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A multi-criteria decision problem based on information uncertainty conveyed by intuitionistic interval-valued fuzzy sets*

by

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Abstract: In a multi-criteria decision problem, represented by intuitionistic interval-valued fuzzy sets that cannot be properly ranked linearly, it is not always possible to adequately estimate the uncertainty of information using a single-degree scalar measure. We analyze four different ways of quantifying information uncertainty, using an overall measure and three types of directional measures. The assessment of different types of information uncertainty is performed in the context of hypothetical and possible changes in the degree of membership and non-membership for a given degree of hesitation of an element in the set (i.e. in terms of permissible changes in the assessment of information uncertainty). This approach allows for taking into account the asymmetric nature of the assessment of changes in information uncertainty, perceived in the real physical world from a human perspective. We propose a ranking method for alternatives/options (separately for each type of information uncertainty), which, additionally, allows for taking into account the level of information certainty in the process of selecting the best alternative. We analyze examples meant to illustrate our approach and show that it allows for the analysis of alternatives from four complementary perspectives based on information uncertainty, something that currently used methods cannot do. We present an example illustrating the comparison of our results with selected results from the literature.

Keywords: multi-criteria decision problem, information uncertainty, interval-valued intuitionistic fuzzy sets, ranking alternatives, *TOPSIS* method

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1. Introduction

A multi-criteria decision-making problem (*MCDM*) belongs to a field of operations research in which one aims at evaluating a limited number of *alternatives / options*, from which a choice is made based on a ranking, taking into account experts' preferences. Stanley Zionts helped to popularize the respective issues in his article, "*MCDM – if not a roman numeral, then what?*", see Zionts (1979). Typically, there is no single, definitive solution to such a problem, and the best-rated solution can be interpreted in various ways.

Unfortunately, experts' preferences are often not precise and certain enough when describing and selecting alternatives, and the computer implementation itself also carries some information uncertainty. In some cases, this contributes to various types of uncertainty and ambiguity. Therefore, the way in which these informational uncertainties are incorporated into deliberations becomes crucial to the effectiveness of the decision-making process.

Solving such a problem could mean, for example, selecting only one alternative from the set of available alternatives, the one that received the "best" rating from the experts in the merit assessment. Another option could be to select a small set of alternatives acceptable to the expert and consider additional information to select the "best" option, for example, for reasons of information certainty, according to the ranking proposed in the paper.

Among the *MCDM* techniques, the *Technique for Order Preference by Similarity to an Ideal Solution* (in short *TOPSIS*) is proposed as a well-known multi-criteria decision-making method. *TOPSIS* was originally developed by Ching-Lai Hwang and Kwangsun Yoon (1981), and further developed by, among others, Yoon (1987), Hwang, Lai and Liu (1993) and Ren et al. (2007). The *TOPSIS* method is primarily preferred for applications that consider information ambiguity. Many researchers have extended and generalized this approach to the fuzzy environment. For example, the method developed by Gorzałczany (1987) is widely used in multi-criteria decision problems based on interval-valued fuzzy sets. An interesting review of the literature in this area can be found in the article by Kokoc and Ersöz (2021).

The basic idea of this method is that the selected best alternative (from a finite set of alternatives, for which the evaluation values are expressed by coefficients) should have the shortest Euclidean distance from the positive ideal solution and the farthest distance from the negative ideal solution*. The algo-

*It should be noted that there is a broad group of MCDM methods, which are based on the general idea of the reference points, corresponding to the best and/or the worst possible, attainable, or preferred solution. A broad stream of work on this subject is due to Andrzej Wierzbicki and his associates (see, e.g., Wierzbicki, 1997), but it must be emphasized that the earliest work, in which such a general approach was proposed, is due to Zdzisław Hellwig (Hellwig, 1968) (eds.).

rithmic process starts by creating a matrix, representing the satisfaction values of each criterion for each alternative. The matrix is then normalized and the values multiplied by the criterion weights. The ideal positive solution and the ideal negative solution are then calculated. Next, the distance of each alternative from these ideal solutions is calculated using a distance measure. Finally, the alternatives are ranked, and the best one is selected.

In this paper, the entire multi-criteria decision problem is expressed by *a decision matrix* in which the coefficients represent the values of each *criterion* satisfied by each alternative and are expressed in the form of intuitionistic fuzzy numbers with interval values. In addition, *the weights* indicate the relative importance of each criterion according to the experts. Each "*alternative-criterion*" pair can be associated not only with an evaluation value in the form of a matrix coefficient, but also with a certain level of information uncertainty conveyed by the interval-valued intuitionistic fuzzy number that describes it. It can also be noted that the degree of certainty of this type of information is a measure of the quality of the information. The authors attempted to describe problems related to decision-making using *a directional information uncertainty assessment framework,* discussed in more detail in the forthcoming article by Szkatuła and Krawczak (in preparation). The method proposed in this article relies on the use of information uncertainty measures to determine the relative coefficients, defining alternatives. The selected alternative, separately for each type of information uncertainty, should provide the highest level of information certainty (in other words, the lowest level of information uncertainty).

This does not mean, of course, that we must take into account all types of information uncertainty in the task under consideration. The approach allows us to choose the one that is appropriate to our expectations and the context of the task under consideration. It is worth remembering that in some applications it may be more important, for example, to consider information uncertainty related to non-membership (risk of rejection), whereas in other ones – uncertainty related to membership (degree of acceptance) may be of primary concern. The proposed directional evaluation framework allows for such a choice to be made.

In the authors' previous work, a single criterion for evaluating alternatives was taken into account and in this paper, we consider a multi-criteria decision-making problem. Four scoring functions are used to transform the matrix representing the values of satisfaction of each criterion for each alternative into four matrices describing the values of information uncertainty. Firstly, based on *the overall evaluation function*, we transform the original decision matrix into the quantification matrix of information uncertainty for alternatives. Secondly, based on *the directional evaluation functions*, we transform this original matrix into three quantification matrices of directional information uncertainty concerning the alternatives. These matrices successively represent: the entire quantification of information uncertainty and the three components with respect to hypothetical, possible changes, respectively, only in the degree of membership, only in the degree of non-membership, or both. This decomposition allows for the analysis of information uncertainty from four complementary perspectives, something that currently used methods cannot do. The ability to incorporate directional information when quantifying information uncertainty allowed us to account for the asymmetric nature of potential changes in information uncertainty perceived in the real physical world from a human perspective.

The remainder of the paper is structured as follows. Section 2 presents a short introduction to the concept of information uncertainty assessment. Section 3 presents a method for ranking alternatives based on the level of certainty of the information they convey. Section 4 presents an illustrative example of how to select the alternative with the highest degree of information certainty.

2. Brief introduction to the concept of information uncertainty assessment

The concept of *a fuzzy set* (in short *FS*) was introduced by Lotfi Asker Zadeh in 1965, such that each element v of a set is characterized by a degree of membership (cf. Zadeh, 1965). It has been expanded to the interval-valued intuitionistic fuzzy set (in short *IVIFS*) – by Krassimir T. Atanassov and George Gargov (1989) – as an object defined in the form of triples. They are a relatively new tool for modeling uncertainty, in particular as a tool for making multi-criteria decisions under conditions of uncertainty, for example in alternative ranking (cf. Selvaraj and Majumdar, 2021). We will briefly repeat the most important concepts of this theory. *The interval-valued intuitionistic fuzzy set* A in a finite set V is an object of the form:

$$A = (v,\ \left[\mu_A^l(v)\,,\ \mu_A^r(v)\right],\ \left[\upsilon_A^l(v)\,,\ \upsilon_A^r(v)\right]\ \mid \forall\ v \in V\ \}, \tag{1}$$

where $[\mu_A^l(v),\ \mu_A^r(v)]$ and $[\nu_A^l(v),\ \upsilon_A^r(v)]$ represent values from closed subintervals of the interval [0,1] and represent, respectively, the membership and non-membership degree of an element $v \in V$ to the fixed set A, respectively,with additional condition $\mu_A^r(v) + \upsilon_A^r(v) \leq 1$. Additionally,

$$\pi_A(v) = \left[\pi_A^l(v)\,,\ \pi_A^r(v)\right] = \quad [1 - \mu_A^r(v)\ -\ \nu_A^r(v),\ 1 - \mu_A^l(v)\ -\ \nu_A^l(v)] \tag{2}$$

is defined as *the degree of uncertainty*, *indeterminacy* or *hesitation*. It expresses the degree of lack of knowledge about whether element v belongs or does not belong to A. The uncertainty of information may range from complete certainty of information to complete lack of knowledge. The bounds of the hesitancy degree satisfy the following condition: $0 \leq \pi_A^l(v) \leq \pi_A^r(v) \leq 1$.

For convenience, Zeshui Xu called each pair of these intervals an *interval-valued intuitionistic fuzzy number* (in short *IVIFN*), see Xu (2007, 2012), which is used to simplify notation. Thus, the interval-valued intuitionistic fuzzy number, for a given $v \in V$ can be written down as the following pair

$$\left(\left[\mu_A^l(v), \mu_A^r(v)\right], \left[\nu_A^l(v), \nu_A^r(v)\right]\right) \tag{3}$$

and for simplicity, for a given $v \in V$, it is sometimes denoted by $\left(\left[\mu_A^l, \mu_A^r\right], \left[\nu_A^l, \nu_A^r\right]\right)$ Comparing and ranking *IVIFS* datasets still remains a challenge. Numerous scoring, accuracy, and evaluation functions have been proposed in the literature to evaluate each set from a single, holistic perspective, but most of them yield inconsistent or counterintuitive results (cf. Xu, 2007; Ye, 2009; Bai, 2013; Selvaraj and Majumdar, 2021; Szkatuła and Krawczak, in preparation).

We propose a new framework for evaluating *IVIFS*s, based on the asymmetric nature of changes in information uncertainty, perceived from a human perspective, discussed in more detail in the forthcoming article by Szkatuła and Krawczak (in preparation). We assume that information uncertainty cannot always be adequately described by a single scalar measure. Depending on the decision-making context and the perspective adopted by the decision-maker, different aspects of uncertainty may play a dominant role. We consider four types of information uncertainty: overall information uncertainty and three types of directional information uncertainty.

We assume that *the information uncertainty area* Ω for a fixed set A covers all possible combinations of hypothetical changes in the degree of membership and non-membership at a fixed hesitation degree. Each element of the information uncertainty area can be represented as a point in the two-dimensional interpretation space of the *IFS* model, as shown in Fig. 1 (in the entire green hatched area). This area is decomposed into three sub-areas (i.e., Area υ, Area $\upsilon\mu$ and Area μ), which allows for directional assessment of changes in information uncertainty.

All of the above areas can be used to characterize the information uncertainty for interval-valued intuitionistic fuzzy sets and to compare them from various perspectives. No previous *IVIFS* model has explicitly performed this three-way decomposition. The proposed overall evaluation function and directional assessment functions offer a simple but effective non-probabilistic measure of information uncertainty and allow the analysis of information uncertainty. The first one is based on *the entire area of information uncertainty* (and will be briefly presented in Case 1), and the other three are based on *sub-areas of directional information uncertainty* (and will be briefly presented in Case 2). A detailed description of information uncertainty in the entire area and sub-areas can be found in the forthcoming article by Szkatuła and Krawczak (in preparation).

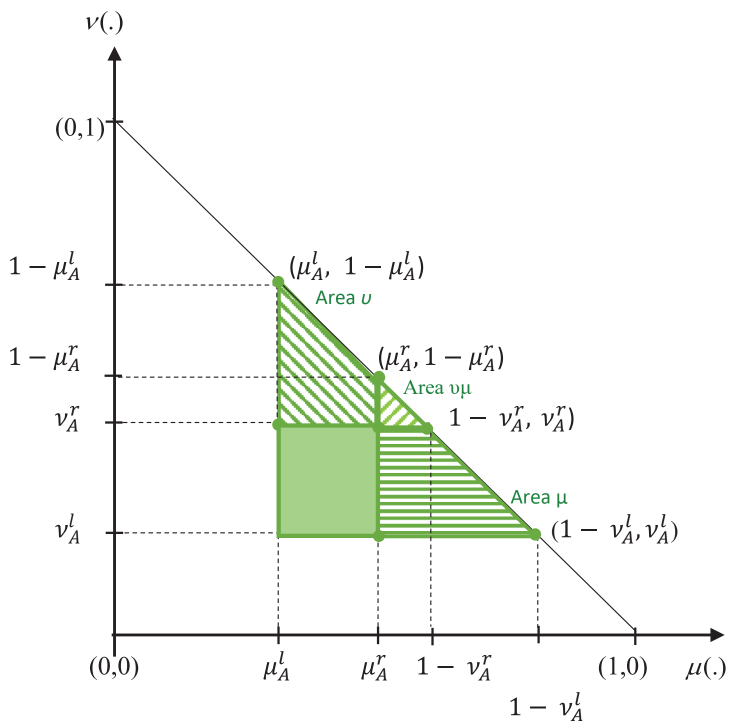

Figure 1. Geometric interpretation of an $A \in IVIFS$ (as a uniform green rectangle) with the three sub-regions of the uncertainty region (hatched green areas)

Case 1 – Overall assessment of information uncertainty (applying the $Area_\Omega$ function)

Let us assume that the assessment of information uncertainty of an interval-valued intuitionistic fuzzy set depends on the assumed overall information uncertainty function. The *overall information uncertainty assessment function* $Area_\Omega(.)$ (i.e., the radical evaluation function) assigns a non-negative degree of information uncertainty to the considered set, ranging from 0 to 1, as follows:

$$Area_\Omega(A) =$$
$$= (1 - \mu_A^l(v) - \nu_A^l(v))^2 - 2(\mu_A^r(v) - \mu_A^l(v)) \cdot (\nu_A^r(v) - \nu_A^l(v)) \qquad (4)$$

and can be treated as non-probabilistic measure of *information uncertainty* conveyed by the interval-valued intuitionistic fuzzy number. The function can be used to characterize and to carry out the comparison of the interval-valued intuitionistic fuzzy sets.

Let us consider A_i and A_j, which are two *IVIFN*s. Based on the function $Area_\Omega(A)$, an order on *IVIFN*s can be defined. Ranking is based on the magnitude of possible hypothetical changes in certainty of information. Let us assume the following notations.

- Notation $A_i \prec_\Omega A_j$ means that A_i is *less certain* than A_j (A_i has more hesitancy than A_j, i.e., has a larger information uncertainty area than the information uncertainty area of the set A_j). In this case the condition $Area_\Omega(A_i) > Area_\Omega(A_j)$ holds.
- Notation $A_i \succ_\Omega A_j$ means that A_i is *more certain* than A_j (i.e., A_i has *less hesitancy* than A_j). In this case the condition $Area_\Omega(A_i) < Area_\Omega(A_j)$ holds.
- Notation $A_i \approx_\Omega A_j$ means that *no difference in certainty is observed* between A_i and A_j , i.e., both sets have equal information uncertainty areas. In this case the condition $Area_\Omega(A_i) = Area_\Omega(A_j)$ holds.

REMARK 1 *It should be emphasized that the value of the function $Area_\Omega(x_{ij})$ defines the degree of information uncertainty of the alternative A_i with respect to the criterion C_j. We can therefore conclude that the lower the value of the function, the greater the certainty of information.*

Let us consider the second case, based on three sub-areas of directional information uncertainty.

Case 2 – Directional assessment of information uncertainty (applying functions $Area_\upsilon$, $Area_{\upsilon\mu}$, $Area_\mu$)

Let us assume that the assessment of information uncertainty of an interval-valued intuitionistic fuzzy set depends on the three assumed directional functions. The *directional information uncertainty assessment functions* for interval-valued intuitionistic fuzzy sets (i.e., the directional evaluation functions) assign to the considered set the non-negative numbers from the range between 0 and 1.

Let the information uncertainty assessment of type 1 be associated with hypothetical changes in the degree of non-membership, and type 2 with simultaneous, hypothetical changes in the degree of membership and non-membership, and type 3 with hypothetical changes in the degree of membership. We will now briefly discuss them individually.

A directional function for assessing the uncertainty of type 1 information is introduced, as follows:

$$Area_\upsilon(A) = (2 - \mu_A^l(v) - \mu_A^r(v) - 2 \cdot \nu_A^r(v)) \cdot (\mu_A^r(v) - \mu_A^l(v)), \qquad (5)$$

and the directional function for assessing the uncertainty of type 2 information, as follows:

$$Area_{\mu\upsilon}(A) = \quad (1 - \mu_A^r(v) - \nu_A^r(v))^2 \tag{6}$$

and finally, the directional function for assessing the uncertainty of type 3 information, as follows:

$$Area_{\mu}(A) = (2 - \nu_A^l(v) - \nu_A^r(v) - 2 \cdot \mu_A^r(v)) \cdot (\nu_A^r(v) - \nu_A^l(v)). \tag{7}$$

These functions can be used to characterize and compare interval-valued intuitionistic fuzzy sets.

Let us consider A_i and A_j, being two *IVIFN*s. The ranking (here: the order of the two) is determined by the magnitude of possible hypothetical changes in information certainty with respect to the degree of non-membership, both degrees and degree of membership, respectively. Let us assume the following notations.

- The notation $A_i \prec_\upsilon A_j$ (similarly for $A_i \prec_{\mu\upsilon} A_j$ and $A_i \prec_\mu A_j$) signifies that A_i is *less certain* regarding the degree of non−membership than A_j (similarly with respect to the degree of non-membership and membership together, as well as with respect to the degree of membership). In other words, it is *more uncertain.*
- The notation $A_i \succ_\upsilon A_j$ (similarly for $A_i \succ_{\mu\upsilon} A_j$ and $A_i \succ_\mu A_j$) signifies that A_i is *more certain* regarding the degree of non−membership than A_j (similarly with respect to the degree of non-membership and membership together, as well as with respect to the degree of membership). In other words, it is *less uncertain.*
- The notation $A_i \approx_\upsilon A_j$ (similarly for $A_i \approx_{\mu\upsilon} A_j$ and $A_i \approx_\mu A_j$) signifies that *no differences* were found with respect to the degree of non-membership between A_i and A_j (similarly with respect to the degree of non-membership and membership together, as well as to the degree of membership).

We can conclude that the lower the value of the directional evaluation functions, the greater the directional certainty of information conveyed by this set.

We will now present two examples in order to illustrate the existing relationships between the concepts presented above.

Example 1 Let us consider five interval-valued intuitionistic fuzzy numbers: A_1, A_2, A_3, A_4 and A_5 – for which the membership degrees are all the same, while the non-membership degrees successively increase. Their corresponding uncertainty degrees π_{A_i} and the obtained values of the uncertainty evaluation functions: $Area_\Omega(A_i)$, $Area_\upsilon(A_i)$, $Area_{\upsilon\mu}(A_i)$ and $Area_\mu(A_i)$ for $i = 1, 2, \ldots, 5$ (corresponding to different types of information uncertainty) are given in Table 1.

Table 1. The interval-valued intuitionistic fuzzy numbers $A_1, \ldots, A_5$ and the evaluation functions values and resulting ranking

A_i	π_{A_i}	$Area_{\Omega}(.)$	$Area_{v}(.)$	$Area_{v\mu}(.)$	$Area_{\mu}(.)$	Rank
$A_1 = ([0.3, 0.5], [0.0, 0.1])$	$[0.4, 0.7]$	0.45	0.20	0.16	0.09	5
$A_2 = ([0.3, 0.5], [0.1, 0.2])$	$[0.3, 0.6]$	0.32	0.16	0.09	0.07	4
$A_3 = ([0.3, 0.5], [0.2, 0.3])$	$[0.2, 0.5]$	0.21	0.12	0.04	0.05	3
$A_4 = ([0.3, 0.5], [0.3, 0.4])$	$[0.1, 0.4]$	0.12	0.08	0.01	0.03	2
$A_5 = ([0.3, 0.5], [0.4, 0.5])$	$[0.0, 0.3]$	0.05	0.04	0.00	0.01	1

It is worth emphasizing that the lower the value of the uncertainty evaluation function, the greater the certainty of the information conveyed by this set. A hierarchy of evaluated fuzzy numbers (so-called ranking) was introduced based on the values of the evaluation functions and the one that should provide the highest level of certainty of information was selected (see the last column in Table 1). For the first evaluation function considered (see the third column in Table 1), we obtain:

$$Area_{\Omega}(A_1) > Area_{\Omega}(A_2) > Area_{\Omega}(A_3) > Area_{\Omega}(A_4) > Area_{\Omega}(A_5),$$

and the following dependencies with respect to the certainty of information are satisfied:

$$A_1 \prec_{\Omega} A_2 \prec_{\Omega} A_3 \prec_{\Omega} A_4 \prec_{\Omega} A_5.$$

The geometric interpretation of selected sets A_1, A_3 and A_5, and their information uncertainty areas are shown in Fig. 2.

For the remaining three directional evaluation functions, that is, $Area_{v}(A_i)$, $Area_{v\mu}(A_i)$ and $Area_{\mu}(A_i)$, we obtain similar dependencies. Therefore, set A_5 is the best alternative with the highest degree of information certainty for any type of information uncertainty and is ranked first in the last column of Table 1.

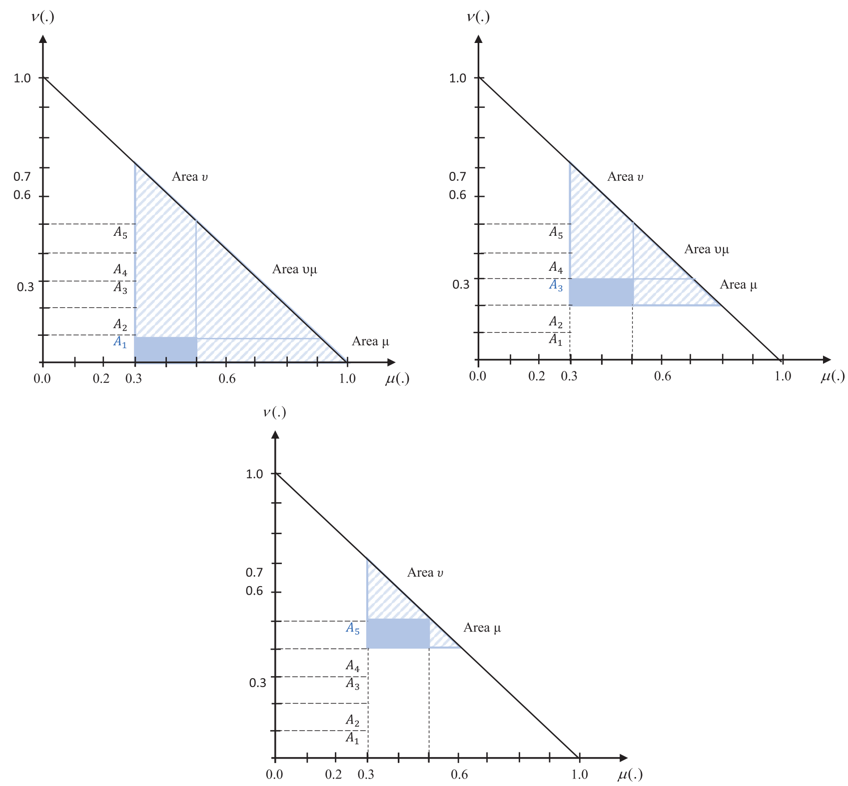

Figure 2. Geometric interpretation of the sets A_1, A_3 and A_5 (shown as uniform blue rectangles) and the corresponding information uncertainty areas (the hatched areas)

Example 2 *Let us consider two different interval-valued intuitionistic fuzzy numbers, denoted A_1 and A_2. Their corresponding equal uncertainty degrees, π_{A_i} and the values of the obtained uncertainty evaluation functions: $Area_\Omega(A_i)$, $Area_\upsilon(A_i)$, $Area_{\upsilon\mu}(A_i)$ and $Area_\mu(A_i)$ for $i = 1{,}2$ (as different types of information uncertainty), are given in Table 2. The values of the directional evaluation functions for the most certain set are shaded. In Table 2 we also present, for comparison, the values of selected evaluation functions for IVIFNs, which are known from the literature: $S()$ – the score function and $H(.)$ – the accuracy function, see Xu (2007).*

The geometric interpretation of the considered sets A_1 and A_2 and their information uncertainty sub-areas are shown in Fig. 3.

The results presented in Table 2 indicate that the area of overall uncertainty of information, associated with the set A_1, is equal to the overall uncertainty of information, associated with set A_2, (i.e., $Area_\Omega(A_1) = Area_\Omega(A_2)$ – see fourth column in Table 2) and therefore no differences are observed between sets A_1 and A_2 (i.e., $A_1 \approx_\Omega A_2$). A similar result was obtained for the function $H(.)$, which does not distinguish between sets because $H(A_1) = H(A_2)$ (see the third column). The function $S()$ specifies the following order: $A_1 < A_2$ (see second column).

Table 2. The interval-valued intuitionistic fuzzy numbers A_1 and A_2 for $\pi_{A_1} = \pi_{A_2} = [0.1, 0.4]$ and the values of the evaluation functions

A_i	$S(.)$	$H(.)$	$Area_\Omega(.)$	$Area_\upsilon(.)$	$Area_{\upsilon\mu}(.)$	$Area_\mu(.)$
$A_1 = ([0.1, 0.3], [0.5, 0.6])$	-0.35	0.75	0.12	0.08	0.01	0.03
$A_2 = [0.5, 0.6], [0.1, 0.3])$	0.35	0.75	0.12	0.03	0.01	0.08

Directional evaluation functions allow us to distinguish between the sets under consideration. For example, the area of information uncertainty with respect to hypothetical changes in the degree of non-membership, associated with the set A_1, is larger than the area associated with the set A_2 (i.e., $Area_\upsilon(A_1) > Area_\upsilon(A_2)$ – see the fifth column in Table 2) and therefore the set A_1 is less certain than the set A_2 (i.e., $A_1 \prec_\upsilon A_2$). Reasoning similarly, the set A_1 is more certain than the set A_2 with respect to hypothetical changes in the degree of membership (i.e., $A_1 \succ_\mu A_2$ – see the last column in Table 2) and no differences are observed between sets A_1 and A_2 related to simultaneous changes in membership, and non-membership is taken into account (i.e., $A_1 \approx_{\upsilon\mu} A_2$ – see the sixth column in Table 2).

It is worth remembering that the resulting ranking of interval-valued intuitionistic fuzzy numbers depends largely on the adopted evaluation function.

The proposed types of information uncertainty were applied to a multi-criteria decision problem. The information uncertainty assessment function $Area_\Omega(.)$ (i.e., the overall measure of information uncertainty) and the directional assessment functions: $Area_\upsilon(.)$, $Area_{\upsilon\mu}(.)$ and $Area_\mu(.)$ (i.e., the directional measures of information uncertainty) determine *the coefficients defining the alternatives*, which will be discussed in more detail in the next section.

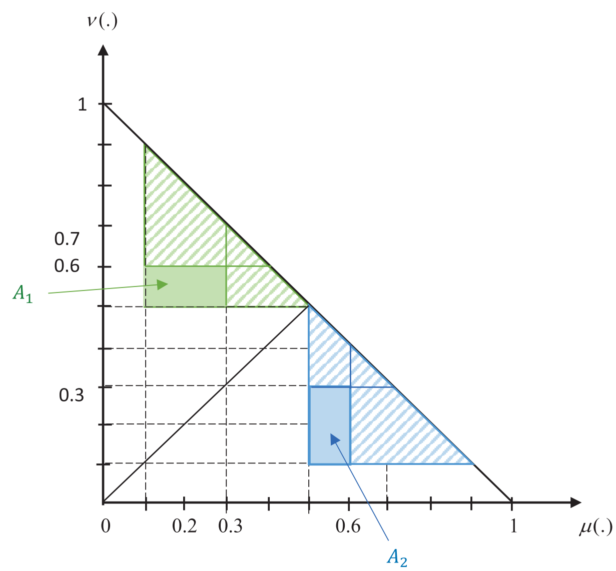

Figure 3. Geometric interpretation of the sets A_1 and A_2 (as uniform rectangles) and their corresponding information uncertainty sub-areas (hatched areas)

3. Ranking of alternatives based on the level of information certainty

3.1. The setting

This section focuses on the ranking of alternatives, described by interval-valued intuitionistic fuzzy sets. After the preferred descending order is obtained, we will choose the alternative with the highest information certainty.

Let us assume that we deal with a set of possible alternatives, described by the N interval-valued intuitionistic fuzzy sets, denoted $\{A_1,\ A_2,\ldots,A_N\}$. Each alternative is assessed from the point of view of K criteria, which are denoted $\{C_1,\ C_2,\ldots,C_K\}$. An evaluation of an alternative A_i with respect to a criterion C_j can be represented by an *IVIFN* value x_{ij}, for $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, K$, as follows:

$$x_{ij} = ([a_{ij},\ b_{ij}]\ ,\ [c_{ij},\ d_{ij}]). \tag{8}$$

Thus, each i-th alternative can be treated as a vector $A_i = \begin{bmatrix} x_{i1} & x_{i2} & \ldots & x_{iK} \end{bmatrix}$ for $i \in \{1, 2, \cdots, \mathrm{N}\}$. Given N alternatives and K criteria, we obtain a decision matrix representing the satisfaction values of each criterion for each alternative, i.e., *the intuitionistic fuzzy matrix with interval values*, defined as

$$\mathrm{X} = \left[\ x_{ij}\ \right] = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1K} \\ x_{21} & x_{22} & \dots & x_{2K} \\ \vdots & \vdots & \dots & \vdots \\ x_{N1} & x_{N2} & \dots & x_{NK} \end{bmatrix}. \tag{9}$$

This matrix X represents the *MCDM* problem, where x_{ij} is the value of the $i-$th alternative with respect to the $j-$th criterion, with x_{ij} being represented in the form of *IVIFN*. Hence, the entries of the matrix $N \times K$ represent the satisfaction value of each criterion C_j $(j = 1, 2, \dots, K)$ for each alternative A_i $(i = 1, 2, \dots, N)$ by an interval-valued intuitionistic fuzzy number.

Let us assume that each entry in the matrix X can be associated with a certain level of uncertainty in the information conveyed by the intuitionistic interval-valued fuzzy number that represents it. Each value x_{ij}, which corresponds to the pair (i-th alternative – j-th criterion) can be evaluated using:

•*overall assessment of information uncertainty* (applying the $Area_{\Omega}(x_{ij})$ function – equation (4)),

•*directional assessments of information uncertainty* (applying the $Area_{\upsilon}(x_{ij})$, $Area_{\upsilon\mu}(x_{ij})$, $Area_{\mu}(x_{ij})$ functions – equations (5), (6) and (7)).

The method for ranking alternatives, based on matrices, created by applying the above functions, will be discussed in more detail in the next sections.

3.2. Ranking of alternatives based on overall information uncertainty assessment

We will now discuss the first of these two above cases in more detail. Based on the overall evaluation function $Area_{\Omega}(.)$ we convert this matrix X into the following *information uncertainty quantification matrix* for the alternatives

$$Area_{\Omega}(\mathrm{X}) = \begin{bmatrix} Area_{\Omega}(x_{11}) & Area_{\Omega}(x_{12}) & \dots & Area_{\Omega}(x_{1K}) \\ Area_{\Omega}(x_{21}) & Area_{\Omega}(x_{22}) & \dots & Area_{\Omega}(x_{2K}) \\ \vdots & \vdots & \dots & \vdots \\ Area_{\Omega}(x_{N1}) & Area_{\Omega}(x_{N2}) & \dots & Area_{\Omega}(x_{NK}) \end{bmatrix}. \tag{10}$$

According to the adopted notations, we obtain the following dependencies for $i = 1, 2, \cdots, N$,

$$Area_{\Omega}(A_i) = \left[\ Area_{\Omega}(x_{i1}) \quad Area_{\Omega}(x_{i2}) \quad \dots \quad Area_{\Omega}(x_{iK})\ \right]. \tag{11}$$

At the beginning, we will be inspired by the concept of the positive ideal solution (*PIS*) and the negative ideal solution (*NIS*) taken from the paper by

Hwang and Yoon (1981). *The hypothetical, positive ideal alternative* A^+ with the smallest level of information uncertainty and the highest level of information certainty (i.e., *complete certainty* – maximum fulfillment of all criteria) is denoted by A^+,

$$\begin{aligned} A^+ &= \begin{bmatrix} \mathrm{x}_1^+ & \mathrm{x}_2^+ & \dots & \mathrm{x}_\mathrm{K}^+ \end{bmatrix} = \\ &= \begin{bmatrix} ([1.0,\ 1.0], [0.0,\ 0.0]) & ([1.0,\ 1.0], [0.0,\ 0.0]) & \dots & ([1.0,\ 1.0], [0.0,\ 0.0]) \end{bmatrix} \end{aligned}$$

This is a situation, in which a hypothetical alternative fully and maximally meets all the criteria.

The hypothetical, negative ideal alternative with the highest level of information uncertainty and the smallest level of information certainty (i.e., *complete uncertainty*) is denoted by A^-,

$$A^- = \begin{bmatrix} \mathrm{x}_1^- & \mathrm{x}_2^- & \dots & \mathrm{x}_\mathrm{K}^- \end{bmatrix} =$$

$$= \left[([0.0,\ 0.0], [0.0\ 0.0]) \quad ([0.0,\ 0.0], [0.0\ 0.0]) \quad \dots \quad ([0.0,\ 0.0], [0.0,\ 0.0]) \right].$$

This is a situation, in which we have no information about the extent to which a hypothetical alternative meets the criteria.

Let us consider the above two cases in turn. For the information uncertainty quantification matrix $Area_\Omega(\mathrm{X})$, the hypothetical positive alternative A^+ can be assessed and written in the form:

$$\begin{aligned} Area_\Omega(A^+) &= \begin{bmatrix} Area_\Omega(\mathrm{x}_1^+) & Area_\Omega\left(\mathrm{x}_2^+\right) & \dots & Area_\Omega(\mathrm{x}_\mathrm{K}^+) \end{bmatrix} = \\ &= \begin{bmatrix} 0.0 & 0.0 & \dots & 0.0 \end{bmatrix}. \end{aligned} \tag{12}$$

The negative ideal alternative A^- can be assessed and written in the form:

$$\begin{aligned} Area_\Omega(A^-) &= \begin{bmatrix} Area_\Omega(\mathrm{x}_1^-) & Area_\Omega\left(\mathrm{x}_2^-\right) & \dots & Area_\Omega(\mathrm{x}_\mathrm{K}^-) \end{bmatrix} = \\ &= \begin{bmatrix} 1.0 & 1.0 & \dots & 1.0 \end{bmatrix} \end{aligned} \tag{13}$$

Next, let us assume that the criteria are not equivalent and the relative importance of each criterion C_j is determined by a set of weights that are normalized to sum to one, i.e., $\sum_{j=1}^{K} w_j = 1$.

To enable the comparison of two *IVIFN*s, a metric method was used. Hence, the proximity of an alternative A_i to the hypothetical, ideal alternatives is defined as the Hamming distance, according to the following general formula, for $i = 1, 2, \dots, N$, as follows:

$$\mathrm{H}_\Omega^+\left(A_i, A^+\right) = \frac{1}{K}\sum_{j=1}^{K} w_j Area_\Omega\left(x_{ij}\right), \tag{14}$$

$$\mathrm{H}_{\Omega}^{-}\left(A_i,\, A^-\right) = \frac{1}{K}\sum_{j=1}^{K} w_j (1 - Area_{\Omega}\,(x_{ij})\,) \tag{15}$$

The overall distance value for alternative A_i can be calculated in the form of

$$H_{\Omega}(A_i) \quad = \quad \frac{\mathrm{H}_{\Omega}^{+}(A_i,\, A^+)}{\mathrm{H}_{\Omega}^{+}\,(A_i,\, A^+) \;+\; \mathrm{H}_{\Omega}^{-}(A_i, A^-)} \tag{16}$$

where $H_{\Omega}(A_i) \in [0,1]$, $\forall i = 1, 2, \cdots N$. This allows us to obtain a preferred order of $H_{\Omega}(A_i)$ distance values and select the alternative that provides the greatest certainty of information.

Remark 2 *It should be emphasized that the smaller the value of the distance* $\mathrm{H}_{\Omega}^{+}\,(A_i,\, A^+\,)$ *the greater the certainty of information concerning the alternative* A_i. *The relationship that the higher the value of* $\mathrm{H}_{\Omega}^{-}\,(A_i,\, A^-\,)$ *the greater the certainty of information relative to the alternative* A_i *is also satisfied. Based on the values of the above distance functions, we can determine the ranking of the considered alternatives and select the alternative with the highest information certainty.*

3.3. Ranking of alternatives based on directional assessments of information uncertainty

Based on the directional evaluation functions, we convert the matrix X into the three directional *information uncertainty quantification matrices* for the alternatives, as follows:

$$Area_{\upsilon}(\mathrm{X}) = \begin{bmatrix} Area_{\upsilon}(x_{11}) & Area_{\upsilon}(x_{12}) & \dots & Area_{\upsilon}(x_{1K}) \\ Area_{\upsilon}(x_{21}) & Area_{\upsilon}(x_{22}) & \dots & Area_{\upsilon}(x_{2K}) \\ \vdots & \vdots & \dots & \vdots \\ Area_{\upsilon}(x_{N1}) & Area_{\upsilon}(x_{N2}) & \dots & Area_{\upsilon}(x_{NK}) \end{bmatrix}, \tag{17}$$

and

$$Area_{\upsilon\mu}(\mathrm{X}) = \begin{bmatrix} Area_{\upsilon\mu}(x_{11}) & Area_{\upsilon\mu}(x_{12}) & \dots & Area_{\upsilon\mu}(x_{1K}) \\ Area_{\upsilon\mu}(x_{21}) & Area_{\upsilon\mu}(x_{22}) & \dots & Area_{\upsilon\mu}(x_{2K}) \\ \vdots & \vdots & \dots & \vdots \\ Area_{\upsilon\mu}(x_{N1}) & Area_{\upsilon\mu}(x_{N2}) & \dots & Area_{\upsilon\mu}(x_{NK}) \end{bmatrix}, \tag{18}$$

and

$$Area_{\mu}(\mathrm{X}) = \begin{bmatrix} Area_{\mu}(x_{11}) & Area_{\mu}(x_{12}) & \dots & Area_{\mu}(x_{1K}) \\ Area_{\mu}(x_{21}) & Area_{\mu}(x_{22}) & \dots & Area_{\mu}(x_{2K}) \\ \vdots & \vdots & \dots & \vdots \\ Area_{\mu}(x_{N1}) & Area_{\mu}(x_{N2}) & \dots & Area_{\mu}(x_{NK}) \end{bmatrix}. \tag{19}$$

We then proceed analogously as in Section 3.1, relying on the information uncertainty quantification matrices created above. We select the "best" alternative for each matrix separately, i.e., for the matrices $Area_{\upsilon}(\mathrm{X})$, $Area_{\upsilon\mu}(\mathrm{X})$ and $Area_{\mu}(\mathrm{X})$ we choose the alternative that provides the highest certainty of information.

4. Illustrative example – alternative with the highest level of information certainty

4.1. The outline of the example

The numerical example, which is considered here, is used to demonstrate the application and effectiveness of our approach in a simple multi-criteria fuzzy decision-making problem. The overall function and directional assessment functions proposed in this article are used in it to select the alternative with the highest degree of information certainty. The concept of evaluation functions is discussed in more detail in the forthcoming article by Szkatuła and Krawczak (in preparation).

Let us assume that we are considering a new project that we want to implement in the future. We want to choose the best project (i.e., the best alternative) from among four possible projects, acceptable to the expert: A_1, A_2, A_3, A_4, evaluated according to three criteria: C_1 : *stability,* C_2 :*development potential*, and C_3 :*reliability*. We want to choose the one that also provides the highest level of information certainty. We therefore want to choose the design that provides the smallest hypothetical deviation from the original design.

The evaluations of projects/alternatives are represented in the form of the intuitionistic fuzzy interval-valued matrix $\mathrm{X} = \left[\ x_{ij}\ \right]$, where $x_{ij} \in IVIFN$, for $i \in \{1, 2, 3, 4\}$, $j \in \{1, 2, 3\}$. Each i-th alternative (assessed using three criteria) can be treated as a vector $A_i = \left[\ x_{i1} \quad x_{i2} \quad x_{i3}\ \right]$. In this way, for four alternatives and three criteria we obtain a fuzzy matrix with *interval-valued intuitionistic numbers* as its entries (taken from the paper by Jun Ye, 2009), as follows:

$$\mathrm{X} = \left[\ x_{ij}\ \right] = \\ = \begin{bmatrix} ([0.4, 0.5], [0.3, 0.4]) & ([0.4, 0.6], [0.2, 0.4]) & ([0.1, 0.3], [0.5, 0.6]) \\ ([0.6, 0.7], [0.2, 0.3]) & ([0.6, 0.7], [0.2, 0.3]) & ([0.4, 0.7], [0.1, 0.2]) \\ ([0.3, 0.6], [0.3, 0.4]) & ([0.5, 0.6], [0.3, 0.4]) & ([0.5, 0.6], [0.1, 0.3]) \\ ([0.7, 0.8], [0.1, 0.2]) & ([0.6, 0.7], [0.1, 0.3]) & ([0.3, 0.4], [0.1, 0.2]) \end{bmatrix}. \tag{20}$$

First, let us assume the hypothetical positive and negative ideal alternatives that represent the lowest level of information uncertainty or the highest level of information uncertainty, as follows:

$$\begin{aligned} A^{+} &= [([1.0,\, 1.0]\,, [0.0,\, 0.0])([1.0,\, 1.0]\,, [0.0,\, 0.0])([1.0,\, 1.0]\,, [0.0,\, 0.0])]\,, \\ A^{-} &= [(\,[0.0,\, 0.0]\,, [0.0\,0.0])(\,[0.0,\, 0.0]\,,\, [0.0\,0.0])\,(\,[0.0,\, 0.0]\,,\, [0.0\,0.0])]\,. \end{aligned}$$

4.2. Creating the information uncertainty quantification matrices for alternatives

According to Eq. 4, we convert the interval-valued intuitionistic fuzzy matrix X into the quantification matrix of information uncertainty as follows:

$$Area_{\Omega}(\mathrm{X}) \quad = \quad \begin{bmatrix} 0.07 & 0.08 & 0.12 \\ 0.02 & 0.02 & 0.19 \\ 0.10 & 0.02 & 0.12 \\ 0.02 & 0.05 & 0.34 \end{bmatrix}. \tag{21}$$

According to Eqs. 5 – 7, we convert the interval-valued intuitionistic fuzzy matrix X into the three directional quantification matrices of information uncertainty as follows:

$$\begin{gathered} Area_{\upsilon}(\mathrm{X}) = \begin{bmatrix} 0.03 & 0.04 & 0.08 \\ 0.01 & 0.01 & 0.15 \\ 0.09 & 0.01 & 0.03 \\ 0.01 & 0.01 & 0.09 \end{bmatrix} \\ Area_{\upsilon\mu}(\mathrm{X}) = \begin{bmatrix} 0.01 & 0.00 & 0.01 \\ 0.00 & 0.00 & 0.01 \\ 0.00 & 0.00 & 0.01 \\ 0.00 & 0.00 & 0.16 \end{bmatrix} \\ Area_{\mu}(\mathrm{X}) = \begin{bmatrix} 0.03 & 0.04 & 0.03 \\ 0.01 & 0.01 & 0.03 \\ 0.01 & 0.01 & 0.08 \\ 0.01 & 0.04 & 0.09 \end{bmatrix} \end{gathered} \tag{22}$$

The geometric interpretation of the sets A_1, A_2, A_3, A_4 and their respective information uncertainty areas (as hatched gray areas) are shown in Fig. 4.

Next, we will be considering the problem of selecting the best alternative on the basis of the overall quantification matrix, given as (21) (see Section 4.3), and then on the basis of the directional information uncertainty quantification matrices, given in (22) (see Section 4.4).

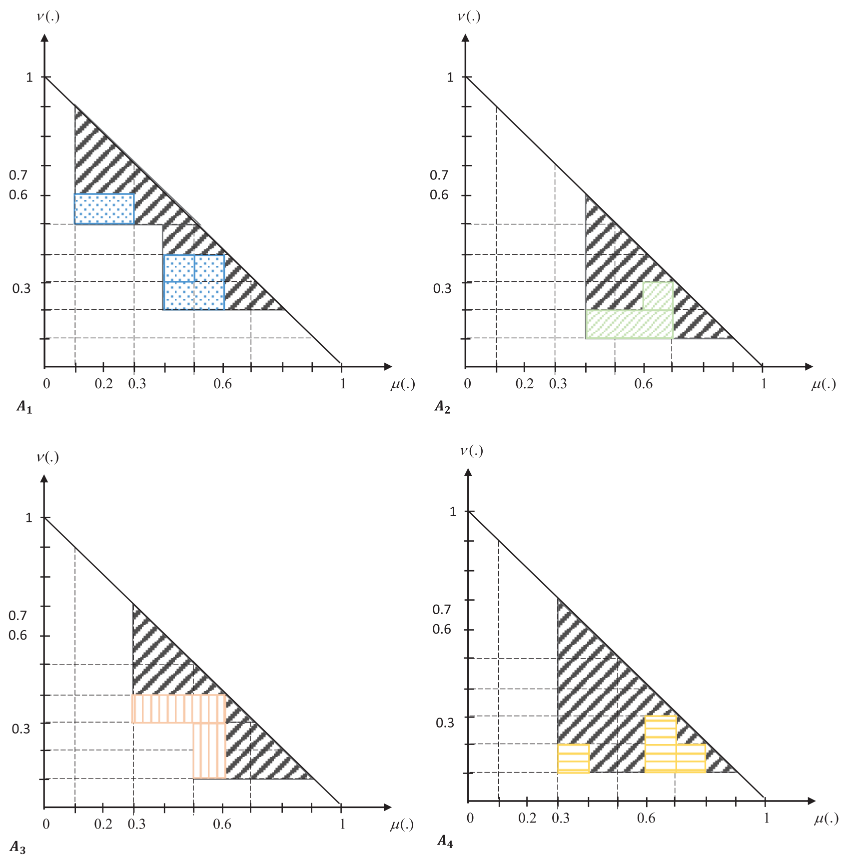

Figure 4. Geometric interpretation of the sets A_1, A_2, A_3, A_4 (as rectangles in different colors) and their information uncertainty areas (as a hatched area)

4.3. Analysis of alternatives based on an overall assessment of information uncertainty

Let us first consider the matrix $Area_\Omega(\mathrm{X})$. Therefore, for the hypothetical positive and negative ideal alternatives we get

$Area_\Omega(\mathrm{A}^+) = \begin{bmatrix} 0.0 & 0.0 & 0.0 \end{bmatrix}$ – i.e., complete information certainty,

$Area_\Omega(A^-) = \begin{bmatrix} 1.0 & 1.0 & 1.0 \end{bmatrix}$ – i.e., complete information uncertainty.

Let us assume that the weight vector W, associated with each criterion, is as follows:

$$W = (w_1,\, w_2,\, w_3) = (0.35, 0.25, 0.40).$$

The Hamming distance, used to calculate the separation measures of alternatives from the positive and negative ideal hypothetical intuitionistic solutions (i.e., A^+ and A^-) is calculated as follows:

$$\mathrm{H}_\Omega^+\left(A_i,\, A^+\right) = \frac{1}{3}\sum_{j=1}^{3} w_j\, Area_\Omega(x_{ij})$$

$$\mathrm{H}_\Omega^-\left(A_i,\, A^-\right) = \frac{1}{3}\sum_{j=1}^{3} w_j(1 - Area_\Omega(x_{ij})).$$

We will also consider the case, in which we ignore the influence of weights in the calculations. Next, the total distance value for alternative A_i can be calculated in the following manner

$$H_\Omega(A_i) = \frac{\mathrm{H}_\Omega^+(A_i,\, A^+)}{\mathrm{H}_\Omega^+(A_i,\, A^+)\ +\ \mathrm{H}_\Omega^-(A_i, A^-)}$$

where $H_\Omega(A_i) \in [0,1]$, $\forall\, i = 1,2,3,4$. The calculation results for the matrix $Area_\Omega$ (X) are shown in Tables 3 and 4.

Table 3. Hamming distance values based on $Area_\Omega$(X) and the resulting rankings of alternatives - without weights

A_i	$\mathrm{H}_\Omega^+(A_i,\, A^+)$	Ranking	$\mathrm{H}_\Omega^-(A_i,\, A^-)$	Ranking
A_1	0.09	3	0.91	3
A_2	0.07(6)	1	0.92(3)	1
A_3	0.08	2	0.92	2
A_4	0.13	4	0.86	4

Table 4. Hamming distance values based on $Area_\Omega$(X) and the resulting rankings of alternatives - with weights

A_i	$\mathrm{H}_\Omega^+(A_i,\, A^+)$	Ranking	$\mathrm{H}_\Omega^-(A_i,\, A^-)$	Ranking
A_1	0.0925	3	0.9075	3
A_2	0.088	1 ex aequo	0.912	1 ex aequo
A_3	0.088	1 ex aequo	0.912	1 ex aequo
A_4	0.1555	4	0.8445	4

The order of the alternatives is as follows: $A_2 \succ_\Omega A_3 \succ_\Omega A_1 \succ_\Omega A_4$, which is equivalent to saying that alternative A_2 provides the greatest certainty of information.

It should be emphasized that in the past various authors proposed new accuracy functions and various aggregation operators meant to help in determining the ranking order of alternatives. The original dataset from the article by Ye (2009) – how to best invest money – has been used in the studies of numerous other authors. The examples thereof are provided, in particular, by the article of Nayagam, Muralikrishnan and Sivaraman (2011) – *which company is the best to invest in*; Zhi-yong Bai (2013) – *how to make a new animation for computer animation competition*; Renato Krohling and André Pacheco (2014) – dataset use as a benchmark, and Daniel Osezua Aikhuele (2017) – *how to select a preferred naval vessel from a group of candidates as a reference point for a new design*.

Next, the obtained calculation results were compared with selected results known from the literature. The detailed results from this comparison are presented in Table 5.

All the above calculation results confirm the selection of alternative A_2 as the best. Our ranking of the above alternatives based on the $Area_{\Omega}(\mathrm{X})$ matrix shows the highest position for the alternative A_2, which is consistent with the information reported in the literature. The remaining positions in the ranking sometimes differ from each other for the same set of alternatives for different authors.

In such ambiguous situations, it is worth trying to use the directional functions to evaluate the alternatives, as this is described in the next section.

4.4. Analysis of alternatives based on the directional assessments of information uncertainty

A more diversified situation occurs when we want to additionally take into account the directionality of information in the process of selecting an alternative, based on the directional quantification matrices: $Area_{\upsilon}(\mathrm{X})$, $Area_{\mu\upsilon}(\mathrm{X})$ and $Area_{\mu}(\mathrm{X})$. The proposed directional approach allows us to evaluate and choose an alternative in cases where we care more about predicting possible changes from the point of view of the degree of non-membership or membership, separately.

Similarly to the case of the matrix $Area_{\Omega}(\mathrm{X})$, as described in Section 4.3, we proceed with three quantification matrices of the directional information uncertainty, each of them separately. The calculation results for the matrix $Area_{\upsilon}(\mathrm{X})$ are presented in detail in Table 6, for the matrix $Area_{\upsilon\mu}(\mathrm{X})$ in Table 7 and for the matrix $Area_{\upsilon\mu}(\mathrm{X})$ in Table 8.

A summary of the obtained rankings of alternatives for the directional matrices contained in Tables 6 through 8 is presented in Table 9.

The results, which are presented in Table 9 clearly indicate that the ranking of alternatives depends on the type of information uncertainty under consideration. The above calculations indicate different ranking results in the three cases considered (see the second, third and fourth columns in Table 9).

Alternative A_2 is the most reliable choice when overall uncertainty or uncertainty related to hypothetical membership changes are considered (see the second and third columns in Table 5, and the third and fourth column in Table 9). Alternative A_4 exhibits the lowest uncertainty with respect to hypothetical changes in the degree of non-membership (see the first place in the ranking in the second column in Table 9). At the same time, no differences are observed for alternatives A_2 and A_3 (i.e., $A_2 \approx_{\mu v} A_3$ in matrix $Area_{v\mu}(\mathrm{X})$), when uncertainty related to simultaneous changes in membership and non-membership is taken into account.

Using the above considerations, the measures here proposed can be effectively applied to compare the alternatives. For example, the alternative A_2 is less certain than the alternative A_4 in terms of the degree of non-membership (i.e., $A_2 \prec_v A_4$), but it is more certain than the alternative A_4 in terms of the membership degree (i.e., $A_2 \succ_\mu A_4$).

The observations presented here show that information uncertainty cannot always be adequately described by a single scalar measure. Depending on the decision-making context and on the perspective,adopted by the decision-maker, different aspects of uncertainty may play a dominant role. Against the background of the nature of information, assumed in the problem here considered, it is worth remembering that something can be better from one point of view and worse from another.

5. Conclusion

Expert evaluations have the disadvantage of being subjective and may suffer from significant information uncertainty. In multi-criteria evaluation and decision-making problems, various types of uncertainty are increasingly being taken into account. This article presents a new approach to expressing and modeling information uncertainty in a multicriteria decision problem for interval-valued intuitionistic fuzzy sets and presents the methods for ranking alternatives based on the level of certainty of the information they convey.

The computational results presented in this paper clearly indicate that the ranking of alternatives depends on the type of information uncertainty under consideration. The proposed uncertainty assessment framework considers four types of information uncertainty: overall information uncertainty and three types of directional information uncertainty. These are: *type 1* – taking into account the uncertainty related to hypothetical changes in the degree of mem-

Table 5. Rankings of alternatives obtained for the dataset from Ye (2009)

Alternative A_i	Own calculations for $Area_\Omega(\mathrm{X})$ without weights	Own calculations for $Area_\Omega(\mathrm{X})$ with weights	Ye (2009)	Bai (2013)	Krohling and Pacheco (2014)	Nayagam, Muralikrishnan and Sivaraman (2011)	Aikhuele (2017) proposed model	Aikhuele (2017) weighted normalized Euclidean	Aikhuele (2017) TOPSIS model
A_1	3	3	4	4	4	4	4	3	4
A_2	1	1 ex aequo	1	1	1	1	1	1	1
A_3	2	1 ex aequo	3	3	3	3	2	2	2
A_4	4	4	2	2	2	2	3	4	3

Table 6. Hamming distance values based on $Area_{\upsilon}(\mathrm{X})$ and the resulting rankings of alternatives

A_i	$\mathrm{H}_{\upsilon}^{+}(A_i, A^{+})$	Ranking	$\mathrm{H}_{\upsilon}^{-}(A_i, A^{-})$	Ranking
A_1	0.05	3	0.95	3
A_2	0.05(6)	4	0.94(3)	4
A_3	0.04(3)	2	0.95(6)	2
A_4	0.03(6)	1	0.96(3)	1

Table 7. Hamming distance values based on $Area_{\upsilon\mu}(\mathrm{X})$ and the resulting rankings of alternatives

A_i	$\mathrm{H}_{\upsilon\mu}^{+}(A_i, A^{+})$	Ranking	$\mathrm{H}_{\upsilon\mu}^{-}(A_i, A^{-})$	Ranking
A_1	0.00(6)	3	0.99(3)	3
A_2	0.00(3)	1 ex aequo	0.99(6)	1 ex aequo
A_3	0.00(3)	1 ex aequo	0.99(6)	1 ex aequo
A_4	0.05(3)	4	0.94(6)	4

bership, *type 2* – taking into account the hypothetical changes in the degree of non-membership, and *type 3* – with simultaneous changes in membership and non-membership being considered. The presented approach allows us to include at least one of the types of information uncertainty, mentioned above, in the calculations.

The proposed directional approach allows us to evaluate and select an alternative in cases where we are more concerned with predicting possible changes from the point of view of the degree of non-membership or membership, separately. Each alternative is evaluated against each criterion based on the level of four types of uncertainty in the information it conveys.

We can then choose the alternative that provides the greatest certainty of information, i.e. has the smallest possible margin, based on the level of the selected type of uncertainty. The proposed directional evaluation framework allows such distinctions to be explicitly taken into account. By decomposing the information uncertainty area into sub-regions and evaluating them separately, the decision-maker gains additional insight into the structure of uncertainty, conveyed by the available information. This enables more informed and context-sensitive decisions, especially in real-world situations, where human perception of uncertainty is inherently asymmetric.

Table 8. Hamming distance values based on $Area_{\mu}$(X) and the resulting rankings of alternatives

A_i	$\mathrm{H}_{\mu}^{+}(A_i, A^{+})$	Ranking	$\mathrm{H}_{\mu}^{-}(A_i, A^{-})$	Ranking
A_1	0.03(3)	2 ex aequo	0.96(6)	2 ex aequo
A_2	0.01(6)	1	0.98(3)	1
A_3	0.03(3)	2 ex aequo	0.96(6)	2 ex aequo
A_4	0.04(6)	4	0.95(3)	4

Table 9. Summary of obtained rankings for the alternatives considered

A_i	Ranking for $Area_{\upsilon}$(X)	Ranking for $Area_{\upsilon\mu}$(X)	Ranking for $Area_{\mu}$(X)
A_1	3	3	2 ex aequo
A_2	4	1 ex aequo	1
A_3	2	1 ex aequo	2 ex aequo
A_4	1	4	4

It may be interesting to compare this issue with the authors' earlier work on the problem of directional comparison of different types of sets, see the book by Szkatuła and Krawczak (2024).

It should always be remembered that the choice of directional interpretation of results depends largely on the needs of the recipients of the information and how they perceive changing uncertainty. It seems that the method based on the directional analysis of the change in information uncertainty provides a completely new approach to calculating the rank of intuitionistic fuzzy alternatives.

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