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USING LINEAR PROGRAMMING WITH INTERVAL COEFFICIENTS IN OPTIMIZATION OF A MILITARY ACTION

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ABSTRACT

In linear programming models, uncertainty is associated with the model coefficients during the formulation stage. In real-life models, also in military field, it is more realistic to represent coefficients as numerical intervals than as exact real numbers. In this case we can use models of linear programming with interval coefficients (LPIC). In this paper we use LPIC to find the optimal solution of a military application.

KEYWORDS:
linear programming, interval coefficients, military optimization

1. Introduction

In military field is more realistic to use models of linear programming with interval coefficients method (LPIC) when we want to find the optimum solution.

In paper 1 we present the LPIC problem:

"Suppose we are given the following LPIC problem:

$$\min Z = \sum_{j=1}^{n} [\underline{c_j}, \overline{c_j}] x_j$$

$$\sum_{j=1}^{n} [\underline{a_{ij}}, \overline{a_{ij}}] x_j \ge [\underline{b_i}, \overline{b_i}], \quad i = 1, \dots, m \qquad (1)$$

$$x_j \ge 0 \ \forall j$$

The best optimum and worst optimum are computed as follows:

(I) For the best optimum:

$$\min \underline{z} = \sum_{j=1}^{n} c'_j x_j$$

$$\sum_{j=1}^{n} a'_{ij} x_j \ge \underline{b_i}, \ \forall i \qquad (2)$$

$$x_j \ge 0 \ \forall j$$

where $c'_j = \underline{c_j}$ and $a'_{ij} = \overline{a_{ij}}$

(II) For the worst optimum:

$$\min \overline{z} = \sum_{j=1}^{n} c''_j x_j$$

$$\sum_{j=1}^{n} a''_{ij} x_j \ge \overline{b_i}, \ \forall i \qquad (3)$$

$$x_j \ge 0 \ \forall j$$

where $c''_j = \overline{c_j}$ and $a''_{ij} = \underline{a_{ij}}$.

So the best optimum solution will be $\underline{z}$, at $(x'_1, x'_2, \dots, x'_n)$ and with coefficient settings c'_j, a'_{ij} and $\underline{b_i}$, $\forall i$ and j and also the worst optimum solution $\overline{z}$ with $(x''_1, x''_2, \dots, x''_n)$ and with coefficient settings c''_j, a''_{ij} $\forall i$ și j.

The general solution of LPIC will be $Z = [\underline{z}, \overline{z}]$ i.e. optimal value of the problem will be between $\underline{z}$ (the best case scenario) and $\overline{z}$ (the worst case scenario)."

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2. Optimal Distribution of Firepower on Objectives

In the military field, there are a multitude of situations that can be "optimized" using various mathematical models.

Problems of distribution of forces and means on objectives refers to: optimal implementation of defense against tanks, optimal implementation of defense against air attacks, optimal choice of combinations of means of combat against different objectives, distribution of resources on different types of weapons and others.

Inspired by Căruțașu (2014) who presented and found an optimal solution to a problem of distribution of firepower on objectives, we create a similar problem, but we used numerical intervals instead of exact real numbers:

The own unit has 5 combat means that use ammunition of the M_1, M_2, M_3, M_4 and M_5 types, and the enemy has 7 types of objectives O_1, O_2, O_3, O_4, O_5, O_6 and O_7 (meaning both equipment and personnel).

Based on the probabilities of hitting each of the 7 enemy targets, the average ammunition consumption of each type necessary to destroy them was established, because of the uncertainty the values are written as intervals (the data is given in the table below).

Each destroyed objective causes the enemy certain percentage losses given in the table below, also these values are intervals because we can't be 100% sure.

Also in the table are given the quantities of ammunition of each type that the own unit has at its disposal as well as the number of objectives of each type that the enemy has.

The problem is to determine the number of enemy objectives of each type that must be destroyed so that the losses caused to the enemy are maximum, taking into account the quantities of ammunition of each type that the own unit has at its disposal.

Table no. 1

Problem data

	O_1	O_2	O_3	O_4	O_5	O_6	O_7	
M_1	[10,12]	[8,10]	[6,8]	[7,9]	[9,11]	[7,9]	[8,10]	[690,720]
M_2	[3,5]	[2,4]	[4,6]	[3,5]	[4,6]	[5,7]	[5,7]	[360,380]
M_3	[6,8]	[3,6]	[5,7]	[8,10]	[7,9]	[4,6]	[7,9]	[520,540]
M_4	[1,3]	[1,2]	[3,5]	[2,4]	[1,2]	[2,4]	[4,6]	[200,220]
M_5	[0,2]	[0,1]	[1,2]	[1,3]	[0,1]	[1,2]	[3,5]	[80,100]
Loss	[0.5%, 1%]	[0.25%, 0.5%]	[1.25%, 1.5%]	[1.75%, 2.5%]	[0.5%, 0.75%]	[0.75%, 1.25%]	[2.5%, 3%]	-
No.objectives	12	15	8	7	13	6	7	-

Let x_i be number of O_i type objectives selected for destruction, $i = \overline{1,7}$.

The function to be maximized has the form:

$$\text{Max } Z=[0.5,1]x_1+[0.25,0.5]x_2+[1.25,1.5]x_3+[1.75,2.5]x_4+[0.5,0.75]x_5+[0.75,1.25]x_6+[2.5,3]x_7$$

For the constrains:

➢ no more ammunition can be used than the one the unit has at its disposal

$$[10,12]x_1+[8,10]x_2+[6,8]x_3+[7,9]x_4+[9,11]x_5+[7,9]x_6+[8,10]x_7 \leq [690,720]$$

$$[3,5]x_1+[2,4]x_2+[4,6]x_3+[3,5]x_4+[4,6]x_5+[5,7]x_6+[5,7]x_7 \leq [360,380]$$

$$[6,8]x_1+[3,6]x_2+[5,7]x_3+[8,10]x_4+[7,9]x_5+[4,6]x_6+[7,9]x_7 \leq [520,540]$$

$$[1,3]x_1+[1,2]x_2+[3,5]x_3+[2,4]x_4+[1,2]x_5+[2,4]x_6+[4,6]x_7 \leq [200,220]$$

$$[0,2]x_1+[0,1]x_2+[1,2]x_3+[1,3]x_4+[0,1]x_5+[1,2]x_6+[3,5]x_7 \leq [80,100]$$

➢ no more objectives of each type can be destroyed than those the enemy has:

$$x_1 \leq 12$$
$$x_2 \leq 15$$

$x_3 \leq 8$
$x_4 \leq 7$
$x_5 \leq 13$
$x_6 \leq 6$
$x_7 \leq 7$

Sign restrictions

-are the sign conditions imposed on decision variables:

$x_1 \geq 0$
$x_2 \geq 0$
$x_3 \geq 0$
$x_4 \geq 0$
$x_5 \geq 0$
$x_6 \geq 0$
$x_7 \geq 0$

We will convert the problem in to a minimization problem and change all "≤" inequalities into "≥" inequalities.

The new model is:

Min(-Z)=[-1,-0.5]x_1+[-0.5,-0.25]x_2+ [-1.5,-1.25]x_3+[-2.5,-1.75]x_4+ [-0.75,-0.5]x_5+[-1.25,-0.75]x_6+[-3,-2.5]x_7

[-12,-10]x_1+[-10,-8]x_2+[-8,-6]x_3+ [-9,-7]x_4+[-11,-9]x_5+[-9,-7]x_6+ [-10,-8]x_7 ≥[-720, -690]

[-5,-3]x_1+[-4,-2]x_2+[-6,-4]x_3+[-5,-3]x_4+ [-6,-4]x_5+[-7,-5]x_6+[-7,-5]x_7 ≥ [-380, 360]

[-8,-6]x_1+[-6,-3]x_2+[-7,-5]x_3+ [-10,-8]x_4+[-9,-7]x_5+[-6,-4]x_6+ [-9,-7]x_7 ≥ [-540, -520]

[-3,-1]x_1+[-2,-1]x_2+[-5,-3]x_3+ [-4,-2]x_4+[-2,-1]x_5+[-4,-2]x_6+[-6,-4]x_7 ≥ [-220, -200]

[-2,0]x_1+[-1,0]x_2+[-2,-1]x_3+[-3,-1]x_4+ [-1,0]x_5+[-2,-1]x_6+[-5,-3]x_7 ≥ [-100, -80]

$x_1 \leq 12$
$x_2 \leq 15$
$x_3 \leq 8$
$x_4 \leq 7$
$x_5 \leq 13$
$x_6 \leq 6$
$x_7 \leq 7$
$x_1 \geq 0$
$x_2 \geq 0$
$x_3 \geq 0$
$x_4 \geq 0$
$x_5 \geq 0$
$x_6 \geq 0$
$x_7 \geq 0$

We obtain:

- The best optimum:

min $-\underline{z}$ =$-x_1$-0.5x_2-1.5x_3-2.5x_4-0.75x_5-1.25x_6-3x_7

-10x_1-8x_2-6x_3-7x_4-9x_5-7x_6-8x_7 ≥-720
-3x_1-2x_2-4x_3-3x_4-4x_5-5x_6-5x_7 ≥ -380
-6x_1-3x_2-5x_3-8x_4-7x_5-4x_6-7x_7 ≥ -540
$-x_1$-x_2-3x_3-2x_4-x_5-2x_6-4x_7 ≥ -220
$-x_3$-x_4-x_6-3x_7 ≥ -100

$x_1 \leq 12$
$x_2 \leq 15$
$x_3 \leq 8$
$x_4 \leq 7$
$x_5 \leq 13$
$x_6 \leq 6$
$x_7 \leq 7$
$x_1 \geq 0$
$x_2 \geq 0$
$x_3 \geq 0$
$x_4 \geq 0$
$x_5 \geq 0$
$x_6 \geq 0$
$x_7 \geq 0$

- The worst optimum:

min $-\overline{z}$ =-0.5x_1-0.25x_2-1.25x_3-1.75x_4-0.5x_5-0.75x_6-2.5x_7

-12x_1-10x_2-8x_3-9x_4-11x_5-9x_6-10x_7 ≥-690
-5x_1-4x_2-6x_3-5x_4-6x_5-7x_6-7x_7 ≥ -360
-8x_1-6x_2-7x_3-10x_4-9x_5-6x_6-9x_7 ≥ -520
-3x_1-2x_2-5x_3-4x_4-2x_5-4x_6-6x_7 ≥ -200
-2x_1-x_2-2x_3-3x_4-1x_5-2x_6-5x_7 ≥ -80

$x_1 \leq 12$
$x_2 \leq 15$
$x_3 \leq 8$
$x_4 \leq 7$
$x_5 \leq 13$
$x_6 \leq 6$
$x_7 \leq 7$
$x_1 \geq 0$
$x_2 \geq 0$
$x_3 \geq 0$
$x_4 \geq 0$

$x_5 \geq 0$
$x_6 \geq 0$
$x_7 \geq 0$

We use LINDO (a linear programming solver) to find the both optimums:

```
File  Edit  Solver  Window  Help

Lindo Model - Lingo1
min -x1-0.5x2-1.5x3-2.5x4-0.75x5-1.25x6-3x7
subject to
    -10x1-8x2-6x3-7x4-9x5-7x6-8x7 >=-720
    -3x1-2x2-4x3-3x4-4x5-5x6-5x7 >=-380
    -6x1-3x2-5x3-8x4-7x5-4x6-7x7 >=-540
    -x1-x2-3x3-2x4-x5-2x6-4x7 >=-220
    -x3-x4-x6-3x7 >=-100
    x1<=12
    x2<=15
    x3<=8
    x4<=7
    x5<=13
    x6<=6
    x7<=7
    x1>=0
    x2>=0
    x3>=0
    x4>=0
    x5>=0
    x6>=0
    x7>=0
end
```

Figure no. 1: *LINDO best optimum*
(Source: Author's compilation)

```
Global optimal solution found.
Objective value:                              -87.25000
Infeasibilities:                               0.000000
Total solver iterations:                              0
Elapsed runtime seconds:                           7.25

Model Class:                                         LP

Total variables:                      7
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                   20
Nonlinear constraints:                0

Total nonzeros:                      53
Nonlinear nonzeros:                   0


                              Variable           Value        Reduced Cost
                                    X1        12.00000            0.000000
                                    X2        15.00000            0.000000
                                    X3        8.000000            0.000000
                                    X4        7.000000            0.000000
                                    X5        13.00000            0.000000
                                    X6        6.000000            0.000000
                                    X7        7.000000            0.000000
```

Figure no. 2: *LINDO solution for best optimum*
(Source: Author's compilation)

```
File  Edit  Solver  Window  Help

Lindo Model - Lingo1
min -0.5x1-0.25x2-1.25x3-1.75x4-0.5x5-0.75x6-2.5x7
subject to
    -12x1-10x2-8x3-9x4-11x5-9x6-10x7 >=-690
    -5x1-4x2-6x3-5x4-6x5-7x6-7x7 >=-360
    -8x1-6x2-7x3-10x4-9x5-6x6-9x7 >=-520
    -3x1-2x2-5x3-4x4-2x5-4x6-6x7 >=-200
    -2x1-x2-2x3-3x4-x5-2x6-5x7 >=-80
    x1<=12
    x2<=15
    x3<=8
    x4<=7
    x5<=13
    x6<=6
    x7<=7
    x1>=0
    x2>=0
    x3>=0
    x4>=0
    x5>=0
    x6>=0
    x7>=0
end
```

Figure no. 3: *LINDO worst optimum*
(Source: Author's compilation)

```
Global optimal solution found.
Objective value:                              -43.75000
Infeasibilities:                               0.000000
Total solver iterations:                              1
Elapsed runtime seconds:                           0.03

Model Class:                                         LP

Total variables:                     7
Nonlinear variables:                 0
Integer variables:                   0

Total constraints:                  20
Nonlinear constraints:               0

Total nonzeros:                     56
Nonlinear nonzeros:                  0


                                Variable           Value        Reduced Cost
                                      X1        0.000000           0.5000000
                                      X2        0.000000           0.2500000
                                      X3        8.000000            0.000000
                                      X4        7.000000            0.000000
                                      X5        13.00000            0.000000
                                      X6        0.000000           0.2500000
                                      X7        6.000000            0.000000
```

Figure no. 4: *LINDO solution for worst optimum*
(Source: Author's compilation)

So the best optimum is $-\underline{z}=-87.25$ (Figure no. 2) for the minimal problem, so for our maximum problem $\underline{z}=87.25$, and also in Figure no. 2 we have the variables. Similar the worst optimum is $\overline{z}=43.75$ for the maximum problem with the fixed variables used to get this solution.

So we can conclude that optimum solution Z of this problem lies between 43.75 and 87.25, so the enemy's losses will be between 43.75% and 87.25%.

3. Conclusions

In recent decades, mathematical modeling of combat operations has developed as an indispensable tool in decision-making at all levels. These models provide the possibility of analyzing different variants of the use of forces and combat means, as well as their dynamics during the conduct of military operations.

Many operational research optimization models, such as linear programming, assume that the data used to build a model is accurate, but in the real world most of these assumptions are only approximately true. So it is more realistic to use models of linear programming with interval coefficients (LPIC).

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