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MATHEMATICAL METHOD TO UNFOLD AN OBLIQUE PYRAMID

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Abstract: *In this paper a mathematical method for calculating the unfolding of an oblique pyramid is shown. The real dimensions of the edges and the angles between the edges of the pyramid, too, are found out with this method. For this, we use the program AutoCAD 2002 and for mathematical calculations we use the program Mathematica 5.0.*

Keywords: *unfolding, oblique pyramid, directing vector, canonical equation of a straight line*

1. INTRODUCTION

Unfolding an Oblique Pyramid has several practical and theoretical applications [1]:

- Education and Learning:
  - Helps in understanding three-dimensional geometry and the relationships between 3D and 2D shapes.
  - It is a useful exercise for students studying geometry and mathematics.
- Design and Architecture:
  - Useful in designing complex structures, where visualization and planning of individual components is required.
  - Helps architects and designers create accurate models and plan construction.
- Manufacturing and Engineering:
  - Used in the processes of cutting and folding materials, such as metal or cardboard, to create three-dimensional shapes.
  - Helps optimize material use and reduce waste.
- Art and Modeling:
  - Useful for artists and modelers in creating complex sculptures and models.
  - Helps in visualization and making three-dimensional shapes from flat materials.

The unfolding of an oblique pyramid is therefore a versatile tool that can be applied in various fields to transform and understand three-dimensional shapes.

Oblique pyramids are used in various types of structures, due to their stability and unique aesthetics. Here are some common examples [1-2]:

- Modern architecture:
  - Commercial and residential buildings: Oblique pyramids are used for roofs and facades, providing a distinctive and modern design.
  - Art structures: Used in innovative architectural projects and public sculptures, oblique pyramids add an artistic and visually interesting element.
- Industrial constructions:
  - Cooling towers: The oblique pyramid shape is used for cooling towers in power plants, due to its efficiency in air circulation.
  - Silos and warehouses: Oblique pyramid structures are used for material storage, providing stability and durability.
- Temporary structures:
  - Pavilions and tents: Oblique pyramids are used in the construction of pavilions and tents for events, due to their ease of assembly and disassembly.
  - Exhibition Structures: Used in exhibitions and trade shows, oblique pyramids offer an attractive and functional design3.

2. THE UNFOLDING OF AN OBLIQUE PYRAMID

We consider the oblique pyramid SABCD (sabcd, s′a′b′c′d’), which has the base ABCD in the horizontal plane, with S(50,60,60), A(65,30,0), B(85,50,0), C(95,50,0), D(105,20,0), like in figure 1.
The coordinates are the expressions in millimeters.

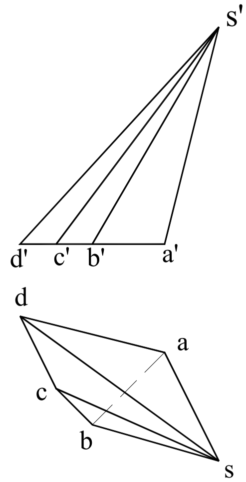

Figure 1. The oblique pyramid

To construct the unfolding of the lateral surface of a pyramid, we must to know the real dimensions of the lateral edges of the pyramid [3-4].

$$\overline{SA} = \sqrt{(x_S - x_A)^2 + (y_S - y_A)^2 + (z_S - z_A)^2} = 68{,}73[mm] \quad (1)$$

$$\overline{SB} = \sqrt{(x_S - x_B)^2 + (y_S - y_B)^2 + (z_S - z_B)^2} = 70{,}178[mm] \quad (2)$$

$$\overline{SC} = \sqrt{(x_S - x_C)^2 + (y_S - y_C)^2 + (z_S - z_C)^2} = 75{,}66[mm] \quad (3)$$

$$\overline{SD} = \sqrt{(x_S - x_D)^2 + (y_S - y_D)^2 + (z_S - z_D)^2} = 90{,}65[mm]$$

where:

x_s, y_s, z_s are the coordinates of the S point;
x_a, y_a, z_a - the coordinates of the A point;
x_b, y_b, z_b - the coordinates of the B point;
x_c, y_c, z_c - the coordinates of the C point,
x_d, y_d, z_d - the coordinates of the D point.

To find out the angles of two lines, we can apply the formula:

$$\cos\varphi = \frac{|\vec{u}_1 \cdot \vec{u}_2|}{|\vec{u}_1| \cdot |u_2|} \quad (4)$$

where:

u_1, u_2 are the directing vectors of the two lines;
φ - the angle among the two lines.

To find out the directing parameters of the straigt line, we been writing the equations of the line, which is determined of two points M_1 (x_1, y_1, z_1) and M_2 (x_2, y_2, z_2).

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \quad (5)$$

We obtain the following expressions:

$$SA: \frac{x - 50}{15} = \frac{y - 60}{-30} = \frac{z - 60}{-60}$$

$$SB: \frac{x - 50}{35} = \frac{y - 60}{-10} = \frac{z - 60}{-60}$$

$$SC: \frac{x - 50}{45} = \frac{y - 60}{-20} = \frac{z - 60}{-60}$$

$$SD: \frac{x - 50}{55} = \frac{y - 60}{-40} = \frac{z - 60}{-60}$$

Thus, to determine the angle α, between the edges SA and SB, the direction vectors of the two lines are:

$$\vec{u}_1 = (15, -30, -60)$$

and $\vec{u}_2 = (35, -10, -60)$

$$\cos\alpha = \frac{|\vec{u}_1 \cdot \vec{u}_2|}{|\vec{u}_1| \cdot |\vec{u}_2|} = 0{,}917 \Rightarrow \alpha = 23{,}46^o$$

To find out the angle β, between SB and SC, the direction vectors of the lines are:

$$\vec{u}_2 = (35, -10, -60)$$

and $\vec{u}_3 = (45, -20, -60)$.

$$\cos\beta = \frac{|\vec{u}_2 \cdot \vec{u}_3|}{|\vec{u}_2| \cdot |\vec{u}_3|} = 0{,}986 \Rightarrow \beta = 9{,}34^o$$

To find out the angle γ, between SC and SD, the direction vectors of the lines are:

$$\vec{u}_3 = (45, -20, -60)$$

and $\vec{u}_4 = (55, -40, -60)$

$$cos\,\gamma = \frac{|\vec{u}_3 \cdot \vec{u}_4|}{|\vec{u}_3| \cdot |\vec{u}_4|} = 0{,}976 \Rightarrow \gamma = 12{,}41^o$$

Finally, we determine the angle θ, between the edges SD şi SA. The directing vectors of the two straight lines are:

$$\vec{u}_4 = (55, -40, -60)$$

and $$\vec{u}_1 = (15, -30, -60)$$

$$cos\,\theta = \frac{|\vec{u}_4 \cdot \vec{u}_1|}{|\vec{u}_4| \cdot |\vec{u}_1|} = 0{,}9023 \Rightarrow \theta = 25{,}54^o$$

Thus, we obtain the unfolding of the oblique pyramid, like în figure 2.

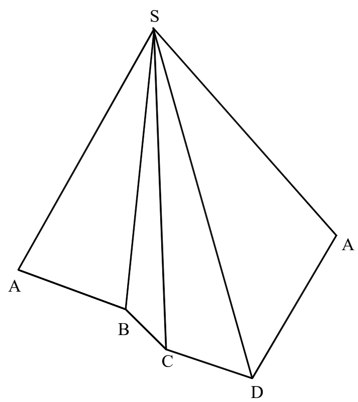

Figure 2. The unfolding of the oblique pyramid

3. CONCLUSIONS

These applications demonstrate the importance of the deployment of an oblique pyramid in engineering, contributing to the efficiency and precision of design and manufacturing processes.

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