

THE INFLUENCE OF THE RIGIDITY OF FLEXIBLE WIRES ON THE PROPER PULSATIONS OF AN ELASTIC SYSTEM WITH ONE DEGREE OF FREEDOM FORMED FROM HOMOGENEOUS WHEELS AND BARS, SPRINGS AND INEXTENSIBLE AND EXTENSIBLE WIRES

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Abstract: The paper aims to study the influence of the stiffness of flexible connecting wires on the free natural pulsations of an elastic system with one degree of freedom formed by wheels and homogeneous bars, springs and flexible wires for two cases:

a) when they are considered inextensible and

b) when they are considered extensible, their flexibility being proportional to their length.

With the help of the professional program MATHCAD, the influence of the relative stiffness of elastic wires subjected to traction, on the natural pulsations, for several variants of flexible and extensible wires, was simulated, using the parametric calculation of the natural pulsations.

Keywords: free vibrations without damping, wheels and homogeneous bars, springs, inextensible and extensible flexible wires

1. INTRODUCTION

To study the vibrations of a system with one or more degrees of freedom, the method of Lagrange's equations of the second kind for conservative systems is used [1], [2], [3]: the differential equation of motion of a system of bodies with respect to the static equilibrium position is obtained using LAGRANGE's equations of the second kind;

-for a conservative system with one degree of freedom, it is written:

$$\left\{ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_1} \right) - \frac{\partial L}{\partial q_1} = 0 \right. \quad (1)$$

-for a conservative system with three degree of freedom, it is written:

$$\left\{ \begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_1} \right) - \frac{\partial L}{\partial q_1} &= 0; \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_2} \right) - \frac{\partial L}{\partial q_2} = 0; \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_3} \right) - \frac{\partial L}{\partial q_3} &= 0 \end{aligned} \right. \quad (2)$$

where:

L is the LAGRANGE function: $L = Ec - V$

Ec - the total kinetic energy of the system of bodies;

V - the potential energy of the elastic elements of the springs written for the elastic elements tensioned relative to the static equilibrium position;

q_1, q_2, q_3 - generalized displacements;

$\dot{q}_1, \dot{q}_2, \dot{q}_3$ - generalized velocities;

2. CASE 1: FLEXIBLE INEXTENSIBLE WIRES

We consider the mechanical system with one degree of freedom formed by two homogeneous wheels of masses m_1 , and m_2 and radius R_1 and R_2 .

The movable axis of homogeneous wheel 1 is connected to the fixed medium by means of a spring of rigidity k_2 . A flexible and inextensible wire is wound over the two homogeneous wheels, connected to the fixed medium at one end directly, and at the other end by means of the spring of rigidity k_1 .

Homogeneous wheels of masses m_1 and m_2 are connected to each other by means of the flexible and inextensible wire and by a spring of rigidity k_3 . A homogeneous bar of mass m_3 and length L_3 is welded to wheel 1, and a homogeneous bar of mass m_4 and length L_4 to wheel 2, as in figure 1.

For the numerical solution in MATHCAD, the following numerical values of the parameters are given:

$m_1=1\text{kg}$; $m_2=2\text{kg}$; $m_3=1.5\text{kg}$; $m_4=2.5\text{kg}$; $R_1=150\text{mm}$, $R_2=100\text{mm}$, $L_3=350\text{mm}$; $L_4=250\text{mm}$; $k_1=200\text{ N/mm}$; $k_2=250\text{ N/mm}$;

Expected results to be determined:

The natural pulsation p_1 of the free vibrations of the system with one degree of freedom

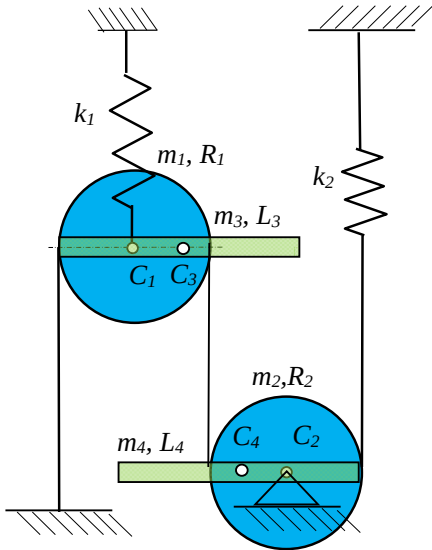


Figure 1

To write the expressions of kinetic energy, in figure 2 we represented the kinematic analysis of the velocities of the system with one degree of freedom, where it was noted with:

q , $\dot{q}$ - generalized displacement and velocity;

ω_1 , ω_2 - the angular velocities of the two homogeneous wheels noted with indices 1, 2;

v_{c1} , v_{c3} , v_{c4} - the velocities of the centers of gravity of the homogeneous wheel and bars noted with indices 1, 3, 4.

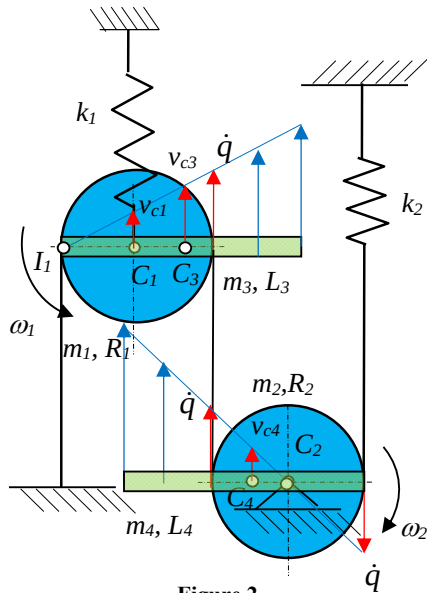


Figure 2

The velocities of the centers of gravity of the homogeneous bars v_{c3} and v_{c4} are written using the proportionality relations from the velocity distributions shown in figure 2:

$$\frac{\dot{q}}{2R_1} = \frac{v_{c3}}{L_3} \Rightarrow v_{c3} = \left(\frac{L_3}{4R_1} \right) \cdot \dot{q};$$

$$\frac{\dot{q}}{R_2} = \frac{v_{c4}}{L_4 - R_2} \Rightarrow v_{c4} = \left(\frac{L_4}{2R_2} - 1 \right) \cdot \dot{q};$$

The expression for the total kinetic energy E_c of the system of bodies is written in parametric form taking into account KOENIG's theorem as follows:

$$\begin{aligned} E_c &= \frac{1}{2} \cdot m_1 \cdot \left(\frac{\dot{q}}{2} \right)^2 + \frac{1}{2} \cdot \frac{m_1 \cdot R_1^2}{2} \left(\frac{\dot{q}}{2R_1} \right)^2 + \\ &+ \frac{1}{2} \cdot \frac{m_2 \cdot R_2^2}{2} \left(\frac{\dot{q}}{R_2} \right)^2 + \frac{1}{2} \cdot m_3 \cdot (v_{c3})^2 + \\ &+ \frac{1}{2} \cdot \frac{m_3 \cdot L_3^2}{12} \left(\frac{\dot{q}}{2R_1} \right)^2 + \frac{1}{2} \cdot m_4 \cdot (v_{c4})^2 + \frac{1}{2} \cdot \frac{m_4 \cdot L_4^2}{12} \left(\frac{\dot{q}}{R_2} \right)^2 \\ &\Rightarrow E_c = \frac{1}{2} A \cdot \dot{q}^2 \end{aligned}$$

where A is the equivalent mass of the system in parametric form:

$$\begin{aligned} A &= \frac{3m_1}{8} + \frac{m_2}{2} + m_3 \cdot \left[\left(\frac{L_3}{4R_1} \right)^2 + \frac{1}{48} \left(\frac{L_3}{R_1} \right)^2 \right] + \\ &+ m_4 \cdot \left[\left(\frac{L_4}{2R_2} - 1 \right)^2 + \frac{1}{12} \left(\frac{L_4}{R_2} \right)^2 \right] \end{aligned}$$

The expression of the total potential energy V corresponding to the deformations of the two springs relative to the static equilibrium position is written in the form:

$$V = \frac{1}{2} k_1 \cdot \left(\frac{q}{2} \right)^2 + \frac{1}{2} k_2 \cdot q^2 \Rightarrow V = \frac{1}{2} K \cdot q^2$$

Where K is the equivalent elastic constant of the system:

$$K = \frac{k_1}{4} + k_2$$

Substituting into Lagrange's equations (1) we obtain the differential equation of vibratory motion::

$$A \cdot \ddot{q} + K \cdot q = 0 \Rightarrow \ddot{q} + \frac{K}{A} \cdot q = 0$$

The natural pulsation p of the free vibrations of the system with one degree of freedom is calculated using the formula:

$$p = \sqrt{K / A}$$

Substituting the numerical values in MATHCAD yields the following result:

$$A = 3,514 \text{ kg}; \quad K = 3 \cdot 10^5 \text{ N/m}$$

$$p = \sqrt{\frac{K}{A}} = 292,191 \text{ rad/s};$$

3. CASE 2: FLEXIBLE EXTENSIBLE WIRES

We consider the mechanical system with one degree of freedom in figure 1 in which, if the rigidities k_3, k_4, k_5 and k_6 corresponding to the four pieces of flexible and extensible wires are introduced, the system with three degrees of freedom in figure 3 is obtained.

Expected results to be determined:

For this system, we will determine: the natural pulsations p_1, p_2 and p_3 of the free vibrations of the system with three degrees of freedom, as well as the natural modes of vibration (the distribution coefficients μ_2 and μ_3 corresponding to the natural pulsations).

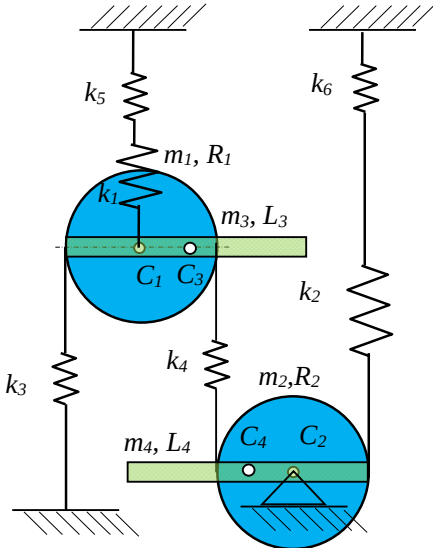


Figure 3

In order to obtain the expressions of kinetic energy in parametric form, we represented in figure 4 the kinematic analysis of the speeds of the system with three degrees of freedom, where it was noted with:

$q_1, q_2, q_3, \dot{q}_1, \dot{q}_2, \dot{q}_3$ the generalized displacements and speeds;

ω_1, ω_2 - angular velocities of the homogeneous wheels;
 v_{c3}, v_{c4} - velocities of the centers of gravity of the homogeneous bars 2-3.

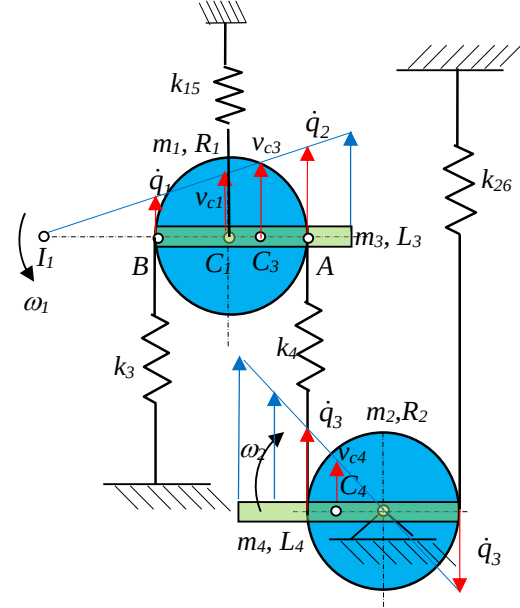


Figure 4

The velocities of the centers of gravity of the homogeneous bars v_{c3} and v_{c4} are written using the proportionality relations from the velocity distributions presented in figure 4:

$$\omega_1 = \frac{\dot{q}_2}{I_1 A} = \frac{\dot{q}_1}{I_1 B} = \frac{\dot{q}_2 - \dot{q}_1}{I_1 A - I_1 B} \Rightarrow \omega_1 = \frac{\dot{q}_2 - \dot{q}_1}{2R_1};$$

$$v_{c1} = \omega_1 \cdot I_1 C_1 \Rightarrow v_{c1} = \frac{\dot{q}_2 + \dot{q}_1}{2};$$

$$\omega_1 = \frac{v_{c3}}{I_1 C_3} = \frac{\dot{q}_1}{I_1 B} = \frac{v_{c3} - \dot{q}_1}{\frac{L_3}{2}} \Rightarrow v_{c3} = \dot{q}_1 + (\dot{q}_2 - \dot{q}_1) \frac{L_3}{4R_1}$$

$$\omega_2 = \frac{\dot{q}_3}{R_2} = \frac{v_{c4}}{\frac{L_4}{2} - R_2} \Rightarrow v_{c4} = \left(\frac{L_4}{2R_2} - 1 \right) \cdot \dot{q}_3;$$

The expression for the total kinetic energy E_c of the system of bodies during vibrations is written using KOENIG's theorem as follows:

$$E_c = \frac{1}{2} \cdot m_1 \cdot (v_{c1})^2 + \frac{1}{2} \frac{m_1 \cdot R_1^2}{2} (\omega_1)^2 +$$

$$+ \frac{1}{2} \frac{m_2 \cdot R_2^2}{2} (\omega_2)^2 + \frac{1}{2} \cdot m_3 \cdot (v_{c3})^2 + \frac{1}{2} \frac{m_3 \cdot L_3^2}{12} (\omega_1)^2$$

$$+ \frac{1}{2} \cdot m_4 \cdot (v_{c4})^2 + \frac{1}{2} \frac{m_4 \cdot L_4^2}{12} (\omega_2)^2$$

Substituting the expressions for the velocities of the centers of gravity of the homogeneous bars v_{c3} and v_{c4} , the parametric formula for the total kinetic energy of the wheel and bar system results:

$$\begin{aligned}
 E_c = & \frac{1}{2} \cdot m_1 \cdot \left(\frac{\dot{q}_2 + \dot{q}_1}{2} \right)^2 + \frac{1}{2} \cdot \frac{m_1 \cdot R_1^2}{2} \left(\frac{\dot{q}_2 - \dot{q}_1}{2R_1} \right)^2 + \\
 & + \frac{1}{2} \cdot \frac{m_2 \cdot R_2^2}{2} \left(\frac{\dot{q}_3}{R_2} \right)^2 + \frac{1}{2} \cdot m_3 \cdot \left(\dot{q}_1 + (\dot{q}_2 - \dot{q}_1) \frac{L_3}{4R_1} \right)^2 + \\
 & + \frac{1}{2} \cdot \frac{m_3 \cdot L_3^2}{12} \left(\frac{\dot{q}_2 - \dot{q}_1}{2R_1} \right)^2 + \frac{1}{2} \cdot m_4 \cdot \left(\frac{L_4}{2R_2} - 1 \right)^2 \cdot \dot{q}_3^2 + \\
 & + \frac{1}{2} \cdot \frac{m_4 \cdot L_4^2}{12} \left(\frac{\dot{q}_3}{R_2} \right)^2
 \end{aligned}$$

The partial derivatives of kinetic energy with respect to generalized velocities are written in parametric form as follows:

$$\begin{cases}
 \frac{\partial E_c}{\partial \dot{q}_1} = \frac{m_1}{4} \cdot (\dot{q}_2 + \dot{q}_1) - \frac{m_1}{8} \cdot (\dot{q}_2 - \dot{q}_1) + \\
 + m_3 \cdot \left(\dot{q}_1 + (\dot{q}_2 - \dot{q}_1) \frac{L_3}{4R_1} \right) \cdot \left(1 - \frac{L_3}{4R_1} \right) - \frac{m_3}{48} \left(\frac{L_3}{R_1} \right)^2 (\dot{q}_2 - \dot{q}_1) \\
 \frac{\partial E_c}{\partial \dot{q}_2} = \frac{m_1}{4} \cdot (\dot{q}_2 + \dot{q}_1) + \frac{m_1}{8} \cdot (\dot{q}_2 - \dot{q}_1) + \\
 + m_3 \cdot \left(\dot{q}_1 + (\dot{q}_2 - \dot{q}_1) \frac{L_3}{4R_1} \right) \cdot \left(\frac{L_3}{4R_1} \right) + \frac{m_3}{48} \left(\frac{L_3}{R_1} \right)^2 (\dot{q}_2 - \dot{q}_1) \\
 \frac{\partial E_c}{\partial \dot{q}_3} = \frac{m_2}{2} \cdot \dot{q}_3 + m_4 \cdot \left[\left(\frac{L_4}{2R_2} - 1 \right)^2 + \frac{1}{12} \left(\frac{L_4}{R_2} \right)^2 \right] \cdot \dot{q}_3
 \end{cases}$$

Denote by A, B, C, D, E, F and G the corresponding equivalent masses from the partial derivatives in LAGRANGE's equations

$$\begin{cases}
 \frac{\partial E_c}{\partial \dot{q}_1} = A \cdot \dot{q}_1 + B \cdot \dot{q}_2; \\
 \frac{\partial E_c}{\partial \dot{q}_2} = C \cdot \dot{q}_1 + D \cdot \dot{q}_2 + E \cdot \dot{q}_3; \\
 \frac{\partial E_c}{\partial \dot{q}_3} = F \cdot \dot{q}_2 + G \cdot \dot{q}_3
 \end{cases}$$

The equivalent masses A, B, C, D, E, F and G are obtained by identifying the expressions of the partial derivative equations:

$$\begin{cases}
 A = \frac{3m_1}{8} + m_3 \cdot \left[\left(1 - \frac{L_3}{4R_1} \right)^2 + \frac{1}{48} \left(\frac{L_3}{R_1} \right)^2 \right]; \\
 B = \frac{m_1}{8} + m_3 \cdot \left[\left(1 - \frac{L_3}{4R_1} \right) \cdot \left(\frac{L_3}{4R_1} \right) - \frac{1}{48} \left(\frac{L_3}{R_1} \right)^2 \right] \\
 C = \frac{m_1}{8} + m_3 \cdot \left[\left(1 - \frac{L_3}{4R_1} \right) \cdot \left(\frac{L_3}{4R_1} \right) - \frac{1}{48} \left(\frac{L_3}{R_1} \right)^2 \right]; \\
 D = \frac{3m_1}{8} + m_3 \cdot \left[\left(\frac{L_3}{4R_1} \right)^2 + \frac{1}{48} \left(\frac{L_3}{R_1} \right)^2 \right]; \quad E = 0; \\
 F = 0; \quad G = \frac{m_2}{2} + m_4 \cdot \left[\left(\frac{L_4}{2R_2} - 1 \right)^2 + \frac{1}{12} \left(\frac{L_4}{R_2} \right)^2 \right]
 \end{cases}$$

The expression of the total potential energy V corresponding to the deformations of the four springs relative to the static equilibrium position is written in this case as a function of the generalized displacements q1, q2 and q3 as follows:

$$V = \frac{k_3}{2} \cdot q_1^2 + \frac{k_{15}}{2} \cdot \left(\frac{q_1 + q_2}{2} \right)^2 + \frac{k_4}{2} \cdot (q_2 - q_3)^2 + \frac{k_{26}}{2} \cdot q_3^2$$

The partial derivatives of the potential energy with respect to the generalized displacements q1, q2, q3 allow the calculation of the corresponding equivalent elastic constants K1, K2 ... K7:

$$\begin{cases}
 \frac{\partial V}{\partial q_1} = k_3 \cdot q_1 + \frac{k_{15}}{4} \cdot (q_1 + q_2) = K_1 \cdot q_1 + K_2 \cdot q_2 \\
 \frac{\partial V}{\partial q_2} = \frac{k_{15}}{4} \cdot (q_1 + q_2) + k_4 \cdot (q_2 - q_3) = K_3 \cdot q_1 + K_4 \cdot q_2 + K_5 \cdot q_3 \\
 \frac{\partial V}{\partial q_3} = -k_4 \cdot (q_2 - q_3) + k_{26} \cdot q_3 = K_6 \cdot q_2 + K_7 \cdot q_3
 \end{cases}$$

By identification, the six equivalent elastic constants (stiffnesses) are obtained:

$$\begin{cases}
 K_1 = k_3 + \frac{k_{15}}{4}; & K_2 = \frac{k_{15}}{4}; \\
 K_3 = \frac{k_{15}}{4}; & K_4 = \frac{k_{15}}{4} + k_4; & K_5 = -k_4; \\
 K_6 = -k_4 & K_7 = k_4 + k_{26};
 \end{cases}$$

The differential equations of motion (2) for the three-degree-of-freedom system can be written as follows:

$$\begin{cases}
 A \cdot \ddot{q}_1 + B \cdot \ddot{q}_2 + K_1 \cdot q_1 + K_2 \cdot q_2 = 0 \\
 C \cdot \ddot{q}_1 + D \cdot \ddot{q}_2 + E \cdot \ddot{q}_3 + K_3 \cdot q_1 + K_4 \cdot q_2 + K_5 \cdot q_3 = 0 \\
 F \cdot \ddot{q}_2 + G \cdot \ddot{q}_3 + K_6 \cdot q_2 + K_7 \cdot q_3 = 0
 \end{cases}$$

The system of second-order linear differential equations admits harmonic solutions of the form:

$$\begin{cases}
 q_1 = a_1 \cdot \cos(pt) \\
 q_2 = a_2 \cdot \cos(pt) \\
 q_3 = a_3 \cdot \cos(pt)
 \end{cases} \Rightarrow \begin{cases}
 \ddot{q}_1 = -p^2 \cdot a_1 \cdot \cos(pt) \\
 \ddot{q}_2 = -p^2 \cdot a_2 \cdot \cos(pt) \\
 \ddot{q}_3 = -p^2 \cdot a_3 \cdot \cos(pt)
 \end{cases}$$

Substituting the solutions into the system of differential equations above, we obtain a homogeneous system of equations with unknown vibration amplitudes a1 a2 and a3:

$$\begin{cases}
 (-p^2 \cdot A + K_1) \cdot a_1 + (-p^2 \cdot B + K_2) \cdot a_2 = 0 \\
 (-p^2 \cdot C + K_3) \cdot a_1 + (-p^2 \cdot D + K_4) \cdot a_2 + (-p^2 \cdot E + K_5) \cdot a_3 = 0 \\
 (-p^2 \cdot F + K_6) \cdot a_2 + (-p^2 \cdot G + K_7) \cdot a_3 = 0
 \end{cases}$$

The homogeneous system of equations admits non-zero solutions if its determinant is zero:

$$\begin{vmatrix} -p^2 \cdot A + K_1 & -p^2 \cdot B + K_2 & 0 \\ -p^2 \cdot C + K_3 & -p^2 \cdot D + K_4 & -p^2 \cdot E + K_5 \\ 0 & -p^2 \cdot F + K_6 & -p^2 \cdot G + K_7 \end{vmatrix} = 0$$

This gives us a third-degree equation in p^2 :

$$\begin{aligned} & (-p^2 \cdot A + K_1) \cdot (-p^2 \cdot D + K_4) \cdot (-p^2 \cdot G + K_7) - \\ & - (-p^2 \cdot C + K_3) \cdot (-p^2 \cdot E + K_5) \cdot (-p^2 \cdot F + K_6) - \\ & - (-p^2 \cdot B + K_2) \cdot (-p^2 \cdot C + K_3) \cdot (-p^2 \cdot G + K_7) = 0 \end{aligned}$$

The cubic equation in p^2 can also be written in parametric form:

$$p^6 \cdot \alpha + p^4 \cdot \beta + p^2 \cdot \gamma + \delta = 0$$

where:

$$\begin{cases} \alpha = -A \cdot D \cdot G + A \cdot E \cdot F + B \cdot C \cdot G \\ \beta = A \cdot D \cdot K_7 + A \cdot G \cdot K_4 + D \cdot G \cdot K_1 - \\ \quad - A \cdot E \cdot K_6 - A \cdot F \cdot K_5 - E \cdot F \cdot K_1 \\ \quad - B \cdot C \cdot K_7 - B \cdot G \cdot K_3 - C \cdot G \cdot K_2 \\ \gamma = -A \cdot K_4 \cdot K_7 - D \cdot K_1 \cdot K_7 - G \cdot K_1 \cdot K_4 + \\ \quad + A \cdot K_5 \cdot K_6 + E \cdot K_1 \cdot K_6 + F \cdot K_1 \cdot K_5 \\ \quad + B \cdot K_3 \cdot K_7 + C \cdot K_2 \cdot K_7 + G \cdot K_2 \cdot K_3 \\ \delta = K_1 \cdot K_4 \cdot K_7 - K_1 \cdot K_5 \cdot K_6 - K_2 \cdot K_3 \cdot K_7 \end{cases}$$

The positive solutions of the cubic equation in p^2 are the three eigenpulsations p_1 , p_2 and p_3 of the system and are obtained using the root command in MATHCAD.

The eigenmodes of vibration are given by the values of the distribution coefficients: μ_{21} and μ_{31} which are obtained from the first two equations of the homogeneous system:

- Mode 1 of vibration:

$$\begin{cases} \mu_{21} = \frac{a_{21}}{a_{11}} = -\frac{p_1^2 \cdot A - K_1}{p_1^2 \cdot B - K_2}; \\ \mu_{31} = \frac{a_{31}}{a_{11}} = -\mu_{21} \cdot \frac{p_1^2 \cdot D - K_4}{p_1^2 \cdot E - K_5} - \frac{p_1^2 \cdot C - K_3}{p_1^2 \cdot E - K_5} \end{cases}$$

- Mode 2 of vibration:

$$\begin{cases} \mu_{22} = \frac{a_{22}}{a_{12}} = -\frac{p_2^2 \cdot A - K_1}{p_2^2 \cdot B - K_2}; \\ \mu_{32} = \frac{a_{32}}{a_{12}} = -\mu_{22} \cdot \frac{p_2^2 \cdot D - K_4}{p_2^2 \cdot E - K_5} - \frac{p_2^2 \cdot C - K_3}{p_2^2 \cdot E - K_5} \end{cases};$$

- Mode 3 of vibration:

$$\begin{cases} \mu_{23} = \frac{a_{23}}{a_{13}} = -\frac{p_3^2 \cdot A - K_1}{p_3^2 \cdot B - K_2}; \\ \mu_{33} = \frac{a_{33}}{a_{13}} = -\mu_{23} \cdot \frac{p_3^2 \cdot D - K_4}{p_3^2 \cdot E - K_5} - \frac{p_3^2 \cdot C - K_3}{p_3^2 \cdot E - K_5} \end{cases}$$

The results of the numerical simulation in MATHCAD are obtained for three cases of wire stiffness values (k_1 , k_2 , ... k_6) having the values given in table 1.

Table 1

Caz	k1 N/mm	k2 N/mm	k3 N/mm	k4 N/mm	k5 N/mm	k6 N/mm
1	200	250	4000	4000	5000	5000
2	200	250	400	400	500	500
3	200	250	40	40	50	50

The numerical results obtained in MATHCAD for the pulsations and eigenmodes of vibration are given in Table 2

Table 2

Caz	p1 rad/s	p2 rad/s	p3 rad/s	μ_{21}	μ_{22}	μ_{23}
1	285	2038	2741	-180	0.55	-0.85
2	238.5	683.5	871.5	-22	0.52	-0.85
3	119.2	239	279	-7	0.48	-0.85

4. CONCLUSIONS

From the analysis of the results presented in tables 1 and 2 of figure 4, the following conclusions can be drawn:

- The fundamental natural pulsation obtained for the case of the wire with the highest stiffness ($p_1=285$ rad/s - case 1) is very close to the natural pulsation of the system with one degree of freedom ($p_1=292$ rad/s) then decreases as the wire stiffness has lower values (cases 2 and 3 in table 2)

- The upper pulsations corresponding to the 2nd and 3rd vibration modes have very high values in the case of the wire 10 times less stiffness ($p_2=2038$ rad/s; $p_3=2741$ rad/s - case 2), respectively in the case of the wire 100 times less stiffness ($p_2=683.5$ rad/s; $p_3=871.5$ rad/s - case 3)

- the distribution coefficients corresponding to the amplitudes q_2/q_1 have such very high values in the case of the rigid wire ($\mu_{21}=-180$ - case 1) which shows that the instantaneous center of rotation I1 is very close to the point B corresponding to the system with one degree of freedom.

- the distribution coefficients corresponding to the amplitudes q_2/q_1 have increasingly lower values in the case of the less rigid wire ($\mu_{21}=-180$ - case 1; $\mu_{21}=-22$ - case 2; $\mu_{21}=-7$ - case 3)

- the very high values of the distribution coefficients corresponding to case 1 show that the instantaneous center of rotation I1 in this case is very close to the point B corresponding to the system with one degree of freedom.

- The parametric method proposed in this work is suitable for numerical simulation using the professional calculation program MATHCAD and allows obtaining results only by modifying the values of the input parameters [5], [6].

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