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DISCRETE REPRESENTATION OF STEEL–CONCRETE COMPOSITE BEAMS UNDER BENDING

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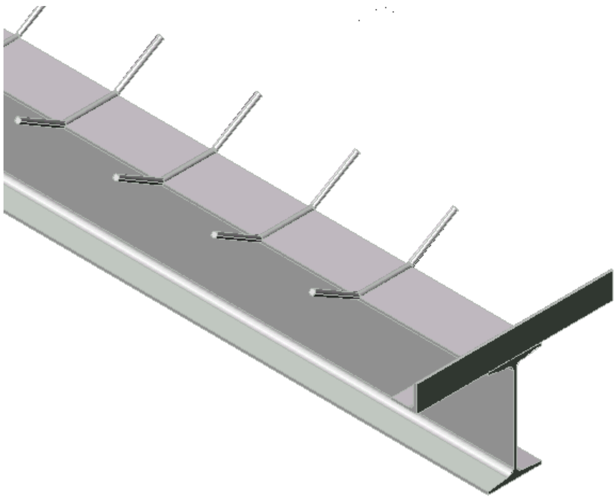
Abstract: For reinforced concrete structures, whose nomenclature and field of application are constantly expanding under modern conditions, the reduction of material consumption in most cases is achieved by decreasing the use of materials such as cement, crushed stone, reinforcing and rolled steel, as well as by searching for new efficient structural forms and design solutions. One of the ways to address this problem is the search for and development of new efficient structural systems, along with the improvement and advancement of calculation methods.
The use of steel–concrete composite beam structures in modern construction is one of the approaches to finding efficient designs, since they are more economical in terms of steel consumption - by 5–20% compared to steel structures. They also allow for a reduction in the structural depth and enable more efficient utilization of the physical and mechanical properties of the materials (concrete and reinforcement), which is particularly important in the construction of buildings and structures, while maintaining their load-bearing capacity and deformability.
Keywords: steel–concrete composite beams, load-bearing capacity, calculation algorithm.

1. INTRODUCTION

Structures in which elements made of different materials work together have long been used in construction (Bouda, 1976, Composite Construction III, 1996, Composite Construction - Conventional and Innovative, 1997). A simple two-element structure can be considered as reinforced concrete. Recent research has also focused on increasing the bearing capacity of RC bent elements using strengthening systems such as FRCM, confirming the ongoing relevance of improving both structural solutions and calculation approaches (Tereshko & Blikharskyy, 2024).Recent experimental studies using digital image correlation (DIC) have enabled detailed assessment of the stress–strain state of RC beams under bending, providing valuable reference points for analytical and numerical modelling (Kopiika et al., 2024). Dense concrete, which resists the main portion of compressive forces, also protects the reinforcing bars from corrosion, while the reinforcement takes up the tensile forces (Blikharskyy & Selejdak, 2021). As modern construction technologies evolve, differences between traditional normative guidelines and actual technological practice become more evident, as shown by discrepancies observed in contemporary field studies (Respondek & Szala, 2024). The selection of appropriate material and construction solutions significantly influences the performance and efficiency of building components, as demonstrated in studies on material optimization in external partitions (Jura & Pawlik, 2024).

A steel-concrete composite beam structure consists of a rolled or built-up steel I-beam, to the upper flange of which a monolithic reinforced concrete slab is attached through a system of short stud connectors (Fig. 1).

In bridge construction (American Association of State Highway and Transportation Officials, 1998, American Institute of Steel Construction, 1994, Anderson, et al., 1973), such structures are also common - they are formed by combining a reinforced concrete slab and a steel beam made of a rolled profile (for example, an I-beam, channel, or T-section). This combination is quite practical from the standpoint of the performance of both elements: the concrete in the upper part of the cross-section effectively resists compressive forces, while the steel beam efficiently carries the tensile forces.



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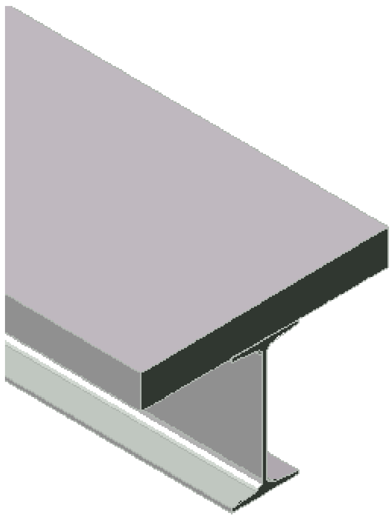

2)

Fig. 1. Steel–concrete composite beam structure: 1) configuration of stud connectors and angle stop (before concreting); 2) steel–concrete composite beam structure (after concreting).

The main aim of this article is to develop and present a numerical algorithm for determining the stress–strain state and load-bearing capacity of steel–concrete composite beams subjected to bending, based on a discrete representation of the cross-section and the use of real stress–strain diagrams of concrete and steel. The novelty of the proposed approach lies in the combined application of a layer-by-layer discrete model of the composite cross-section, an iterative successive-approximation procedure accounting for material nonlinearity, and direct incorporation of experimentally obtained σ–ε relationships. This allows a more realistic description of structural behavior compared to conventional linear-elastic or simplified transformed-section methods.

2. METHODOLOGY OF RESEARCH

The technological process of manufacturing steel–concrete composite structures conventionally consists of two production stages: the formation of the reinforcement anchoring framework and the fabrication of the composite steel–concrete structure Klymenko, et al., 2002).

Quite often, in such structures, each layer is designed separately, and their joint action is not taken into account, which leads to increased material consumption and overall structural weight.

In the strength of materials (Pysarenko, et al., 2004), problems are typically solved using simple mathematical methods with predefined assumptions, and the main element considered in the analysis is a straight bar.

The design formulas for determining normal stresses in bending are generally derived from the conditions of pure plane bending, which is characterized by the fact that out of the six internal forces, only *Mx* is nonzero (*N = Qx = Qy = 0; My = Mz = 0*). Thus, the equilibrium condition that relates stresses and internal forces in the beam's cross-section can be expressed as:

$$\int_F \sigma y dF = M. \tag{1}$$

The development of deformations on the lateral surface of a conventional rectangular beam supported at two ends exhibits a clear pattern: under pure bending, the longitudinal lines curve along circular arcs, while the contours of the cross-sections remain plane curves intersecting the longitudinal lines at right angles. In other words, under pure bending, the cross-sections remain plane and, as they rotate, stay normal to the deflected axis of the beam. In this case, the fibers located above the neutral axis are subjected to compressive forces, while those below are subjected to tensile forces; the length of the neutral axis remains unchanged.

Based on experimentally obtained real stress–strain diagrams of materials (concrete and reinforcement), to evaluate the stress–strain state of composite steel–concrete structures in the pure bending zone, we employ the hypothesis of plane sections.

The numerical model of the composite steel–concrete structure is represented in a discrete form convenient for computation (Fig. 2), by dividing it into a finite number of spatial parallelepipeds (Barabash, 1988, Klymenko, et al., 1994, Barabash, 1995, Klymenko, et al., 2001, Klymenko, et al., 2007a, Klymenko et al., 2007b, Klymenko, et al., 2008, Shevchuk, et al., 2010). Along its length, the composite steel–concrete beam is divided into *n* segments (*i = 1, n*), and along the height of the cross-section - into *m* layers (*j = 1, m*). It is assumed that within each *i*-th segment, the external design loads are constant and applied to the midsection.

Each elementary segment *j* is characterized by its own geometric parameters: width $t_{i,j}$, height $b_{i,j}$, and cross-sectional area $A_{i,j}$. In addition, it possesses its own physical and mechanical properties - computed for the mid-plane of each layer - namely, the sectional modulus of elasticity $E_{i,j}$, the tensile or compressive strains $\varepsilon_{i,j}$, and the corresponding stresses $\sigma_{i,j}$.

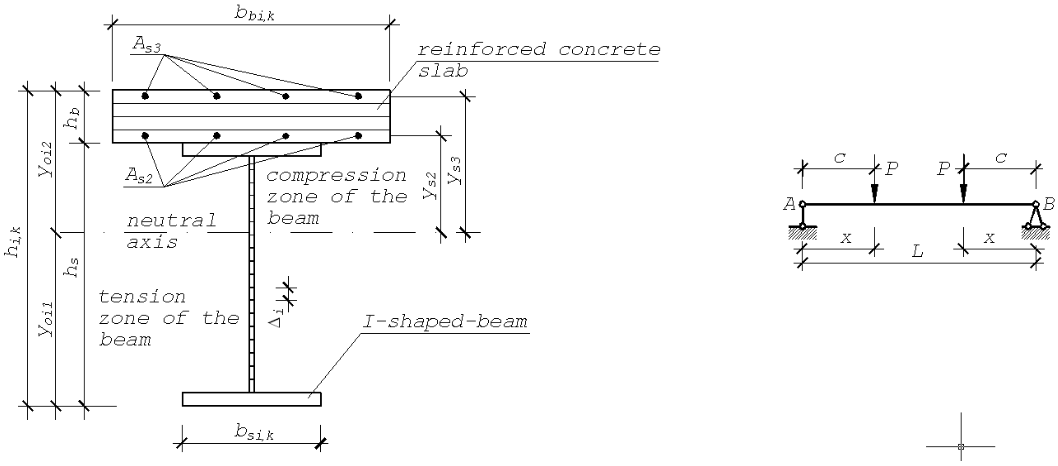

Fig. 2. Discrete representation of a steel–concrete composite beam element

The relationship between the stresses and strains of concrete and reinforcement is represented by the experimentally obtained σ–ε diagrams (CEB-FIP Model Code, 1990). In the calculations, the integration in equation (1) is replaced by summation over the layers and segments. Considering the nonlinearity of the actual material diagrams (for concrete, steel beam, and reinforcing bars), the problem was solved using an iterative (successive approximation) method.
At the first stage, when the modulus of elasticity of the concrete and the steel beam is assumed constant ($E_{si} = E_{s0}$; $E_{bi} = E_{bo}$), the reduced (equivalent) area of the composite steel–concrete cross-section is calculated as follows:

$$(EA)_i = \sum_{j=1}^{m} E_{sij} \cdot b_{ij} \cdot \Delta + \sum_{j=m+1}^{m+1+p} E_{bij} \cdot b_{ij} \cdot \Delta + E_{s_2,i} \cdot A_{s_2,i} + E_{s_3,i} \cdot A_{s_3,i}, \qquad (2)$$

where $E_{sij}, E_{bsij}, E_{s_2i}, E_{s_3i}$– sectional moduli of elasticity of the steel beam, concrete, and reinforcing bars in section i and layer j, kN/cm²;
b_{ij}, Δ – width and height of the elementary segment ij, cm;
$A_{s_2,i}, A_{s_3,i}$ – area of reinforcing bars in the i-th section, cm².
Let's compute the transformed first moment of area about the bottom face of the cross-section:

$$(ES)_i = \sum_{j=1}^{m} E_{sij} \cdot b_{ij} \cdot \Delta \cdot y_{ij} + \sum_{j=m+1}^{m+1+p} E_{bij} \cdot b_{ij} \cdot \Delta \cdot y_{ij} + E_{s_2,i} \cdot A_{s_2,i} \cdot y_{s_2,i} +$$
$$+ E_{s_3,i} \cdot A_{s_3,i} \cdot y_{s_3,i}, \qquad (3)$$

where y_{ij}– distance from the bottom face of the composite section i to the centroid of the elementary layer j;
$y_{s_2,i}, y_{s_3,i}$– distance from the centroids of the upper and lower reinforcing bar sections to the bottom face of the composite cross-section.

Then, the centroid of the transformed cross-section will be located at a distance from its bottom face:

$$y_{oi} = \frac{(ES)_i}{(EA)_i}. \qquad (4)$$

Transformed moment of inertia *(EI)ᵢ* of the composite beam cross-section with respect to its centroid:

$$(EI)_i = \sum_{j=1}^{m} E_{sij} \cdot b_{ij} \cdot \Delta \cdot y_{ij}^2 + \sum_{j=m+1}^{m+1+p} E_{bij} \cdot b_{ij} \cdot \Delta \cdot y_{ij}^2 + E_{s_2,i} \cdot A_{s_2,i} \cdot y_{s_2,i}^2 + $$
$$+E_{s_3,i} \cdot A_{s_3,i} \cdot y_{s_3,i}^2, \tag{5}$$

where y_{ij}^2 – square of the distance from the centroid of the transformed section *i* to the midpoint of the elementary layer *j*.

The value of the external bending moment in the pure bending zone is calculated using the following formula:

$$M_i = F \rightleftarrows \times c, \tag{6}$$

де *c* – distance from the support to the point of application of the external load *F.*

The relative elongation of an arbitrary fiber located at a distance *u* from the neutral layer under pure bending can be determined by considering the deformation of a beam segment of length d_z:

$$\varepsilon_x = \frac{(\rho+y)d\theta - d_z}{d_z} = \frac{(\rho+y)d\theta - \rho d\theta}{\rho d\theta} = \frac{y}{\rho}, \tag{7}$$

where ρ - the radius of curvature of the neutral laye.

Substituting (7) into Hooke's law

$$\varepsilon = \frac{\sigma}{E}, \tag{8}$$

we express the normal stress σ_x in terms of the curvature $1/\rho$:

$$\sigma = \frac{E}{\rho} y, \text{ hence } \frac{1}{\rho} = \frac{\sigma}{Ey}. \tag{9}$$

Substituting this expression into (1), we obtain the formulation of Hooke's law for bending:

$$\frac{1}{\rho} = \frac{M}{EI_x}. \tag{10}$$

If the left-hand sides of equations (9) and (10) are equal, we can equate the right-hand sides as well, from which we obtain Navier's formula:

$$\sigma = \frac{My}{I_x}. \tag{11}$$

The relative strains in an arbitrary *j*-th elementary layer of the *i*-th cross-section are calculated using the following formula:

$$\varepsilon_{ij} = M_i \times \frac{h_{ij} \times (j - 0{,}5) - y_{0i}}{(EI)_i}. \tag{12}$$

The stresses σ_{ij} in the elementary layers are determined using the actual material stress–strain *σ–ε* diagrams. The highest calculation accuracy is achieved when the description is based on experimentally obtained stress–strain diagrams of the materials (concrete and rolled reinforcement). This highlights the critical role of reliable material input; recent data-driven approaches, including artificial neural networks, are increasingly used to predict steel mechanical properties and can support the parametrization of computational models (Ivković et al., 2024.)

3. RESULTS AND DISCUSSION

Let us attempt to implement such an algorithm, supplementing it with a description of the actual stress–strain diagrams for concrete and reinforcing steel. The physical nonlinearity of concrete and rolled steel materials is described using the method of successive approximations.

Input data formation. Before starting the general calculation of the composite steel–concrete structure, all the necessary initial input data must be prepared and there are quite a few of them: b_i, h_i - geometric dimensions of the cross-section; L - design span of the beam subjected to bending; distances from the support to the point of application of the external concentrated load; strength characteristics of materials (concrete and reinforcement) (control points of the σ–ε diagram): R_b, R_{bt}, ε_{bu}, ε_{bf}, ε_{but}, ε_{bft}, R_s, R_{si}, ε_{si}; initial sectional moduli of elasticity of materials - E_b and E_s.

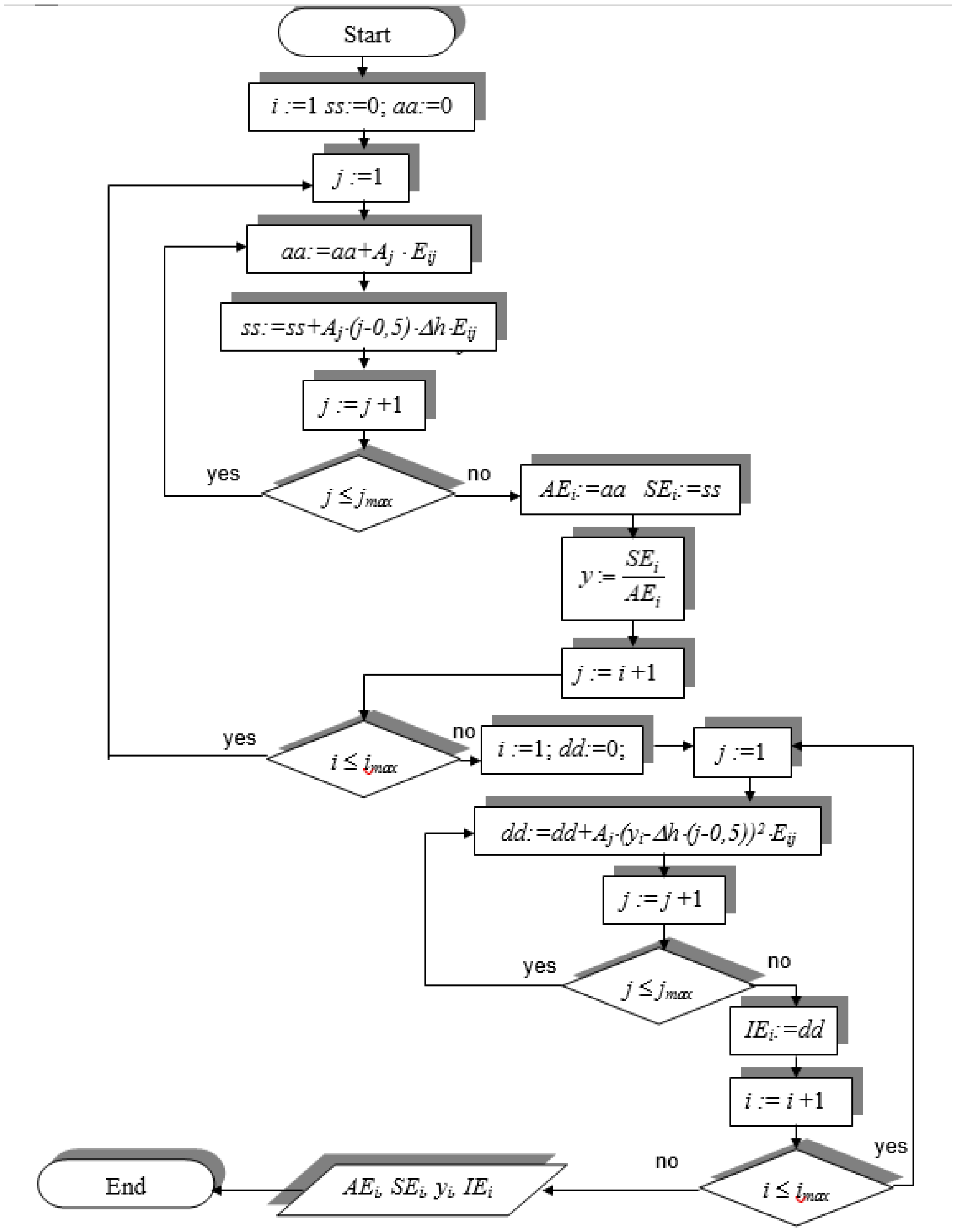

Fig. 3. Block diagram for determining the geometric characteristics AE_i, SE_i, y_i, IE_i of the composite steel–concrete cross-section.

To describe the geometric scheme of the beam in the computer model, it is conventionally divided into i segments along the span (with i ranging from 1 to i_{max}) and j layers along the height of the composite cross-section (with j ranging from 1 to j_{max}). The larger the values of i_{max} and j_{max}, the higher the accuracy of the calculation.

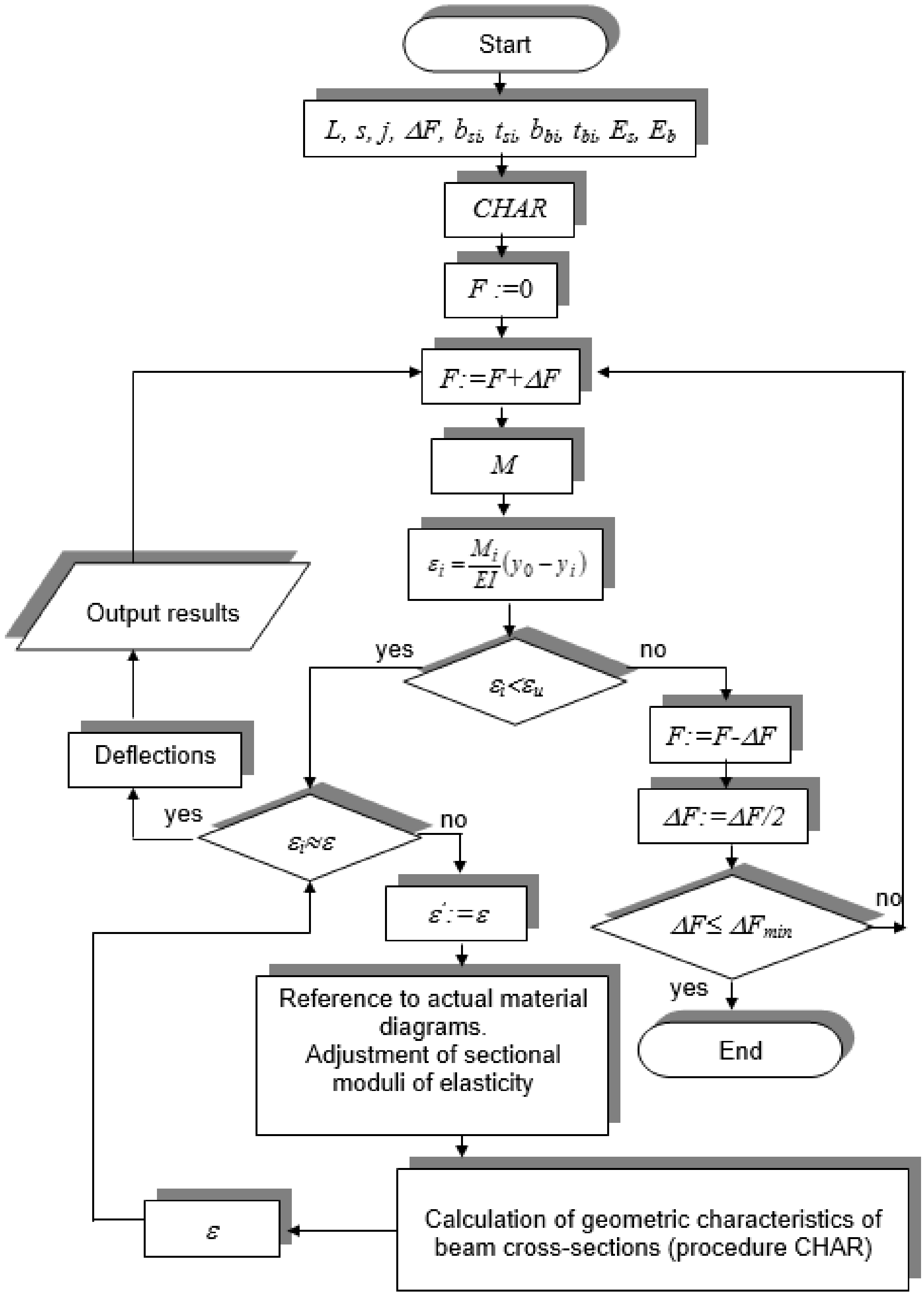

Fig. 4. Block diagram for determining the stress–strain state of steel–concrete beams in the pure bending zone

Within each segment, the bending moment is assumed to be constant and equal to its value at the mid-section. The same applies to the layers along the height of the composite cross-section, where for each layer the physical and mechanical properties of the materials, the sectional modulus of elasticity, strains, and stresses are taken as constant.

Determination of the transformed geometric characteristics. This is the first and most important stage at the beginning of the calculation (Fig. 3). First, the area AE_i of the composite cross-section is determined as the sum of the elementary segments multiplied by the initial sectional moduli of elasticity of the materials.

With respect to the bottom face of the composite section, the first (static) moment of area SE_i is determined as the sum of the products of the areas of the elementary layers and the distances from the centroid of each elementary layer to the bottom face of the section.

The obtained values of AE_i and SE_i make it possible to determine y_i знайти $y_0 = \frac{SE_i}{AE_i}$- the centroid of the *i*-th cross-section, that is, the height of the neutral axis.

With respect to the neutral axis, the moment of inertia IE_i of the composite cross-section is then determined. It is calculated as the sum of the products of the areas of the elementary segments and the squares of the distances from the centroid of the *j*-th layer to the neutral axis.

Application of external load. The external load is applied in the form of two concentrated forces on the top surface of the composite section, at a distance *c* from the left and right supports. The value of the bending moment in the *i*-th segment is easily defined using the principles of structural mechanics.

The force F varies from 0 to the ultimate (failure) load F_{max}. The load increment is applied as $F = F + \Delta F$ using the variable ΔF.

To determine the exact value of the ultimate load, it is proposed to gradually reduce the increment ΔF. That is, when failure occurs, the value of the external load is decreased by the last increment $F = F - \Delta F$, effectively returning to the previous step, then the increment itself is halved $\Delta F = \Delta F / 2$, and the calculation is repeated.

The program terminates when the increment ΔF becomes smaller than a predefined minimum value ΔF_{min}, that is, when the condition $\Delta F < \Delta F_{min}$ is satisfied.

Determination of strains in the elementary layers. The strains along the height of the composite cross-section are determined using the well-known formula based on the hypothesis of plane sections:

$$\varepsilon_{ji} = \frac{M_i}{EI_i}\left(delta\,(j-0{,}5) - y_i\right). \tag{13}$$

After obtaining the strain values (Fig. 4), they are checked to ensure that they do not exceed the allowable limits. If the condition is satisfied, the real material stress–strain diagrams σ–ε are used to determine the stresses in the elementary layers and to refine their sectional moduli of elasticity. Using the updated sectional moduli, the transformed geometric characteristics of the beam's cross-sections are recalculated, and new strain values are obtained. The new results are then compared with the previous ones. If the deviation is small and within the predefined accuracy range, the program outputs the results for strains, stresses, external load value, and deflections at the current load stage. If the condition is not satisfied, the calculation within the iterative (successive approximation) cycle is repeated.

Increment of external load. If the deviation of deformations in successive cycles lies within the prescribed accuracy, the new deformation values are stored. This marks the completion of the internal iterative cycle - the cycle of successive approximations. The next step is to increase the external force F by an increment ΔF.

Determination of actual deflections. The displacement of the beam axis is calculated based on the well-known Mohr's integral, in which the integration is replaced by summation:

$$f_i = \sum_{i=1}^{max} \frac{M_i \cdot M_{i,1}}{I_{red,i}}, \tag{14}$$

where $M_{i,1}$ - the bending moment value in section *i* produced by a unit load.
Then, the external compressive load is incrementally increased by ΔF (that is, $F=F+\Delta F$), and the calculation is repeated starting from **Application of external load**. The external load continues to increase until one of the materials reaches its ultimate strain limit. To improve the accuracy of determining the ultimate load, a successive load-step refinement is applied. For this purpose, the previously obtained values $A_{red,i}$, $S_{red,i}$, y_i, E_{in}, E_{ik} and $I_{red,i}$, corresponding to the stage just before failure, are restored. The load increment ΔF is then halved $\Delta F=\Delta F/2$, and the calculation is repeated. This refinement continues until the condition $\Delta F < \Delta F_{min}$ is satisfied. **Selection of the material type.** The transition from the concrete diagram to the reinforcing steel diagram is quite straightforward. If the numbers of the elementary layers along the height of the composite cross-section do not exceed a certain predetermined value j_b, these layers correspond to the reinforcing steel zones, and their own input parameters are used E_s, R_s, ε_s. The stresses in these layers are determined according to a special computational algorithm. If, however, the number of the elementary layer along the section height exceeds the value j_b, these layers correspond to the concrete zones, which are characterized by their own physico-mechanical properties.

4. CONCLUSIONS

In this study, a numerical method for determining the stress–strain state and load-bearing capacity of steel–concrete composite beams subjected to bending was developed. The method is based on the hypothesis of plane sections, a discrete layer representation of the composite cross-section, and an iterative successive-approximation procedure that accounts for the physical nonlinearity of concrete and steel using real, experimentally obtained stress–strain diagrams.
The developed mathematical model and calculation algorithm made it possible to reproduce the actual behavior of steel–concrete composite beams under short-term loading with sufficient accuracy. The results of numerical simulations showed good agreement with available experimental data. The calculated ultimate bending moments differed from the experimental values by only a few percent, which confirms the reliability of the proposed approach and its applicability for engineering analysis and design of steel–concrete composite beam structures.
The obtained convergence between numerical and experimental results indicates that the adopted discretization of the cross-section and the use of actual σ–ε material relationships allow for an adequate description of stress redistribution between concrete, reinforcement, and the steel beam during the loading process.
The main limitations of the presented method are related to its sensitivity to the accuracy of input data, including geometric discretization parameters and material characteristics, as well as to the assumptions adopted in the model. In its current form, the algorithm is limited to pure bending conditions and short-term loading and does not explicitly account

for long-term effects such as concrete creep and shrinkage, degradation of shear connectors, or dynamic loading.

Future research should therefore focus on extending the proposed model to include time-dependent material behavior, alternative loading schemes, and more complex boundary conditions. Further experimental investigations are also recommended to validate the method for a wider range of cross-section geometries, material combinations, and structural configurations, as well as to improve the robustness and universality of the numerical procedure.

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