

PHYSICAL PRINCIPLES IN REVEALING THE WORKING MECHANISMS OF BRAIN. PART V

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Abstract. Still after the right concept of universe, we extract from the Einstein-de Sitter debate presented previously the physical characteristics of a continuum charge background to serve for the description of brain. This time we settle for the cosmological metric of the universe, which appear to be well suited for the brain, and the content in events of a generic universe serving for the definition of events in the brain. The hologram is also defined as a general characteristic of a universe, in order to serve in describing the process of memory. The idea of coherence is used, in order to define the frequency in an invariant way.

Keywords: the metric of charge background, Hebbian assembly, the holographic memory, Kepler's second law, the concept of phase, the concept of coherence, frequency by coherence.

1. Insights on the Theory of Charge

It is symptomatic in matters of physics, that in the Einstein's method of projection (see V, §2.3) the four-dimensional Euclidean manifold was always considered just ancillary. A typical methodology [1] sees it as just a method of embedding the space in the Robertson-Walker cosmological spacetime. Considering the interpretation, this methodology can even go as far as considering the ancillary a way of describing the matter content of such a universe [2]. Our point is that such ancillary is just a particular case: in fact the Einstein's method means embedding of the world in a continuum of charge. With the present part of the work we just start exposing some consequences of this point of view.

A provisional framework of interpretation is the following:

(1) the static properties generating the Newtonian forces in matter – gravitational, electric, magnetic, inertial, etc. – necessary in the representation of the poles of a static structure, are assigned to Hertz's material particles (see III, §2.3). These are incidentally associated into dipoles, thus providing the optical properties of the matter of universe: a Maxwell fish-eye.

(2) the interpretation proper in the case of brain universe is provided by particles of arbitrary charge, tentatively satisfying the Louis de Broglie's duality principle. They are moving along the geodesics of the Maxwell fish-eye metric at a scale of time that appears as instantaneous in terms of the time of our experience.

(3) the (sub)structure thus generated is an *instanton* [3b].

Let us get into some mathematical details of this construction.

1.1. The Doctrine of Charge vs. the Doctrine of Field

Notice that the Einsteinian cosmological doctrine involves a certain duality in the consideration of the metric (2.3.1) of IV: taken as such it may count as geometrically abstract, just as well as phenomenologically necessary (according to Katz's charge phenomenology). The intervention of general relativistic precepts, regards just the transformation of the continuum characterized by this quadratic form, into an optical medium supporting the 'light phenomenon' of the universe it describes. In this respect, the Einstein equations can also have a conceptual connotation regarding the idea of a universe: they select the proper solutions of the concept to be targeted.

These solutions, for the historical case of Einstein-de Sitter debate, are specifically addressed to the *vacuum field equations* as conceived by Einstein, both in his initial general-relativistic theory [4b], and in the *Cosmological Considerations* [5] where he included the celebrated cosmological

constant. As we have already mentioned in V, §2.3, Einstein himself started regretting the introduction of this cosmological constant, and with good reasons at that. Fact is that he has been trying, subsequently, to eliminate that cosmological term from the curriculum of general relativity. In one of his almost immediate attempts to dispense with the cosmological constant [6], which appeared to him as “gravely detrimental to the formal beauty of the theory”, Einstein suggested [*loc. cit.* §2, equation (1a)] new field equations, describing the gravitation along his initial ideas on general relativity, having the form

$$G_{\mu\nu} - \frac{1}{4} g_{\mu\nu} G = -\kappa T_{\mu\nu} \quad (1.1.1)$$

These, he thought, may be able to ‘free us from the necessity of introducing a special constant λ ’. Here T is the energy tensor which, if determined exclusively by electromagnetic fields, satisfies the identity $\text{tr}(\mathbf{g}^{-1} \cdot \mathbf{T}) = 0$. This identity is also naturally satisfied by the tensor from the left hand side of equation (1.1.1), so that this equation can have a touch of rational basis, just like the original field equations introduced by Einstein. He noticed, nevertheless, that the equations from (1.1.1) are, indeed, a consequence of the original field equations with the tensor T determined exclusively by electromagnetic fields, “but not conversely”.

In carrying the analysis further, Einstein reached what we cogitate as an ‘ultimate conclusion’, namely that one has to follow the ideas of Gustav Mie, along the lines indicated by David Hilbert in 1915, in describing the matter formations of the universe. Parenthetically: we have followed all these works in their English translations, printed in [7]. The essential work of Gustav Mie is only reproduced in fragments there, but one can also consider the (still partial) translation of the Mie’s theory of matter due to D. H. Delphenich, in order to fill in for the missing parts of the translation of Renn and Schemmel, or *vice versa*. We are interested to see why Einstein *wanted to avoid*, though, “the lines of Mie’s theory”, and to this end let us describe this theory just in its broad strokes.

The fundamental one, from among the points raised by Mie in his theory, is the existence of a space-extended electron having an *electromagnetic internal structure* in the sense of Maxwell. Mie even reached the idea that the inertial and gravitational masses of such an electron *are different*, which, however, could not grow into a proper fruition, due to the eternal problem of physics: *the interpretation as a concept, always deals with figments of our imagination, not with real physical structures*. Fact is, that on this occasion Einstein found an opportunity of (re)appraise his own theory of general relativity, with the declared prospect of getting rid of the cosmological constant. After a careful analysis of his original field equations of gravitation [4b], he concludes:

... if we wish to contemplate *the possibility that gravitation may take part in the structure of the fields which constitute the corpuscles*, we cannot regard equation (1) [*equation originally proposed by Einstein in his 1916 theory of general relativity, a/n*] as confirmed ([6]; *emphasis added, a/n*).

Let us say it once again: ‘*the fields which constitute the corpuscles*’, if it is to conclude on a corpuscle structure by the case of an electron, are to be understood as electromagnetic fields described, according to Gustav Mie, by the Maxwell equations. So, Einstein has modified the field equations as shown in (1.1.1) along the lines of his proposed cosmological equations [5], and carried the analysis accordingly. The conclusion of this analysis is altogether encouraging:

The scalar of curvature (*the trace of tensor $\mathbf{G}$, denoted here G , a/n*) plays the part of a negative pressure which, outside of the electric corpuscles, has the constant value G_0 . In the interior of every corpuscle there subsists a negative pressure (positive $G - G_0$), the fall (*gradient, a/n*) of which maintains the electrodynamic force in equilibrium. The *minimum of pressure*, or, respectively, the maximum of the scalar of curvature, *does not change with time in the interior of the corpuscle* ([6]; *emphasis added, a/n*).

Obviously, admitting the equations (1.1.1), – which is essentially the equation cosmologically modified by Einstein, only with a special value of the cosmological constant included in it – Mie’s ideas can be accommodated *without further hypotheses*: the ‘lines of Mie’s theory’ can, therefore, *be avoided altogether*, in favor of a proper theory of general relativity. The internal pressure might be of electromagnetic nature after all, but there is a catch: “the *problem of the constitution of elementary quanta* cannot yet be solved on the immediate basis of the given field equations”. Why?

The ‘elementary quanta’ here are *quanta of electricity*: they necessitate Planck’s quantization, indeed, *but within the matter, not in the light* like the original one. Besides, there is no restrictive condition for the charge of a sphere in equilibrium, in order to be able to introduce a quantization condition for charge, the way this condition is currently understood in physics. Incidentally, notice that there cannot be such conditions: the very static Newtonian force, which the relativity tries to avoid, is the essential condition of this kind of quantization. Interestingly enough, Gustav Mie himself touched this issue and, more importantly for what we have to say here, he did that from the very point of view of the constitutive element of matter used by Planck himself: *the dipole* (see Chapter Four in the Delphenich’s translation of the Mie’s theory). However, he reached

the conclusion of the today's physiology of the brain: such a dipole should be a *substantially different electromagnetic structure* from what we know as dipole of charges – the case considered by Planck himself – and may even have the properties of a magnet, for instance. So, again in line with the physics of the epoch, that is, by having a physical structure in view, Gustav Mie concludes:

It is extremely difficult to establish more exact conditions for *the possibility of existence of such elementary dipoles*, but it should be of little use in view of *our total ignorance of the nature of the world function*. For once, I will assume *that there are such elementary dipoles*, and I will draw some consequences from this assumption. ([8], *our translation from German; emphasis added, a/n*).

Mie's world function is an extension of the energy in the form of a Hamiltonian, involving, therefore the geometry of the background in its calculation, *i.e.* calculated *via* the metric of this background. This metric is, in the views of the general relativity, always assumed, algebraically speaking, *a quadratic form*, and here Edward Kasner has entered the stage on the case presented by this issue, with a conclusion worth considering.

1.2. The Admissible Metric of a Universe

The notable undertake of Kasner is referring to the possible *quadratic* metrics for the *vacuum field* equations corresponding to original Einstein field equations and those from (1.1.1), that is:

$$G_{\mu\nu} = 0 \quad \text{and} \quad G_{\mu\nu} - \frac{1}{4} g_{\mu\nu} G = 0 \quad (1.2.2)$$

The results of Kasner regard the metrics allowed by these equations *assuming* that they are a sum of quadratic forms [9]. Notice, again, that the first equation here implies the second one, but not *vice versa*. The essential note of Kasner's intervention, is that equations (1.2.2) are exclusively referring to the four-dimensional case, not at one or another of its Einsteinian projections or embeddings. One may say that the idea is implicit to Kasner, that the abstract four-dimensional metrics postulated by Einstein and de Sitter are justifiable as solutions of the equations (1.2.2).

The first significant one, from among his results along this line [10a] is about the field allowed by *the first* of the equations (1.2.2). It states that the equation

$$(ds)^2 \equiv (dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (dx_4)^2 = 0 \quad (1.2.3)$$

cannot take place if there is a *permanent gravitational field* in the sense of Einstein: the first of equations (1.2.2) does not provide solutions for this case. Here x_1, x_2, x_3, x_4 are the coordinates in the Einsteinian four-manifold. Kasner translates this result into a kind of incompatibility between light and gravitation, of the nature of that which led to the rejection of the Lorentz metric in the cosmological case. It is significant, in our context, that the ‘permanent gravitational field’ is represented here by a conformally flat metric of the form

$$(ds)^2 \equiv F(x_1, x_2, x_3, x_4)[(dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (dx_4)^2] \quad (1.2.4)$$

This represents, from the point of view of the scale transition, the condition that the gravitational field is the same for *any* value of the infrafinite distance representing the propagation of light. Interestingly enough the Einstein’s equations would tell us that the ‘permanent gravitation field’ thus conceived is perfectly determined at a finite scale. Indeed, Kasner has found that the solution of Einstein’s field equations $G_{\alpha\beta}=0$ is given, in this case, by a nonhomogeneous quadratic form (1.2.4), where F is given by:

$$F = \{a_0[(x_1)^2 + (x_2)^2 + (x_3)^2 + (x_4)^2] + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5\}^{-2} \quad (1.2.5)$$

provided the center of this quadric is located such that:

$$(a_1)^2 + (a_2)^2 + (a_3)^2 + (a_4)^2 - 4a_0a_5 = 0 \quad (1.2.6)$$

In this case, there is, possibly, an admissible transformation of coordinates that reduces (1.2.4) to a sum of squares, therefore to a set of coordinates where the metric tensor is a *constant* diagonal matrix. Therefore, in such a case, the permanent gravitational field is inexistent, indeed, and the metric is flat. Another immediate result of Kasner, shows that the Einstein’s original gravitational field cannot be *embedded in a five-dimensional Euclidean flat* [10b]. In other words, one cannot use Einstein’s procedure of embedding for it. This may count as a solid reason for introducing the cosmological term after all.

Indeed, this does not mean that the de Sitter’s philosophy does not work for embedding, since it *is not referring* to the first of the equations (1.2.2), but to the second. Thus, on a positive note [10c], the field characterized *by the second equation* from (1.2.2), is embeddable in a six-flat, given, in a geometrical form, by *two* unit spheres. It is this result that, in our opinion, is of real help in a theory of brain as a universe: it liberates us from the Planck’s proviso consisting from description of

the dipole components as vibrating along the axis of dipole. Indeed, this liberation gives us the opportunity “to assume the vibrations *as taking place in space instead of in a straight line only*”, as Planck himself would say (see [11], Chapter 1). Let us elaborate a little on this issue.

The necessity of such a liberation comes just naturally, according to our developments thus far in the present work, from the idea of a general optical medium having the characteristics of a Maxwell fish-eye: if the material particles are moving along the geodesics of such a medium, these particles, as well as the ‘elementary quanta’ of Einstein, for that matter, must have *afortiori* three degrees of freedom. The dipole structure should then be an Ampère current element (see [12], Chapter 5, especially §5.1) confined along the ‘axis’ between two portions of surfaces, that must have a negative curvature if the charges are involved in the physical structure of the universe containing it. This structure can then be geometrically explained by the infinitesimal deformation of surfaces, a phenomenon induced through the presence of charges in the medium: the charges at the ends of Ampère element get an extra displacement as a result of the infinitesimal deformation. This phenomenon, considered from the perspective of the concept of instanton, can offer us the possibility to explain the classical structure of a planetary atom, as follows [3b].

Assume, indeed, the following, more realistic image – at least we see it that way! – of a planetary atom: a spatially extended spherical nucleus of positive charge, and a spatially extended spherical electron of negative charge. A kinematic image can be assumed for the uniform charge of each sphere, based on the concept of instanton: *each one of these spheres is an instanton*. This means that the charges, considered attached to the interpretative particles, move so fast inside spheres, that these regions appear, *in each and every instant of time reckoned at the time scale of the Kepler motion of electron around nucleus*, as uniformly charged with positive and, respectively, negative charges. In other words, we have a static ensemble of interpretative material particles in each one of the two regions, but the charge attached to particles moves so fast on the space at the disposal of this ensemble, that this space appears as uniformly charged at the time scale of the Kepler motion. In a notorious Schrödinger phrasing, the charge is ‘simultaneously in all kinematically possible states, however not equally likely in all of them’. The ‘likelihood’ of the presence of the particle inside the sphere is measured here by a special probability density, as we shall show later. The physics resides upon what we would like to call the *Feynman’s interpretative concept*. Quoting, indeed:

The wave function $\psi(\mathbf{r})$ for an electron in an atom, does not describe a smeared-out electron with a smooth charge density. *The electron is either here, or there, or somewhere else, but wherever it is, it is a point charge*. On the other hand,

think of a situation in which there are an enormous number of particles in exactly the same state, a very large number of them with exactly the same wave function. Then what? One of them is here and one of them is there, and the probability of finding one of them at a given place is proportional to $\psi\psi^*$. But since there are so many particles, if I look in any volume $dx dy dz$ I will generally find a number close to $\psi\psi^* dx dy dz$. So in a situation in which ψ is the wave function for each of an enormous number of particles which are all in the same state, $\psi\psi^*$ can be interpreted as the density of particles. If, under these circumstances, each particle carries the same charge q , we can, in fact, go further and interpret $\psi^* \psi$ as the density of *electricity* (original emphasis here, a/n). Normally, $\psi\psi^*$ is given the dimensions of a probability density, then ψ should be multiplied by q to give the dimensions of a charge density. For our present purposes we can put this constant factor into ψ , and take $\psi\psi^*$ itself as the electric charge density. With this understanding, $\mathbf{J}$ (the current of density of probability I have calculated) becomes directly the electric current density.

So in the situation in which we can have very many particles in exactly the same state, there is possible a new physical interpretation of the wave functions. The charge density and the electric current can be calculated directly from the wave functions and the wave functions take on a physical meaning which extends into classical, macroscopic situations ([13], Volume III, §21–4; our emphasis, except as mentioned, n/a).

We reproduced this lengthy passage first of all in order to get the gist of a classical interpretation: a kinematics is needed in order to describe the particle as being ‘here, or there, or somewhere else’. The prototype of it was, indeed, constructed by Richard Feynman himself in his celebrated approach of the wave mechanics [14, 15]. However, the passage above turns out to be useful in suggesting that the Schrödinger’s interpretation of the wave function (see I, §2.3) is by no means in disagreement with the general idea of interpretation involving the concept of material particle. The wave function of Schrödinger even gains a bonus of clarity here, regarding its meaning: it represents *that instantaneous density of the charge*, which, obviously, is different from a Newtonian density, and was described in physics of the last century as a *probability density*. Thus, in order to update the Feynman’s interpretation, we can say that ‘the interpretative particles themselves are, at any instant, everywhere inside our spheres’, with a probability described by a density given through

the wave function. However, this wave function is here suggested as a construction *upon waves exclusively propagated with speeds superior to a certain experimental limit*, for instance the speed of light, as in the case of de Broglie's wave groups. Putting this issue aside for a later deep analysis, the fact remains that the two instantons can be described as continuous static charges represented as one continuous manifold of dipoles. Each one of these dipoles is *instantaneously* established – with 'instantaneity' defined in terms of time scale of the Kepler motion – by the connection of a material particle from one region with a material particle from the other region. One can say that both the electron and the nucleus of the planetary model are two Maxwell fish-eye mediums instantaneously connected by a Lorentz transformation. The result is an Ampère current element, which can be imagined as a continuous congruence of straight lines, representing instantaneous dipoles of an arbitrary physical nature: electric, magnetic or mixed, depending on what kind of charges are connected within an instant.

This is a situation geometrically described by Edward Kasner, and according to this description, it can be associated only with the Einstein's second set of equations (1.2.2). More to the point, Kasner's results can be summarized as follows ([10c]; see also [9]): if the metric form is necessarily a quadratic differential in four variables, then the Einstein equations proper [the first one of the equations (1.2.2)] allow for only an Euclidean or a conformal-Euclidean metric. On the other hand, the Einstein's equations with a cosmological term [the second one of the equations (1.2.2)] allow only for a solution which is either Euclidean or conform-Euclidean (the de Sitter's case), or *a sum of two surface metrics of constant negative curvature*. This last case can be represented geometrically as an Einstein embedding of a four-dimensional manifold into a six-flat represented by the union of two unit spheres. As we just illustrated right above, this is the case of classical planetary atom with spatially extended components.

In equations, this important result can be expressed as follows: if on this Einsteinian manifold, the second of the equations (1.2.2) is valid, and the metric is necessarily a quadratic form, then we must have, up to a constant:

$$(ds)^2 = \frac{(dx_1)^2 + (dx_2)^2}{(x_2)^2} + \frac{(dx_3)^2 + (dx_4)^2}{(x_4)^2} \quad (1.2.7)$$

This represents the universe as two 'blades' given by surfaces of negative curvature, each of which represents an instanton (see §2.3 of IV). The metric (1.2.7) can be considered as the cosmological metric of an Einsteinian four-manifold embedded in a six-dimensional flat described by the equations:

$$(X_1)^2 + (X_2)^2 + (X_3)^2 = 1, \quad (X_4)^2 + (X_5)^2 + (X_6)^2 = 1 \quad (1.2.8)$$

in coordinates $(X_1, X_2, X_3, X_4, X_5, X_6)$. Indeed, each one of the two terms from equation (1.2.7) can be considered as the Cayley-Klein metric of the geometry having as absolute a regular unit sphere (see §2.3 of IV). In this case, physics must consider undertaking the task of describing the connection of the geometries of the two ‘blades’, by a mathematically proper procedure, compatible with the two Beltrami-Poincaré metrics entering the formula (1.2.7). This is what we like to call the mathematical physics of Kasner’s geometry of the universe, and it constitutes just another point in our program of development of the theory of brain along the present work. On a final note concluding this section, we need to mention this overall result of Kasner: the only fields that satisfy the second set of equations (1.2.2), in a universe with quadratic metric, correspond to metrics of *constant Riemannian curvature* [10c]. And, from now on we dedicate the physics exclusively to the problems of brain.

2. The Events of a Brain Universe

In deciding on the identity of the two fundamental concepts of the universe in the case of brain – which, by analogy with the known physical universe of our experience, we may call *matter* and *light* – we need to notice that the fundamental physical property of the brain matter is the charge: the brain produces charges. This is how we take the physiological *event of firing*: judging by its consequences, such an event can be described as the *instantaneous production of charge*. According to the facts of our experience, this event can be cogitated as a kind of *reverse lightning*, so to speak, since, according to our experience, we can take the lightning as an annihilation of an already existing charge. The problem is: to what extent can this phenomenon be framed in an *interpretation*? It is the answer to this question that brings in the *concept of holography* in the first place, and the idea of a hologram to represent the memory.

2.1. A Mathematical Description of the Concept of Memory

In view of the results presented before (see IV, §2.1), one can say that the equation of motion, as a physical concept, is ‘common’, in fact, for the free particle and harmonic oscillator, since in the two cases *the equations of motion only differ by an Arnold transformation*. By this theorem, the charge is interpretable as a free particle migrating from a pole to another, in an instantaneous Maxwell fish-eye structure. If we apply this ‘Arnold scenario’ as it were, to a *genuine free particle* itself – i.e. to a particle whose motion is described by an equation linear in ‘time’, as in equation (2.1.14) of IV

–then we find that the Arnold transformation enjoys the properties of a special group action in two variables of a group having the $SL(2, \mathbb{R})$ type structure (see I, §2.4, equation (2.20) *ff*; also [12], Chapter 4, §4.2):

$$t = \frac{\alpha\tau + \beta}{\gamma\tau + \delta}, \quad x = \frac{\xi}{\gamma\tau + \delta} \quad (2.1.1)$$

Here (x, t) suggest the ideas of the coordinate and the time of the original uniform motion of the particle. This action has, indeed, a group structure, which gives us a *possibility of interpretation* by an invariant function which we found previously (see I, §2.4):

$$\frac{\xi^2}{a\tau^2 + 2b\tau + c} \quad (2.1.2)$$

This suggests that the group (2.1.1) is referring to some kind of ‘radial motion’, as it were, of the free particles, whereby ξ plays the part of a radial coordinate, measuring, in fact, a standard deviation on their ensemble.

However, we still do not find any trace of the *concept of frequency* in this approach of the description of these free particles serving for the interpretation of the harmonic oscillators. If we really need to reach this concept here, we must also consider the case of a genuine oscillator, whereby the frequency comes with a second order differential equation [see IV, equation (2.1.12)] involving, apparently, the elastic and inertial properties of the continuum to be interpreted. This idea was instated in physics even from the times when Augustin Fresnel added the *diffraction phenomenon* to the phenomenology of light, which classically – that is, as inherited from Newton and Hooke – included just the phenomena of *reflection* and *refraction*. One can say that, by ‘updating’ the classical phenomenology of light this way [16-18], Fresnel simply gave a chance of interpretation to the old Hooke’s ideas on the description of light phenomenon in material media ([19], pp. 53 – 69). And when addressing the issue along this line of thought, a thing becomes obvious, indicating the insufficiency of the diffraction phenomenon in completing the phenomenology of light: the essential concept in describing the light phenomenon is that of a *phase*, which is just as essential in the case of charges as it is in the case of light, and the only one, in fact, to transcend the different concepts of a Wien-Lummer enclosure realizing a universe. Then the frequency comes out of this game only as an invariant, specifically related to an $SL(2, \mathbb{R})$ type action of the group of 2×2 matrices with real entries $(\alpha, \beta, \gamma, \delta)$, as indicated in equation (2.1.1) for the time and ‘radial motion’.

Now, we have to settle for the meaning of a hologram in the brain and, in view of the phenomenological developments revealed by physiology, we are entitled to assume, for the moment, that the best approach is by using the previous observations in associating a time dependence to the *concept of phase* (see I, §§2.3 to 3.1). For this we shall use the idea of a *neuronal assembly*, also called sometimes *Hebbian assembly*, since it was introduced to the science of physiology by Donald Olding Hebb, by the middle of the 20th century (see the newest edition of this work, [20]). This idea underwent substantial modifications ever since it was proposed in 1949: one can say that the concept grew up with the science of physiology itself [21]. Nowadays it acquired notoriety as a kind of interesting catch-phrase that says all about the concept [22]: “*neurons wire together if they fire together*”. The neurons that ‘fire together’ will be ‘wired together’ in an ensemble serving for the deposit of the memory. We shall try to explain the meaning of this phrase, representing, in our opinion, the quintessential experience of humans, actually: the memory is always built by a repetitive use of the neurons. In this case, the repetitive use simply establishes a Hebbian assembly serving for depositing the memories in the form of holograms.

To start with, in a mathematical reasoning, recall that a typical *complex signal* corresponding to a charge as in equation (2.1.10) of IV, *a priori* satisfying the Schrödinger equation (see I, §2.3) in *any position* in the brain space must be of the form:

$$q(t) = A(t)e^{i\phi(t)} \quad (2.1.3)$$

Here, the *amplitude* $A(t)$ and the *phase* $\phi(t)$ are to be considered as real random variables. These two characteristics may be supposed mathematically continuous, in order to make our argument easier. We shall try to associate a frequency to such a complex signal, and this amounts to an association of $q(t)$ with a solution of the second order differential equation *dynamically* describing the harmonic oscillator of a given frequency, ω_0 say. So, we must establish an equivalence, which, in real terms, comes down to a system of two equations for amplitude and phase:

$$\ddot{q} + \omega_0^2 q = 0 \quad \therefore \quad \begin{cases} \frac{\ddot{A}}{A} + \omega_0^2 = \dot{\phi}^2 \\ 2\frac{\dot{A}}{A} + \frac{\ddot{\phi}}{\dot{\phi}} = 0 \end{cases} \quad (2.1.4)$$

The second one of the equations from the right hand side here, gives right away:

$$A^2 \cdot \dot{\phi} = \text{constant} \quad (2.1.5)$$

which is a kind of Kepler's second law, usually known as the *area law*. This is an incident susceptible to suggest a periodic motion – obviously represented just by its equation of motion, having nothing to do with a trajectory, as a second differentia of the concept – for the amplitude itself, when compared to the details of the classical Kepler problem. Of course, here this means that we shall need a kind of interpretation of this amplitude, and such an interpretation involves, as we have indicated previously, the classical idea of free particle. In theoretical physics this was, indeed, already understood and realized on the occasion of a description of the collective motions in the realm of nuclear matter [23].

Therefore, taking for guidance the second of the classical Kepler laws, we may need to revise even the concept of particle of the physics, in order to make it fit for a universe in general. For this, having in mind the absence of the equation of motion in the general case, we just need to notice that the equation (2.1.5) is referring to a *full orbit*, and a particle serving for interpretation can be located *in any point of this orbit*, 'equally likely', so to speak. In other words, not only the position in the *whole space* serving as a host for matter, but even the position of the particle serving for interpretation *on the trajectory* is actually a random process of the kind described by Schrödinger on the occasion of offering an interpretation for his newly introduced, at that time, wave function. Quoting the very words of Schrödinger, the wave function would aptly represent the 'strength' of existence of a particle in a position:

... If we like paradoxes, we may say that the system exists, as it were, *simultaneously in all the positions kinematically imaginable*, but not "equally strongly" in all... ([24], p. 120; *our emphasis, a/n*).

In matters of memory, this 'strength' should be, somehow, measured by the charge persisting on a certain Hertz material particle, playing the part of a 'pole' in a Maxwell fish-eye structure, with a well-defined, even quantitatively, idea of 'persistence', of course. However, according to the view enticed by our development thus far, the second of Kepler laws must be a first incentive for the introduction of the stochastic element in the very classical mechanics in general: those simultaneous positions could not be 'kinematically imaginable' but by a dynamics of Newtonian kind, based on static Newtonian forces. The 'equal strength' would then be described through the wave function as introduced by Schrödinger himself. It is important to keep this idea in mind, but reserve it for later purposes.

For now, though, using the equation (2.1.5), it can be shown right away that the first equality from the right hand side of (2.1.4) gives an Ermakov-Pinney equation for the amplitude of charge:

$$\ddot{A} + \omega_0^2 A = \frac{R_0^2}{A^3} \quad (2.1.6)$$

where R_0 is a real constant. The connection with the periodic motion *per se* can then be established as follows. Let A be the composite amplitude of a two-dimensional harmonic oscillator, described by a quadratic form in the partial amplitudes of component signals varying in time according to the equation of $q(t)$ from (2.1.4), *i.e.*, in particular:

$$A^2 = q_1^2 + q_2^2 \quad (2.1.7)$$

where q_1 and q_2 are two independent solutions of the equation from the left hand side of (2.1.4). They can be taken as the two components – electric and magnetic – of a charge. This amplitude satisfies the equation (2.1.6) with R_0 the constant from (2.1.5). Thus, the frequency ω_0 is associated to the components of the vector $|A\rangle$ in an obvious way, inasmuch as they are classical oscillators.

It is in these conditions, however, that one can conclude directly from (2.1.7) that the square of amplitude of the signal from equation (2.1.3) itself – that is, practically, the *intensity of signal* it represents, if it is to speak in optical terms – satisfies a linear third-order differential equation of known type, that can be gotten right away, based on the Ermakov-Pinney equation (2.1.6):

$$\frac{d^3}{dt^3} A^2 + 4\omega_0^2 \frac{d}{dt} A^2 = 0 \quad (2.1.8)$$

When comparing this equation with the equation of a ray in an optical description (see [3], §2.1) a host of conclusions come over us, that impose by themselves, as it were. First of all, we have the striking conclusion, namely that the square of charge must be *in the brain* what the intensity of light is in the physical optics: in other words, we just got here, indeed, the *counterpart of light* of the Einsteinian, or Planckian universe. Indeed, the equation (2.1.8) is characteristic for a Maxwell fish-eye medium (see previously, IV, §2.2), and therefore the brain matter must be considered such a medium for the *propagation of charge*.

A warning sign on this issue in the case of physical universe, is the existence *Hanbury Brown-Twiss effect*: there are *intensity correlations* of the rays issuing from the same distant source of light [25, 26]. Indeed, the square of the amplitude means an intensity in the optical realm. And, if an equation like (2.1.8) proves to be universal according to the general mathematical structure of a signal, then we can conclude that a medium having the refraction index of a Maxwell fish-eye is that necessary all-pervading medium of the classical ether type, support of every phenomenon in the world

we inhabit. Anyway, such a medium can, at least, serve for a guidance in our proceedings. For once, we need to follow the idea of a meaning of the refraction index as suggested by this ray optics, and carry it out in the realm of charge. But then, more importantly, we need to follow the track of an equation like (2.1.8). It is particularly important to know if such an equation appears anywhere else in physics at all, and in what conditions: in the case of brain it means charge-square correlations, stochastically quantifying the intensity of ‘wiring’ by the events of ‘firing’.

In this respect we have shown previously ([11], Chapter 3, §3.4), that such a linear equation is of essence in the theory of regularization of the Kepler motion. We read in this incident the applicability of equation (2.1.8) for the realm of nuclear matter, by excellence a realm of charge. Therefore, based on this conclusion one can say that the equation (2.1.8) is of essence in the problem of interpretation in general, securing the invariance of this interpretation with respect to the scale transition. In this case, the Maxwell fish-eye medium is, indeed, a universal coordinate space describing the nuclear matter. This conclusion ensues along the lines of a natural classical condition of analyticity, imposed by the theory of regularization of the Kepler motion (see also [27], Chapter 4).

A major problem still remains to be solved here, though, for it is directly connected to the equation (2.1.7), which, in turn, is conditioning any result declared thus far: how can we define the frequency in a proper way, that is, in such a way as *to include the phenomenon of holography* from the very beginning?! And, why should we consider the holography, in the first place?! The answer – in this instance, that is at the level of interpretation particles – stays in the fact that the holography imposes by itself through an implementation of the *idea of coherence*, and this can be represented right away using the ‘Kepler’s second law’ (2.1.5). Thus this equation seems to be an apt universal mathematical fact, endorsed by the theory of regularization, therefore apt to be used for the description of the behavior of matter ([12]; §3.4). Taking, therefore, for the amplitude as a function of phase, the very definition provided by the Kepler’s second law (2.1.5), will be consistent with the *holographic principle* defined according to the original Dennis Gabor’s ideas on coherence, but with a little tweak on formulation: *only coherent signals can assume the same frequency*. This is the case instituted in physical optics, anyway. This approach of the concept of frequency allows us to consider it as a statistic *associated with an assembly of phases*, and will also allow a proper *association of frequency with the phenomenon of charge creation in the brain*, which seems overwhelmingly important for the physical theory of brain.

For once, such a definition means that *the time variation of phase must be physically recognizable in the intensity of a certain wave*, which is the basic physics’ requisite in matters holographical [28, 29]: the intensity is, in fact, not independent on the phase of signal. Then,

proceeding just mathematically, we are able to transform the Keplerian condition (2.1.5) into a second-order differential equation for the amplitude of the harmonic signal:

$$q = \frac{C}{\sqrt{\dot{\phi}}} \quad \therefore \quad \ddot{q} + \frac{1}{2}\{\phi, t\}q = 0 \quad (2.1.9)$$

where C is a constant. Indeed, by comparison with the oscillator's equation of motion, the right hand side of this equation *defines the frequency in terms of the phase* of a general signal like that from the equation (2.1.3), by the equation

$$\omega_0^2 \stackrel{\text{def}}{=} \frac{1}{2}\{\phi, t\} \quad (2.1.10)$$

So, we can take this formula as *defining the frequency when the phase is known*, thus including the holographic phenomenon into the definition of a general optical medium. This incorporates, harmoniously, we should say, the Longuet-Higgins' concept of temporal recall into the structure of the optical medium [30]. Then our problem is reduced to the mathematical description of this phenomenon, and this definition of frequency provides a natural way for the very definition of memory.

Everything thus revolves now around the definition of the Schwarzian derivative [see, for the relevant details and a comprehensive presentation of this operation [31], Chapter 5, §§X, XI, XII]. There is an outstanding property in this definition, of which we shall make much use in this work: any solution of the equation (2.1.10) *is defined up to a homographic transformation*. This would mean that the manifold of solutions of equation (2.1.10) corresponding to a general phase is three-dimensional, not in the sense of the linear superposition rule, though, but in the sense that it can be *surveyed* by locating its points with three parameters. In the superposition rules' phrasing, we rather have here a *nonlinear superposition rule* with three basic solutions of the equation (see [32], §§2 and 3, especially equations (3.51–53)). More precisely, knowing three solutions of the equation (2.1.10), a fourth one can be found right away, without any integration, because it must have a constant cross ratio with those three. In order to prove this statement, we use the general relation of transformation of the Schwarzian (see [31], especially Chapter 5, §XII, Ex. 19(iii)):

$$\{\theta, t\} = \{\phi, t\} + \{\theta, \phi\} \cdot \dot{\phi}^2 \quad (2.1.11)$$

where $\{\theta, \phi\}$ is the Schwarzian derivative of the phase θ with respect to the phase ϕ . If this derivative is null, the two phases are connected by a homographic relation [*ibidem*, Ex. 19(v)], *i.e.*:

$$\{\theta, \phi\} = 0 \quad \therefore \quad \theta(\phi) = \frac{\alpha\phi + \beta}{\gamma\phi + \delta} \quad (2.1.12)$$

so that equation (2.1.11) becomes

$$\{\theta, t\} = \{\phi, t\} \quad (2.1.13)$$

Therefore, these phases are coherent in ‘the optical way’: they define the same frequency as defined by equation (2.1.10). The homographic action of the matrices 2×2 with real entries, can cover the whole ensemble of solutions of the equation (2.1.10). According to this theorem, the general form of the solution of equation (2.1.10) depends on three parameters: it can be obtained from any particular solution by the group formula (2.1.12). In other words, we can construct the whole system of phases of a signal having a definite frequency – in technical terms: a ‘frequency-coherent signal’, *i.e.* the kind of signals used in the implementation of holography in Gabor’s original acceptance – starting from a particular one. The whole system of phases corresponding to the same frequency – this one being defined by the equation (2.1.9), with an amplitude as in equation (2.1.7) – is the orbit through a particular phase θ of the group of real homographies. This is a continuous group with three infinitesimal generators, locally described as a $\mathfrak{sl}(2, \mathbb{R})$ Riemannian space. Therefore, this kind of Riemannian space *is the local expression of the holographic phenomenon*, which here has a precise meaning: the whole system of phases corresponding to the same frequency. This gives us a possibility of interpretation – and speaking of interpretation here, we mean interpretation in the wave-mechanical sense, *whereby the phase can be associated to a particle* [33] – mathematically describable in the terms that follow, mirroring the actual construction of a hologram.

First, one needs to find a ‘seed phase’, as it were, a phase that remains the same during the local hologram construction. This is the hard part of the mathematical description of the holographic phenomenon according to the previous definition: finding the phase whose information content is carried over into any other phase. One can thus better appreciate the definition (2.1.10) of the frequency, which allows us a firm mathematical characterization of the coherence, and thereby a clearcut characterization of the phenomenon of holography. Let us, therefore, assume for the moment that we have found that ‘seed phase’, and denote it by θ . The whole system of phases ϕ carrying the very same information is described by equation (2.1.12), with $\theta = \text{constant}$. Thus, any phase ϕ coherent with θ in the sense of our definition of frequency, is mathematically describable by the solutions of a differential equation of Riccati type, that *we take as defining an instanton* (see I, §2.4):

$$d\phi = \omega^1 \phi^2 + \omega^2 \phi + \omega^3 \quad (2.1.14)$$

This means that, in cases where the phase θ defines a time, the equation $\theta = \text{constant}$ defines ‘an instant’ of that time, and this instant is spatially described by equation (2.1.14). This differential equation correlates the variation of phase *at the same frequency* and the same seed phase with the variations of the three parameters describing the holographic phenomenon. Here the differentials (ω^k) are the components of the standard $\mathfrak{sl}(2, \mathbb{R})$ coframe [see I equation (2.27)]. If we are able to transform this equation into an ordinary differential equation with respect to a certain ‘time parameter’, then it gives us an expression of the phase rate to be used in equation (2.1.9), in order to define the amplitude.

Now, we choose to disregard the general mathematical theory, which frames the facts logically, in their utmost generality. We think it more informative, and convincing at the same time, to report that in most cases which *we have encountered* thus far in our study, this transformation to an ordinary differential equation, is an easy task facilitated by the metrics of the $\mathfrak{sl}(2, \mathbb{R})$ -type. Namely, as a rule, these metrics possess three *Killing vectors*, for which the dual rates (ω^k/dt) are *constants along their geodesics* (see [34], for mathematical details). It is known, indeed – and we shall repeat the procedure in the due time here, for a typical case of interest in the problem of brain – that the differential forms of the $\mathfrak{sl}(2, \mathbb{R})$ -type coframe are projections of the momentum forms generated *via* the metric Lagrangian, along the Killing vectors. Therefore, in such cases, the equation (2.1.14) becomes an ordinary Riccati differential equation along the geodesics:

$$\dot{\phi} = a^1 \phi^2 + 2a^2 \phi + a^3, \quad a^k \cdot d\tau \equiv \omega^k \quad (2.1.15)$$

where (a^1, a^2, a^3) are three constants characterizing the $\mathfrak{sl}(2, \mathbb{R})$ -type geodesics in question, and a dot over means differentiation with respect to the arclength $d\tau$ of the geodesics. This means that a geodesic becomes a point in the $\mathfrak{sl}(2, \mathbb{R})$ -type Riemannian space. So, according to the holographic principle formulated above, *i.e.* based upon the idea of coherence, only along such geodesics the physical theory may happen to be properly interpretable in a wave-mechanical sense. Of course, the process asks for an inversion of the amplitude defined by the rate of phase (2.1.15), so that the inverse of the amplitude will appear, by Arnold’s theorem, as describing a free particle. Indeed, using the combination of the Kepler law (2.1.5) with the equation (2.1.15) gives:

$$\frac{r^2}{a^1 \phi^2 + 2a^2 \phi + a^3} = \text{constant}, \quad A^2 \equiv r^{-2} \quad (2.1.16)$$

This is invariant function of the action (2.1.1), and represents a *radial motion* of a free particle, whose kinematics is described in a time provided by the phase ϕ . Notice that the amplitude and the radial distance are ‘reciprocal’, as it were, which seems to be a normal situation in the case of regularization ([11], Chapter 3, §3.4).

We have strong reasons to believe that, physically speaking, this should be the case: for once, according to *Wagner’s theorem* (see [12], Chapter 4, §4.3) this ‘holographic space’ is the realm of the free particles realizing the oscillators. The most important of these clues, though, is the fact that the holographic definition of the frequency characterizes indeed the nucleus of a planetary atom in theoretical physics. This statement seems to us sufficiently proven as a consequence of the classical dynamical problem associated to Kepler problem [35]. However, in the theory of nuclear matter *per se*, this idea comes associated with an idea of *interpretation via* the concept of *collective coordinates* [23]. So, we need to insist on the physical aspect of the problem from the perspective of these two natural philosophical concepts. For this we need first some special geometrical considerations regarding the realizations of homographies, for they are the basis of definition of the holographic phenomenon. Inasmuch as a homography is simply defined as an action of the 2×2 matrices, specifically on phases in this case, the realizations in question involve some properties of these very matrices, as determined by their actions.

2.2. The Firing and the Memory Deposit

Our basic idea is that there is a close physical connection between the concept of motion and that of memory, which, from natural-philosophical point of view goes deep down to the very definition of the forces in modern physics. This connection can be intuitively understood by an analogy currently used in theoretical physiology, in order to associate the connections inside the brain with the behavior of the bearer of this brain, *i.e.* the man. As this behavior is dictated by the environment and, as this environment is, in turn, dictated by a limited portion of the Earth’s surface, it is just natural to think that this very surface is the memory of a moment from the ‘life of Earth’, as it were. This point of view is taken up in any evolutive theory of the Earth as a planet: an originally spherical body of molten lava, slowly cooling, has a surface retaining the local shapes of a certain moment of its evolution. Then the coming to being of memory is suggested by that fact of experience that initiated the modern physics while arousing the Galilean relativity: *the motion of ships on the face of sea*. On a quiet sea, such a motion leaves a trace for a limited period of time depending, obviously, on the properties of sea water, and thus witnessing for a limited portion of the ship’s

trajectory. In modern terms, this idea of memory can even be improved by the modern technological realizations, allowing us to think, for instance, of a submarine ship, ‘making memories’ not on the surface, but within the very body of water.

Our results in physically developing this analogy are extracted from the evolution of the theoretical physics of the last century, and gathered in a work soon to be published, regarding the concept of instanton [3b]. Based on the analogy just sketched above, they amount to the following: a particle moving in the continuum background of charge leaves a trace of dipoles in that continuum, thus making a memory. This philosophy seems to be confirmed to a high degree, even by the experimental physiology [36], so that we just have here a relatively firm intuitive basis for our concept of brain as a universe. In this respect, we just have to encourage such a physiological idea, by constructing the mathematical theory supporting it, which we do indeed. The basis of that mathematics is the *idea of surface* brought, front and center, as it were, by Gerald Edelman and his collaborators, in the modern science of physiology [37], as the *cell surface*.

Thus, we just summarize these results here, in terms of the definition of the holographic phenomenon from the previous section. Start with the observation that, according to the description of the concept of holography, the construction of a 2×2 matrix must be of essence. Then a dipole can be described by a particular type of matrix, whose action is exerted on the phases of the two component-charges of the dipole, in a certain way to be figured out later. One particular kind of action can this way be figured out intuitively: if the matrix gives an oriented correlation from a charge to the other one of the dipole, when used it in the reverse orientation, it should reproduce the initial phase. This means that the matrix should represent an *involution*: applied twice it gives the initial result. Mathematically, this means that the representative matrix has the essential property of *null trace*. Indeed, according to the Hamilton-Cayley theorem, a 2×2 matrix, V say, must satisfy the *algebraic identity*:

$$V^2 - \text{tr}(V) \cdot V + \det(V) \cdot \mathbf{1} = \mathbf{0} \quad (2.2.1)$$

where $\mathbf{1}$ is the identity matrix and $\mathbf{0}$ is the matrix with null entries. Also the symbols $\text{tr}(\dots)$ and $\det(\dots)$ represent the sum of diagonal entries – the trace – and, respectively, the determinant of the matrix. From this identity we have right away:

$$\text{tr}(V) = 0 \quad \Leftrightarrow \quad V^2 = -\det(V) \cdot \mathbf{1} \quad (2.2.2)$$

The notation V is not just casual here: it is intended to show that these matrices have the property of the vectors in linear space. First, they can always be written as a linear combination of three basic matrices $\mathbf{I}, \mathbf{J}, \mathbf{K}$:

$$\mathbf{V} = v_1 \mathbf{I} + v_2 \mathbf{J} + v_3 \mathbf{K} \quad (2.2.3)$$

which is the essential property of the vectors in linear space. The three null-trace matrices from the right hand side of this equation thus form a basis of linear independent vectors in the space of 2×2 null-trace, *i.e. involutive* in any of their actions, matrices.

There are, however, substantial differences between these vectorial matrices and the regular vectors in space, due to the fact that they are assumed to act, either in one dimension – by the homography, as in the case of the holographic property – or in two dimensions – by regular linear action on two-dimensional vectors – or even in four dimensions, by multiplication on 2×2 matrices. From this point of view, the three basic matrices $\mathbf{I}, \mathbf{J}, \mathbf{K}$, must satisfy the following non-commutative algebra:

$$\mathbf{IJ} + \mathbf{JI} = \mathbf{JK} + \mathbf{KJ} = \mathbf{KI} + \mathbf{IK} = \mathbf{0} \quad (2.2.4)$$

with the following multiplication table:

$$\begin{array}{c} \text{The First Factor} \\ \times \begin{array}{ccc} \mathbf{I} & \mathbf{J} & \mathbf{K} \end{array} \\ \begin{array}{l} \mathbf{I} \quad -\mathbf{1} \quad -\mathbf{K} \quad \mathbf{J} \\ \mathbf{J} \quad \mathbf{K} \quad \mathbf{1} \quad \mathbf{I} \\ \mathbf{K} \quad -\mathbf{J} \quad -\mathbf{I} \quad \mathbf{1} \end{array} \end{array} \quad (2.2.5)$$

Based on these algebraic properties, one can prove that the algebra of ‘vector-matrices’ is closed, for they satisfy the linear independence relation:

$$m_1 \mathbf{I} + m_2 \mathbf{J} + m_3 \mathbf{K} = \mathbf{0} \Leftrightarrow m_1 = m_2 = m_3 = 0 \quad (2.2.6)$$

i.e. the null vectors have indeed null components as in the case of regular vectors. Further on, the involutive property can be expressed in the form:

$$\det(\mathbf{V}) \equiv v_1^2 - v_1^2 - v_1^2 = -\mathbf{V} \cdot \mathbf{V} \quad (2.2.7)$$

This relation shows, by an obvious association (see II, Chapter 4, especially §4.2), that the null-trace matrices are prone to represent the charges in the static case of ensembles of particles. Our task here is to show how the process of memory deposit on a hologram can be accomplished, and

what are the characteristics of the charges issued by the fundamental physiological processes in the universe represented by brain.

There is a particularly important *apriori* choice of the basis $\mathbf{I}, \mathbf{J}, \mathbf{K}$, whereby the charge can be represented as in the case of classical Yang-Mills fields of theoretical physics (see II, §5.4). In other words, our contention is that the fundamental fields in the universe of brain are indeed Yang-Mills fields, not just plain electromagnetic fields, as the physiologists regularly consider them. The choice in question is the following:

$$\mathbf{I} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{J} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{K} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.2.8)$$

With this choice for the basis, we can represent the charges ‘quantum-mechanically’, as it were, by the particular matrices

$$\mathbf{Q}(\mathbf{x}) \stackrel{\text{def}}{=} x_1 \mathbf{J} + x_2 \mathbf{K} \quad \therefore \quad \mathbf{Q}(\mathbf{x}) \cdot \mathbf{Q}(\mathbf{x}) = (x_1^2 + x_2^2) \cdot \mathbf{1} \quad (2.2.9)$$

The quantum-mechanical idea comes here with the consideration of the eigenvalues of this matrix, which are equal in magnitude and opposite in sign, exactly like the charges in a classical electric or magnetic dipole:

$$q_{1,2} = \pm \sqrt{x_1^2 + x_2^2} \quad (2.2.10)$$

According to our adopted natural philosophy (see II, §5.2), this analogy goes deeper: the vector $\mathbf{x}$ defined by the ket $|x\rangle$ according to the rule $\langle x| \equiv (x_1, x_2)$, is prone to represent a charge with electric part x_1 and magnetic part x_2 , or *viceversa*. The whole matrix (2.2.9) though, quantum-mechanically represents a classical dipole with the characteristic charges given by its eigenvalues from equation (2.2.10).

Now, what is more important, from the point of view of the holographic principle, this dipole defines itself two fixed phases – the fixed points of the homographic action of matrix (2.2.9) – which are given by the tangent of the *phase of charges*, according to formulas:

$$\phi_{1,2} = -\frac{x_2}{x_1} \pm \sqrt{1 + \frac{x_2^2}{x_1^2}} \quad \therefore \quad \phi_{1,2} = -\tan\theta \pm \frac{1}{\cos\theta} \quad (2.2.11)$$

with an obvious notation for that phase. These fixed points allow us to define the deposit of memory. In order to do that, we must get into some details of particular representation of the homographic action of a certain matrix [3].

The homographic action of a 2×2 matrix, $\mathbf{M}$ say, is uniquely determined by three numbers strictly calculable knowing its entries and the *characteristic cross-ratio* of the homography. This cross-ratio is a constant for a given matrix and a given range of action. It is defined as the cross-ratio of two points corresponding to one another through the homography, and calculated with reference to the fixed points of homography, according to usual formula

$$k \stackrel{\text{def}}{=} (\phi, \phi_0; \phi_1, \phi_2) = \frac{\phi - \phi_1}{\phi - \phi_2} : \frac{\phi_0 - \phi_1}{\phi_0 - \phi_2} \quad (2.2.12)$$

For a given homographic action determined by the matrix $\mathbf{M}$ and the phase ϕ_0 , this cross-ratio is a constant when ϕ runs through its possible values. These values may be real or complex numbers, but we assume they real, just in order to keep things simple, and settle our ideas. Thus, by using the relation (2.2.12), we can write the entries of the family of matrices whose homographic action is ‘centered’ on ϕ_0 . These are

$$\frac{A}{\phi_1 - \phi_2 k} = \frac{B}{(k - 1)\phi_1 \phi_2} = \frac{C}{1 - k} = \frac{D}{\phi_1 k - \phi_2}; \quad \mathbf{M} \stackrel{\text{def}}{=} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad (2.2.13)$$

and can be arranged as a special linear family of matrices:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \sim \begin{pmatrix} \phi_1 & -\phi_1 \phi_2 \\ 1 & -\phi_2 \end{pmatrix} - k \begin{pmatrix} \phi_2 & -\phi_1 \phi_2 \\ 1 & -\phi_1 \end{pmatrix} \quad (2.2.14)$$

The base matrices of this family are two singular matrices, with the entries depending exclusively on the fixed points of homography, and its parameter is the characteristic cross-ratio k of the homographic action. This will help us in proving that between the two fixed phases, ϕ_l and ϕ_2 *there is always a homographic relation*.

To start with, let us notice that in *coordinates* (A, B, C, D) a point represents a matrix. The quadratic form representing a quadric, more specifically, a hyperboloid. That is:

$$AD - BC = 0 \quad (2.2.15)$$

which is simply the locus of points of coordinates (A, B, C, D) , representing the ensemble of singular matrices. Geometrically, we have here a one-sheeted hyperboloid, which is a *doubly ruled* surface: it has *two systems* of generators described by the parameters ϕ_1 and ϕ_2 . This can be ascertained as follows: take, for instance, the first singular matrix from the linear family (2.2.14). The representation (2.2.13) gives the point of coordinates

$$\frac{A}{\phi_1} = \frac{B}{-\phi_1\phi_2} = \frac{C}{1} = \frac{D}{-\phi_2} \quad (2.2.16)$$

One can notice right away that there are two systems of planes whose intersections can count as two straight lines – the generators, or the rulings of our quadric – at the intersection of which the point (2.2.16) is located, namely:

$$\begin{array}{l} A\phi_2 + B = 0 \\ C\phi_2 + D = 0 \end{array} \quad \text{and} \quad \begin{array}{l} A - \phi_1 C = 0 \\ B - \phi_1 D = 0 \end{array} \quad (2.2.17)$$

So these equations describe the two systems of generators of the hyperboloid, whose parameters are the invariant phases ϕ_1 and ϕ_2 . Now, assume a generic point of coordinates $(\alpha, \beta, \gamma, \delta)$ say, in this four-dimensional space, located either inside or outside the quadric (2.2.15), but not on the quadric itself; to wit: $\alpha\delta - \beta\gamma \neq 0$. In other words, such a point represents a *non-singular* matrix. Its *polar plane* with respect to the quadric (2.2.15), meets this very quadric along a hyperbola having the equation of representation

$$-\alpha\phi_2 + \delta\phi_1 - \beta + \gamma\phi_1\phi_2 = 0 \quad \therefore \quad \phi_1 = \frac{\alpha\phi_2 + \beta}{\gamma\phi_2 + \delta} \quad (2.2.18)$$

In other words: an arbitrary non-singular 2×2 matrix represents a transformation which gives *a correspondence between the two phases* representing the fixed points of given matrix. The last equality in this equation can be taken as a *generalized relation* between the two phases defining a *doublet*, that can be taken as characteristic to a *dipole* by the procedure of holography: a phase for one of the charges, the other phase for the accompanying charge of the dipole, with the dipole itself represented by a non-singular matrix correlating the two fixed phases.

To conclude: the correlation (2.2.18) means that any pair of phases of the form:

$$\phi, A(\phi) \quad (2.2.19)$$

where $(\alpha, \beta, \gamma, \delta)$ are arbitrary real numbers satisfying the condition $A(\phi) = (\alpha\phi + \beta) / (\gamma\phi + \delta)$ with $\alpha\delta - \beta\gamma \neq 0$, can *always be considered a pair of fixed phases* characterizing a dipole according to the holographic principle. In other words, in the holographic definition of frequency by coherence of phases as in equation (2.1.10), any pair of phases corresponding to the same frequency can be considered as characterizing a dipole. The differential equations (2.1.14) and (2.1.15) are nothing more than the equations describing a system of phases of a possible hologram.

3. Conclusions

The mathematics just developed until this moment allows for an intuitive image of the firing event. More to the point, a scenario can be framed as follows: along the geodesics of the Maxwell fish-eye structure, charges circulate as particles. They can be considered stochastic processes, described by the probabilities of presence of the some electric particles along the geodesics of the Maxwell fish-eye. The poles are interpretative Hertz material particles, providing the ensembles from which the dipoles are constructed as incidentals, and electric particles circulating along the geodesics of the Maxwell fish-eye ‘organizations’. Such electric particles are giving the structure of an instanton. Their occurrence is stochastic: they appear with equal probability in any position of a geodesic of the Maxwell fish-eye structure, and can migrate from a geodesic to another by ‘quantum leaps’ as in the classical planetary model of atoms. Some of these occurrences are firing events: the problem is to decide which ones. In order to do this we need to consider the Planck’s quantization process more closely.

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