

Modelling and predicting coastal zone depth profile evolution: a survey

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Abstract

We survey results concerning the problem of identifying depth profiles at coastal zone, which evolve in time due to natural as well as anthropic activities. This issue is relevant to control the modifications of the environment occurring near sea coastlines, but also in river's estuaries and harbors. One of the main goals is to predict the time evolution of the depth profile in the long-term (i.e., over years or decades, say), and to do this on the basis of *real* observed and measured *data*, available in several databases. Most mathematical models are formulated in terms of partial differential equations of the diffusive type, in one or two space dimensions. Consequently, from the mathematical standpoint, the aforementioned identification problem takes on the form of an inverse problem for some given parabolic equation associated with suitable initial and boundary conditions.

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1. Introduction

Natural phenomena and nowadays even more human activities (like constructing groynes, artificial islands, and so on) strongly affect the time evolution of coastal zones, and may have eventually an unpredictable impact on the environment. Just think of the dramatic effects on coastlines in countries like The Netherlands.

It is therefore of paramount relevance to analyze the various phenomena which play a role in the coastlines evolution, as well as to consider which action to undertake to remedy the damage represented, e.g., by coastal erosion. An important issue is definitely that of making reasonable and possibly reliable predictions on the resulting profiles in few years ahead, in order to plan suitable restorations.

Clearly, even the case of the borderlines of bays and rivers's estuaries is of a similar interest, and collecting data 'on the field' should be considered an useful task to accomplish, in all cases.

Because of the importance of these problems, many research papers have been written concerning several related issues. Most of them are devoted to the evolution of coastal *lines*, but the dynamics of the *depth profile* is also being under investigation. Most of such papers deal with measured data, obtained at various geographic locations. As a rule, some sort of empirical formulae are proposed. Matching with the observations is achieved by choosing a number of parameters in the specific formula under consideration.

In some papers, a driving model governing the equations is introduced to describe the process' dynamics. After an appropriate model calibration, the solution to the model, obtained numerically, will describe the measurements with a certain precision. These two part of the modeling, that is, resorting to some empirical formulae and using suitable governing equations, are not in contrast to each other, but are, rather, complementary.

The rest of the paper is organized as follows. In section 2, various models which have appeared in the

literature to describe depth profile evolution are recalled. In section 3, a reaction-diffusion-type model, some approach to analyze it, the structure of the (real) available data, and the perspectives of making predictions on future evolution are presented. In section 4, the present approach, which is a combination of several techniques, is discussed. In the final section 5, some future directions concerning 2D models are debated.

2. The models describing depth profile evolution

In order to keep as low as possible the negative ecological impact on the coastal zone during the implementation of engineering activities, as well as to solve problems arising from the natural modifications of the coast's morphology itself, it is necessary to have available data and models, which allow to simulate the morphodynamical behavior on various time and space scales.

A variety of models do exist in the current literature, capable of describing processes in the coastal zone. One can find a sort of classification of the topographic evolution of erosional and depositional systems, e.g., in [1]. The authors there distinguish between local and nonlocal models, depending on the way the sediment's flux is accounted for. It is noted, in particular, that "a simple linear diffusion model for the sediment flux may not be adequate", see [2] for details.

This is an example of the use of exact formulae with a certain number of parameters, adopted to describe process' dynamics. A number of publications exist where processes at river's estuaries are studied, e.g., in [3], which are not considered in this survey.

Among alternative approaches, one can mention that followed in [4], where the influence of the angle of waves approaching the coast is discussed, an issue important since this has an impact on sediments' transport; see, e.g., [5], where a nonlinear diffusion-type model, associated with the formation of sedimentary ocean deltas is presented.

Depending on the desired scale of forecasting and on the level of reliability and accuracy required, the suitable models, appropriate for the description of the evolution of coastal profiles, can be classified as process-oriented models, behavior-oriented models, parametric equilibrium models, geomorphological models, and stochastic models. Note that a specific model may share features characteristic of more than one of such classes. An alternative classification was proposed in [6]. Here the author categorized some numerical models of climate change, related to coastal change, into the following groups: simple physics-based models, scale-aggregated semi-empirical models, behavior-oriented models, process-based models, and reduced-complexity models. Many references were provided there.

In particular, a behavior-based model could be defined as a phenomenological model which is capable of covering a set of observed phenomena, and that can then be used for forecasting. It is however neither intended to be able to explain the specific physical principles underlying the phenomena nor to relate them to general laws. Rather, the processes involved in the coastal evolution can be simplified using semi-empirical averaged formulae [7,8].

Among the most recent papers, we mention [9] (and the references therein), where calibration and verification of a number of the so-called one-line models, based on high quality field experiments, were conducted up to distances of 200 m from the shoreline. Such models consist of few analytic relations among the parameters under consideration which should be identified.

Behavioral models are widely used to depict the long-term evolution of coastal profiles, due to the difficulties of identifying which processes are actually relevant on the scale of several years. Most of these models use a variety of simplifications and heuristic estimates of the relevant parameters, as well as of initial and boundary conditions.

For instance, the Shoreface Translation Model (STM) developed in [10,11] estimates parameters and initial bathymetric profiles (based on measured data) to predict the evolution. More precisely, it estimates the time and space evolution of the profiles, basing on the sediment budget, that is the ratio between the amount of sediment entering the coastal zone and that exiting from it, the composition of bottom sediments, and the rise of a sea level.

Another example, but concerning rivers, is provided by the model developed in [12] to analyze the long-

term evolution of the coastal profile associated with changes in the sea level and exchange of precipitation between coastal strip, barrier, and estuary. This model studies coastal depth evolution over the time span of millennia. This shows the effects observed over extensive ranges of time for which a behavior-based model can be applied, as opposed to the case of process-based models, which are usually valid only in the short term.

3. A Reaction-Diffusion model

Among the models available in literature, a rather simple “averaged” one seems to be the most promising. Having a few parameters in the governing equation, these models are expected to be useful in the *prediction* of the evolution of the depth profile, after a proper calibration made against the measured data. Such prediction should be valid for a reasonably long time, at the scale of a few years ahead, and with an acceptable accuracy.

The diffusion type model equation, proposed in [7], satisfies these requirements. The simplified version of such a model, considered in [13], has just two coefficients, which could be regarded as representing diffusion and drift in the model equation.

If the model describes reasonably well the measured data, it should be possible to predict the behaviour of the depth profile exploiting much less data, which is the case when resorting to a Neural Network approach, as it has been done in [13].

3.1. The basic model

In [7], the following diffusion equation was proposed for modelling the process of long-term evolution of the depth profile in coastal zones:

$$(1) \quad \frac{\partial(\delta X)}{\partial t} = D(z) \frac{\partial^2(\delta X)}{\partial z^2} + \Psi\left(t, z, \delta X, \frac{\partial(\delta X)}{\partial z}\right),$$

where $\delta X(z, t)$ is the depth change at distance z from the shoreline, $D(z) > 0$ is a diffusion coefficient, and Ψ is a certain function accounting for external forces. Note that, since *changes* of the depth profile are under study, this model deals with the *difference* between the current depth profile function and its values at a certain time. Therefore, the sought function, $\delta X(z, t)$, is identically zero for $t = 0$. According to [7], the term Ψ makes it possible to account for the effects of a number of influences, various processes of transport, and interference due to anthropic activities, such as mining sand and minerals. One of the key-points of this approach is the necessarily careful calibration of the coefficients of the governing equation (1) which depend on space and time. After this is done, one can retrieve the time evolution of the depth profile. Needless to say that the previous equation should be accompanied by suitable initial and boundary data.

The diffusion model with a “drift” term,

$$\frac{\partial y}{\partial t} + V \frac{\partial y}{\partial x} = \varepsilon \frac{\partial^2 y}{\partial x^2},$$

was earlier introduced in [14].

In [15], the authors suggested to consider, alternatively, a model for the distance from the shore as a function of depth changes, $z(X, t)$. This could be described by the same type of equations as in (1). In this case, the peak of the profile of the diffusion coefficient could be associated to the position where the wave breaks down, see [15].

A diffusion model with “a forcing term” on the right-hand side of the governing equation as above was also considered in [16]. Here, a prediction was made concerning the East coast of England. However, the evaluation of the forcing term (which, in fact, is responsible for the coastline evolution) is unclear.

An attempt to predict the coastal evolution on the scale of 50–100 years ahead was made in [17], but no equations were given to substantiate their model.

An overview of the numerical models describing the coastal evolution, available up to around the year 2000, could be found in [18]. A distinction was made there between equilibrium and non-equilibrium models, as well as between longshore uniform and non-longshore uniform models. All these models were discussed in terms of their general assumptions, approaches, and applicability.

In [19], a model for the cross-shore profile evolution, based on a diffusion-type formulation combined with a data-driven approach, was presented and discussed. In such a formulation, the space-varying diffusion coefficient and a source function describing the aggregation of all non-diffusive morphodynamic processes, allow to predict the time evolution of the beach profile. This model requires measurements backwards in time of beach profiles and waves, in order to determine the unknown parameters. It was validated using cross-shore beach profiles and waves measured over two decades in the Milford-on-Sea beach, located in the Christchurch Bay beach system, in the UK.

3.2. Time-independent coefficients

In [13,20] the linear function $\Psi := B(z)\delta X + f(t)$ was chosen for the right-hand side of equation (1), and thus the rather simple diffusion equation

$$(2) \quad u_t = D(z)u_{zz} + B(z)u + f(t), \quad 0 \leq z \leq H, \quad t > 0$$

was studied. Here, the parameter H is the so-called “distance of closure” (similar to the “depth of closure,” mentioned in some references), which is the distance from the shoreline where the considered transport phenomena do not have any longer any impact on the depth’s modifications. The function $f(t)$ may simulate the impact of the surface waves (wind waves) on the depth profile. As a rule, we suppose that $f(t) \equiv 0$. In order to solve numerically the equation (2), some measured profiles (obtained by sonars and altimetric assessments as in [21]) were used to provide initial and boundary values to be used in the system

$$(3) \quad \begin{aligned} u(z, 0) &= 0, & u(0, t) &= v_0(0, t) \\ u_z(0, t) &= v_1(0, t), & u(H, t) &= 0. \end{aligned}$$

Here, $u(z, t)$ represents the change of depth at the distance z from the shoreline, hence it is identically zero at the initial time, $t = 0$. The prescribed value of the z -derivative, $u_z(0, t)$, is used as a Neumann-type boundary condition, since this allows constructing a Cost Functional (CF, for short, see below), useful to calibrate the model, having a simple structure.

Since the basic information needed to describe the evolution of the depth profile is embodied in the coefficients $D(z)$ and $B(z)$, the calibration problem consists in selecting them in such a way that the solution to equation (2) with the initial and boundary conditions (3), “optimally” describes the measured data, that is, ultimately, the evolution of the depth profile. The term “optimal” means that the difference between the measured profile, $u^{meas}(z, t)$, and the computed depth profile, $u^{comp}(z, t)$, is the smallest possible.

This result is achieved by minimizing the following CF:

$$(4) \quad \Phi(D(z), C(z)) = \int_z |u^{meas}(z, t) - u^{comp}(z, t, D(z), C(z))| dz.$$

Assuming that the underlying model might be represented by the diffusion equation (2), and starting from some “initial guess” for the coefficients $D(z)$ and $C(z)$, we use a suitable inversion algorithm, based on the minimization of the CF (4), to obtain iteratively a better approximation, say D_n and C_n , at the n -th iteration, for the two coefficients, simultaneously.

Such a minimization procedure is well-studied in Geophysics, see [22], e.g., and therefore, we will not discuss it here in detail.

3.2.1. Reducing the problem dimension

In [13], the coefficients in the governing equation (2) were assumed to be piecewise constant and time independent.

This allows one to use formal Fourier transform and therefore to study an ordinary instead of a partial differential equation. As a solution of ordinary differential equations with *constant* coefficients is known, some calculation could be done in an analytic form. Such an approach is widely used in geophysics, where many researchers are perform simulation in the “frequency domain”.

A formal Fourier-type transform (or a formal Laplace-type transform as in [23]) was used to reduce the problem dimension. More precisely, to solve the original problem, the Fourier-type (or Laplace-type) representation

$$(5) \quad U(z, \omega) = \int_0^T u(z, t) e^{-i\omega t} dt, \quad \left(U(z, \omega) = \int_0^T u(z, t) e^{\omega t} dt \right)$$

was used, with a sort of the discrete version of the inverse formula and the proper CF characterizing the L^2 norm of the difference between measure $U^{meas}(z, \omega)$ and $U^{comp}(z, \omega)$.

3.2.2. Structure of measured data

The long-time evolution of the cross-shore position of coastal profiles was monitored during 10 years, from 1981 to 1991, at Duck, North Carolina (USA). A visualization of the initial profile (at the beginning of the observation time) and of the final profile of the cross-shore position in this region is shown in Figure 1 (the initial profile is shown as a solid line, the final profile as a dashed line).

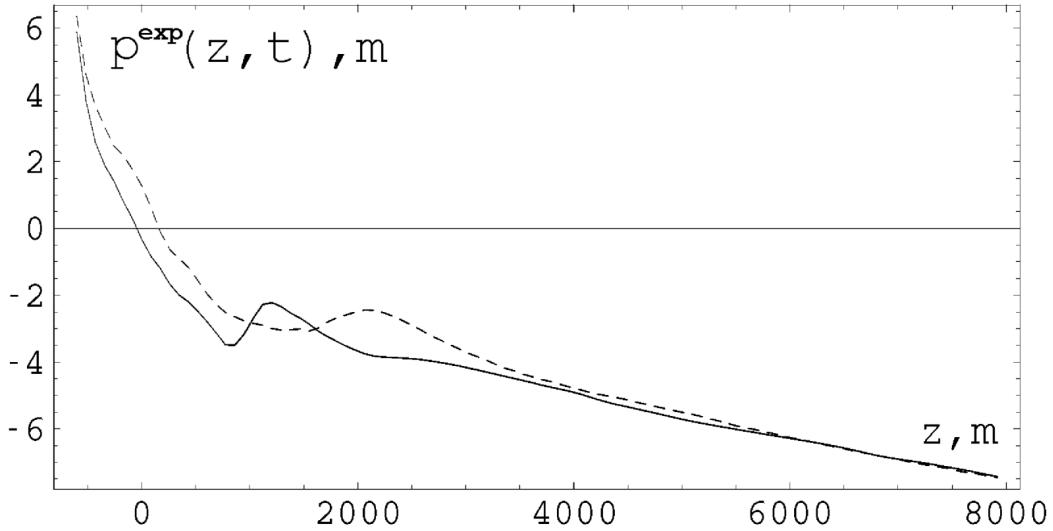


Figure 1. Initial profile (solid line) and final profile (dashed line) of the cross-shore position, obtained by measurements made at Duck (USA), during 10 years, from 1981 to 1991. Here $p^{exp}(z, t) = u^{meas}(z, t) + p(z, 0)$ with $p(z, 0)$ being the depth profile at the initial time moment.

Such real data for the cross-shore position consists of a 100×250 array, corresponding to 100 observation points in space (i.e., z), each representing an average over 80 meters, and 250 observation times, each representing an average over 15 days. The source function, $f(t)$, in the model equation below, is given by the average wave height, and has been measured at the same observation times.

A large body of data, similar to those obtained at Duck, has been collected over 60 years in a number of places, in The Netherlands (JARKUS dataset). The format of these data is different compared to those from Duck. The differences are as follows:

- (1) In The Netherlands, measurements have been taken once a year, from 1963 through 2023, while at Duck the depth profiles were obtained every two weeks.
- (2) Second, no source term ($f(t)$ in (2)) was included in the governing equation for the Delfland data, whereas the source term representing a forcing term associated with the wave conditions, was part of the model. While in the latter case the numerical results were almost unchanged when setting $f(t) \equiv 0$, it turned out that using a nonzero $f(t)$ resulted in a more stable algorithms. Overall good numerical results, obtained in both, the Duck and the Delfland case, seem to indicate that the source term seemed to play a little role in determining the long-time behavior, at least at these locations.

An explanation can perhaps be found in the fact that $f(t)$ is a rather rapidly oscillating function, on the scale at which the profiles vary, and has a time-average of almost zero.

- (3) Last (but not least), the measurements at Duck were accomplished up to a distance of 8,000 m from the coastline. With 100 observation points, this corresponds to a space step-size of 80 m. At Delfland, instead, the depth profiles were studied only within a distance of 500 m from the coastline. With the same number of observation points, the space step-size now is 5 m, that is a factor 16 shorter than at Duck.

Nevertheless, it turns out that the diffusion model (2) reasonably describes both sets of data.

3.2.3. Calibration results

In Figure 2, a 3D plots of the profiles, $p^{dir}(z, t) = u^{comp}(z, t) + p(z, 0)$ and $p^{exp}(z, t) = u^{meas}(z, t) + p(z, 0)$ ($p(z, 0)$ being the depth profile at the initial time moment) for the Delfland data are shown.

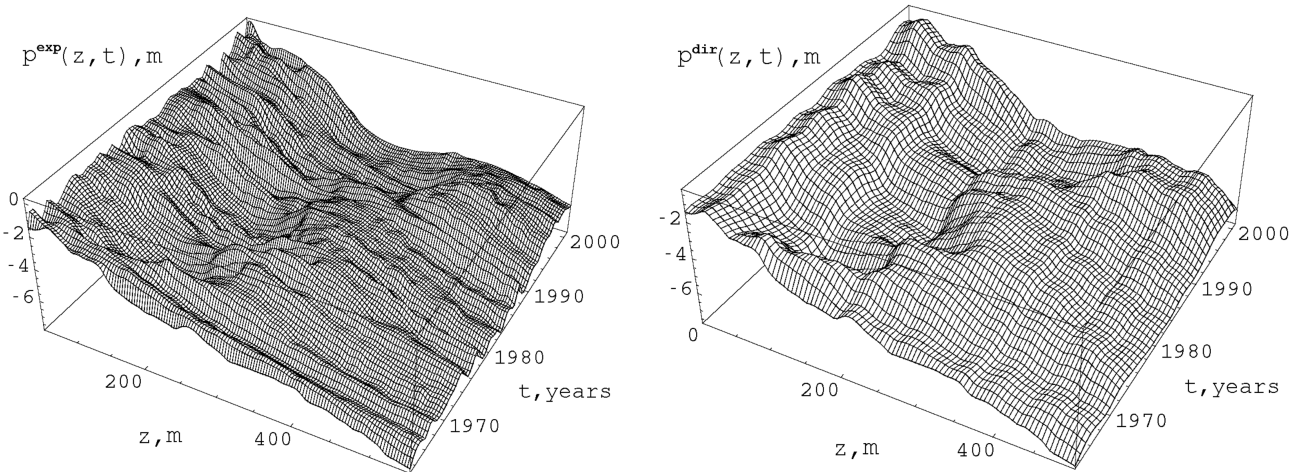


Figure 2. Experimental data obtained at Delfland, extrapolated (to obtain the missing observation points) and then interpolated on a regular mesh over the space-time plane (left). Depth profile, numerically simulated with the model piecewise constant coefficients $D(z)$ and $B(z)$ taking 40 values (right). Here $p^{exp}(z, t) = u^{meas}(z, t) + p(z, 0)$ with $p(z, 0)$ being the depth profile at the initial time moment, $p^{dir}(z, t) = u^{comp}(z, t) + p(z, 0)$.

Summarizing, one can conclude that (see [13,23] for details):

1. Even the very rough assumption of merely *constants* parameters of the model equation seems to reproduce already the correct pattern for the measured data obtained at Duck. Even though the time-dependent profiles (measured and obtained as the solution to the diffusion equation with constant coefficients) look a little different, the general trend of the measured profile can be recognized in the computed solution.
2. Under the more realistic assumption that the coefficients $D(z)$ and $B(z)$ are *piecewise constant* (using a certain number of layers of equal thickness), the diffusion type model (2) better describes the measured data. As one can expect, the quality of the solution improves when the number of

layers increases from 1 to 25. The average relative error, $\delta(z)$ in (6), is about 10.8% when only one pair of constant values are used for B and D , and reduces to about 5.4% when 25 pairs of constants are used instead (i.e., in the “25 layers model”).

3. The time-dependent relative error, defined by

$$(6) \quad \delta(z, t) = 100\% \cdot \left| \frac{u_{\min}^{\text{comp}}(z, t) - u^{\text{meas}}(z, t)}{u_{\min}^{\text{comp}}(z, t)} \right|,$$

correspondingly to four values of time, $t = 0.2T, 0.4T, 0.6T$, and $0.8T$ does *not* grow in time.

3.2.4. Prediction

A prediction of future coastal profiles from past and present data can be made on the basis of the model described above.

After determining the coefficients $D(z)$ and $B(z)$, one could use equation (2) and the appropriate IV and BVs to compute the profile $u^{\text{comp}}(z, t)$ on $\mathcal{D}' \equiv \{0 \leq z \leq H, 0 \leq t \leq T'\}$, with $T' > T$, provided that the boundary data are given on the entire boundary of $\mathcal{D}'$.

In order to test the performance of the model concerning the prediction, we suppose to know all IV, BVs, and the internal data in $\mathcal{D}$, up to time $T'' < T$, and use the BVs on the boundary of $\mathcal{D}$ up to time T , obtained extrapolating the BVs at previous times. The IV to start with will be given by the *experimental* (i.e., measured) values, $u^{\text{meas}}(z, T)$, rather than the values $u^{\text{comp}}(z, T)$ computed in $\mathcal{D}$. Therefore, the numerical solution to equation (2) will be obtained in the domain $\mathcal{D}_{T'', T} \equiv \{0 \leq z \leq H, T'' \leq t \leq T\}$. The resulting (computed) values, say $\tilde{u}(z, t)$, for $T'' \leq t \leq T$, are then compared with $u^{\text{meas}}(z, t)$.

Focusing again on the case of Delfland, for the purpose of illustration, we solved the *direct* problem (2) with the coefficients $B(z)$ and $D(z)$ previously identified. We first extrapolated the boundary data on $z = 0$ and $z = H$ from $T'' = 36$ year (i.e., measured in 1998) up to $T = 37$ (i.e., in 1999). Extrapolation was made by cubic polynomials, constructed using data measured before T'' .

The error made between predicted and measured profiles amounts to about 50 cm (12%) in some location, but it is much less (about 10 – 20 cm, i.e., 2.5 – 5%) everywhere else, see Figure 3.

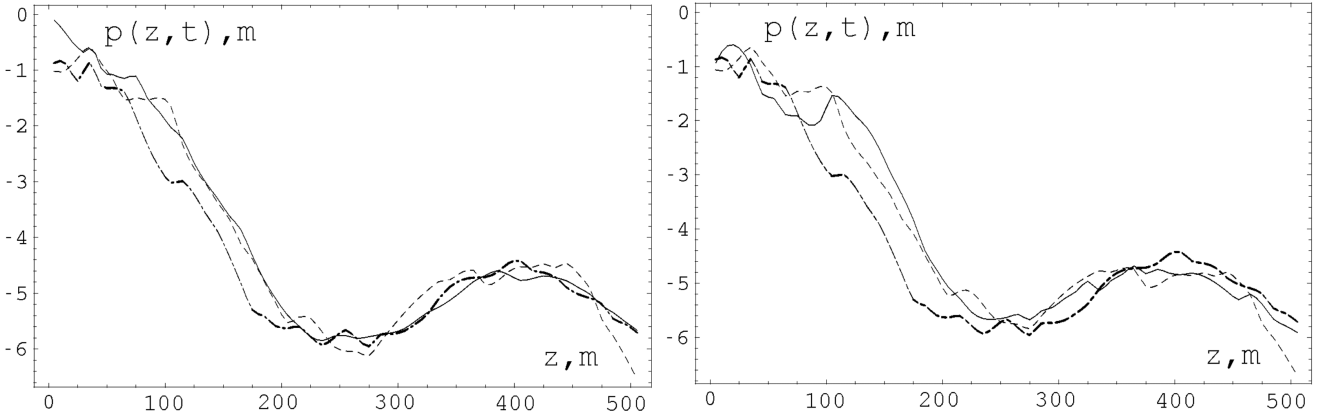


Figure 3. Comparison between the one-year (left) and the two-year (right) predicted profiles, $p(z, t) = u^{\text{comp}}(z, T) + p(z, 0)$ (dashed lines) and the experimentally measured profile, $u^{\text{meas}}(z, T) + p(z, 0)$ (solid lines). The point-dashed lines show the “initial” profile, $p(z, 0)$; m stands for meters.

As one could expect, results are not so good for two or three years prediction ahead.

In [24], the authors addressed the question of using the minimal number of profile measurements in time, required for a reasonably good prediction of the depth profile evolution for 2–3 years ahead. A calibration was made according to that model, using the JARKUS dataset (G. de Boer, Jarkus dataset description. URL <https://publicwiki.deltares.nl/display/OET/Dataset+documentation+JarKus>). At least

20 years of measurements were needed for a prediction of just 3 years ahead within a 10% integrated relative error in the “frequency domain” (that is using Fourier type of Laplace type transform).

Tables 1 and 2 present the values of Integrated Relative Error (IRE) for the three years depth profile evolution prediction after model (2) calibration within 27 years (Table 1) and 19 years (Table 2). Different periods of calibration are presented. The IRE has the form

$$\delta = 100\% \frac{1}{T} \frac{1}{H} \frac{1}{N_m} \sum_{i=1}^{N_m} \int_0^T \int_0^H \left| \frac{u_i^{meas}(z, t) - u_i^{comp}(z, t)}{u_i^{meas}(z, t)} \right| dz dt$$

Table 1. Integrated Relative Error: 27 years model calibration and 3 years prediction

Calibration Prediction period	1965 - 1994	1966 - 1995	1967 - 1996	1968 - 1997	1969 - 1998
Relative Error	7,11	5,57	4,77	5,16	5.14

Table 2. Integrated Relative Error: 19 years model calibration and 3 years prediction.

Calibration Period	1965 - '86	1967 - '88	1969 - '90	1971 - '92	1973 - '94
Relative Error	11,32	12,04	12,18	13,00	13,00
Calibration Period	1974 - '95	1975 - '96	1976 - '97	1977 - '98	
Relative Error	13,73	13,83	13,59	13,38	

The structure of the CF was also studied in [24]. It was proposed to consider a certain part of the space interval and to minimize the sum of the difference between the measured and computed profiles in two points $z = z_1$ and $z = z_2$. Numerical experiments show that prediction results are better (in terms of the values of IRE) if the near-shore zone is excluded from consideration.

One can expect it as the near-coast zone is typically not subject to precise measurement (if any), as phenomena such as wave beaking may have strong impact on it.

3.3. Time-dependent coefficients

To make the prediction period shorter, in [25,26] the authors proposed to consider *time dependent* coefficients in the governing equation. More precisely the following model was considered

$$(7) \quad u_t(z, t) = \frac{\partial}{\partial z} (D(z, t) u_z) + C(z, t) u_z,$$

with the same initial and boundary conditions as in (3).

The coefficients $D(z, t)$ and $C(z, t)$ of the model, were chosen *piecewise linear* functions of space and time. Uniform partitions of space and time intervals were used.

The same calibration strategy as that followed for the time independent model coefficients was adopted, that is, the CF was taken in the form (4).

The minimization of the CF to calibrate the model was accomplished independently on each time interval $t \in [t_{i-1}, t_i]$.

As it was described above, an initial approximation for the coefficients of the equation, $D(z, t)$ and $C(z, t)$, that is the corresponding set of numbers, was first made, then these were changed in such a way that the value of the CF in (4) decreases. To update the current values of $D(z, t)$ and $C(z, t)$, an open source version of some *Genetic Algorithm* was implemented, see [27].

A comparison was made in terms of the IREs, Δ , defined as

$$(8) \quad \Delta = 100\% \frac{\int_z^z \int_t^t |v(z, t) - u(z, t)| dz dt}{\int_z^z \int_t^t |v(z, t)| dz dt}.$$

In order to predict the long-term evolution of coastal profiles, in [25,26], the authors proposed a regression model which describes the computed coefficients of the diffusion equation, $D(z, t)$ and $C(z, t)$, and then used the resulting model to extrapolate the coefficients in time.

A regression analysis for the present work has been carried out using a Support Vector Machine (SVM) (which provides machine learning algorithms) [28], implemented using the open source machine learning library “scikit-learn” [29] in Python programming language. As a SVM kernel function, a radial basis function (RBF) was used.

Having many parameters to choose (values of the model coefficients $D(z_i, t_j)$ and $C(z_i, t_j)$), it becomes possible to describe rather well the entire array of the measured data. Values of IRE (8) are given in Table 3.

Table 3. IRE (in Percent) Describing the Measured Depth Profile for Different Space Intervals and Time Intervals.

	[1991, 2000]	[1996, 2005]	[2001, 2010]	[2006, 2015]
Space Interval $0 \leq z \leq H$	3.87	3.42	2.86	3.55
Space Interval $H/10 \leq z \leq H$	2.96	3.08	2.33	2.40
Space Interval $H/4 \leq z \leq H$	1.87	1.74	1.05	1.32

The values of IRE are larger in case we consider the whole space interval, starting from the coast. We conjecture that the diffusion and drift may be affected either by wave breaks or by other phenomena occurring in the near-shore zone, that are difficult to quantify.

The values of IRE, in the form (8), that have been obtained for a two and three years prediction are summarized in Tables 4 and 5. Here the values of IRE for different space intervals, namely, $z \in [0, H]$, $z \in [H/10, H]$, and $z \in [H/4, H]$. The calibration periods are the years [1991, 2000] (interval [0, 10] in the tables below), [1996, 2005] (interval [5, 15]), [2001, 2010] (interval [10, 20]), and the years [2006, 2015] (interval [15, 25]) are given.

Table 4. IRE Made in the Prediction of the Depth Profile evolution for 2 Years for Different Space Intervals and Time Intervals.

	[1991, 2000]	[1996, 2005]	[2001, 2010]	[2006, 2015]
Space Interval $0 \leq z \leq H$	19.88	6.30	9.25	10.85
Space Interval $H/10 \leq z \leq H$	12.35	5.37	8.44	10.33
Space Interval $H/4 \leq z \leq H$	10.49	4.57	8.08	9.12

Table 5. IRE Made in the Prediction of the Depth Profile Evolution for 3 Years for Different Space Intervals and Time Intervals.

	[1991, 2000]	[1996, 2005]	[2001, 2010]	[2006, 2015]
Space Interval $0 \leq z \leq H$	18.92	6.82	9.97	10.10
Space Interval $H/10 \leq z \leq H$	12.89	5.81	9.30	9.83
Space Interval $H/4 \leq z \leq H$	10.50	5.09	8.01	8.20

4. Discussion

Models based on diffusion equations are promising to describe the long-term time evolution of coastal depth profiles. Their numerical solutions are delicate, since *real data* are exploited. These are available on many geographical sites, spanning few decades.

Using a combination of several techniques, it turns out that it is possible to describe the measured data with a good accuracy. Namely, Finite Element Methods to solve the direct problem (2) – (3), minimization of the Cost Functional (4) for the model calibration according to the measured data, using Genetic Algorithms and Regression Analysis to predict the depth profile evolution. The measured data can be described rather well by standard numerical methods, and an IRE (8) between measured and computed data turned out to lie within a few percent, see the Table 3. According to the literature,

meaningful predictions are indeed possible over 1 to 3 years ahead (the measurements were taken once a year in the case under study), clearly with a different degree of accuracy.

Predictions of the depth profile evolution improve if a certain near coast area is disregarded. The corresponding data are collected in Tables 4 and 5. Ignoring near-shore regions should be considered as rather natural, since a number of short-term factors, such as, e.g., wave breaks (due to storms), may have a strong impact there.

In order to predict the evolution of the depth profiles' at river estuaries, the alongshore behavior should be included. This action would increase the computational cost of the model by an order of magnitude, and hence, using massive parallel computing for the numerical treatment would be mandatory.

5. Future directions

In [23,30], a first attempt was made to address the 2D problem, that is to study the evolution of both, the shoreline and the depth profiles. Several 1D problems, treated as was described in Section 3 for a single 1D case, at several fixed y_i 's, for $i = 1, 2, \dots, m$, y denoting a coordinate along the shoreline, were considered separately for the governing equation (2) with $f(t) \equiv 0$. Then, some averaging over these " y_i profiles" was obtained.

In [23] the problem for equation (2) was considered applying a discrete version of the Laplace type transform (see (5)). First, the coefficients, $D(z)$ and $B(z)$, were supposed equal at all 30 observation points along the coast. The following version of misfit functional in (4) was considered:

$$\Phi(D, B) = \sum_{i=1}^{30} \int_{\omega} (U_i(\omega)|_{z=0} - U_i^{comp}(\omega, D, B)|_{z=0}) d\omega$$

The numerically obtained minimal value of this functional was found

$$\Phi(D, B) = 41.5.$$

Then the coastal line was treated as that one consisting of three different parts of 10 observation points along shore each. Three pairs of coefficients, $D_i(z)$ and $B_i(z)$, have been numerically reconstructed to minimize the following CF

$$\begin{aligned} \Phi(D_1, B_1, D_2, B_2, D_3, B_3) &= \sum_{i=1}^{10} \int_{\omega} (U_i(\omega)|_{z=0} - U_i^{comp}(\omega, D_1, B_1)|_{z=0}) d\omega \\ &+ \sum_{i=11}^{20} \int_{\omega} (U_i(\omega)|_{z=0} - U_i^{comp}(\omega, D_2, B_2)|_{z=0}) d\omega + \sum_{i=21}^{30} \int_{\omega} (U_i(\omega)|_{z=0} - U_i^{comp}(\omega, D_3, B_3)|_{z=0}) d\omega. \end{aligned}$$

As was obtained, a minimal value of the last CF found

$$\Phi(D_1, B_1, D_2, B_2, D_3, B_3) = 32.3,$$

that is smaller compared to the minimal one computed for just one value of equation coefficients along-shore.

In closing, it seems that diffusion-transport models are promising to predict coastal and depth profile evolution. Formulating more precise 2D models should definitely be useful to obtain reliable results on prediction based on real data.

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