



Construction of the Principle of Adaptive Position-Trajectory Control of Moving Objects (On the Example of a Multi-Link Industrial Robot Manipulator)

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Abstract: This article presents the principle of adaptive position-trajectory control for moving objects, illustrated using a multi-link industrial robot manipulator. The principle is described through several stages derived from its main control objectives. A SCARA robot with adaptive position-trajectory control is chosen as the study object, and mathematical models are provided to compute its kinematic structure and dynamic characteristics. These models make it possible to determine the manipulator's position in coordinates, the trajectory and position of the end-effector, and to analyze real-time trajectory generation. Based on the proposed principle, boundary conditions for the robot's dynamic properties are formulated, enabling the development of control models, algorithms, and robust-adaptive position-trajectory control structures. The approach follows a classical PD controller framework using filtered and approximated tracking errors. Computer visualization of the robust-adaptive control structures is used to examine dynamic behavior. The proposed structures ensure high-accuracy and high-speed motion along specified trajectory positions.

Keywords: *Adaptive control principle, Position-trajectory control, Trajectory and position determination, Multi-link manipulator, Specified coordinate and position.*

1. Introduction

The introduction of digital technologies and the Industry 4.0 concept in industrial enterprises is becoming increasingly important in automating production processes, increasing product quality and labor productivity, and reducing the human factor. In such conditions, the use of multi-link industrial robot manipulators allows for the implementation of technological processes with high accuracy and speed [1]. In particular, the movement of robot manipulators along specified coordinates, accurate trajectory tracking during interaction with objects, and adaptability to external influences are key factors ensuring production efficiency [2].

In recent years, many studies on the control of industrial robots have used kinematic and dynamic models, trajectory planning methods, Proportional-Integral-Differential (PID) and adaptive control algorithms [1, 2]. In particular, approaches aimed at determining the position of the robot manipulator, forming and tracking the movement trajectory, ensuring stable operation under external influences and parametric uncertainties have been widely researched [3]. However, existing control methods do not fully address the issues of the interaction of the robot's executive body with the external environment, friction factors, coordination of velocity and acceleration parameters, and complex processing of sensor data in real time [4]. In this regard, improving the principle of adaptive-position trajectory control of moving objects is one of the urgent tasks [5]. The purpose of this study is to develop the principle of adaptive-positional control, which provides precise control of the movement of a multi-link industrial robot-manipulator according to coordinates, real-time tracking of the trajectory, and flexibility to external influences [6].

In the study, the kinematic and dynamic characteristics describing the movement process of the robot manipulator were analyzed, and the algorithms for planning the sequence of movements by coordinates and positions were formed. Also, a method of dynamically adapting control parameters based on the integration of information from sensors and the operator is proposed [6]. The scientific novelty of the proposed approach lies in the development of stable control criteria based on the organization of adaptive-position trajectory control, filtering and approximation of the tracking error, taking into account all the dynamic characteristics of the robot manipulator simultaneously and taking into account boundary conditions.

During the control process, the state of each link of the robot manipulator is analyzed, and the trajectory parameters are changed in accordance with external

influences and current coordinates. This ensures that the robot arm moves with high precision and speed. By processing data received from sensors in real time and integrating it with a database, control reliability and dynamic stability are increased. Based on the research results, it is shown that the proposed adaptive-position trajectory control principle allows industrial robot manipulators to increase the accuracy of movement, achieve stable trajectory tracking, and quickly adapt to external influences.

2. Related works

Positional and trajectory control of multi-link industrial robot manipulators are dynamic systems with complex, delicate, and variable characteristics. Ensuring high accuracy, adaptability, and stability in the control of such systems is one of the urgent tasks. In recent years, control methods based on adaptive, intelligent technologies have been widely studied in this direction.

In previous studies, the position and trajectory control of industrial robot manipulators was mainly implemented using classical PID controllers or models based on robot dynamics. The modeling method based on Lagrangian dynamics served as an important foundation for the analysis of multi-link manipulators. They were used to develop control algorithms taking into account the dynamics of robot motion.

Adaptive control [7] developed algorithms that allow for adaptive position-trajectory control of the system in relation to uncertainties [7] in robot parameters and external influences. The adaptive hybrid control algorithms they proposed provided effective control of the robot, taking into account real-time parametric changes [8].

In recent studies, interest in intelligent technologies, especially control strategies based on neural networks, fuzzy logic, and self-learning technologies, has increased. These approaches provide the ability for the robot system to adapt to previously unknown situations. In particular, self-learning models allow the robot to independently form a control strategy based on its interaction with the environment [9]. In addition, in multi-link manipulators, accurate and stable control signals are generated by filtering data from various sensors.

In the case of trajectory planning, B-spline and polynomial interpolation methods, as well as genetic algorithms, are used to optimize the manipulator movement [10]. Through them, the manipulator movement is controlled in a fast and efficient manner.

The approach proposed in our article is aimed at increasing the efficiency and stability of existing control methods in uncertain conditions when building the principle of adaptive position-trajectory control of moving objects, in particular, on the example of a multi-link industrial robot manipulator [11]. Adaptive control algorithms, by integrating optimal trajectory planning and tracking mechanisms based on real-time sensor data, increase the accuracy, speed, and flexibility of the proposed manipulator gripper movements [12]. This contributes to the scientific and practical development of adaptive position-trajectory control of multi-link manipulators and brings the level of existing technologies to a new level.

3. Materials and methods

The principle of adaptive position-trajectory control of moving objects is aimed at determining the approximation of a multi-link industrial robot manipulator to a specified coordinate and position, and mathematically accurate descriptions of the kinematic and dynamic characteristics of each link, taking into account the state, position, and trajectory of the coordinates [13]. Typically, the kinematic structures, coordinate systems, and dynamic characteristics of a multi-link industrial robot manipulator are composed of mathematical expressions that describe external influences and control signals [14]. The kinematics of a robot manipulator with adaptive position-trajectory control is not related to degrees of motion, but to determining its state and position along a given coordinate [15]. The basic concepts include the change of position coordinates and include the A matrices of the coupling variable, the T matrix of the manipulator, the Cartesian coordinates of the link length, and the direct and inverse problems of kinematics [16]. The adaptive positional control system is aimed at approaching the specified coordinate trajectory or position of an object moving along the coordinate systems of industrial robots, taking into account the state, position, and trajectory of the object [17]. Here, it is possible to mention the moving units in the cylindrical coordinate system, spherical coordinate system, angular coordinate system, cylindrical coordinate system, and Cartesian coordinate system [18].

SCARA robot manipulators are usually chosen for the assembly, sorting, and placement operations of the production process (Fig. 1).

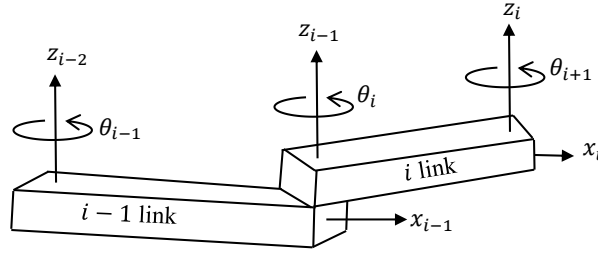


Fig. 1. Coordinate serial connection diagram of a SCARA robot according to Denavit and Hartenberg parameters

Each link i has a joint variable q_i , which is the angle (e.g., θ_i , in degrees) for Rotary (R) links and the length (e.g., d_i , in length) for Prismatic (P) or extendable links. The joint vector of an n -link robot is defined as $q = [q_1 \ q_2 \ \dots \ q_n]^T \in \mathbf{R}^n$; links are conventionally numbered from the base to the last link, with link 0 being considered fixed [19].

The joint vector $q = [\theta \ h \ r]^T$ is the degree of freedom of the link, i.e. $n = 3$. Thus, for a linked robot with n degrees of freedom, a six-link manipulator requires complete position and coordinate states in the three-dimensional space $\mathbf{R}^3$ is required. For the analysis, a coordinate frame is assumed to be attached to each link. The base frame is attached to the 0-th support (base) of the manipulator. The location of the coordinate frame in the link is often chosen according to the Denavit-Hartenberg convention (parameters) [20].

The relationship between the links shown in Fig. 2 is given by the matrix A for the i -link as follows:

$$(1) \quad A_i(q_i) = \begin{bmatrix} R_i & p_i \\ 0 & 1 \end{bmatrix},$$

where: $R_i(q_i)$ is the 3×3 rotation matrix of the link ($R_i^{-1} = R_i^T$); $p_i(q_i) = [x_i \ y_i \ z_i]^T \in \mathbf{R}^3$ is the transformation vector; R_i – represents the rotation of the coordinate frame at the i -th link relative to the coordinate frame at the $i = 1$ link; p_i represents the transformation of the coordinate frame at the i -th link relative to the coordinate frame at the $i = 1$ link. Here, the 4×4 uniform transformation A_i represents the coordinate transformation of the i -th link relative to the $i = 1$ link [2, 21, 22].

The matrix A is a function of the joint variable $A_i(q_i)$ such that when q_i changes with the robot motion, A_i changes accordingly. A_i also depends on the link rotation and link length parameters set for each link.

The position of the actuator is defined as the union of the matrices T and is given in terms of the coordinate state as follows:

$$(2) \quad T(q) = A_1(q_1) A_2(q_2) \dots A_n(q_n) \equiv \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}.$$

This 4×4 uniform variable matrix is a function of the joint variable vector q . The sum of the 3×3 rotation matrices is given by $R(q) = R_1(q_1) R_2(q_2) \dots R_n(q_n)$.

An n -joint manipulator has n degrees of freedom, and once the joint variables q_i are determined, the position of the last link is completely fixed. This position can be described in joint coordinates or Cartesian coordinates [23, 24]. The joint coordinate state of the last link is simply given by the value of the n -vector q . The Cartesian position of the last link is given by specifying the direction and change of the coordinate state associated with the last link in terms of the reference frame. This is exactly what $T(q)$ represents, i.e., $T(q)$ represents the position of the last link in the Cartesian coordinate system.

4. The experiment and results

The inverse problem of the kinematics of a multi-link industrial robot manipulator with adaptive position-trajectory control consists in determining the rotation angles q_i required to set the positions of its last link in a given Cartesian coordinate. This corresponds to the solution (2) for $q \in \mathbf{R}^n$ and R denotes the displacement p of the last link. Suppose that, like the link variable $q(t) \in \mathbf{R}^n$, there is another trivial variable $y(t) \in \mathbf{R}^p$, which is given by:

$$(3) \quad y(t) = h(q(t)),$$

where $y(t)$ represents the position of the manipulator in Cartesian coordinates. Taking the particular derivatives, we write the following equation:

$$(4) \quad \dot{y} = \frac{\partial h}{\partial q} \dot{q} \equiv J(q) \dot{q},$$

where $J(q)$ is the associative relation of the Jacobian to $h(q)$. This equation shows how the combined velocities $\dot{q}$ are converted into the velocities $\dot{y}$.

According to the change in velocity (4), the incremental motion is changed according to $\Delta y = J(q) \Delta q$.

The differential acceleration change in expression (4) takes the following equation

$$(5) \quad \ddot{y} = J \ddot{q} + \dot{J} \dot{q}.$$

Using the concept of substitution, it can be shown that forces expressed in terms of q can be transformed into forces expressed in terms of y as

$$(6) \quad \tau = J^T(q) F,$$

where τ is the force in the joint space (given as a torque vector for an n -rotating robot), and F is the force vector of the manipulator in the y coordinate. If y is the position of the manipulator in the Cartesian coordinate, then F is the force vector $[f_x \ f_y \ f_z]^T$ for the three coordinates, and $[\tau_x \ \tau_y \ \tau_z]^T$ for the moment. It

follows that the energy consumption of the manipulator in all coordinates cannot be calculated if $J(q)$ is not taken into account [21].

Therefore, the dynamic model of a multi-link manipulator expresses the relationship between the movements and the forces and torques. The dynamic equation of a multi-link industrial robot manipulator with adaptive position-trajectory control can be written as follows:

$$(7) \quad M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = \tau,$$

where $M(q)$ is the inertia matrix, $V_m(q, \dot{q})$ is the Coriolis/non-centrifugal force (centrifugal force), $F(\dot{q})$ is the friction coefficient, $G(q)$ is the drag vector, and $\tau_d(t)$ represents the distortion. The control input vector $\tau(t)$ has torque components for rotary links and force components for prismatic links. It is often convenient to write the dynamics of an industrial robot manipulator with adaptive control in the following form:

$$(8) \quad M(q)\ddot{q} + N(q, \dot{q}) + \tau_d = \tau,$$

where

$$(9) \quad N(q, \dot{q}) = V_m(q, \dot{q})\dot{q} + F(\dot{q}) + G(q),$$

represents a vector of trivial coefficients.

The dynamics can be calculated for a given manipulator from the matrices of the link A and the inertias of the link [25]. At the same time, they can be used to obtain the first-order Lagrange equation of motion:

$$(10) \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau.$$

Based on the Kinetic energy K and Potential energy P , the Lagrange equation is written as

$$(11) \quad L = K - P.$$

It can be seen that the required values of each link $q_d(t)$ can be achieved by determining the vector trajectory $q_{d_i}(t)$ containing the required values of time $q_d(t)$. This required trajectory vector $q_{d_i}(t)$ is determined by decomposing the high-level tasks based on high-level trajectory planning and then transmitted to the real-time motion control system [6, 23]. In particular, based on the dynamic characteristics of multi-link robots, its adaptive position-trajectory control principle is based on basic boundary conditions. The boundary conditions of the dynamic characteristics of the multi-link industrial robot manipulator based on the adaptive position-trajectory control principle are given in Table 1.

Based on the principle of adaptive position-trajectory control, the calculation of the total kinetic energy, potential energy, and acceleration of each link of the robots in motion according to the boundary conditions of the dynamic characteristics of the multi-link industrial robot manipulator is carried out based on the Lagrange equation (Fig. 2).

Table 1. Boundary conditions of dynamic characteristics of a multi-link industrial robot manipulator based on the principle of adaptive position-trajectory control

P1	The inertia matrix $M(q)$ is symmetric, positive definite, and bounded such that all $M(q) \leq \mu_2 I$. For links with rotational motion, the unique values of the connection variables q_i depend on the functions $\sin(q_i)$, $\cos(q_i)$. For manipulators without prismatic links, the boundaries μ_1 , μ_2 are constant
P2	The Coriolis/centrifugal vector $V_m(q, \dot{q})$, $\dot{q}$ is the quadratic expression of $\dot{q}$; V_m is bounded as follows: $\ V_m\ \leq v_B \ \dot{q}\ $, or equivalently $\ V_m \dot{q}\ \leq v_B \ \dot{q}\ ^2$
P3	The Coriolis/centrifugal force matrix can always be chosen such that the matrix $S(q, \dot{q}) \equiv \dot{M}(q) - 2V_m(q, \dot{q})$ is symmetric. Therefore, for all x vectors $x^T S x = 0$. This means that the spurious forces in the robot system do no work
P4	The friction conditions have the following approximate form: $F(\dot{q}) = F_v \dot{q} + F_d(\dot{q}),$ where F_v is a diagonal matrix with constant coefficients representing viscous friction and $F_d(\cdot) = K_{d_i}$ is a vector with elements of the form $\text{sgn}(\dot{q}_i)$. Here $\text{sgn}(\cdot)$ is a sinuous function and K_{d_i} are the dynamic friction coefficients. These friction limits are limited such that $\ F(\dot{q})\ \leq f_B \ \dot{q}\ + k_B$ are constant coefficients, f_B, k_B
P5	The friction vector is bounded as follows: $\ G(q)\ \leq g_B$. For links with rotational motion, the link variables q_i participate only in the form of $\sin(q_i)$ and $\cos(q_i)$. For manipulators with rotational motion, g_B is a constant boundary
P6	The boundedness of the perturbations is $\ \tau_d(t)\ \leq d_B$

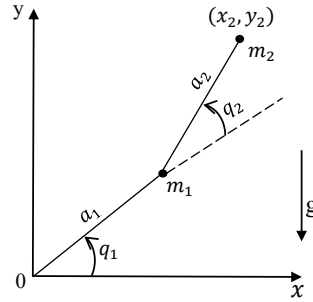


Fig. 2. Kinematic diagram representing the dynamic characteristics of a multi-link industrial robot manipulator

According to Fig. 2, it is possible to see how to derive the dynamics of multi-link manipulators [26, 27].

In addition, the trivial state variable equation for multi-link industrial robot manipulators is $\dot{x} = f(x, u)$, where $x(t)$ is the internal state and $u(t)$ is the input control signal; such state variable descriptions allow the development of control laws for the multi-link manipulator and its computer simulation. There are three state variable descriptions for the boundary conditions of the dynamic characteristics of the multi-link industrial robot manipulator, which are given in Table 1. In the form of a position/velocity state variable, the $2n$ -vector is denoted as $x \equiv [q^T \quad \dot{q}^T]^T$ and is written as

$$(12) \quad \dot{x} = \begin{bmatrix} \dot{q} \\ -M^{-1}(q)N(q, \dot{q}) \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}(q) \end{bmatrix} \tau,$$

where $u(t) \equiv \tau(t)$ is given as the state variable.

The linear state-variable equation $\dot{x} = Ax + Bu$ can be defined as follows:

$$(13) \quad \dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} u,$$

and the control signal is of the form $u(t) \equiv -M^{-1}(q)N(q, \dot{q}) + M^{-1}(q)\tau$.

The third form of the state variable description is the Hamiltonian form, which follows from Hamilton's equations of motion [4, 28]. Here the state is defined as a $2n$ -vector $x \equiv [q^T \ p^T]^T$, where $p(t) \equiv M(q)\dot{q}$ is the generalized momentum, so the state variable equation becomes:

$$(14) \quad \dot{x} = \begin{bmatrix} -M^{-1}(q)p \\ -\frac{1}{2}(I_n \otimes p^T) \frac{\partial M^{-1}(q)}{\partial q} p \end{bmatrix} + \begin{bmatrix} 0 \\ I_n \end{bmatrix} u.$$

The control input signal is assumed to be $\tau = G(q)$. These equations relate the velocities in Cartesian coordinates and are defined as $J(q) \equiv \frac{\partial h(q)}{\partial q}$, so

$$(15) \quad \begin{bmatrix} v \\ \omega \end{bmatrix} = \dot{y} = J(q)\dot{q},$$

where $v(t)$ is the linear velocity of the last link of the industrial robot manipulator, and $\omega(t)$ is the rotational velocity. Both velocities are 3-vectors. By differentiating this equation, the acceleration variable can be determined as follows: $\ddot{y} = J\ddot{q} + \dot{J}\dot{q}$. Substituting these expressions into the Equation (7), the dynamics can be written in the form of a Cartesian coordinate system:

$$(16) \quad \bar{M}\ddot{y} + \bar{N} + f_d = F,$$

where $\bar{M} \equiv J^{-T}MJ^{-1}$, $\bar{N} \equiv J^{-T}(N - MJ^{-1}\dot{J}\dot{y})$ and the disturbance signal $f_d \equiv J^{-T}\tau_d$ are defined. The control input signal for the dynamics of an industrial robot moving along a Cartesian coordinate is F , which has three force and three torque components.

The boundary conditions of these kinematic and dynamic characteristics serve to develop the principle of adaptive position-trajectory control of a multi-link manipulator. For example, the desired trajectory of the manipulator $q_d(t)$ is determined by the trajectory planner [25, 29]. When using inverse linear control elements, the control error can be determined as

$$(17) \quad e(t) = q_d(t) - q(t),$$

and by differentiating twice, the canonical coordinate Equation (13) can be written in terms of the expression $x \equiv [e^T \ \dot{e}^T]^T$ as

$$(18) \quad \frac{d}{dt} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u,$$

with it

$$(19) \quad u \equiv \ddot{q}_d + -M^{-1}(q)(N(q, \dot{q}) + \tau).$$

After stabilizing the resulting error tracking system, the linear system differentiation method in Equation (18) is used to select the feedback control $u(t)$, and then the required moments of the lever for each state are calculated using Equation (19) as follows:

$$(20) \quad \tau \equiv M(q)(\ddot{q}_d - u) + N(q, \dot{q}).$$

When calculating the dynamics of an industrial robot manipulator with an adaptive position-trajectory control system, it is possible to develop its control algorithms using mathematical methods based on linearization [26, 30]. Control algorithms allow the development of Proportional-Integral-Differential (PID) controllers and classical approaches and adaptive position-trajectory digital control structures.

Control algorithms and control structures represent the criterion for controlling the nonlinear integrator inverse kinematics, which ensures the tracking of the desired trajectory. It is based on calculating the moment τ , which makes the dynamics of a finite element in the Equation (7) equivalent to the linear dynamics in Equation (18), which is called the linearization method of inverse kinematics.

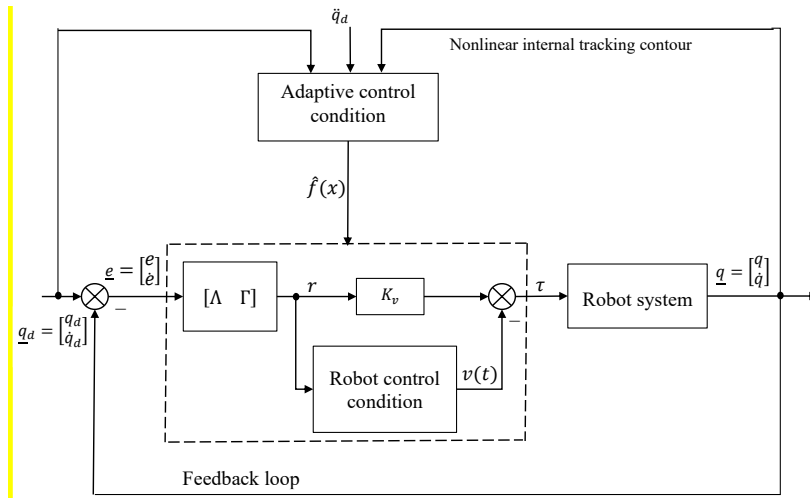


Fig. 3. Adaptive control system aimed at ensuring the stability of moving objects

These integrators are placed in the outer tracking loop and allow ensuring the stability of the object according to the specified coordinate trajectory and position through reference models based on the approximation of filtered and tracking errors [31]. As a result, it is possible to adjust the parameters of the object and develop optimal control criteria depending on the change of the boundary conditions of the dynamic characteristics. In fact, the selection of the control signal $u(t)$ forms an adaptive control system (Fig. 3) aimed at ensuring the stability of objects moving with a calculated-momentum PID controller and is expressed by the equation

$$(21) \quad \begin{aligned} \dot{e} &= e, \\ \tau &= M(q)(\ddot{q}_d + K_v \dot{e} + K_p e + K_i \varepsilon) + N(q, \dot{q}). \end{aligned}$$

It has its own dynamics, which ensures stable tracking without choosing the integral coefficient K_i too large [32, 33]. The adjuster also calculates the desired trajectory $q_d(t)$, the tracking error, and the moments of the multi-link industrial robot manipulator. Here, $q_{d1}(t) = g_1 \sin\left(\frac{2\pi t}{T}\right)$ and $q_{d2}(t) = g_2 \sin\left(\frac{2\pi t}{T}\right)$ are chosen (Fig. 5).

Table 2. Control models and algorithms based on the approximation of filtered errors

A dynamic model of a multi-link industrial robot manipulator: $M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = \tau$
Filtered observation error model $r(t)$: $\begin{aligned} e &= q_d - q, \\ r &= \dot{e} + \Lambda e, \end{aligned}$ where Λ is the positive definite constructive parameter matrix and $q_d(t)$ is the required trajectory
Calculated torque control model: $\tau = K_v r + M(q)(\ddot{q}_d + \Lambda \dot{e}) + V_m(q, \dot{q})(\dot{q}_d + \Lambda e) + F(\dot{q}) + G(q)$
Adaptive control model: $\tau = W(x)\phi + K_v r, \quad \dot{\phi} = \Gamma W^T(x)r,$ where $W(x)$ is the regression matrix, and Γ is the matrix of the positive definite adjustment rate
Resilience-based robust control model: $\begin{aligned} \tau &= \hat{f} + K_v r - v(t), \\ v(t) &= \begin{cases} -r \frac{F(x)}{\ r\ }, & \ r\ \geq \varepsilon, \\ -r \frac{F(x)}{\varepsilon}, & \ r\ < \varepsilon, \end{cases} \end{aligned}$ where ε is a small positive design parameter and $F(x)$ is a known scalar function that bounds the unknowns $\hat{f} = f - \hat{f}$, such that $\ \hat{f}\ \leq F(x)$
A model of stable control of variable structures: $\tau = \hat{f} + K_v r - v, \quad v = -(F(x) + \eta) \text{sgn}(r),$ where η is a small positive design parameter and $F(x)$ is a known function of the form $\ f - \hat{f}\ $ that bounds these uncertainties
Control model of neural network training: $\tau_l = \hat{f}_l + K_v r - v, \quad v = -(K_p e + K_s \ e\ ^2 r), \quad \hat{f}_l = \hat{f}_{l-1} + K_L r,$ where l is the iteration number, K_v, K_p, K_s are the positive diagonal design matrices of the gains and K_L is the positive diagonal training gain matrix

The adaptive control system based on the PID controller, shown in Fig. 3, aimed at ensuring the stability of moving objects, ensures that the tracking error e and the speed error $\dot{e}$ in the control process asymptotically approach 0. This makes it possible to monitor the time-dependent changes in the filtered error $r(t)$ and the position tracking error $e(t)$ in the dynamics of the controlled object, and they are mainly selected as follows:

$$(22) \quad \begin{aligned} \dot{\varepsilon} &= e, \\ \tau &= K_v \dot{\varepsilon} + K_p e + K_i \varepsilon. \end{aligned}$$

In addition, many control methods based on the classical approach are based on control algorithms based on the observation of the control process, that is, on the approximation of filtered errors (Table 2).

We perform a mutual analysis of mathematical models in the MATLAB 2024a program, according to the methods for calculating the proportional-differential torque for the control signal $u(t)$ and the control scheme of the PID controller based on the classical approach, with all the nuances of the multi-link industrial robot manipulator, and we can present its results in five main cases of the PID controller based on the classical approach (Fig. 4).

The situation in Fig. 5 shows, on the one hand, that the control system can maintain stability even under the influence of external disturbances, and on the other hand, it is difficult to fully achieve ideal accuracy (zero error). At the same time, it emphasizes the importance of dynamic compensation methods in uncertain environments and variable load conditions, despite the successful implementation of control.

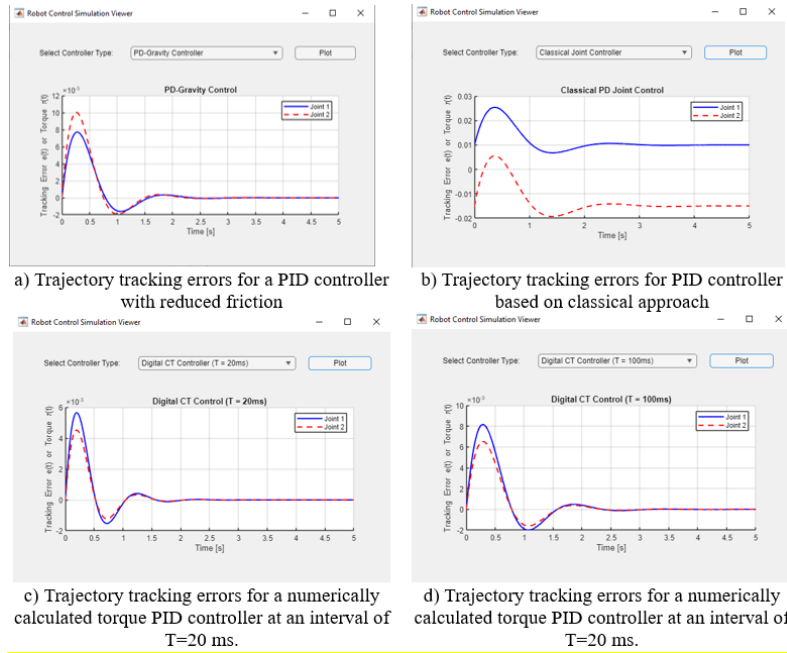


Fig. 4. Characteristics of error analysis in robot manipulator trajectory tracking based on calculated and numerical PID control methods

In the adaptive position-trajectory control of a multi-link industrial robot manipulator, given a trajectory $q_d(t)$, the tracking error $\tau(t)$ and the filtered tracking error $r(t)$ are determined and are expressed as follows:

$$(23) \quad e = q_d - q,$$

$$(24) \quad r = \dot{e} + \Lambda e,$$

where Λ is a positive definite design parameter matrix, and it is usually recommended to choose Λ as a diagonal matrix with large positive values [24]. In this case, if the filtered error $r(t)$ is boundedly captured, it can be conveniently expressed by introducing the following equation:

$$(25) \quad \|e\| \leq \frac{\|r\|}{\sigma_{\min}(\Lambda)}, \quad \|\dot{e}\| \leq \|r\|,$$

where, the minimum singular value of the matrix $\sigma_{\min}(\Lambda)$ and the 2nd norm are used.

According to Table 3, the trajectory defined by control algorithms based on the approximation of filtered errors can be limited as

$$(26) \quad \begin{Bmatrix} q_d(t) \\ \dot{q}_d(t) \\ \ddot{q}_d(t) \end{Bmatrix} \leq q_B,$$

where q_B is expressed as a known scalar Boundary.

The filtered error of the robot dynamics as a result of differentiating Equations (7) and (9) can be seen from the following expression:

$$(27) \quad M\dot{r} = -V_m r + f(x) + \tau_d - \tau,$$

where the nonlinearity function of the robot is defined as

$$(28) \quad f(x) = M(q)(\ddot{q}_d + \Lambda \dot{e}) + V_m(q, \dot{q})(\dot{q}_d + \Lambda e) + F(\dot{q}) + G(q).$$

The vector of the x coordinate contains all the time signals necessary to calculate $f(\cdot)$, for example, it can be determined as follows: $x \equiv [e^T \quad \dot{e}^T \quad q_d^T \quad \dot{q}_d^T \quad \ddot{q}_d^T]^T$. The important point is that $f(x)$ contains all possible uncertain parameters of the multi-link robot manipulator, except for $V_m r$ in expression (27), in which case the control stability according to the Lyapunov criterion is not required. The boundary conditions for the characteristics of the dynamics of the multi-link industrial robot given in Table 1 must preserve the asymmetry property P3. From this, the general type of control based on the approximation of filtered errors is obtained as

$$(29) \quad \tau = \hat{f} + K_v r - v(t),$$

where $\hat{f}$ is the value of $f(x)$, $K_v r = K_v \dot{e} + K_r \Lambda e$ is the external PID adjustment loop, and $v(t)$ is an auxiliary signal that provides reliability against disturbances and modeling errors [19, 28]. In adaptive control, robust control, neural network-based control, and point-based logic-based control, the value of $\hat{f}$ and the reliability-providing signal $v(t)$ are determined in various ways (Fig. 5). Using the nonlinearity stabilization method, it is possible to show that the stability of the observation error is ensured by choosing various precise adjusters [29]. Adjusters are generated, and the stability results are given based on the most important

closed-loop error dynamics. The closed-loop error dynamics are determined by substituting the Equation (29) into the Equation (27):

$$(30) \quad M\dot{r} = -V_m r - K_v r + \tilde{f} + \tau_d + v(t),$$

where the error detection through the approximation function is given by

$$(31) \quad \tilde{f} = f - \hat{f}.$$

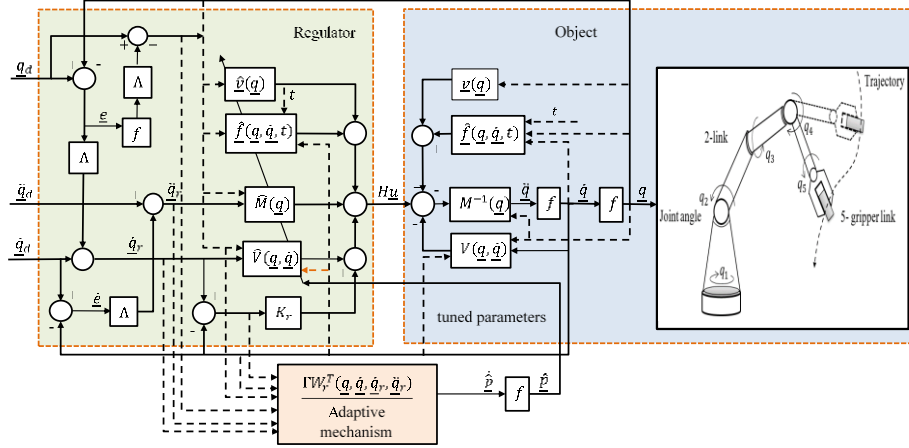


Fig. 5. Structural scheme of the system of adaptive position-trajectory control of a multi-link robot manipulator based on the approximation of filtered errors

To implement the described adaptive control, it is first necessary to determine the regression matrix $W(x)$. When calculating the matrix $W(x)$, it is necessary to use a method that satisfies the asymmetry property P3 of the matrix $V_m(q, \dot{q})$ [27]. The vector of unknown parameters contains the masses of the robot links and the parameters $\varphi = [m_1 \ m_2]^T$, where m_2 represents the mass of the load acting on the manipulator. The tracking errors are determined by the following relations: $e_1 = q_{d1} - q_1, e_2 = q_{d2} - q_2$. Based on the characteristics of the dynamics of a multi-link robot manipulator, the elements of the 2×2 matrix $W(x)$ [28] are expressed as

$$\begin{aligned} W_{11} &= a_1^2(\ddot{q}_{d1} + \lambda_1 \dot{e}_1) + a_1 g \cos q_1, \\ W_{12} &= (a_2^2 + 2a_1 a_2 \cos q_2 + a_1^2)(\ddot{q}_{d1} + \lambda_1 \dot{e}_1) + \\ &+ (a_2^2 + a_1 a_2 \cos q_2)(\ddot{q}_{d2} + \lambda_2 \dot{e}_2) - a_1 a_2 \dot{q}_2 (\dot{q}_{d1} \sin q_2 + \lambda_1 e_1) - \\ &- a_1 a_2 (\dot{q}_1 + \dot{q}_2) (\dot{q}_{d2} \sin q_2 + \lambda_2 e_2) + \\ &+ a_2 g \cos(q_1 + q_2) + a_1 g \cos q_1, \\ W_{21} &= 0, \\ W_{22} &= (a_2^2 + a_1 a_2 \cos q_2)(\ddot{q}_{d1} + \lambda_1 \dot{e}_1) + a_2^2(\ddot{q}_{d2} + \lambda_2 \dot{e}_2) + \\ &+ a_1 a_2 (\dot{q}_{d1} + \lambda_1 e_1) \sin q_2 + a_2 g \cos(q_1 + q_2). \end{aligned}$$

In adaptive control systems based on filtered error approximation, the main design work is aimed at choosing a dynamic approximation $\hat{f}$, which should provide the ability to self-adapt online [29].

This adaptive approach is important in analyzing the dynamics of SCARA robots with adaptive links. Based on the equations (12)-(20) and the control models based on the approximation of filtered errors presented in Table 2, a dynamic model for the SCARA robot is presented in Fig. 2. The robot is composed of flexible links, and the effect of flexibility is taken into account in the model through a single generalized coordinate q_e . This coordinate represents the torsional deformation of the link connected to the rotating link. Taking $N^T = [0 \ 1 \ 0]$, the dynamic equations are used to derive the control relations for the input forces F and M and the reference trajectory of the bending coordinate. Based on these relationships, the adaptive coordinate dynamics is described by the following equation:

$$-A_{12} \sin q_e \dot{q}_e \dot{q}_{sr} + A_{22} \ddot{q}_{vr} + A_{23} \ddot{q}_{er} - A_{22}(q_v - q_e) \dot{q}_e \dot{q}_{er} + k_f q_{vr} - k_f q_{er} + k_r \dot{q}_{vr} = 0.$$

In cases where the deformation of the link is small, it can be assumed that $q_v - q_e \approx 0$. This significantly reduces the value of the limit $A_{22}(q_v - q_e) \dot{q}_e \dot{q}_{er}$. Therefore, this term is considered as a small quantity of higher order and is excluded from the equation. As a result, the following simplified equation is formed, which is convenient for the analysis of the control algorithm:

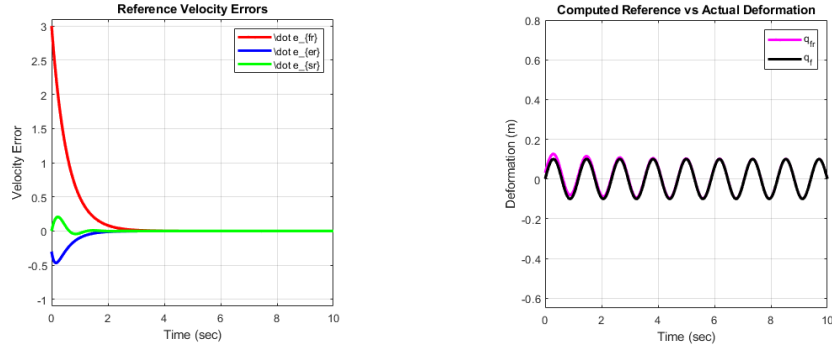
$$A_{23} \ddot{q}_{er} - k_f q_{er} = A_{12} \sin q_e \dot{q}_e \dot{q}_{sr} - A_{22} \ddot{q}_{vr} - k_f q_{vr} - k_r \dot{q}_{vr}.$$

The bounded solution for q_{er} cannot be determined by direct integration because this second-order differential equation is unstable. Therefore, the roles of the coordinates q_e and q_v are interchanged. In this case, the trajectory is determined by the following output equation $q_{ed} = q_e(t)$, and q_{vd} is replaced by q_{ed} . Based on the assumption of small link deformation, this substitution reflects the real motion characteristics of the robot manipulator with sufficient accuracy. Therefore, the reference trajectory q_{vr} for the rotating link coordinate q_v is determined by the equation

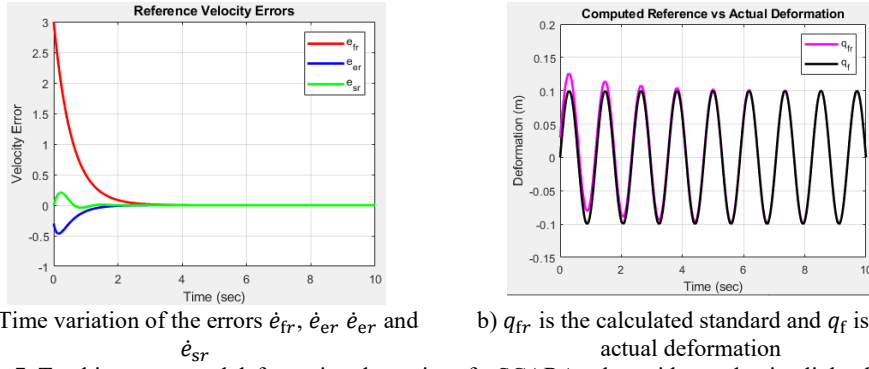
$$A_{22} \ddot{q}_{vr} + k_r \dot{q}_{vr} + k_f q_{vr} = A_{12} \sin q_e \dot{q}_e \dot{q}_{sr} - A_{23} \ddot{q}_{er} + k_r \dot{q}_v + k_f q_{er}.$$

The simulation results of the control system with different coefficients of different levels of flexibility are presented in Fig. 6. Fig. 7 shows the system dynamics for the cases $EI = 1 \text{ N.m}^2$, Fig. 8 shows the system dynamics for the cases $EI = 10 \text{ N.m}^2$, and Fig. 9 shows the system dynamics for the cases $EI = 100 \text{ N.m}^2$. In the left graphs of these figures, the deviation of the rotational speed from the reference speed $\dot{e}_{sr}$ is depicted by a red line, the deviation of the adaptive coordinate speed from the reference value $\dot{e}_{er}$ is depicted by a blue line, and the change in the time of the deviation errors of the deformation speed from the

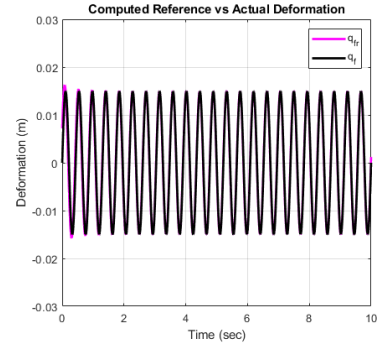
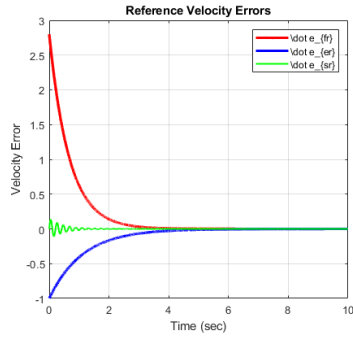
reference value $\dot{e}_{fr}$ is depicted by a green line, and the calculated reference value q_{fr} is depicted by a pink line, and the change in the actual deformation q_f is depicted by a black line. The results show that the deformation amplitude and vibration frequency are strongly dependent on the value of the various coefficients of the degree of flexibility of the links k_f . As the flexibility of the link increases, the deformation decreases, but high-frequency control moments are generated. This is clearly shown in Fig. 9. Also, the structural diagram of the adaptive position-trajectory control system based on the approximation of filtered errors also reflects the time variation of the filtered error $r(t)$ and the position tracking error $e(t)$ of the robot arm (Fig. 10).



a) Time variation of the errors $\dot{e}_{fr}$, $\dot{e}_{er}$ and $\dot{e}_{sr}$ b) q_{fr} is the calculated standard and q_f is the actual deformation
Fig. 6. Tracking errors and deformation dynamics of a SCARA robot with an adaptive link when $EI = 1 \text{ N.m}^2$



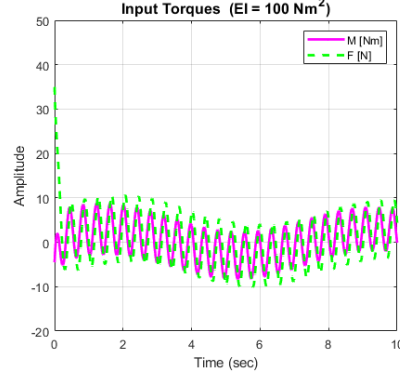
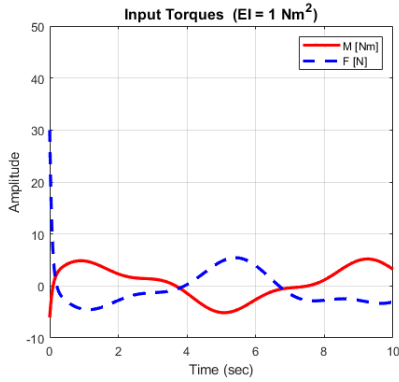
a) Time variation of the errors $\dot{e}_{fr}$, $\dot{e}_{er}$ and $\dot{e}_{sr}$ b) q_{fr} is the calculated standard and q_f is the actual deformation
Fig. 7. Tracking errors and deformation dynamics of a SCARA robot with an adaptive link when $EI = 10 \text{ N.m}^2$



a) Time variation of the errors $\dot{e}_{fr}$, $\dot{e}_{er}$, $\dot{e}_{sr}$ and $\dot{e}_{sr}$

b) q_{fr} is the calculated standard and q_f is the actual deformation

Fig. 8. Tracking errors and deformation dynamics of a SCARA robot with an adaptive link when $EI = 100 \text{ N.m}^2$



a) Time variation of the errors $\dot{e}_{fr}$, $\dot{e}_{er}$, $\dot{e}_{sr}$ and $\dot{e}_{sr}$

b) q_{fr} is the calculated standard and q_f is the actual deformation

Fig. 9. Tracking errors and deformation dynamics of a SCARA robot with an adaptive link when $EI = 1 \text{ N.m}^2$ and $EI = 100 \text{ N.m}^2$

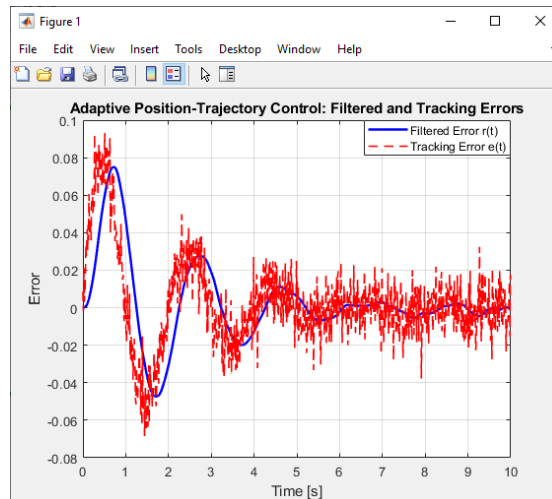


Fig. 10. Dynamics of filtered and tracking errors in approximation-based control: $r(t)$ – how the filtered error changes over time; $e(t)$ – how the tracking error changes over time

It can be seen from the graph that although the error is large at the initial time, over time the filtered error and the real error reach equilibrium, that is, the system reaches a stationary state. This demonstrates the stability and robustness of the control algorithm [3, 11, 30]. If the system is likely to have external influences and parametric uncertainties, the adaptive position-trajectory control principle is the most effective solution. Because in this control method, the filtered error $r(t)$ decreases rapidly and the tracking error $e(t)$ stabilizes at $\mu \approx 0$ with small oscillations. In this case, according to the P3 condition, it is not required to ensure Lyapunov stability based on the asymmetry of S , which makes the control more effective. Also, in cases where it is necessary to obtain a fast and stable response in the control process, the classical PD control itself under ideal conditions may be sufficient.

5. Conclusion

The article proposes to build a principle of adaptive position-trajectory control of moving objects. The principle of adaptive position-trajectory control is considered on the example of a multi-link manipulator. Based on the main objectives of the control principle, several of its stages are described. Also, a SCARA with adaptive position-trajectory control was selected as the example of the research object, and mathematical models for calculating its kinematic structure and dynamic characteristics were developed. The developed mathematical models will serve as the basis for studying the state of a multi-link industrial robot manipulator with adaptive position-trajectory control in terms of coordinates, the trajectory and position of the manipulator executive body, and the process of trajectory generation in real time. Also, based on the formation of boundary conditions for the dynamic characteristics of a multi-link industrial robot manipulator based on the principle of adaptive position-trajectory control, control models and algorithms were developed. The algorithms developed based on the boundary conditions of the dynamic characteristics allowed the development of robust-adaptive, position-trajectory control structures for a multi-link industrial robot manipulator based on a PD adjuster, based on the classical approach, filtered, and based on the approximation of tracking errors. Computer-aided visualization of control structures, the correct selection of the boundary conditions of the dynamic characteristics, taking into account the physical characteristics of the object, provides high accuracy and speed of movement of the multi-link robot manipulator along the specified trajectory and position.

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