also in the sense of classical logic, as it is an instance of a valid schema of propositional logic, namely $(\varphi \rightarrow \psi, \psi \rightarrow \chi)/\varphi \rightarrow \chi$). It is presumably an example of what Aristotle mentions as "syllogisms based on a hypothesis"²⁷ [An. Pr. 46a32–b25] and what has been later developed by Theophrastus as hypothetical syllogisms.²⁸ According to Aristotle's claim (N), the argument (Ex) is probably an example of what "necessary results", but is not a real "deduction (syllogism)" in the sense of the A-definition. However, Aristotle does not say that this is a different (broader) kind of deduction, but that this is *not a deduction at all*: "but as yet the conclusion *has not been deduced*; for the propositions are not in the shape we described" [An. Pr. 47a10–31, emphasis mine]. It seems that, according to Aristotle, the proper syllogistic shape "in one of the three figures" is an indispensable attribute of deduction, mere following by necessity is not enough. Aristotle expresses explicitly that "every demonstration and every deduction must be formed by means of the three figures mentioned above", and that "it is evident that deductions *per impossibile* also will be made through these figures. Likewise all the other hypothetical deductions" [An. Pr. 41a21–b6].²⁹

²⁶ The modal aspect of the two premises and the conclusion is omitted in the formalisation on the grounds that (as will hopefully be shown) in Aristotle's conception, the necessity expressed is presumably part of conditional statements that claim some connection between their antecedents and consequents.

²⁷ Susanne Bobzien warns that translating Aristotle's wording 'syllogismos ex hypotheseos' as 'hypothetical syllogism' can be misleading—Bobzien [2002: 360].

²⁸ See Barnes [1985] or Ierodiakonou [2020].

²⁹ Cf. Lear [1980: 11].

Perhaps in the example (Ex), the correct syllogistic deduction *would* be possible to formulate—but for this purpose, it would be necessary to find what is missing and get the argument into a suitable “shape”, according to Aristotle’s aforementioned instructions (such as “we must posit” the necessary one which “has been omitted” [An. Pr. 47a10–31]). But where is the problem with (Ex)? The conclusion follows necessarily, is “new”, and both premises have something to do with it: the argument (Ex) seems to meet the Λ -definition.

Two points are to be mentioned. The first point is how Aristotle considers “syllogism based on a hypothesis” or “from a hypothesis”:³⁰ it is a deduction which is based on some initial assumption; however, though every premise can in a sense be considered an assumption, a “hypothesis” is usually a more complex kind of assumption, which is not expressible by a simple proposition and maybe does not work as a premise at all.

In a typical argument from a hypothesis, the hypothesis states or presupposes some connection between what can be stated by (simple) propositions, for example that something implies something, which can be expressed by means of a conditional (as was in the example (Ex) the first proposition ‘If some M is E, then some A is E’). The conditional presents some preliminary agreement that the consequent would be accepted if the antecedent were proved: “... a preliminary agreement must be reached if one is to accept the conclusion” [An. Pr. 50a29–39]. Susanne Bobzien expresses how Aristotle understands this presupposition in the following way: “One then makes an advance agreement (...) that if something else, say p, has been proved (or is accepted), q needs to be accepted too.” [Bobzien 2002: 369]³¹ In an argument based on a hypothesis, something (p) can be deduced by syllogistic steps as a conclusion, but something (q) is only a result of a presupposition expressed as a

³⁰ Cf. Bobzien [2002: 366–72] or Malink [2020: 10].

³¹ Cf. also Patzig [1968: 155–6]: “According to Aristotle one and the same proposition q can be proved both ‘simply’, that is ‘deictically’, by proving q itself, and also hypothetically, by proving a proposition p and proceeding by means of an implication $p \rightarrow q$ to q . The implication $p \rightarrow q$ need be neither proved nor evident, in which case q is only hypothetically proved—that is, proved for all who are willing to *grant* the implication. If the implication is not explicitly proved but can be assumed as *evidently valid*, we have the case presented by *reductio ad impossibile*. Here the implication which effects the proof has the form $(not-not-q) \rightarrow q$; that q follows from *not-not-q*, is an *evident* hypothesis, and its evidence allows us to regard *reductio ad impossibile*, *hypothetical* though it is, as a *valid* proof of q .”

hypothesis and agreed in advance, and therefore is not deduced.³² Aristotle illustrates how it works with an example:

For instance if a man should suppose that unless there is one faculty of contraries, there cannot be one science, and should then argue that not every faculty is of contraries, e.g. of what is healthy and what is sickly; for the same thing will then be at the same time healthy and sickly. He has shown that there is not one faculty of all contraries, but he has not proved that there is not a science. And yet one must agree. But the agreement does not come from a deduction, but from a hypothesis. This argument cannot be reduced; but the proof that there is not a single faculty can. The latter argument no doubt was a deduction; but the former was a hypothesis. [An. Pr. 50a17–28]

Aristotle probably wants to show by this example that if we suppose ‘if p, then q’, and then deduce p, we can accept q, but this acceptation is based on the *presupposition agreed in advance* (‘if p, then q’), not deduced in the sense of producing a sequence of deductive steps (syllogisms).

And (now we come to the second point), a conditional ‘if p, then q’ is not to be understood as a common premise of a (potential) syllogism. Unlike classical logic, Aristotle does not understand conditionals as simply expressing a combination of truth values of contained propositions (the same is highly probably true for the stoics³³), as it is understood by a logical analysis using material implication.³⁴ A conditional is not a simple proposition, but a complex expressing that something follows from something. In some cases, a conditional can be seen as an “abbreviated” formulation of a (possibly quite complex) deduction: e.g. the conditional ‘If a man is running, then an animal is running’ can be considered a so called *enthymeme*, i.e. a shorter expression of a syllogism, in which one premise is not stated (by adding the premise ‘Every man is an animal’, a *Disamis* syllogism can be formulated). A conditional ‘if p, then q’ is therefore not at the level of (simple) proposition asserting

³² Cf. Ebrey: “We agree in advance that successfully arguing for p is sufficient to establish q (hence agreeing that if p then q) and then go on to prove p using a syllogism. From this we reach our conclusion, q. This conclusion, we are told, is not reached from a syllogism, but rather from a hypothesis (41a38–41, 50a17–9, 50a25–6)” [Ebrey 2015: 196].

³³ See, for example, Mates [1961: 42–51]; Frede [1987: 103]; Bobzien [2003: 85–123]; Kneale & Kneale [1984: 134–5].

³⁴ Nor, for example, does Lewisian strict implication seem suitable for formalizing the Aristotelian conditional.

“something of something” [An. Pr.40b30–41a20], but, so to speak, at the “meta-level” of (possible) deducing; the conditional thus does not represent a common premise.³⁵ This may be the reason why Aristotle does not consider a syllogism based on a hypothesis to be a genuine syllogism,³⁶ but only an argument which could be based on proper syllogism(s), with additional analysis needed:

Thus, if something results from certain assumptions, one should not try to reduce it right away, but first find the two premisses, then divide them into their terms, and take as middle term the one that is said in both premisses; for that the middle term occurs in both premisses is necessary in all the figures. [An. Pr. 47a35–39]

A proper syllogism reveals how what is stated in the conclusion comes about through what is stated by premises (“certain things being laid down, something other than these necessarily comes about through them” [Top. 100a25–101a4]). If it were the case that every A is B and, at the same time, it were the case that every B is C, it is apparent that it would necessarily be the case that every A is C. If, on the other hand, the argument based on a hypothesis ‘If (if every A is B, then every C is D; and every A is B), then every C is D’ should be apparent, there has to be a (possible) fact, a state of affairs corresponding to the hypothesis formulated in the first premise, namely ‘if every A is B, then every C is D’. However, does it really express a state of affairs? The conditional expresses, in fact, *two*

³⁵ Cf. “...Aristotle uses the conditional ‘If it has been shown that there is a single power of contraries then also that the science is the same’ (50a34–5). The antecedent and the consequent of this last conditional speak about what has been shown or proved. The whole conditional therefore seems to be concerned with what inferential steps are allowed. Apparently, it is not used to make a statement (what one says by using it cannot be evaluated as true or false), but to lay down an inferential rule which enables one to move from one inferential step to another” [Crivelli 2011: 147].

³⁶ Suzanne Bobzien [2002: 372] expresses it in the following way: “A syllogism from a hypothesis is thus an argument in which the demonstrandum is not directly the conclusion of a probative syllogism, and is hence not (properly) deduced (*sylogizesthai*), but is inferred indirectly via an agreed hypothesis, which connects the *demonstrandum* with another proposition which in turn is (properly) deduced. Thus Aristotle’s other syllogisms from a hypothesis—whatever else they may be—are certainly not arguments that are valid because of their logical form; nor were they regarded as such”.

states of affairs, and these do not need to have anything in common.³⁷ The conditional states a relation between them, but this relation is presupposed, claimed, not apparent from both the propositions expressed.³⁸ In a genuine syllogism, the conclusion is “given” by both premises, by their “being so”. In an argument from a hypothesis, the conclusion may follow “by necessity” (if both premises are “true”, then necessarily the conclusion is true), but the truth of the conclusion is not simply given by the states of affairs expressed by both premises, because at least one of the premises is not a simple proposition, but a complex presupposition expressing not a state of affairs, but an accepted hypothesis.³⁹

This may be the reason why Aristotle probably does not consider syllogisms from hypothesis to be genuine syllogisms, *i.e.* deductive steps. The “premise” ‘if p, then q’ is not a genuine premise but a presupposition of certain connection between two propositions,⁴⁰ for example that it is possible to deduce from the premise p and one other premise which must be added (like *Every man is an animal* from the previous example) the conclusion q

³⁷ If there is something in common, such as ‘if every A is B, then every C is A’, it is not enough to reveal a genuine deduction step (“when one thing is taken to hold of one thing, nothing follows of necessity” [An. Pr. 40b34–35]). If there are two terms in common, as in the case of conversion (no A is B, therefore no B is A), it is not a deduction, as both propositions seem to express only *one* state of affairs.

³⁸ Cf. “I argue that Aristotle does not allow conditionals, or anything else that links whole propositions, in syllogisms because they do not indicate how the propositions are linked through predications between terms” [Ebrey 2015: 196].

³⁹ Ebrey gives an example showing that we can have a “necessary following” conclusion without seeing the connection between described states of affairs:

“If humans are animals, dogs are mammals.
Humans are animals.

Dogs are mammals.

(...) we are given no indication why dogs’ being mammals is connected to humans’ being animals. That connection is stipulated without any explanation.” [Ebrey 2015: 195-196]

⁴⁰ Cf. for example Crivelli: “Certain formulations occurring in T12 [An. Pr. A50a16–28] appear to confirm what other evidence gave us reason to conjecture: that syllogisms from a hypothesis are not syllogisms. Specifically, in T12 we are told that the conclusions of the inferences under consideration ‘have not been shown through a syllogism, but are all agreed through a compact’ ...” [Crivelli 2011: 144]. Cf. also Glezer [2007: 323–34].

(‘if p, then q’ can be thus considered to be an *enthymeme*).⁴¹ The presupposition is therefore not a claiming “something about something” but a “meta-presupposition” that there is a deduction.⁴²

Imagine that there is a deduction, a syllogism $(\varphi, \chi)/\psi$. If χ is accepted in advance for some reason, for example, if it is a generally known truth, then it can be omitted and the complex proposition $\varphi \rightarrow \psi$ can be used as an abbreviation for the syllogism. Now the argument $(\varphi \rightarrow \psi, \varphi)/\psi$ can be seen as saying something like: we presuppose that there is a deductive way from φ to ψ ; if we can state φ (we are able to prove φ by some deductive step(s)), we should accept ψ as deduced *under the presupposition* of the existence of a deductive way from φ to ψ . Therefore, arguments such as *modus ponens* (as well as other hypothetical arguments) can then be considered “meta-arguments” based on “meta-presuppositions” of the existence of a genuine syllogistic deduction.⁴³

5 Deduction *per impossibile*

A slightly different case is an argument *per impossibile*, which Aristotle also seems to count (at least in some places) among syllogisms from a hypothesis. If a conclusion c should be proved from premises p_1 and p_2 by deduction *per impossibile*, p_1 and p_2 are posited as true

⁴¹ Cf. Alexander from Aphrodisias: “Someone, reaching here, might ask how the account given is still the definition of syllogism if there are also other arguments in which ‘some things being posited something different from what is posited follows by necessity...’. In fact these arguments fail to satisfy the condition ‘because these things are posited’, as was said previously. For the necessity in these arguments is not through the things posited, but derives from the fact that the universal, which is omitted, is true, and when this is added, this argument also becomes a syllogism. A totally hypothetical argument would be distinguished from a syllogism by the word ‘posited’; for it also has its conclusion dependent on what is posited, but not ‘because these things are the case’” [in An. Pr. 350.9–20].

⁴² Cf. the following statement by Smiley (albeit in a slightly different context): “...for to say that Q follows from P is equivalent to saying that there exists a deduction of Q from P (...)” [Smiley 1973: 139].

⁴³ The hypothetical argument (Ex) can therefore be understood as follows: if there is a deductive way from the premise ‘Some man is existing’ to the conclusion ‘Some animal is existing’ (a generally known proposition (p_1) ‘Every man is an animal’ can be added to create a proper syllogism), and if there is a deductive way from the premise ‘Some animal is existing’ to ‘Some substance is existing’ (the premise (p_2) ‘Every animal is a substance’ can be added), then there is a deductive way from the premise ‘Some man is existing’ to the conclusion ‘Some substance is existing’ (‘Every man is a substance’ can be added, which is the conclusion resulting from premises (p_2) and (p_1) in *Barbara* syllogism).

and it is supposed as a “hypothesis” that $\neg c$. Then it is deduced from (say) p_1 and $\neg c$ that $\neg p_2$ by a proper syllogism in an appropriate shape, *i.e.* in one of the three figures. Now because it is “something impossible” that both p_2 and $\neg p_2$ were true, the hypothesis, namely $\neg c$, cannot be true (*cf.* An. Pr. 45a23–b12). Generally, the impossible conclusion can also be something evidently false, as “odd numbers are equal to evens” [An. Pr. 41a21–b6]. The point is that in an argument *per impossibile*, a proper syllogism is also used, but its conclusion is something impossible or relatively impossible regarding the assumptions: “For this we found to be deducing *per impossibile*, *viz.* proving something impossible by means of a hypothesis conceded at the beginning” [An. Pr. 41a21–b6]. This leads to the (desired) rejection of the hypothesis $\neg c$. The deduction (deductive steps, this means proper syllogisms) is performed also in this case in the three figures.

Compared to the argument from a hypothesis, it is even more apparent that the argument *per impossibile* as a whole is not a proper deduction in the sense that it would show how a (new) conclusion “necessarily comes about through” premises, *i.e.* “certain things being laid down”. The structure of the argument *per impossibile* is more complicated: if it were so-and-so (hypothesis), something impossible would be deduced by a proper syllogism; therefore, it is not so-and-so. The direct deduction of the desired conclusion from the given premises is simply not shown; what are presupposed in an argument *per impossibile* are not premises, but the existence of a syllogism proving $\neg p_2$ from $\neg c$ and p_1 .⁴⁴ As, according to Aristotle, “Everything which is concluded probatively can be proved *per impossibile*, and that which is proved *per impossibile* can be proved probatively, through the same terms” [An. Pr. 62b40–63a24], arguments *per impossibile* are convertible to proper syllogisms.⁴⁵

⁴⁴ *Cf.* Crivelli: “The adjective ‘ostensive’ translates ‘deiktikos’, which is connected with the verb ‘deiknúnai’. This verb may be rendered by ‘to show’, ‘to make known’, perhaps even ‘to explain’. Aristotle’s use of ‘deiktikos’ might be intended to suggest that the inferences it is applied to are apt to provide an explanation of what is stated by their conclusion. Now, syllogisms through the impossible are inadequate to provide explanations: if you ask me why so-and-so is the case, you will hardly be satisfied by my telling you that it is because if so-and-so were not the case then such-and-such an impossibility would follow. Inferences that progressively ‘construct’ their conclusions by appealing to the mereological relations between the terms contained in the propositions involved are instead eminently suited to explain why so-and-so is the case.” [Crivelli 2011: 172].

⁴⁵ *Cf.* An. Pr. 2.11–4, see also Lear [1980: 9].

Both syllogisms based on a hypothesis (if the hypothesis is satisfied⁴⁶) and syllogisms *per impossibile* are thus theoretically reducible to a (chain of) proper syllogisms.

6 Deduction

To sum up: Aristotle tries to prove and show thoroughly that “every demonstration and every deduction must be formed by means of the three figures” [An. Pr. 41a21–b6]. So every real deduction or, better to say, every deductive step must meet the A-definition and thus be in the proper syllogistic shape – two simple propositions as premises, one as a conclusion, three terms, one middle term in both premises. Arguments having a different shape do not meet the A-definition in combination with Aristotle’s conception of propositions: for example, although the conclusion in a hypothetical argument can necessarily follow from “premises” (from what is presupposed, better to say), it does not “follow of necessity from their being so”, because the “premise(s)” do not express a state of affairs, but a meta-presupposition, a preliminary agreement. Arguments in a non-syllogistic shape thus do not represent genuine deduction, but rather can be seen as a kind of abbreviation which is ultimately based on (supposition of existence of) corresponding proper syllogisms.

Compare Jonathan Lear’s formulation:

Aristotle has provided (...) an argument that the three syllogistic figures are adequate for the expression of all non-formal deductions. If that argument were valid, it would follow that any deductive consequence of any set of premisses can be reached by a series of obviously valid inferences. For any deduction could, in theory, be expressed as a chain of syllogistic inferences and those formal inferences could be perfected. In one’s actual deductive practice one may move quickly, making large inferential steps with, perhaps, a passing reference to theorems already proved. But this practice is licensed, for Aristotle, not by an analysis of consequence, but by the guarantee that, in doubtful cases, any non-formal deduction can be formalized, and any formalized deduction can be perfected—transformed into an argument in which every step follows obviously. [Lear 1980: 13]

The A-definition thus covers, in this conception, *all* deduction: every genuine deduc-

⁴⁶ This means that there is a proper syllogism (a chain of syllogisms) connecting the antecedent and the consequent of the given conditional. If it is not the case, the argument based on a hypothesis is not analysable into deductive steps, even though it may appear valid according to the current conception.

tive step should be, according to Aristotle, not only necessary truth-preserving, but also relevant⁴⁷ and non-repetitive⁴⁸ (its “in virtue of form” is a result of these demands). Both requirements are incorporated into the formal pattern of the Aristotelian system: it is the structure of proper syllogisms, *i.e.* deductive steps, that ensures the relevance of premises and non-repetition.

I want to emphasize here two important points that seem to me to be under-recognized: Aristotle's logic not only “could be interpreted” as “something like a relevance logic” [Smith 2022: 3.2], but it cannot be properly understood without such an interpretation; the requirement of relevance (along with the requirement of novelty) is what determines the structure of valid deductive steps (and hence questions like ‘why not three premises?’, ‘why not four terms?’ indicate a misunderstanding). 2) Understanding conditionals not as something akin to truth-functional implications but rather as abbreviated deductive reasoning makes the interpretation of Aristotle's notion of deduction much more plausible and consistent.

There is, therefore, a significant difference between the modern notion of logical consequence/inference and Aristotle's one: the difference consists not so much in the degree of formalization, but in the way in which the notion of consequence/inference is aptly captured. Aristotle's definition is narrower, but closer to intuition: intuitively valid arguments in common discourse are standardly expected to be relevant and non-repetitive; and deducing does not mean simply repeating the premises or claiming something unrelated to the premises as the conclusion. At the same time, the whole syllogistic system is rather simple and can be (in its categorical form) interpreted in an extensional way.

However, the syllogistic shape of every deduction is a result not only of the A-definition, but of its combination with Aristotle's conception of propositions with his strictly monadic understanding of predicates. It is therefore theoretically possible to accept A-definition without being obliged to accept the Aristotelian basic syllogistic structure of every deductive step; if Aristotle admitted propositions with more subjects and a relational predicate, his basic deductive step might have a different structure. It is therefore possible to be sceptical regarding Aristotle's monadic logic, but, at the same time, appreciate his highly intuition-friendly definition of logical consequence/inference, which results in very sophisticated

⁴⁷ This is the conclusion that Philipp Steinkrüger (slightly hesitantly) proposes in his article [2015].

⁴⁸ While the requirement of relevance is intensely discussed in logic today, this requirement of non-repetition is now standardly considered strange rather than intuitive.

demands on the formal shape of proper deduction.⁴⁹

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