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Intelligent maximum likelihood self-adjustable block roots assignment for a class of MIMO stochastic systems*

by

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Abstract: This paper presents a new self-adjustable block roots assignment control scheme for multivariable stochastic systems, via the use of conventional intelligent MIMO maximum likelihood identification algorithm, handled with an adaptive neural based fuzzy inference system (ANFIS). The proposed state-space self-tuning control methodology can be applied to the multivariable stochastic system without requiring prior knowledge of system parameters and noise properties. Illustrative examples demonstrate the effectiveness of the proposed approach.

Keywords: solvents, self-adjustable block roots, matrix polynomial, self-tuning, MIMO systems, ANFIS

1. Introduction

A critical consideration in modern control system design is the ability to maintain robust performance in the presence of uncertainties, including both stochastic disturbances and unknown system parameters. In this regard, Disturbance Observers (DOs) have received increasing attention in both academic research and industrial applications, due to their effectiveness in estimating and compensating for unmeasured inputs and model mismatches. Recent studies include the observer-type Kalman innovation filter for uncertain linear systems (Guo Shieh and Coleman, 2001), the online observer/Kalman filter identification method for unknown stochastic systems (Wu et al., 2014), and the observer-based minimum variance control of uncertain piecewise affine systems (Razavi et al., 2016). Other contributions address the design of observers for fuzzy stochastic systems

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(Guo, Zhu and Xu, 2015), constrained uncertain nonlinear discrete-time systems (Hassan and Kar, 2015), and flight vehicle dynamics (Mohamed and Kar, 2015). For example, the fading-memory Kalman filter has been successfully applied to the estimation of the states and unknown inputs of an electric vehicle driveline (Rahimi, Mousavi and Boulet, 2016).

In parallel, system identification has become a central topic in adaptive control, enabling online estimation of system dynamics from observed data. Several techniques have been developed for this purpose, including least-squares estimation (Ljung, 1999; Bao et al., 2011), gradient-based algorithms (Liu, Sheng and Ding, 2010), maximum likelihood estimation (Agüero et al., 2010), and step-response-based methods (Ahmed, Huong and Shah, 2007). Additionally, polynomial transformation techniques have been employed to address dual-rate sampled-data systems and cases with missing observations (Bao et al., 2011). Building on these developments, state-space self-tuning control (STC) strategies, introduced in the early 1980s (Warwick, 1981; Tsay and Shieh, 1981), have shown strong potential in the adaptive control of linear multivariable stochastic systems (Shieh, Bao and Chang, 1989). These approaches incorporate the Kalman state estimation algorithm (Åström and Wittenmark, 1989) within online identification schemes, often relying on innovation-based models. Such methods offer several advantages, as they:

- enable successive estimation of internal system states;
- ensure precise control of both stable and unstable, as well as minimum and non-minimum phase systems;
- provide simple, reliable, and robust self-tuning behavior;
- adaptive Kalman gains can be computed in real time based on the evolving system dynamics.

Recent developments in MIMO control have focused on adaptive and intelligent techniques to handle uncertainty and structural complexity. Liu and Zhu (2021) proposed a neural network-based strategy for stochastic nonlinear systems, while Gao (2023) addressed output feedback pole configuration for systems with varying dimensions. Iterative learning methods have been applied by Trojaola (2021) to hydraulic MIMO control, and He (2020) developed an analytical MPC tuning approach for fractional dead-time systems. Advanced fuzzy scheduling (Barbosa and Silvestre, 2025), switching MIMO architectures (Schröders, 2020), and virtual system transformations (Pan, 2023) have further extended the design space. Complementary to these efforts, Bekhiti (2024, 2025), and Bekhiti et al. (2025a,b) have contributed polynomial-based methods for block root assignment, order reduction, and spectral factorization in large-scale MIMO systems, with applications in robotics, power drives, and missile guidance.

The present work introduces a new self-adjustable block roots assignment algorithm, designed for a class of MIMO stochastic systems. The proposed framework modifies the conventional Kalman filtering process by integrating a refined maximum likelihood (ML) parameter estimation technique tailored to MIMO systems. In contrast to the standard STC approach, which relies on covariance matrices estimated via the recursive extended least squares (RELS) method, our formulation uses the ML-based parameter estimates directly to improve adaptation and estimation performance under uncertainty. This paper is organized as follows: In Section 2, theoretical preliminaries are introduced. Section 3 focuses on the proposed intelligent maximum likelihood stochastic block observer-estimator. Section 4 discusses the self-adjustable block roots assignment for unknown MIMO stochastic linear systems, and illustrative examples are given all along the design steps. Finally, comments and conclusions will finish the paper.

2. Theoretical preliminaries

2.1. MIMO dynamic system representation

The term *system description* generally refers to a mathematical formulation that captures the relationship between the physical variables of a system and its constituent components. This formulation defines the mathematical model of the system, from which one can derive, in a large proportion of cases, a set of high-order coupled differential equations, or more generally, an ℓ^{th} degree, m^{th} order vector linear differential (or difference) equation with constant matrix coefficients. Such equations represent a natural extension of scalar models to the multivariable case, and ultimately lead to matrix transfer function representations, as discussed in Bekhiti (2015) and Bekhiti et al. (2015). A dynamical system can be modeled using either an internal description, commonly known as the state- space representation, or an external description, which corresponds to the input-output formulation.

2.1.1. State space description

Linear time-invariant m-input p-output system can be described by a state equation in general coordinates:

$$\left\{\begin{array}{ll} \dot{X} = & AX(t) + BU(t) \\ Y = & CX(t) + DU(t) \quad \text{with} \quad X(0) \quad \textit{constant} \end{array}\right. \tag{1}$$

where $A \in R^{n\times n}, B \in R^{n\times m}$, $C \in R^{p\times n}, D \in R^{p\times m}$, the state $X(t) \in R^{n\times 1}$, output $Y(t) \in R^{p\times 1}$, and input $U(t) \in R^{m\times 1}$, $n = ml$ or $n = pl$ with l being an integer, n: is the total number of states, m: is the number of input-output channels, ℓ: is the order of sub-model, while $X(0)$ is an initial $n \times 1$ vector.

2.1.2. Matrix transfer function description

When a dynamic system is described by a state equation in general coordinates, it is often transformed to specific canonical forms, so that analysis and synthesis of a MIMO system can easily be performed (Akroum and Hariche, 2008, 2009, 2010). The very conventional canonical representation in frequency domain is the so called matrix fraction description (MFD), which can be either right, RMFD, or left, LMFD.

$$\begin{aligned} H(\lambda) &= C\,(\lambda I - A)^{-1} B + D \\ &= N_r(\lambda) {D_r}^{-1}(\lambda) \\ &= {D_l}^{-1}(\lambda) N_l(\lambda) \end{aligned} \tag{2}$$

where: N_r, D_r, N_l and D_l are matrix polynomials and "λ" stands for $\left(\dfrac{d}{dt}\right)$ operator. This fact has led to an active research effort in matrix polynomials theory, see, for instance, Yaici and Hariche (2014) and Yaici, Hariche and Clark (2014).

2.2. Matrix polynomials

In this section, we attempt to present some of important results obtained in the theory of matrix polynomials. Stronger emphasis will be placed on the latent structure of these matrix polynomials, which consists mainly of the latent roots and latent vectors, as well as solvents. The algebraic theory of matrix polynomials has been investigated by Dennis, Traub and Weber (1976, 1978) and by Gohberg, Kaashoek and Rodman (1978) and Gohberg, Lancaster and Rodman (1982). Spectral factors of a lambda matrix and the right (left) solvents, for a right (left) characteristic matrix polynomial have been defined. The different transformations between right (left) solvents and spectral factors are mainly proposed by Shieh and Tsay (1981).

Definition 1 *Given the set of $m \times m$ complex matrices $A_0, A_1, ..., A_l$, the following matrix valued function of the complex variable λ is called a matrix polynomial of degree l and order m:*

$$A(\lambda) = A_0\lambda^l + A_1\lambda^{l-1} + ... + A_{l-1}\lambda + A_l. \tag{3}$$

Definition 2 *The complex number λ_i is called a latent root of the matrix polynomial $A(\lambda)$ if it is a solution of the scalar polynomial equation $det(A(\lambda)) = 0$. The nontrivial vector V_i, solution of $A(\lambda_i)V_i = 0_m$, is called a primary right latent vector, associated with λ_i. Similarly, the nontrivial vector W_i, solution of $W_i^T A(\lambda_i) = 0_m$, is called a primary left latent vector, associated with λ_i.*

Remark 1 *If $A(\lambda)$ has a singular leading coefficient (A_l) then $A(\lambda)$ has latent roots at infinity. From the definition we can see that the latent problem of a* *matrix polynomial is a generalization of the concept of eigenproblem for square matrices. Indeed, we can consider the classical eigenvalues/vector problem as the one of finding the latent root/vector of a linear matrix polynomial* $(\lambda I - A)$.

We can also define the spectrum of a matrix polynomial $A(\lambda)$ as being the set of all its latent roots (notation: $\sigma(\lambda)$). It is essentially the same definition as the one of the spectrum of a square matrix.

Definition 3 *A right (left) block root is also called solvent of* λ*-matrix* $A(\lambda)$ *and is an* $m \times m$ *real matrix* R *(*L*) such that:*

$$A_R(R) = \sum_{i=0}^{l} A_i R^{l-i} = O_m \qquad A_L(L) = \sum_{i=0}^{l} L^{l-i} A_i = O_m. \tag{4}$$

The following key results regarding solvents of matrix polynomials were established by Shieh and Tsay (1982a):

- *Solvents of a matrix polynomial do not necessarily exist in general.*
- *The generalized right (or left) eigenvectors, associated with a right (or left) solvent are identical to the generalized latent vectors of the matrix polynomial.*

Definition 4 *A matrix* R *(respectively:* L*) is called a right (respectively: left) solvent of the matrix polynomial if and only if the binomial* $(\lambda I - R)$ *(respectively:* $(\lambda I - L)$*) divides exactly* $A(\lambda)$ *on the right (respectively: left).*

Theorem 1 (Yaici and Hariche, 2014) *Let* $\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ *be a set of eigenvalues of an* $n \times n$ *block controllable system in SSD, described by Eq(1), and* $\{V_1, V_2, \cdots, V_n\}$ *be its corresponding eigenvectors (we propose right). Let* Λ *be an* $n \times n$ *matrix, whose diagonal is constituted by the eigenvalues* $\Lambda = diag\ ([\lambda_1, \lambda_2, \cdots, \lambda_n])$ *and a matrix* V *composed of right eigenvectors* $V = [V_1, V_2, \cdots, V_n]$*, let* W *be the matrix of left eigen vectors or* $W = V^{-1}$*, the state matrix of the system, can be rewritten as* $A = V \Lambda W$*. For each right eigenvector* V_i $(i = 1, \cdots, n)$ *its corresponding latent vector is* $v_i = T_{c1} V_i$*, where* $T_{c1} = [O_m \cdots I_m] \left[B, AB, \cdots A^{l-1}B\right]$*. By duality,* $w_i = W_i T_{o1}$*.*

Remark 2 The proposed process consists in assigning a desired latent structure by placing the block roots. So, for a set of pl (p, l being integers) desired latent roots $\{\lambda_1, \cdots, \lambda_{pl}\}$ and a set of corresponding latent vectors (left or right) $\{v_1, \cdots, v_{pl}\}$, we can construct l block roots if there exist p groups of l linearly independent latent vectors (see Shieh and Tsay, 1981)

$$R_i = [v_{i1}, \cdots, v_{ip}] \begin{pmatrix} \lambda_{i1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_{ip} \end{pmatrix} [v_{i1}, \cdots, v_{ip}]^{-1}. \tag{5}$$

2.3. Coprime factorization

In the analysis and synthesis of multivariable control systems, the concept of coprimeness of the fractions of transfer function matrices is equivalent to the concepts of controllability and observability in dynamic equations. In the latter case the right coprime factorization of H is

$$H(\lambda) = N_r(\lambda)D_r^{-1}(\lambda) \tag{6}$$

where $N_r(\lambda)$ and $D_r(\lambda)$ are stable coprime transfer functions. The stability implies that $N_r(\lambda)$ should contain all the RHP zeros of $H(\lambda)$, and $D_r(\lambda)$ should contain as RHP zeros, all the RHP poles of $H(\lambda)$. The coprimeness implies that there should be no common RHP zeros in N_r and D_r, resulting in pole zero cancellations when forming $N_rD_r^{-1}$. Mathematically coprimeness means that there exist stable $U_r(\lambda)$ and $V_r(\lambda)$, such that the following Bezout identity is satisfied:

$$U_rN_r + V_rD_r = I. \tag{7}$$

Similarly, the left coprime factorization of H is

$$H(\lambda) = D_l^{-1}(\lambda)N_l(\lambda). \tag{8}$$

Here, N_l and D_l are stable and coprime, that is, there exist stable $U_l(\lambda)$ and $V_l(\lambda)$ such that the following Bezout identity is satisfied

$$N_lU_l + D_lV_l = I. \tag{9}$$

Remark 3 *In the discrete-time case, stability is ensured when the modulus of each eigenvalue is strictly less than one.*

Definition 5 *Let a system $H(\lambda)$ be described in RMFD as in eq.(6), then we have the following:*

- *$N_r(\lambda)$ and $D_r(\lambda)$ are right-coprime if they have unimodular common right divisors.*
- *$N_r(\lambda)$ and $D_r(\lambda)$ are right-coprime if they have no common latent roots and associated latent vectors.*
- *If $N_r(\lambda)$ and $D_r(\lambda)$ are right-coprime then $H(\lambda)$ is said to be irreducible. The same definition can be applied to the left coprimeness and also in discrete systems.*

2.4. Reconstructibility aspects

Specifically, we want to investigate to what extent it is possible to reconstruct the state x when the input u and the output y are known. The motivation is that we often can measure the output and prescribe (and hence know) the input, whereas the state variable is hidden.

Definition 6 *Two states, x_0 and x_1, in the space $\mathcal{X}$ are called indistinguishable on the interval $[0,\ T]$ if for any input u we have $y_u(t, x_0) = y_u(t, x_1)$ for $0 \leq t \leq T$. Hence, x_0 and x_1 are indistinguishable if they give rise to the same output values for every input u.*

We notice that the input function is of no relevance to distinguishability, i.e. if one u is able to distinguish between two states, then any input is. In fact, x_0 and x_1 are indistinguishable if and only if

$$Ce^{At}x_0 = Ce^{At}x_1, \quad \text{for } 0 \leq t \leq T.$$

Obviously, x_0 and x_1 are indistinguishable if and only if

$$v := x_0 - x_1$$

is indistinguishable from 0. It follows that

$$Ce^{At}x_0 = Ce^{At}x_1 \quad \text{if and only if} \quad Ce^{At}v = 0, \quad \text{for } 0 \leq t \leq T,$$

and hence if and only if

$$CA^k v = 0 \quad \text{for } k = 0, 1, \dots .$$

Using the Cayley-Hamilton theorem (Kalman, 1963), we can show that we only need to consider the first n terms,

$$\mathcal{O}v = 0 \qquad \text{where} \qquad \mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}. \tag{10}$$

As a consequence, the distinguishability of two vectors does not depend on T. The space of vectors v, for which $\mathcal{O}v = 0$ holds, is denoted $\langle kerC \mid A \rangle$, and called the unobservable subspace. It is equivalently characterized as the intersection of the spaces $kerCA^k$ for $k = 0, \cdots, n-1$, i.e.

$$\langle kerC \mid A \rangle = \bigcap_{k=0}^{n-1} kerCA^k. \tag{11}$$

One can also say: $\langle kerC \mid A \rangle$ is the largest A-invariant subspace contained in $kerC$. Still another characterization is: $v \in \langle kerC \mid A \rangle$ if and only if $y_0(t, v)$ is identically zero, where the subscript refers to the zero input.

Definition 7 *The system $\mathcal{S}(A, B, C, D)$ is called observable if any two distinct states are distinguishable.*

The previous considerations immediately lead to the following result:

Theorem 2 *The following statements are equivalent:*

- *the system* $\mathcal{S}(A, B, C, D)$ *is observable,*
- *every state is distinguishable from the origin,*
- $\langle kerC \mid A\rangle = 0$,
- $Ce^{At}v = 0 \quad (0 \le t \le T) \Rightarrow v = 0$,
- *rank*$(\mathcal{O}) = n$.
- *the* n *columns of* Ce^{At} *are linearly independent on* $[0, \infty)$ *over the field of complex numbers, or alternatively, the* n *columns of* $C(\lambda I - A)^{-1}$ *are linearly independent over the field of complex numbers,*
- *the rank* $\begin{pmatrix} \lambda_i I - A \\ C \end{pmatrix} = n$ *for all complex numbers* λ_i*, or alternatively, for all eigenvalues of* A.

Definition 8 (State observability) *The dynamical system* $\dot{x} = Ax + Bu$*;* $y = Cx + Du$ *(or the pair* (A, C)*) is said to be state observable if, for any time* $t_1 > 0$*, the initial state* $x(0) = x_0$ *can be determined from the time history of the input* $u(t)$ *and the output* $y(t)$ *in the interval* $[0,\ t_1]$. *Otherwise, the system, or* (A, C)*, is said to be state unobservable.*

A system is state observable if we can obtain the value of all individual states by measuring the output $y(t)$ over some time period. However, even if a system is state observable, it may not be observable in a practical sense. For example, obtaining $x(0)$ may require taking high order derivatives of $y(t)$, which may be numerically poor and sensitive to noise.

Let us consider the following system, described by its transfer matrix, defined as:

$$\begin{aligned} H(\lambda) &= N(\lambda)D(\lambda)^{-1} \\ &= C(\lambda I - A)^{-1}B + D \\ &= C\mathbb{V}(\lambda I - \Lambda)^{-1}\mathbb{V}^{-1}B + D \\ &= C\mathbb{V}(\lambda I - \Lambda)^{-1}\mathbb{W}B + D, \end{aligned} \tag{12}$$

where:

$$\mathbb{V} = \begin{bmatrix} \mathbb{V}_1 & \mathbb{V}_2 & \cdots & \mathbb{V}_l \end{bmatrix}, \quad \mathbb{W} = \mathbb{V}^{-1} = \begin{bmatrix} \mathbb{W}_1^T \\ \mathbb{W}_2^T \\ \vdots \\ \mathbb{W}_l^T \end{bmatrix}, \ \Lambda = \begin{pmatrix} R_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_l \end{pmatrix}$$

$A \in R^{n\times n}, B \in R^{n\times m}, C \in R^{p\times n}, D \in R^{p\times m}$
$\mathbb{V}_i$: right block eigenvector of A corresponding to R_i
$\mathbb{W}_i$: left block eigenvector of A corresponding to R_i
R_i : the block eigenvalues of A (right solvent).

If $A \in R^{n\times n}$ is block diagonalizable, then it can be rewritten as:

$$A = \mathbb{V}\Lambda\mathbb{W} = \sum_{i=1}^{n} \mathbb{V}_i R_i \mathbb{W}_i^T. \tag{13}$$

Rewriting down the preceding matrix transfer function in terms of its block eigenvalues, we obtain:

$$H(\lambda) = \left(\sum_{i=1}^{l} C\mathbb{V}_i \left(\lambda I_m - R_i\right)^{-1} \mathbb{W}_i^T B \right) + D \tag{14}$$

The contribution of the block root R_i in the matrix transfer function is cancelled when $C\mathbb{V}_i = 0$ or $\mathbb{W}_i^T B = 0$, or both. Matrices $E_i = \mathbb{V}_i \mathbb{W}_i^T$ are called block projectors and satisfy the following properties:

1. $\sum\limits_{i=1}^{l} E_i = \sum\limits_{i=1}^{l} \mathbb{V}_i \mathbb{W}_i^T = I_n$ 2. $\mathbb{W}_j^T \mathbb{V}_i = \begin{cases} O_m & i \neq j \\ I_m & i = j \end{cases}$

3. $E_i = \mathbb{V}_i \mathbb{W}_i^T$ and ${E_i}^2 = E_i$ 4. $E_i E_j = O_m$ iff $i \neq j$.

Remark 4

1. Multi-input, multi-output systems generally lead to matrix fraction descriptions (MFD) of rational matrices and/or block partial fraction expansions expressed in terms of projectors contribution.

2. In the SISO case, if the transfer function of a system involves pole-zero cancellations, then the system is either uncontrollable or unobservable or both. If the transfer function does not involve any pole-zero cancellation, then the system is both controllable and observable, However, in the MIMO case, block pole-zero cancellation no longer necessarily indicates problems with controllability and/or observability. MIMO systems can have more zeros than poles, and yet be strictly proper, also can have poles and zeros at the same place and still may or may not be perfectly controllable and observable.

3. MIMO deterministic and stochastic observers design

In the design of a closed-loop system using modern control techniques, the applied control strategy typically consists of a feedback loop based on the system state vector $X(t)$. Examples of such strategies are provided by the application of the state feedback law $U(t) = F.X(t) + G.R(t)$ for pole assignment, decoupling, and model matching. This means that for this type of feedback law to be applicable, the entire state vector $X(t)$ must be available (measurable). In practice, however, it happens very often that not all state variables of a system are accessible to measurement. This obstacle can be circumvented if a mathematical model for the system is available, in which case it is possible to estimate the state vector. A widely known method for state-vector estimation or reconstruction is that of using an observer.

3.1. State-vector reconstruction using a block Luenberger observer

3.1.1. The setting

Consider the MIMO deterministic system, described by equation (1), then an observer for a dynamic system $S(X, Y, U)$ with state $X(t) \in R^{n\times 1}$, output $Y(t) \in R^{p\times 1}$, and input $U(t) \in R^{m\times 1}$ is another dynamic system $\hat{S}(\hat{X}, Y, U)$, having the property that the state $\hat{X} \in R^{n\times 1}$ of the observer $\hat{S}$ converges to the state $X(t)$ of the process S, independently of the input $U(t)$ or the state $X(t)$. An estimator of the full state $X(t)$ can be constructed in the following manner. We consider the system

$$\dot{\hat{X}}(t) = A\hat{X}(t) + BU(t) + L(Y(t) - \hat{Y}(t)) \tag{15}$$

where $\hat{Y}(t) = C\hat{X}(t) + DU(t)$. Note that (15) can be written down as

$$\dot{\hat{X}}(t) = (A - LC)\,\hat{X}(t) + [B - LD,\ L]\begin{bmatrix} U(t) \\ \\ Y(t) \end{bmatrix}. \tag{16}$$

The error between the actual state $X(t)$ and the estimated state $\hat{X}(t)$, $\varepsilon(t) = X(t) - \hat{X}(t)$, is governed by the differential equation

$$\dot{\varepsilon}(t) = \dot{X}(t) - \dot{\hat{X}}(t) = [AX + BU] - \left[A\hat{X} + BU + LC(X - \hat{X})\right], \tag{17}$$

or

$$\dot{\varepsilon}(t) = [A - LC]\,\varepsilon(t). \tag{18}$$

By solving (18), we obtain

$$\varepsilon(t) = \exp[(A - LC)t]\varepsilon(0). \tag{19}$$

Now, if the matrix of $A_d = (A - LC)$ is stable, then $\varepsilon(t) \to 0$ as $t \to \infty$, independently of the initial condition $\varepsilon(0) = X(0) - \hat{X}(0)$. This asymptotic state estimator is known as the Luenberger observer.

Lemma 1 *There exists a matrix $L \in R^{n\times p}$ such that the block roots of $(A - LC)$ can be assigned to arbitrary real or complex conjugate locations if and only if (A, C) is block observable.*

3.1.2. Observer gain design ($L \in R^{n\times p}$)

Obtaining the observer gain matrix L is equivalent to relocating the block poles of the observer, that is, the latent structure of $A_d = (A - LC)$ in Equation (18), which is an alternative to solving the algebraic Riccati equation. Hence, specification of the block poles of A_d uniquely determines the gain matrix L. A number of algorithms can be used to determine the gain matrix, some of which are incorporated into the popular control system design software packages. Some of the algorithms have been found to be numerically ill-conditioned; so caution should be exercised in using the results.

We found the block roots placement formula to be effective in most applications. The desired matrix of the observe system is given by $A_d = A - LC$, and taking the similarity transformation $X_o = T_o X$ we obtain

$$\begin{aligned} {T_o}^{-1} A_d T_o &= {T_o}^{-1}(A - LC)T_o \\ &= A_o - L_o C_o \\ &= \begin{pmatrix} I_p & O_p & \cdots & -(A_l + L_{ol}) \\ \vdots & \ddots & \cdots & \vdots \\ O_p & \cdots & I_p & -(A_1 + L_{o1}) \end{pmatrix}, \end{aligned} \tag{20}$$

where $L = T_o L_o = T_o \left[L_{ol}^T \cdots L_{o1}^T\right]^T$ and $L_{oi} \in R^{p\times p}$ for $i = 1 \cdots l$. Now, matching the desired block observable matrix to the one found in equation (20) we get: $L_{oi} = A_{di} - A_i$. The observer matrix gain L is constructed from the corresponding latent structure $(S_i,\, i = 1, \cdots, l)$ according to the equation

$$L = T_o \left(A_o \begin{pmatrix} O_p \\ O_p \\ \vdots \\ I_p \end{pmatrix} - \begin{pmatrix} I_m & S_1 \cdots {S_1}^{l-1} \\ I_m & S_2 \cdots {S_2}^{l-1} \\ \vdots & \vdots \ \ddots \ \vdots \\ I_m & S_l \cdots {S_l}^{l-1} \end{pmatrix}^{-1} \begin{pmatrix} {S_1}^l \\ {S_2}^l \\ \vdots \\ {S_l}^l \end{pmatrix} \right) \tag{21}$$

$$T_o = \left[\Omega_o^{-1} J^T,\, A\Omega_o^{-1} J^T,\, \cdots,\, A^{l-1}\Omega_o^{-1} J^T\right], \quad J = [O_p,\, \cdots,\, I_p]$$

$$\Omega_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{l-1} \end{bmatrix}.$$

Remark 5

1. The designed block observer has more degrees of freedom, because instead of assigning only eigenvalues we are able to assign the whole set of eigen-structure (block poles).

2. It is concluded that even if the original model is unstable, the designed state observer can still reconstruct the states of the system satisfactorily, provided that a suitable observer gain vector is designed. Of course, the original system, although unstable, has to be fully observable.

3. The same procedure can be followed in the case of MIMO discrete systems.

4. This method does not require prior knowledge of the open-loop block poles, and the observer does not impose any restrictions on the position of the desired block poles or their nature.

5. Finally, the proposed observer is of deterministic nature, which can not resolve the stochastic problem, and this obstacle is the unique limitation of the given observer, so, in order to ensure its more effective utilization we propose a new stochastic block observer, see Section 3.3.

Example 1 The jet model during cruise flight at $MACH = 0.8$ and $H = 40,000ft.$ is given by state equation (Stevens and Lewis, 2003):

$$A = \begin{pmatrix} -0.8307 & 2.4626 & -1.2339 & -1.6184 \\ 0.2426 & -2.2210 & 1.6156 & -0.8875 \\ 0.1925 & 0.7818 & -1.8056 & -0.5714 \\ -0.3678 & -0.7244 & 3.0466 & -3.1428 \end{pmatrix}, B = \begin{pmatrix} 1.2462 & 0.9837 \\ 0.6972 & 0.3929 \\ 0.5485 & 0.3379 \\ 1.4538 & 0.2110 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.2857 & 0.8338 & 0.5075 & -0.4782 \\ 0.9046 & -0.1944 & -0.7934 & 0.2872 \end{pmatrix}, D = \begin{pmatrix} 0.0000 & 0.0000 \\ 0.0000 & 0.0000 \end{pmatrix}$$

with $X = (beta, yaw, roll, phi)^T \in R^{4\times 1}$, $U = (rudder, aileron)^T \in R^{2\times 1}$. The system is block observable, because rank $\Omega_o = n$, and then it can be transformed to its block observable form as $A_o = T_o^{-1}AT_o$, $B_o = T_o^{-1}B$, $C_o = CT_o$, where:

$$A_o = \begin{pmatrix} 0 & 0.0000 & -23.4627 & 22.3872 \\ 0.0000 & 0.0000 & -26.8259 & 25.3165 \\ 1.0000 & 0.0000 & -16.5390 & 9.6207 \\ 0.0000 & 1.0000 & -17.1067 & 8.5389 \end{pmatrix}, B_o = \begin{pmatrix} -1.4890 & 5.0156 \\ -1.5036 & 6.0537 \\ 0.5205 & 0.6792 \\ 0.9741 & 0.6060 \end{pmatrix}.$$

The desired eigen-structure (eigenvalues-eigenvectors) is selected as:

$$\{-40.00 - 9.99 - 3.00 - 2.01\}, \left(\begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \\ -0.0862 & 0.2002 & -0.2782 & -0.2860 \\ 0.4685 & -0.5874 & 0.6273 & 0.6289 \\ 0.1260 & 0.0953 & -0.2213 & -0.2322 \\ 0.8702 & -0.7783 & 0.6929 & 0.6847 \end{array}\right).$$

According to Theorem 1, $v_i = T_{c1}V_i$, and we get the right desired block roots (solvents):

$$S_i = [v_{i1} \quad v_{i2}] \begin{bmatrix} \lambda_{i1} & 0 \\ 0 & \lambda_{i2} \end{bmatrix} [v_{i1} \quad v_{i2}]^{-1}$$

$$S_1 = \begin{pmatrix} -0.9215 & -28.4744 \\ 0.3439 & -11.0785 \end{pmatrix}, \qquad S_2 = \begin{pmatrix} -44.9882 & -3.6255 \\ 57.7706 & 1.9882 \end{pmatrix}.$$

The observer gain matrix in its block observable form is:

$$L_o = \begin{pmatrix} 7.7248 & -34.2110 \\ -42.1855 & 130.1446 \\ 0.6849 & -8.9148 \\ -24.1563 & 46.3150 \end{pmatrix}.$$

3.2. Stochastic Kalman filter

This section deals with noise signals, which are random or stochastic processes, that affect the plant itself or the available measurements. As mentioned earlier, the state estimator will be here a filter. The optimal linear filter, if optimization is understood as minimization of the estimation error variance, is the well known Kalman filter (Kalman, 1963; Kalman, Falb and Arbib, 1969). A linear system with white noise added to the input and, often but not always, white noise added to the output, is the most common model for consideration of randomness in control systems. There are many reasons why people so commonly use a linear system driven by white noise as a model of randomness, despite the fact that no system is truly linear and no noise is truly white. One of the reasons is that such a model is both elementary and tractable. The calculations are easy. The results can be understood without a deep knowledge of the mathematics of stochastic processes. The second reason is that a large class of stochastic processes can be effectively represented as the output of a linear system driven by white noise.

Kalman Filtering (or optimal state estimation in the sense of minimum variance), allows the user to estimate the state X of a dynamic system from a noise contaminated input output pair (U, Y).

Let us consider the m-inputs p-outputs stochastic system, described by the following discrete state space equations:

$$\begin{cases} X(k+1) = AX(k) + BU(k) + M\omega(k) \\ Y(k) = CX(k) + DU(k) + \nu(k) \end{cases} \tag{22}$$

where: $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$ are the system, input and output matrices, respectively. $D \in R^{p\times m}$, $M \in R^{n\times q}$ are the output-input and state-noise matrices, respectively. $X \in R^n$, $U \in R^m$, $Y \in R^p$ are the state, input and output vectors, respectively. $\omega \in R^q$ is zero-mean Gaussian white noise processes with covariance matrix $W = W^T > 0$, $E[\omega(k)\omega^T(k+j)] = W\delta(k)$, $E[\omega(k)] = 0$. $\nu \in R^p$ Is zero-mean Gaussian white noise process with covariance matrix $V = V^T > 0$, $E[\nu(k)\nu^T(k+j)] = V\delta(k)$, $E[\nu(k)] = 0$. ν and ω are independent of each other. $E[\omega(k)\nu^T(k+j)] = 0$. Here, E is the expectation (mean) value of random process, and $\delta(k)$ is Dirac (delta) function.

The mean $m(k) = E[\bullet]$ is obtained by solving the recurrence equation

$$E[X(k+1)] = AE[X(k)] + ME[\omega(k)] \Leftrightarrow m(k+1) = Am(k). \tag{23}$$

Solving this difference equation using eigen-function method yields

$$m(k) = A^{(k-k_0)}m(0). \tag{24}$$

Now, from equations (22) and (23) we obtain

$$X(k+1) - m(k+1) = A\left(X(k) - m(k)\right) + M\omega(k). \tag{25}$$

Let us define the covariance matrix $P(k) = E[(X(k) - m(k))\,(X(k) - m(k))^T]$. And we consider the following proposition: $X(k)$ and $\omega(k)$ are independent $\forall k$

$$E\left[\omega(k)(X(k) - m(k))^T\right] = 0 \ \text{ and } \ E[\omega(k)\omega^T(k)] = W\delta(0).$$

By solving the Lyapunov equation, we obtain

$$P(k+1) = AP(k)A^T + MWM^T. \tag{26}$$

Consider now a discretized system of the compact form:

$$\left\{\begin{array}{rcl} X(k+1) & = & AX(k) + [B \quad M]\left[\begin{array}{c} U(k) \\ \\ E[\omega(k)] \end{array}\right] + M\,[\omega(k) - E[\omega(k)]] \\ \\ Y(k) & = & CX(k) + DU(k) + \nu(k). \end{array}\right. \tag{27}$$

The observer to be designed has the following structure

$$\hat{X}(k+1) = A\hat{X}(k) + BU(k) + L(k)(Y(k) - C\hat{X}(k) - DU(k)). \tag{28}$$

The error is $\varepsilon(k) = X(k) - \hat{X}(k)$ and its equation is given by:

$$\varepsilon(k+1) = (A - L(k)C)\,\varepsilon(k) + M\omega(k) - L(k)\nu(k) \tag{29}$$

with

$$\left[\left[\begin{array}{c} \omega(k) \\ \\ \nu(k) \end{array}\right]\left[\begin{array}{cc} \omega(k) & \nu(k) \end{array}\right]\right] = \left[\begin{array}{cc} W_{q\times q} & O_{q\times p} \\ O_{p\times q} & V_{p\times p} \end{array}\right]\delta(j).$$

By applying the Lyapunov equation (26) to the error equation (29), we get

$$\begin{array}{rl} P(k+1) = & (A - L(k)C)\,P(k) + P(k)\,(A - L(k)C)^T \\ & + \left[M \,\vdots - L(k)\right]\left[\begin{array}{c} W \\ \cdots \\ V \end{array}\right]\left[M \,\vdots - L(k)\right]^T. \end{array} \tag{30}$$

Discrete time Kalman filter is designed for minimizing the following criterion, $\mathcal{J}(k)$, with respect to $L(k)$

$$\mathcal{J}(k) = \sum_{i=1}^{n} E[\varepsilon_i^2(k)] = \text{trace}\left(E[\varepsilon(k)\varepsilon^T(k)]\right) = \text{trace}(P). \tag{31}$$

In other words, the Kalman filter determines the optimal gain $L(k)$ that ensures observer stability while providing the best possible state estimation. This corresponds to minimizing the cost function $\mathcal{J}(k)$ as much as possible. To minimize the trace of the estimation error of covariance matrix P with respect to $L(k)$, it is sufficient to minimize the trace $trace(P(k+1))$, thereby achieving the smallest possible estimation error:

$$\min_{L(k)} \mathcal{J}(k) \Leftrightarrow \frac{\partial \operatorname{tr}\big(P(k+1)\big)}{\partial L(k)} = 0. \tag{32}$$

$$(32) \Leftrightarrow L(k)CP(k)C^T + L(k)V - AP(k)C^T = 0. \tag{33}$$

By solving for the Kalman gain matrix, we obtain

$$L(k) = \left(AP(k)C^T\right)\left(CP(k)C^T + V\right)^{-1}. \tag{34}$$

Finally, the discrete time observer is governed by:

$$\begin{cases} \hat{X}(k+1) = A\hat{X}(k) + BU(k) + L(k)(Y(k) - C\hat{X}(k) - DU(k)) \\ L(k) = \left(AP(k)C^T\right)\left(CP(k)C^T + V\right)^{-1} \\ P(k+1) = AP(k)A^T + MWM^T - AP(k)C^T\left(CP(k)C^T + V\right)^{-1} CP(k)A^T. \end{cases} \tag{35}$$

We summarize the preceding steps in the following algorithm:

Algorithm 1

```
1. Initialization
$E\left[M\omega(k)\omega^T(k)M^T\right] = MWM^T$
$E\left[\omega(k)\nu^T(k)\right] = 0$
$E\left[\nu(k)\nu^T(k)\right] = V$
$E[(X(0) - m_0)(X(0) - m_0)^T] = P_0$
2. Kalman gain estimation
    For $k = 0 : N$
    $L(k) = \left(AP(k)C^T\right)\left(CP(k)C^T + V\right)^{-1}$
    $P(k+1) = AP(k)A^T + MWM^T - AP(k)C^T\left(CP(k)C^T + V\right)^{-1} CP(k)A^T$
    $\hat{X}(k+1) = A\hat{X}(k) + BU(k) + L(k)(Y(k) - C\hat{X}(k) - DU(k))$
    End
```

3.3. The proposed intelligent maximum likelihood stochastic block observer-estimator

The well-known Kalman filter can be used to optimally estimate the state of the dynamic systems. However, the Kalman gain depends on the parameters and the variations of the system noises and the observation noises. For some advanced control objectives, like the self-tuning feedback control, the system parameters and the covariance of noises are very likely to be unknown, and must be identified first (Shieh, Bao and Chang, 1989). In order to solve this problem we have derived a joint algorithm for state estimation and MIMO parameter identification. As a result, the Kalman gain matrix and the states can be estimated without utilising the solution of discrete Riccati equation as in the standard state estimation algorithm.

Consider the following dynamic system, represented in block observable form:

$$\begin{cases} X(k+1) = A_o X(k) + B_o U(k) + \omega(k) \\ Y(k) = C_o X(k) + \nu(k) \end{cases} \tag{36}$$

where: $A_o = \begin{pmatrix} I_p & O_p & \cdots & -A_1 \\ \vdots & \ddots & \dots & \vdots \\ O_p & \cdots & I_p & -A_l \end{pmatrix}$, $B_o = \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_l \end{pmatrix}$, $C_o = \begin{pmatrix} O_p \\ O_p \\ \vdots \\ I_p \end{pmatrix}^T$.

The observer to be designed has the following structure:

$$\begin{cases} \hat{X}(k+1) = A_o \hat{X}(k) + B_o U(k) + L_o(k) e(k) \\ Y(k) = C_o \hat{X}(k) + e(k) \end{cases} \tag{37}$$

with $L_o = \left[L_{o1}^T(k), L_{o2}^T(k), \cdots, L_{ol}^T(k)\right]^T \in R^{n \times p}$ and $e(k) = Y(k) - C_o \hat{X}(k) = C_o \tilde{X}(k) + \nu(k)$.

The input output relationship of the steady state observer representation can be reformulated and rewritten in the ARMAX model form as:

$$\begin{aligned} Y(k) &= C_o \left[I_n - A_o q^{-1}\right]^{-1} \left[B_o q^{-1} U(k) + L_o q^{-1} e(k)\right] + e(k) \\ &= [A(q^{-1})]^{-1} [B(q^{-1})] U(k) + [A(q^{-1})]^{-1} [D(q^{-1})] e(k) \end{aligned} \tag{38}$$

$A(q^{-1}) = I_p + A_1 q^{-1} + A_2 q^{-2} + \cdots + A_l q^{-l}$
$B(q^{-1}) = B_1 q^{-1} + B_2 q^{-2} + \cdots + B_l q^{-l}$
$D(q^{-1}) = I_p + D_1 q^{-1} + D_2 q^{-2} + \cdots + D_l q^{-l}$
and
$L_{oi} = D_i - A_i$ for $i = 1, \cdots, l$.

The zeros of $det[D(q^{-1}]$ are inside the unit circle.

Eq. (38) constitutes the MIMO ARMAX model. If the parameter matrices A_i, B_i, C_i in Eq. (38) are known, and the covariance matrix is available, then the recursive algorithm in equations of Eq. (35) can be applied to determine the Kalman gain $L_o(k)$. Thus, the state $X(k)$ can be optimally estimated.

When the parameter matrices A_i, B_i, C_i are unknown, and the covariance matrix is not available, we utilize the dynamic observer description in conjunction with the MIMO Maximum Likelihood parameter estimation algorithm to determine the adaptive Kalman gain $L_o(k)$ and the estimated state $\hat{X}$. This is achieved using the estimate parameter matrices $\hat{A}_i(k)$, $\hat{B}_i(k)$ and $\hat{D}_i(k)$, which approximate the true parameters $A_i(k)$, $B_i(k)$ and $D_i(k)$ for $i = 1, \cdots, l$. The estimated Kalman gain $\hat{L}_{oi}(k)$ becomes

$$\hat{L}_{oi}(k) = \hat{D}_i(k) - \hat{A}_i(k) \qquad i = 1, \cdots, l \tag{39}$$

and the estimated state vector in the block observable form is

$$\begin{cases} \bar{X}(k+1) = \hat{A}_o(k)\bar{X}(k) + \hat{B}_o(k)U(k) + \hat{L}_o(k)\bar{e}(k) \\ \bar{e}(k) = Y(k) - \hat{C}_o(k)\bar{X}(k) \end{cases} \tag{40}$$

where: $\hat{A}_o(k)$, $\hat{B}_o(k)$, $\hat{L}_o(k)$ contains the estimated parameter matrices $\hat{A}_i(k)$, $\hat{B}_i(k)$, $\hat{L}_{oi}(k)$ for $i = 1, \cdots, l$. When the estimated parameter converges to the real parameter matrices, the $\bar{X}(k)$ and $\bar{e}(k)$ converge to the optimal state $\hat{X}(k)$ and the innovation process $e(k)$, respectively.

To estimate the parameter matrices $A_i(k)$, $B_i(k)$ and $D_i(k)$, the MIMO Maximum Likelihood parameter estimation algorithm can be applied. For the previously given MIMO ARMAX model, the equation (38) can be developed to yield

$$\begin{aligned} D(q^{-1})e[k] = \quad & (y[k] + A_1 y[k-1] + \cdots + A_{n_a} y[k-n_a]) \\ & - (B_1 u[k-1] + \cdots + B_{n_b} u[k-n_b]) \end{aligned} \tag{41}$$

with $n_a = n_b = n_d = l$, and using the Kronecker operator we can rewrite it as:

$$e[k] = I_p \otimes y^T[k] col(I_p) - [\eta_y, \eta_u, \eta_e]\,\theta \tag{42}$$

where:

$$\begin{aligned} &\theta = [\theta_A, \theta_B, \theta_D]^T \\ &\eta_y = I_p \otimes \left(y^T[k-1] + \cdots + y^T[k-n_a]\right), \quad \theta_A = \left[col(A_1^T)^T \cdots col(A_{n_a}^T)^T\right] \\ &\eta_u = -I_p \otimes \left(u^T[k-1] + \cdots + u^T[k-n_b]\right), \quad \theta_B = \left[col(B_1^T)^T \cdots col(B_{n_b}^T)^T\right] \\ &\eta_e = -I_p \otimes \left(e^T[k-1] + \cdots + e^T[k-n_d]\right), \quad \theta_D = \left[col(D_1^T)^T \cdots col(D_{n_d}^T)^T\right]. \end{aligned}$$

The best estimate of the parameter vector $\hat{\theta}$ can be obtained using a numerical minimization algorithm such as:

- Steepest descent method: $\theta_{k+1} = \theta_k - \lambda \nabla^T E$
- Gauss Newton method: $\theta_{k+1} = \theta_k - (\nabla^T \nabla)^{-1} \nabla^T E$

with

$$\nabla = \begin{bmatrix} \dfrac{\partial e[m+1]}{\partial \theta^T} \\ \vdots \\ \dfrac{\partial e[N]}{\partial \theta^T} \end{bmatrix}, \qquad E = \begin{bmatrix} e[m+1] \\ \vdots \\ e[N] \end{bmatrix}.$$

Algorithm 2 MIMO ML Algorithm

```
Step 1:
For $k = m + 1$ to $N$
    ■ Compute the prediction error
    $\hat{e}[k] = \hat{A}(q^{-1})y[k] - \hat{B}(q^{-1})u[k] - D_1\hat{e}[k-1] \cdots - D_{n_d}\hat{e}[k - n_d]$
    ■ Compute the partial derivatives of $e[k]$: $\frac{\partial e[k]}{\partial \theta^T}$
    The elements of $\frac{\partial e[k]}{\partial \theta^T}$ can be computed through MIMO IIR (Infinite Impulse Response) digital filtering using the updated matrix coefficients estimates $\hat{D}_i$ of the matrix polynomial $\hat{D}(q^{-1})$
Step 2: Estimate the parameter vector $\theta$ using
    $\theta_{k+1} = \theta_k - \lambda \nabla^T E$ or $\theta_{k+1} = \theta_k - (\nabla^T \nabla)^{-1} \nabla^T E$
    with $m = n_a$ , $0 < \lambda < 1$ and $N$ being the number of $I/O$ data.
Step 3: If no convergence, go to step 1.
```

Comments: The use of the Kronecker operator, block filtering using MIMO IIR (Infinite Impulse Response) and digital filtering using the updated matrix coefficients estimates are the main features of the MIMO Maximum Likelihood algorithm. Better process and noise dynamics estimates can be achieved by increasing the number of samples or increasing the signal to noise ratio.

To enhance the performances of the proposed observer we suggest the use of ANFIS (Adaptive Neuralbased Fuzzy Inference System) network architecture and its hybrid learning rule. Inspired by the idea of founding the fuzzy inference procedure on a feed forward network structure, Dr J.-S. R. Jang proposed a fuzzy neural network model, whose architecture is shown in Fig. 1. He reported that the ANFIS architecture can be employed to model nonlinear functions, identify nonlinear components on-line in a control system, and predict a chaotic time series (Jang, 1993). It is a hybrid neuro-fuzzy technique that brings learning capabilities of neural networks to fuzzy inference system. The learning algorithm tunes the membership functions of a Sugeno type fuzzy inference system using the training input-output data. This network can be extended to include more input-output membership functions.

Example 2 Let us consider the 2-input 2-output process (i.e. $p = m = 2$) described in LMFD by its polynomial matrices as

$$A(q^{-1}) = I_p + A_1 q^{-1} + A_2 q^{-2}, \quad A_i \in R^{2\times 2}$$
$$B(q^{-1}) = B_1 q^{-1} + B_2 q^{-2}, \quad B_i \in R^{2\times 2}$$

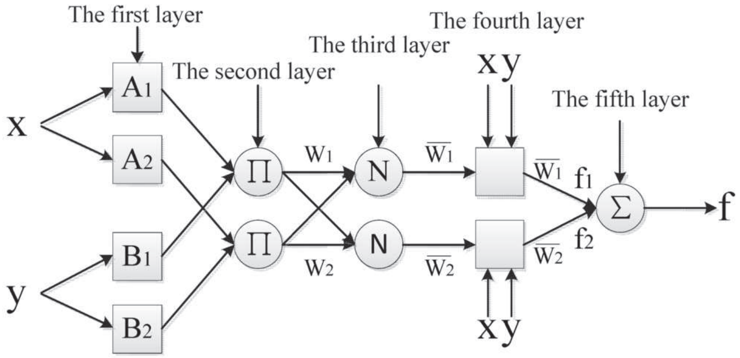

Figure 1. ANFIS structure for TS system with two inputs and one output

$$D(q^{-1}) = I_p + D_1q^{-1} + D_2q^{-2}, \qquad D_i \in R^{2\times 2}$$

with

$$\begin{bmatrix} A_1 \\ \\ A_2 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.4 \\ 0.3 & -0.6 \\ -0.1 & -0.3 \\ 0.2 & 0.3 \end{bmatrix}, \begin{bmatrix} B_1 \\ \\ B_2 \end{bmatrix} = \begin{bmatrix} -0.1 & -0.9 \\ 0.2 & 0.3 \\ -0.8 & -0.3 \\ 0.1 & 0.7 \end{bmatrix}, \begin{bmatrix} D_1 \\ \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & -0.9 \\ 0.3 & 0.4 \\ -0.5 & 0.7 \end{bmatrix}.$$

The equivalent state-space innovations (see Bekhiti, 2018) representation of the system becomes:

$$\begin{aligned} \hat{X}(k+1) &= \begin{bmatrix} O_2 & -A_1 \\ \\ I_2 & -A_2 \end{bmatrix} \hat{X}(k) + \begin{bmatrix} B_1 \\ \\ B_2 \end{bmatrix} U(k) + \begin{bmatrix} D_1 - A_1 \\ \\ D_2 - A_2 \end{bmatrix} e(k) \\ Y(k) &= [O_2 \quad I_2]\,\hat{X}(k) + e(k). \end{aligned}$$

The aim is to estimate the matrix polynomials, $A(q^{-1})$, $B(q^{-1})$ and $D(q^{-1})$ from I/O data, contaminated by white noise. A Pseudo-Random Binary Sequence (PRBS) data sequence of length $N = 1000$ is used to excite the system. A simulation experiment has been performed for signal to noise ratio equal to $20\,db$ for both outputs.

$$\frac{\partial e[k]}{\partial(\theta_A)} = \left[\frac{\partial e[k]}{\partial(col(A_1^T))^T}, \frac{\partial e[k]}{\partial(col(A_2^T))^T}\right] \Leftrightarrow$$

$$\frac{\partial e[k]}{\partial(\theta_A)} = \left[D(q^{-1})^{-1}\left[I_p \otimes y^T[k-1]\right], D(q^{-1})^{-1}\left[I_p \otimes y^T[k-2]\right]\right]$$

$$\frac{\partial e[k]}{\partial(\theta_B)} = \left[\frac{\partial e[k]}{\partial(col(B_1^T))^T}, \frac{\partial e[k]}{\partial(col(B_2^T))^T}\right] \Leftrightarrow$$

$$\frac{\partial e[k]}{\partial(\theta_B)} = -\left[D(q^{-1})^{-1}\left[I_p \otimes u^T[k-1]\right], D(q^{-1})^{-1}\left[I_p \otimes u^T[k-2]\right]\right]$$

$$\frac{\partial e[k]}{\partial(\theta_D)} = \left[\frac{\partial e[k]}{\partial(col(D_1^T))^T}, \frac{\partial e[k]}{\partial(col(D_2^T))^T}\right] \Leftrightarrow$$

$$\frac{\partial e[k]}{\partial(\theta_D)} = -\left[D(q^{-1})^{-1}\left[I_p \otimes e^T[k-1]\right], D(q^{-1})^{-1}\left[I_p \otimes e^T[k-2]\right]\right]$$

and finally we can form $\frac{\partial e[k]}{\partial(\theta)^T}$ as:

$$\frac{\partial e[k]}{\partial(\theta)^T} = \left[\frac{\partial e[k]}{\partial(\theta_A)}, \frac{\partial e[k]}{\partial(\theta_B)}, \frac{\partial e[k]}{\partial(\theta_D)}\right].$$

The elements thereof can be computed through MIMO IIR (Infinite Impulse Response) digital filtering using the updated matrix coefficients estimates $\hat{D}_1$,$\hat{D}_2$ of the matrix polynomial:

$$\hat{D}(q^{-1}) = I_2 + \hat{D}_1 q^{-1} + \hat{D}_2 q^{-2}.$$

Then, using the Gauss Newton method to update the parameter vector θ, we obtain the results shown below:

$$\hat{\theta}_A^T = \begin{bmatrix} 0.4932 \\ -0.4055 \\ 0.2963 \\ -0.6029 \\ -0.1022 \\ -0.2968 \\ 0.1975 \\ 0.3040 \end{bmatrix}, \qquad \hat{\theta}_B^T = \begin{bmatrix} -0.1056 \\ -0.8996 \\ 0.1954 \\ 0.3014 \\ -0.8017 \\ -0.2929 \\ 0.1018 \\ 0.7053 \end{bmatrix}, \qquad \hat{\theta}_D^T = \begin{bmatrix} 0.7225 \\ 0.1946 \\ 0.3023 \\ 0.9205 \\ 0.3202 \\ 0.4200 \\ -0.4972 \\ 0.7518 \end{bmatrix}.$$

After 1000 iterations or the time step $k = 1000$, the estimated parameter matrices are:

$$\hat{A}_1 = \begin{bmatrix} 0.4932 & -0.4055 \\ 0.2963 & -0.6029 \end{bmatrix}, \qquad \hat{A}_2 = \begin{bmatrix} -0.1022 & -0.2968 \\ 0.1975 & 0.3040 \end{bmatrix}$$

$$\hat{B}_1 = \begin{bmatrix} -0.1056 & -0.8996 \\ 0.1954 & 0.3014 \end{bmatrix}, \qquad \hat{B}_2 = \begin{bmatrix} -0.8017 & -0.2929 \\ 0.1015 & 0.7053 \end{bmatrix}$$

$$\hat{D}_1 = \begin{bmatrix} 0.7225 & 0.1946 \\ 0.3023 & 0.9205 \end{bmatrix}, \qquad \hat{D}_2 = \begin{bmatrix} 0.3202 & 0.4200 \\ -0.4972 & 0.7518 \end{bmatrix}.$$

The adaptive Kalman gain matrix at the time step $k = 1000$ is

$$\hat{L}_o = \begin{bmatrix} \hat{D}_1 - \hat{A}_1 \\ \\ \hat{D}_2 - \hat{A}_2 \end{bmatrix} = \begin{bmatrix} 0.2293 & 0.6001 \\ 0.0060 & 1.5234 \\ 0.4224 & 0.7168 \\ -0.6947 & 0.4478 \end{bmatrix}.$$

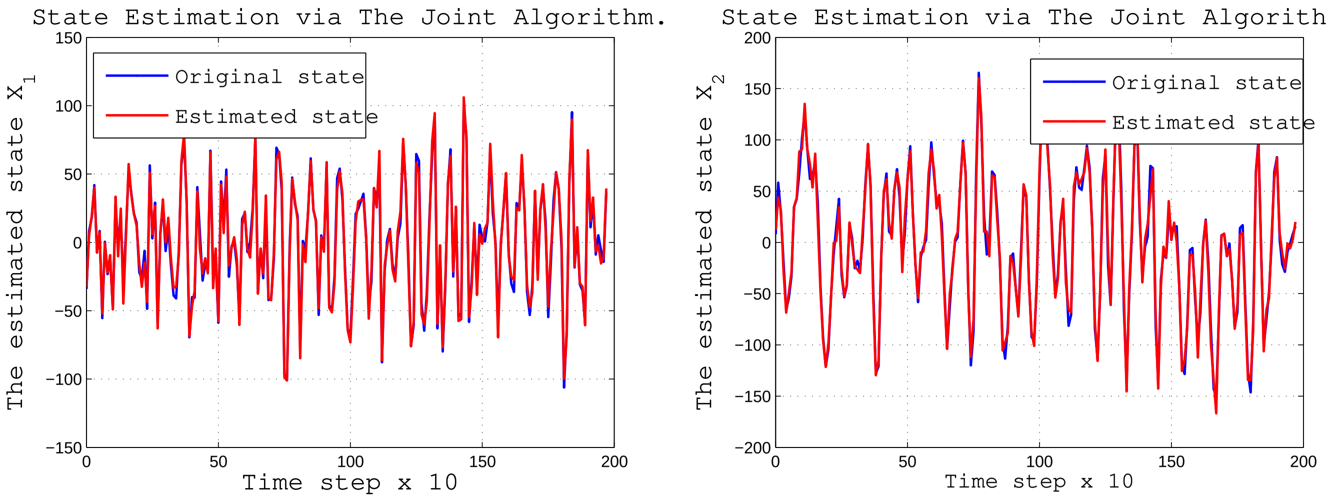

Figure 2. Estimated (X_1, X_2) vs. the original system using the Joint Algorithm

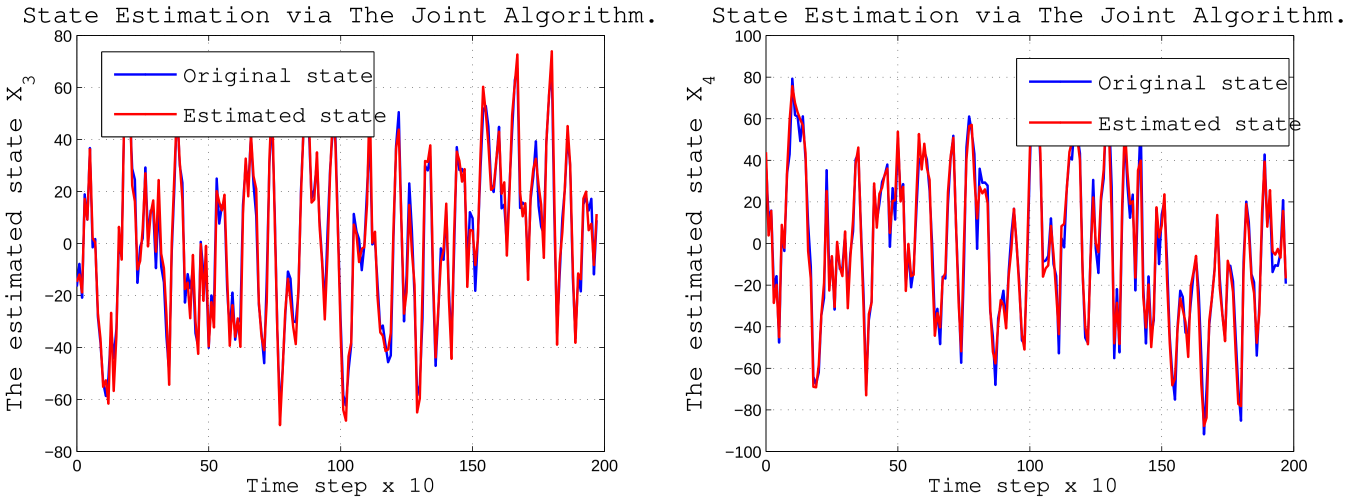

Figure 3. Estimated (X_3, X_4) vs. the original system using the Joint Algorithm

4. Self-adjustable block roots assignment

4.1. The setting

A large-scale MIMO system, described by state equations, is often decomposed into small subsystems, from which the analysis and design of the MIMO system can be easily performed. Such systems can be studied via the eigenstructure (i.e. eigenvalues and eigenvectors), of the state matrix A. The eigenvalues and eigenvectors can determine system performance and robustness far more directly and explicitly than other indicators (Hariche, 1987). Hence, their assignment should improve feedback system performance and robustness distinctly and effectively.

In this part of the paper, the feedforward gain will be determined in order to provide steady state tracking. Consider the MIMO system of equation (36), the control law of a MIMO square system (m-inputs, m-outputs) that is given by:

$$U(k) = -\hat{K}_{FB}\hat{X}(k) + \hat{K}_{FF}r(k) \tag{43}$$

where $r \in R^{m\times 1}$ is a reference input vector, $\hat{K}_{FF} \in R^{m\times m}$ is the feedforward or input gain matrix at time step k and $\hat{K}_{FB} \in R^{m\times n}$ is the feedback gain matrix at time step k. The closed loop controlled system using the previous control law becomes:

$$\left\{ \begin{array}{r} \hat{X}(k+1) = \left(\hat{A}_o - \hat{B}_o\hat{K}_{FB}\right)\hat{X}(k) + B_o\hat{K}_{FF}r(k) + \hat{L}_o(k)e(k) \\ Y(k) = C_o\hat{X}(k) + e(k). \end{array} \right. \tag{44}$$

4.2. Feedback gain design

The design of state feedback control in MIMO systems leads to the so-called matrix polynomials assignment (Bekhiti et al., 2015a). The use of block poles constructed from a desired set of closed-loop poles offers the advantage of assigning a characteristic matrix polynomial rather than a scalar one, also the feedback gain matrix permitting the assignment of the desired set of poles is not unique; this is due, in the case of solvent placement method, to the fact that different block poles can be constructed from the same set of eigenvalues, see Pereira (2003), Shieh and Tsay (1982a,b), Shieh, Chang and McInnis (1986). Then, the degree of freedom offered by the choice of the feedback gain matrix could be exploited to satisfy some desired closed loop performance qualities (the system response characteristics, robustness, tracking, decoupling, etc...). This can be done by choosing the structure of the solvent to be placed, which gives the best feedback gain matrix that achieves the desired objectives:

$$\hat{F}_{FB}(k) = \left\{ \hat{A}_o(k) \begin{pmatrix} O_p \\ O_p \\ \vdots \\ I_p \end{pmatrix} - \begin{pmatrix} I_m & S_1 \cdots {S_1}^{l-1} \\ I_m & S_2 \cdots {S_2}^{l-1} \\ \vdots & \vdots \ \ddots \ \vdots \\ I_m & S_l \cdots {S_l}^{l-1} \end{pmatrix}^{-1} \begin{pmatrix} {S_1}^{l} \\ {S_2}^{l} \\ \vdots \\ {S_l}^{l} \end{pmatrix} \right\}^{BT} \tag{45}$$

where S_i are the desired solvents to be relocated, and they characterize completely the system, since they carry the whole set of spectral data. The notation "BT" stands for the block transpose.

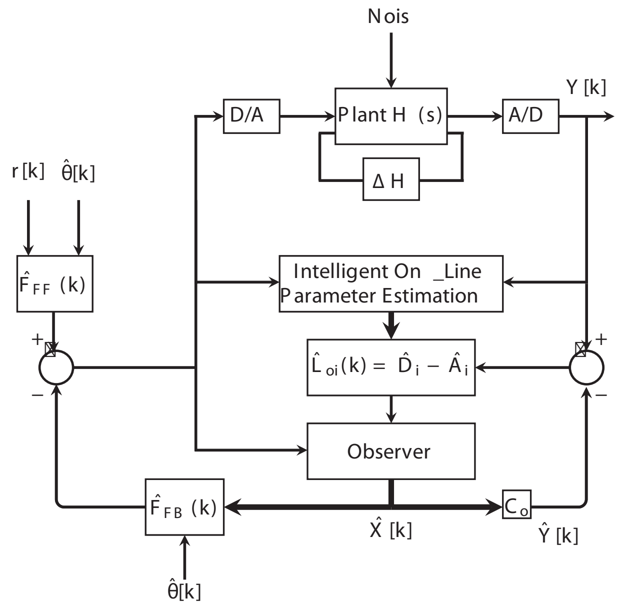

Figure 4. Self-adjustable block roots assignment

4.3. Feedforward gain design

Another fundamental design problem in systems engineering is to control a system so that a specified output follows a given nonzero reference trajectory $r(t)$. This is called the tracking or servodesign problem. For this purpose, the regulator control law must be modified. The fundamental issue here is that for tracking, some additional feedforward terms must be added to the control input besides the state feedback.

For a constant steady state input vector $r(t)$, the expectation of the steady state output can be adjusted to be equal to the $r(t)$, or $\lim_{k\to\infty} E[Y(k)] = r(k)$.

The mean of the steady state output can be determined from (44) as:

$$\lim_{k\to\infty} E[Y(k)] = \lim_{z\to 1} \left\{ \begin{array}{c} C_o(zI_n - \hat{A}_o + \hat{B}_o\hat{F}_{FB})^{-1}\hat{B}_o\hat{F}_{FF}r(k) \\ +[I_n + C_o(zI_n - \hat{A}_o + \hat{B}_o\hat{F}_{FB})^{-1}\hat{L}_o].E[e(k)] \end{array} \right\} \quad (46)$$

$$\lim_{k\to\infty} E[Y(k)] = r(k) \Rightarrow r(k) = C_o(I_n - \hat{A}_o + \hat{B}_o\hat{F}_{FB})^{-1}\hat{B}_o\hat{F}_{FF}r(k). \quad (47)$$

Then, the value of feedforward gain is

$$\hat{F}_{FF} = [C_o(I_n - \hat{A}_o + \hat{B}_o\hat{F}_{FB})^{-1}\hat{B}_o]^{-1},$$

and the control law becomes:

$$U(k) = -\hat{K}_{FB}\hat{X}(k) + [C_o(I_n - \hat{A}_o + \hat{B}_o\hat{F}_{FB})^{-1}\hat{B}_o]^{-1}r(k). \tag{48}$$

REMARK 6 $\hat{F}_{FB}$ is the DC gain of the closed-loop system

$$\hat{F}_{FB} = \text{DC-gain}(H_{cl}(z^{-1})).$$

$$\begin{aligned} \hat{F}_{FB} &= \lim_{z\to 1} H_{cl}(z^{-1}) \\ &= \left[C_o\left(I_n - \hat{A}_o + \hat{B}_o\hat{F}_{FB}\right)^{-1}\hat{B}_o\right]^{-1} \end{aligned} \tag{49}$$

EXAMPLE 3 Here a discrete time controller with feedforward will be designed for the two-link robot from Hendricks, Jannerup and Sorensen (2008). The design will be based on the model linearized around the state, given by the two link angles, $\theta_1 = 45°$ and $\theta_1 = 30°$. Prior to the controller design the model must be discretized with the sample period desired for the controller. The robot is a relatively fast mechanical system, so $T = 0.05$ s will be used. This means that it will be possible to handle frequencies up to about 5 Hz (one tenth of the sample frequency) with reasonable accuracy. If further investigations show that this is not adequate, the design must be revised with a different sample period selection. The discrete time transfer matrix becomes:

$$H(z) = \frac{1}{\Delta(z)}\left\{N_1z^3 + N_2z^2 + N_3z + N_4\right\} + \delta H(z)$$

where $\delta H(z)$ represents the system uncertainties, which are below $4\%\, H(z)$;

$$\Delta(z) = z^4 - 3.135z^3 + 3.67z^2 - 1.901z + 0.3679$$

$$N_1 = \begin{pmatrix} 0.1348 & 0.0577 \\ 0.1298 & 0.0817 \end{pmatrix}, \qquad N_2 = \begin{pmatrix} -0.3078 & -0.1368 \\ -0.2998 & -0.1928 \end{pmatrix}$$

$$N_3 = \begin{pmatrix} 0.2331 & 0.1075 \\ 0.2293 & 0.1506 \end{pmatrix}, \qquad N_4 = \begin{pmatrix} -0.0585 & -0.0280 \\ -0.0581 & -0.0389 \end{pmatrix}.$$

The aim of the present example is to design a complete self adjustable controller-observer meeting the desired objectives via the block pole relocation for the (joint-estimator/controller), as shown in Fig. 4. The complete eigenstructure is given below:

$$S_1 = \begin{pmatrix} 11.0266 & -8.1373 \\ 14.5933 & -10.8266 \end{pmatrix}, \qquad S_2 = \begin{pmatrix} -1.5329 & -0.4294 \\ 1.0802 & -0.1671 \end{pmatrix}.$$

The simulation results of the self-adjusted outputs and reference inputs are shown in Fig. 5. The convergence of the system parameters, the input gain and steady state feedback gain matrices are tested and validated using the tolerance error criteria $\left(\xi = \| \theta - \hat{\theta} \|^2 / \| \theta \|^2\right)$, with ξ less than 0.1%. After 1000 iterations or time step $k = 1000$, the estimated LMFD coefficients (i.e. parameter matrices) are

$$\hat{A}_2 = \begin{bmatrix} -1.6639 & 0.2385 \\ -0.0561 & -1.4708 \end{bmatrix}, \qquad \hat{A}_1 = \begin{bmatrix} 0.6676 & -0.1747 \\ 0.0371 & 0.5414 \end{bmatrix}$$

$$\hat{B}_2 = \begin{bmatrix} 0.1348 & 0.0577 \\ 0.1298 & 0.0817 \end{bmatrix}, \qquad \hat{B}_1 = \begin{bmatrix} -0.0786 & -0.0324 \\ -0.0914 & -0.0601 \end{bmatrix}$$

$$\hat{D}_2 = \begin{bmatrix} -0.1423 & 0.0037 \\ 0.0306 & -0.0537 \end{bmatrix}, \qquad \hat{D}_1 = \begin{bmatrix} 0.0050 & -0.0001 \\ -0.0024 & -0.0006 \end{bmatrix}.$$

The estimated state feedback gain, input gain, observer gain matrices after the time step $k = 1000$ are

$$\hat{L}_o = \begin{bmatrix} \hat{D}_2 - \hat{A}_2 \\ \\ \hat{D}_1 - \hat{A}_1 \end{bmatrix} = \begin{bmatrix} -0.6626 & 0.1746 \\ -0.0395 & -0.5420 \\ 1.5216 & -0.2348 \\ 0.0867 & 1.4171 \end{bmatrix}$$

$$\hat{F}_{FB} = \begin{bmatrix} -0.5080 & 0.5648 & 2.5564 & 0.3220 \\ 1.0988 & -0.6095 & 1.6336 & 2.0783 \end{bmatrix}, \hat{F}_{FF} = \begin{bmatrix} 5.2476 & -1.8459 \\ -4.1446 & 8.9198 \end{bmatrix}.$$

5. Conclusion

In this paper, a new self-adjustable block roots assignment control method is developed by appropriately modifying the conventional state-space self-tuning control scheme. The modified self-tuning control scheme, which utilizes the intelligent MIMO maximum likelihood identification algorithm, yields a better performance than that with the conventional STC scheme for the control of stochastic systems. The simulation results show that the proposed method can deal with the changeable stable/unstable and min/nonmin-phase cases, and adaptively can control stochastic processes.

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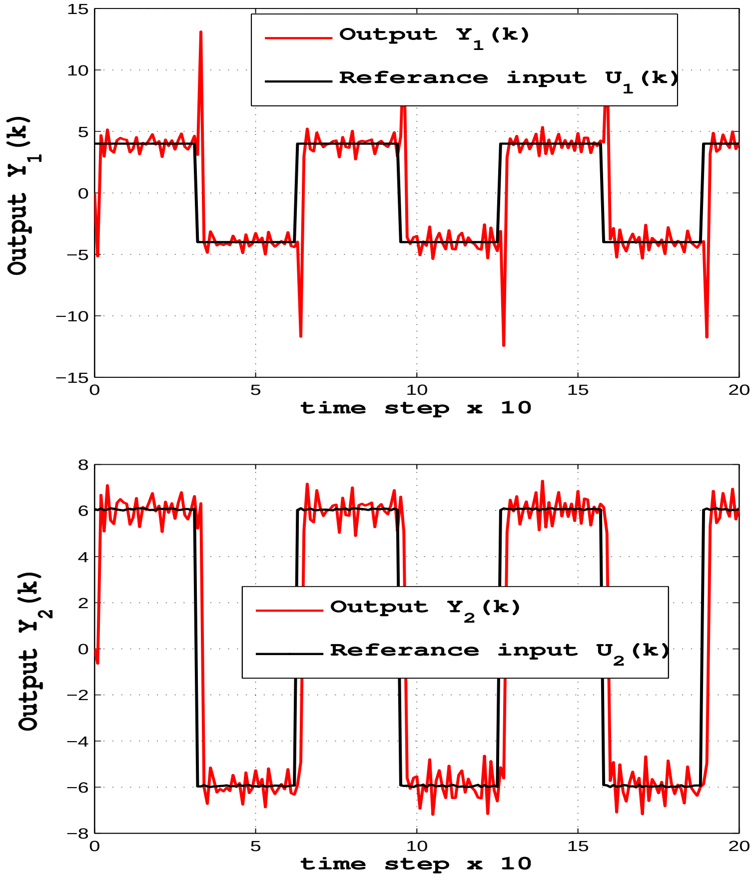

Figure 5. Trajectory tracking control using the Joint Algorithm

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