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PERTURBATION ANALYSIS OF KINEMATIC TRAJECTORIES USING FRACTAL DYNAMICS FOR THE DETECTION OF EXPERIMENTAL ERRORS IN PATIENTS WITH RHEUMATOID ARTHRITIS

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Abstract. In the context of finger biomechanics and the development of biomechatronic systems, joint motion trajectories exhibit nonlinear variations and local instabilities that cannot always be adequately described by classical deterministic models; therefore, this study proposes a mathematical framework based on the introduction of controlled fractal perturbations generated through Mandelbrot-type iterative dynamics. The time series of MCP, PIP, and DIP joint angles were processed using a graphical user interface developed in MATLAB, designed for the kinematic analysis of the index finger, modeled as a serial kinematic chain with three degrees of freedom, allowing the reconstruction of the distal phalanx trajectory and its comparison with a reference trajectory derived from healthy-subject data. Through the automatic computation of Cartesian errors, normal deviations, and discrepancies in the joint space, the proposed method highlights motion variability and local instabilities, contributing to data validation and to the optimization of objective

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biomechanical assessment in rheumatoid arthritis.

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1. Introduction

Finger movement represents a complex biomechanical process resulting from the coordinated interaction between joints, tendons, muscles, and neuromotor structures (Lv *et al.*, 2022). In the case of the index finger, fine motor control is essential for performing functional hand activities, such as grasping, object manipulation, writing, or executing precision gestures. For this reason, the objective analysis of joint trajectories is of major importance both for understanding hand biomechanics and for assessing functional alterations associated with various musculoskeletal disorders (Henderson *et al.*, 2021).

In rheumatoid arthritis, involvement of the small joints leads to progressive alterations in mobility, coordination, and movement stability (Duduca *et al.*, 2026). Joint pain, stiffness, inflammation, and structural deformities may alter the way patients perform movements, generating deviations from physiological trajectories (Şerban *et al.*, 2026). In addition, the functional limitation of one joint may promote the development of compensatory mechanisms, resulting in overloading of other joints or segments in order to maintain the motor task (Fraenkel *et al.*, 2021). These alterations are not always evident through subjective clinical assessment; therefore, quantitative methods capable of accurately characterizing the kinematic behavior of the fingers are required (Hamy *et al.*, 2023).

Deterministic models used in biomechanics play an important role in the mathematical description of movement, as they allow the representation of the relationships between anatomical segments, joints, and the resulting trajectories (Turnea *et al.*, 2024). In the case of the index finger, for example, movement can be described through a serial kinematic chain composed of the metacarpophalangeal, proximal interphalangeal, and distal interphalangeal joints (Turnea *et al.*, 2025; Wu *et al.*, 2021). By defining segment lengths and joint angles, the position of the distal phalanx can be reconstructed, and the contribution of each joint to the final finger trajectory can be analyzed (Naeem and Faidh-Allah, 2021).

However, these models generally rely on simplified assumptions. They assume that the segments behave as rigid elements, that the joints have well-defined axes of rotation, and that movement can be described through stable and reproducible mathematical relationships (Wu *et al.*, 2021). In reality, human movement is not perfectly regular. Even in healthy subjects, the execution of the same movement may exhibit small variations in amplitude, velocity, coordination, or trajectory. These variations are determined by individual

anatomical characteristics, neuromuscular control, fatigue, execution accuracy, tissue elasticity, and motor adaptation to the performed task (Asghari *et al.*, 2021). Thus, the finger trajectory no longer deviates only pointwise from the theoretical model, but may present local alterations, instabilities, and regions of irregularity that reflect either functional impairment or compensatory strategies (Henderson *et al.*, 2021).

In patients with rheumatoid arthritis, these variations may become more pronounced and more difficult to interpret (Munteanu *et al.*, 2023). Inflammation, pain, joint stiffness, progressive deformities, and reduced mobility can significantly alter the manner in which movement is performed (Phutane *et al.*, 2021). An affected joint may present a reduced range of motion, and the patient may compensate for this limitation through the excessive use of other joints (Matei *et al.*, 2025).

Furthermore, real trajectories may also be influenced by experimental factors. The recording of finger movement may depend on sensor quality, measurement devices, the stability of the acquisition system, or the noise present in the data (Henderson *et al.*, 2021). An observed deviation between the patient's trajectory and the reference trajectory does not automatically indicate a pathological alteration; it may also be caused by improper calibration, measurement artifacts, or variations occurring during the experimental protocol. For this reason, direct comparison with a fixed nominal trajectory may lead to incomplete or even erroneous interpretations (Farrell *et al.*, 2022).

Therefore, a reference trajectory should not be regarded as an ideal, unique, and rigid line, but rather as a central representation of a movement considered to be normal, around which a certain degree of acceptable physiological variability exists (Phutane *et al.*, 2021). This approach is closer to biomechanical reality, as it acknowledges that human movement contains natural fluctuations and cannot be fully reduced to a perfectly deterministic path. In this regard, simply measuring the distance between the real trajectory and the nominal trajectory is not always sufficient; it is also necessary to assess how the deviation is locally distributed, whether it is constant or irregular, and whether it occurs within a critical phase of the movement (Asghari *et al.*, 2021).

The integration of fractal perturbations into kinematic analysis provides a more flexible framework for describing these variations (Faini *et al.*, 2024). By introducing controlled Mandelbrot-type perturbations around the reference trajectory, the model enables the generation of a local variability region that can more realistically simulate the natural irregularities of movement. Thus, the reference trajectory is no longer treated as an ideal curve, but rather as a possible kinematic domain, within which small local fluctuations are accepted without being automatically interpreted as pathological alterations or measurement errors (Faini *et al.*, 2024).

This fractal variability region can be interpreted as a biomechanical tolerance space. If the patient's real trajectory remains within or close to this

region, the deviation may be considered compatible with normal movement variability or with minor experimental variations (Arsac, 2021). Conversely, if the real trajectory significantly exceeds this domain, especially in specific movement segments, this may indicate local instability, a compensatory strategy, or a relevant functional alteration. At the same time, abrupt, isolated, or incoherent exceedances may suggest the presence of experimental errors, such as acquisition noise or incorrect sensor positioning (Veronez *et al.*, 2024).

2. Materials and methods

For movement reconstruction, the index finger is modeled as a serial kinematic chain with three degrees of freedom, in which each phalanx is considered a rigid segment and each joint is represented by a rotation angle (Naeem and Faidh-Allah, 2021). This representation enables the geometric description of the distal phalanx position as a function of the anatomical segment lengths and the values of the joint angles measured over time. Thus, for each acquisition time point, the system calculates the position of the fingertip or distal phalanx in the plane, obtaining a kinematic trajectory that reflects the actual manner in which the movement is performed (Li *et al.*, 2023).

The study cohort included kinematic data acquired from three clinically healthy subjects and from one patient diagnosed with rheumatoid arthritis, who was evaluated at two distinct time points: before and after the implementation of the kinetic rehabilitation program. The data acquisition protocol consisted of performing a series of successive flexion–extension movements of the index finger, with the aim of highlighting the kinematic behavior of the articular chain involved in movement execution. Data obtained from the healthy subjects were used to construct a reference trajectory, considered representative of a physiological movement pattern. This reference trajectory served as the basis for comparison in the analysis of the movement performed by the patient with rheumatoid arthritis, allowing the identification of deviations from normal kinematic behavior both before and after the rehabilitation program. According to the experimental protocol applied in this study, the database included recordings of successive flexion–extension movements of the index finger, and the purpose of the processing stage was to transform the acquired data into an interpretable format for the functional assessment of the patient with rheumatoid arthritis (Phutane *et al.*, 2021).

Inertial sensors were used to capture index finger movement, mounted at the level of each phalanx in order to record the kinematic changes occurring during flexion–extension movements (Henderson *et al.*, 2021). Sensor positioning was performed by identifying the midpoint of each phalanx, considered an optimal placement site for reducing the influence of undesired local displacements and for obtaining a more stable representation of the movement of the analyzed segment. Through this distribution, the motion capture system

enabled coordinated tracking of the finger segments and provided the data required for reconstructing the kinematic trajectory of the distal phalanx (Farrell *et al.*, 2022).

At this stage of development, the experimental validation of the acquisition system was performed exclusively at the level of the index finger, as this segment provides a suitable model for assessing the feasibility of sensor placement, signal acquisition, and kinematic trajectory reconstruction. Nevertheless, the proposed configuration is not limited to a single digit. Owing to its modular design, the system can be adapted for whole-hand motion analysis by extending the same sensor-placement methodology to the remaining fingers and by adding the necessary reference units.

Such an expansion would enable the acquisition of simultaneous kinematic data from multiple digits and would allow a more detailed evaluation of hand biomechanics. In particular, it would facilitate the analysis of inter-segmental and inter-digital coordination, global movement organization, compensatory strategies, and functional grasping patterns. Therefore, the index-finger setup presented in this study should be interpreted as a preliminary implementation of the proposed acquisition framework, providing the methodological basis for future applications dedicated to complete hand kinematic assessment.

Kinematic modeling enables the transformation of acquired joint data, expressed as angular values, into a more intuitive and functionally relevant spatial representation. Instead of limiting the analysis to the separate variation of the MCP, PIP, and DIP angles, the method allows the combined effect of these joints on the overall movement of the finger to be observed (Naeem and Faidh-Allah, 2021). This approach is particularly important in patients with rheumatoid arthritis, where the involvement of one joint can modify the entire motor strategy, and compensation may occur through an increased contribution of other joints (Nava-Téllez *et al.*, 2022).

The real trajectory obtained from the experimental data is compared with a reference trajectory derived from recordings performed in healthy subjects. This reference represents a model of movement considered to be normal and is used as a basis for identifying kinematic deviations. In this way, the analysis is not related solely to an ideal theoretical formula, but to a biomechanical behavior experimentally observed in subjects without functional impairment. Comparison with such a reference enables the identification of differences between physiological movement and movement altered by pathological, compensatory, or experimental factors (Phutane *et al.*, 2021).

The fractal cloud generated around the reference trajectory enables the identification of regions in which the patient's movement deviates significantly from the estimated biomechanical behavior (Veronez *et al.*, 2024). In this regard, the proposed model has not only a descriptive role but also contributes to differentiating between actual functional deviations and measurement artifacts

generated by incorrect sensor positioning, acquisition noise, or instability of the experimental system (Faini *et al.*, 2024).

To describe the kinematic behavior of the finger in rheumatoid arthritis, the proposed model integrates the reference trajectory, patient deviation, biomechanical instability, and Mandelbrot fractal dynamics into a unified mathematical system (Arsac, 2021; Veronez *et al.*, 2024). The aim of this system is to characterize how functional alterations in movement influence the shape and density of the fractal cloud generated around the healthy trajectory.

The reference trajectory is considered the physiological kinematic axis against which the patient's movement is evaluated (Li *et al.*, 2023; Phutane *et al.*, 2021). It is defined by the position vector:

$$r_H(t_n) = \begin{bmatrix} x_H(t_n) \\ y_H(t_n) \end{bmatrix} \quad (1)$$

Where, $x_H(t_n)$ și $y_H(t_n)$ represent the coordinates of the healthy trajectory at the discrete time point t_n . This trajectory is not interpreted as an absolutely ideal curve, but rather as a central reference around which a controlled region of biomechanical variability may exist.

The deviation of the real trajectory from the reference trajectory is quantified by the local kinematic distance (Phutane *et al.*, 2021):

$$D(t_n) = \|r_{\text{actual}}(t_n) - r_H(t_n)\| \quad (2)$$

Where, $r_{\text{actual}}(t_n)$ represents the measured trajectory of the patient. Low values of $D(t_n)$ indicate movement close to physiological behavior, whereas increased values may indicate a functional deviation, local instability, or a possible acquisition error (Veronez *et al.*, 2024).

Kinematic stiffness is introduced through a term $R(t_n)$ which describes the functional resistance of movement and the loss of trajectory smoothness (Lv *et al.*, 2022):

$$R(t_n) = \|K_{RA}(t_n)\| \quad (3)$$

Where, $K_{RA}(t_n)$ represents the stiffness matrix associated with rheumatoid involvement. From a biomechanical perspective, an increase in this term suggests restricted mobility, a more rigid movement pattern, and reduced joint adaptability (Nava-Téllez *et al.*, 2022).

To describe the local instability of movement, the term $T(t_n)$ is introduced, which quantifies the rapid variations of the trajectory relative to the estimated mean behavior (Asghari *et al.*, 2021):

$$T(t_n) = \|r_{\text{actual}}(t_n) - \bar{r}_{\text{actual}}(t_n)\| \quad (4)$$

Where, $\bar{r}_{actual}(t_n)$ represents a filtered form of the real trajectory. This parameter highlights micro-oscillations and local irregularities of movement, which may occur in the context of joint instability or impaired neuromuscular control (Veronez *et al.*, 2024).

Dynamic kinematic variability is expressed through the term $V(t_n)$, defined based on the difference between the real velocity and the estimated mean velocity (Faini *et al.*, 2024):

$$V(t_n) = \|\dot{r}_{actual}(t_n) - \bar{\dot{r}}_{actual}(t_n)\| \quad (5)$$

This term reflects velocity fluctuations along the trajectory. An increased value may indicate jerky movement, lack of coordination, functional tremor, or motor compensations (Arsac, 2021).

Based on these kinematic and biomechanical components, the biomechanical Mandelbrot coefficient is defined as follows (Faini *et al.*, 2024):

$$c_{bmc} = [a_0 + \alpha_1 D(t_n) + \alpha_2 R(t_n)] + i[b_0 + \beta_1 T(t_n) + \beta_2 V(t_n)] \quad (6)$$

This coefficient is used to control the shape and dispersion of the fractal cloud. The real component of the coefficient, $a_0 + \alpha_1 D(t_n) + \alpha_2 R(t_n)$, describes the influence of global deviation and stiffness on the longitudinal displacement of the fractal cloud (Veronez *et al.*, 2024). The imaginary component, $b_0 + \beta_1 T(t_n) + \beta_2 V(t_n)$ is associated with transverse dispersion, micro-instability, and local trajectory irregularity.

The fractal dynamics are generated through the Mandelbrot relation (Arsac, 2021):

$$z_{n+1} = z_n^2 + c_{bmc}(t_n), \quad z_0 = 0 \quad (7)$$

Where, z_n represents the fractal state at iteration n . Through this relation, the patient's biomechanical variations are transformed into a fractal behavior that can generate irregular and self-similar distributions around the reference trajectory (Faini *et al.*, 2024).

To integrate the fractal, result into the kinematic space, the local fractal index is defined as follows (Faini *et al.*, 2024):

$$M_n = \frac{|z_n|}{1 + |z_n|} \quad (8)$$

This index normalizes the fractal amplitude and limits the numerical effect of the Mandelbrot iteration. Thus, M_n can be interpreted as a local measure of the intensity of the fractal perturbation (Veronez *et al.*, 2024).

The density of the fractal cloud is modeled by the following relation (Faini *et al.*, 2024):

$$\rho_n = \rho_0 [1 + X |c_{bmc}(t_n)|] \quad (9)$$

Where, ρ_0 represents the baseline density of the fractal cloud, and X is the sensitivity coefficient of the model to kinematic changes. This formulation shows that the cloud density increases proportionally with the magnitude of the biomechanical Mandelbrot coefficient. Therefore, a more unstable or more rigid movement generates a denser and more dispersed fractal cloud (Veronez *et al.*, 2024).

In the comparative analysis performed before and after the implementation of the kinetic rehabilitation program, the density of the fractal cloud can be used as an indicator of the changes occurring in movement quality (Phutane *et al.*, 2021). Since the fractal cloud describes the region of kinematic variability generated around the reference trajectory, changes in its density provide information regarding the degree of stability, regularity, and motor control of the analyzed trajectory. Thus, an increased fractal density before the kinetic rehabilitation program may indicate a more irregular movement pattern, characterized by local instability, stiffness, abrupt trajectory variations, or possible compensatory mechanisms. Conversely, a reduction in fractal density after the kinetic rehabilitation program may suggest an improvement in kinematic control and a closer approximation of the patient's movement to the reference behavior (Henderson *et al.*, 2021).

The fractal perturbation introduced around the reference trajectory is expressed as follows (Arsac, 2021):

$$P_{fract} = \rho_n M_n \begin{bmatrix} \cos(\omega t_n + \varphi) \\ \sin(\omega t_n + \varphi) \end{bmatrix} \quad (10)$$

Where, ω controls the local frequency of the variations, and φ represents the phase of the perturbation. This term generates the spatial distribution of fractal points around the reference trajectory and enables the simulation of controlled local variations of movement (Faini *et al.*, 2024).

Within the proposed model, an important property of the fractal cloud is biomechanical self-similarity (Faini *et al.*, 2024). This describes the ability of the fractal structure to reproduce, at a local scale, variation patterns similar to those observed at the global level of the trajectory (Arsac, 2021). From the perspective of kinematic analysis, this principle allows movement to be interpreted not merely as a pointwise deviation from the reference trajectory, but as a complex spatial structure in which large trajectory variations may be composed of repeated local perturbations.

In the analyzed joint trajectories, self-similarity reflects the fact that global kinematic deviations may contain local micro-oscillations, while major movement deviations may be composed of fine perturbations distributed along

the trajectory. Thus, the fractal cloud generated through Mandelbrot dynamics enables the simultaneous modeling of global movement instability and the local instabilities that occur in specific segments of the trajectory. This approach is relevant in rheumatoid arthritis, as the patient's movement may exhibit both evident deviations from the healthy reference and subtle irregularities that are difficult to identify through a strictly deterministic kinematic analysis.

By introducing self-similarity into the model, fractal perturbations are not considered simple random variations, but rather the expression of a geometric organization of biomechanical instability. Global trajectory oscillations may include local micro-oscillations, while fine variations may contribute to the formation of an irregular yet structured kinematic pattern. In this regard, the fractal cloud functions as a representation of the biomechanical uncertainty region around the reference trajectory, allowing differentiation between acceptable movement variations and significant deviations.

Methodologically, this property allows the Mandelbrot model to be used for the simultaneous description of multiple levels of instability: global trajectory instability, local instability of movement segments, biomechanical tremor, and fine kinematic errors. Thus, the analysis is not limited to measuring an average distance from the reference, but rather examines how deviations are distributed, repeated, and organized along the trajectory. This characteristic is important for identifying the regions in which the patient's movement exceeds the variability domain estimated by the fractal cloud.

The biomechanical perturbation associated with rheumatoid arthritis is introduced through the following term:

$$P_{RA}(t_n) = \alpha K_{RA}^{-1}(t_n)F(t_n) + \beta \dot{r}_{actual}(t_n) + \gamma \ddot{r}_{actual}(t_n) + \delta I(t_n) \quad (11)$$

This term integrates the influence of joint stiffness, applied forces, velocity, acceleration, and the inflammatory component (Henderson *et al.*, 2021). From a medical perspective, $P_{RA}(t_n)$ describes the pathological contribution to the alteration of the functional trajectory (Phutane *et al.*, 2021).

The simulated patient trajectory is obtained by superimposing the healthy trajectory, the pathological perturbation, the fractal perturbation, and experimental noise (Farrell *et al.*, 2022):

$$r_{sim}(t_n) = r_h(t_n) + P_{RA}(t_n) + P_{fract}(t_n) + \sigma \xi(t_n) \quad (12)$$

Where, $\sigma \xi(t_n)$ represents the experimental noise. This component allows the inclusion of errors introduced by the measurement system, such as sensor noise, marker instability, or calibration errors (Henderson *et al.*, 2021).

The error between the real trajectory and the simulated trajectory is defined as follows (Phutane *et al.*, 2021):

$$E(t_n) = \|r_{actual}(t_n) - r_{sim}(t_n)\| \quad (13)$$

This error quantifies the extent to which the patient's movement can be explained by the proposed fractal model. Low values indicate that the real trajectory falls within the estimated variability region, whereas increased values suggest a significant deviation (Veronez *et al.*, 2024).

To synthesize the deviations, the Fractal Anomaly Index (FAI) is introduced:

$$FAI = \frac{1}{N} \sum_{n=1}^N |E(t_n) - p_n M_n| \quad (14)$$

This index compares the real trajectory error with the estimated fractal perturbation (Faini *et al.*, 2024). If the *FAI* is low, the patient's movement is compatible with the variability generated by the model. If the *FAI* is high, the trajectory exceeds the kinematic tolerance region and may indicate significant functional impairment or an experimental error (Veronez *et al.*, 2024).

In the comparative analysis performed before and after the kinetic rehabilitation program, the density of the fractal cloud can be used as an indicator of the kinematic response. Before the intervention, an increased density of the fractal cloud may reflect an irregular trajectory characterized by local instability, stiffness, and uncontrolled movement variations. After the implementation of the kinetic rehabilitation program, a reduction in fractal density may indicate improved movement stability and a closer approximation of the patient's trajectory to the reference behavior.

Global kinematic stiffness is evaluated as follows:

$$KSI = \frac{1}{N} \sum_{n=1}^N \left\| \frac{d^2 r_{actual}}{dt^2} \right\| \quad (15)$$

The *KSI* index expresses the mean level of acceleration variations along the trajectory (Hirose *et al.*, 2020). An increased value may be associated with jerky movement, stiffness, reduced motor control, or biomechanical instability (Veronez *et al.*, 2024).

The Fractal Anomaly Index (FAI) defined in Eq. 14 is further used as a classification variable for the interpretation of the kinematic behavior according to predefined threshold values. An increased value may be associated with jerky movement, stiffness, reduced motor control, or biomechanical instability (Veronez *et al.*, 2024):

$$C = \begin{cases} \text{stable movement,} & FAI < \theta_1 \\ \text{moderate kinematic deviation,} & \theta_1 < FAI < \theta_2 \\ \text{unstable / experimental error,} & FAI \geq \theta_2 \end{cases} \quad (16)$$

3. Results and Discussions

The analysis of the results was performed using the graphical interface developed for the processing and visualization of the kinematic trajectories of the index finger. The obtained results were interpreted comparatively, before and after the implementation of the kinetic rehabilitation program, by relating the patient's trajectory to the reference trajectory derived from healthy-subject data.

In the graphical representations of the trajectories, the reference curve describes the kinematic behavior considered physiological, while the associated fractal cloud delineates the biomechanical tolerance domain. The patient's real trajectory is superimposed onto this domain, allowing direct observation of the regions in which the movement remains close to the reference or, conversely, exceeds the estimated variability region. In the evaluation performed before the kinetic rehabilitation program, the patient's trajectory shows a more evident deviation from the healthy reference, with a more pronounced separation particularly in the descending portion of the movement. This difference suggests the presence of kinematic instability and less controlled execution, compatible with the functional impairment specific to rheumatoid arthritis.

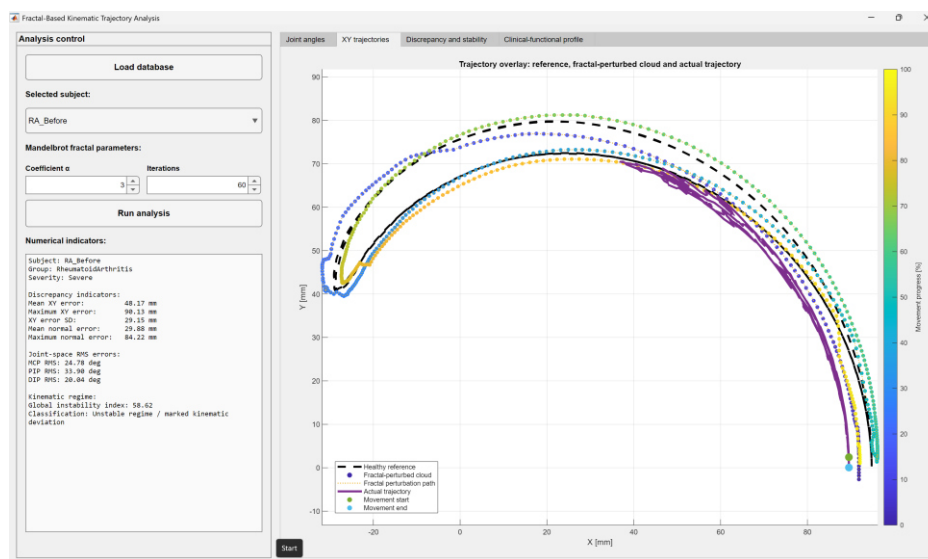


Fig. 1 – Representation of the kinematic trajectory of the patient with rheumatoid arthritis before the implementation of the kinetic rehabilitation program, in comparison with the reference trajectory and the perturbed fractal cloud.

After the kinetic rehabilitation program, the patient's trajectory becomes closer to the region delineated by the healthy reference and the fractal cloud. This

change indicates a reduction in movement dispersion and an improvement in kinematic control. From a biomechanical perspective, the approximation of the real trajectory to the fractal tolerance domain may be interpreted as an increase in functional stability and as a reduction in local deviations caused by stiffness, pain, or compensatory mechanisms. However, the persistence of the classification within a marked kinematic deviation regime indicates that recovery is partial and that the trajectory does not fully return to physiological behavior.

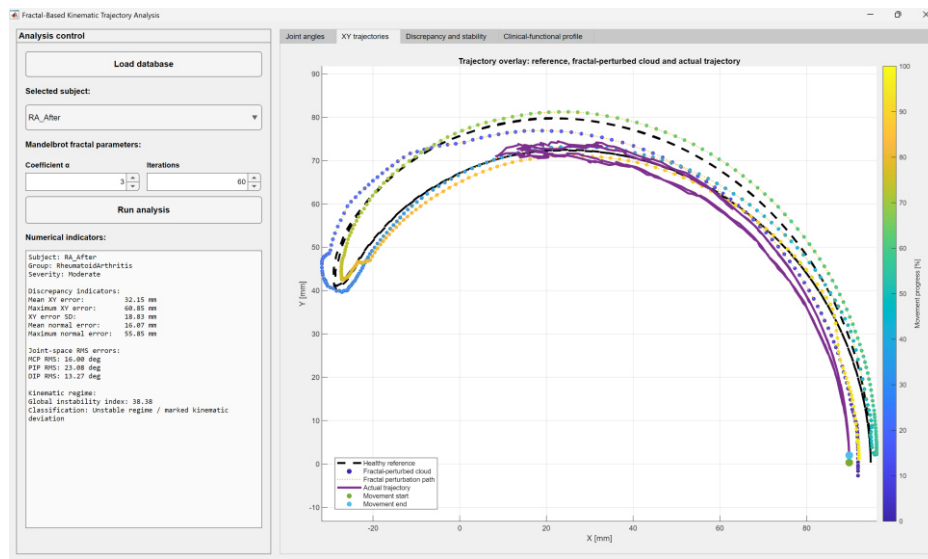


Fig. 2 – Representation of the kinematic trajectory of the patient with rheumatoid arthritis after the implementation of the kinetic rehabilitation program, in comparison with the reference trajectory and the perturbed fractal cloud.

An important result is related to the change in the density and distribution of the fractal cloud. Before the kinetic rehabilitation program, the fractal cloud is associated with a more irregular trajectory, in which local deviations and oscillations are more evident. This increased density of the fractal cloud reflects more pronounced local instability and high kinematic variability. After the intervention, the distribution of the trajectory in relation to the fractal cloud becomes more compact, suggesting a reduction in local instabilities and better organization of movement. Thus, the density of the fractal cloud can be interpreted as an indirect indicator of the quality of motor control: a high density suggests unstable and dispersed movement, whereas a reduced density indicates a smoother trajectory that is closer to the reference.

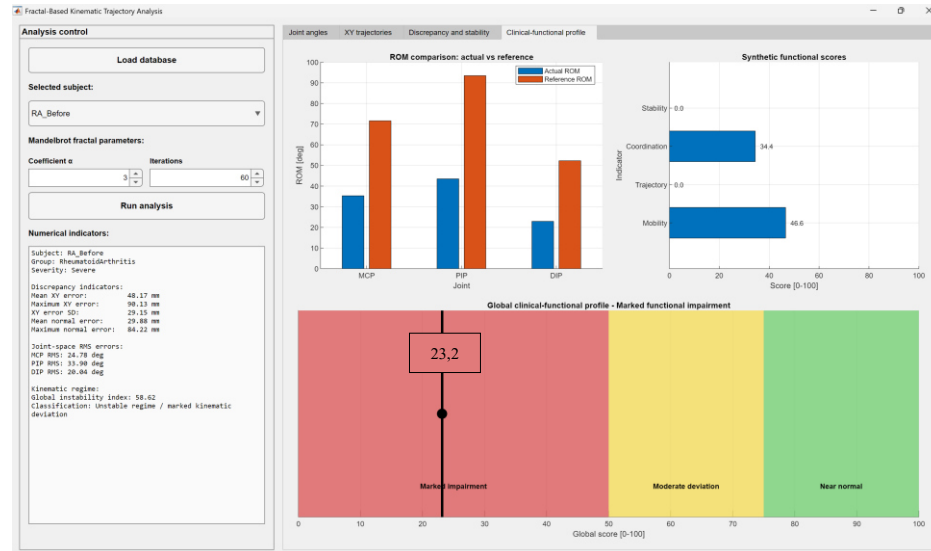


Fig. 3 – Clinical-functional profile of the patient before the kinetic rehabilitation program.

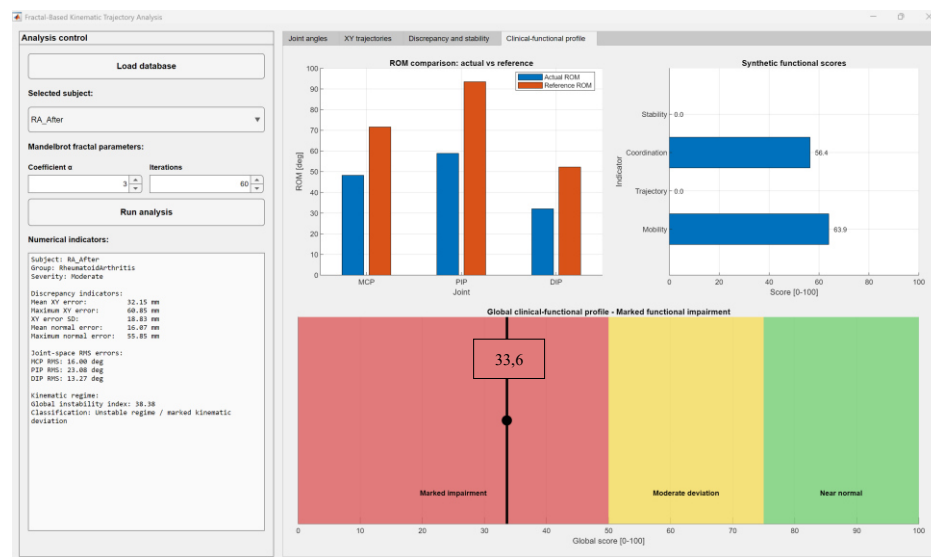


Fig. 4 – Clinical-functional profile of the patient after the kinetic rehabilitation program.

The numerical indicators provided by the interface support this visual interpretation. In the initial assessment, the global kinematic instability index had a value of approximately 54.62, whereas after the kinetic rehabilitation program it decreased to approximately 39.38. This reduction indicates a decrease in the

global instability of movement and an improvement in trajectory regularity. In addition, the global clinical-functional profile increased from 23.2 before the kinetic rehabilitation program to 32.6 after the intervention. Although both values remain within the range of marked functional impairment, the increase in the score indicates measurable functional progress.

The comparative analysis of movement amplitudes highlights an increase in ROM values after the kinetic rehabilitation program. In the bar charts, the patient's actual ROM values remain lower than the reference values; however, the difference between the initial and final states indicates functional improvement. The increase in amplitude is visible for all three analyzed components, suggesting that the kinetic intervention contributed to improving the patient's ability to perform the movement.

The functional scores presented in the interface provide an integrated interpretation of movement by combining several dimensions: mobility, trajectory, coordination, and stability. In the initial assessment, the scores associated with trajectory and stability are reduced, indicating a significant deviation from the physiological model and substantial movement instability. After the kinetic rehabilitation program, the mobility score increases to approximately 67.9, and the coordination score to approximately 56.4, compared with the lower initial values. This evolution suggests that the patient acquired an improved ability to perform the movement, with more efficient coordination of the involved segments.

The global clinical-functional profile, represented on a 0–100 scale, is useful for the physiotherapist because it rapidly summarizes the results in a visual form that is easy to interpret. The red zone indicates marked functional impairment, the yellow zone indicates moderate deviation, and the green zone suggests movement close to normal. In the analyzed case, the patient's score remains within the marked impairment zone both before and after the kinetic rehabilitation program; however, the shift from 23.2 to 32.6 indicates a tendency toward improvement. This representation is important because it allows the specialist to observe the patient's progress without separately interpreting all curves, errors, and automatically calculated indices.

From the physiotherapist's perspective, the results are relevant not only through the numerical values obtained, but also through the visual localization of deviations. Superimposing the real trajectory onto the reference trajectory and the fractal cloud allows the identification of movement segments in which the patient loses kinematic control. These regions may guide the adaptation of the rehabilitation program by introducing stabilization exercises, motor control training, progressive mobilization, or correction of compensatory mechanisms. In this regard, the graphical interface functions not only as an analytical tool, but also as decision-support for the individualization of the physiotherapeutic intervention.

Another important aspect is the differentiation between actual functional deviation and possible experimental error. If the patient's trajectory gradually and coherently exceeds the fractal tolerance region, the deviation may be interpreted as the expression of a biomechanical limitation or a compensatory strategy. Conversely, abrupt, isolated exceedances that are not correlated with the rest of the movement may suggest acquisition artifacts, such as sensor noise, unstable marker positioning, or variations in the experimental protocol. Therefore, fractal modeling contributes to data validation and reduces the risk of erroneous trajectory interpretation.

The results support the usefulness of the fractal cloud as a tool for evaluating the response to the kinetic rehabilitation program. A reduction in trajectory dispersion relative to the fractal region, associated with a decrease in the instability index and an increase in the clinical-functional score, indicates a favorable evolution. However, the persistence of the global score within the range of marked impairment shows that the patient still presents significant functional limitations. This observation is clinically relevant because it demonstrates that progress should not be interpreted solely through an increase in range of motion, but also through improvements in trajectory regularity, stability, and quality. By integrating the reference trajectory, the fractal cloud, instability indices, and the clinical-functional profile, the proposed method provides a more comprehensive perspective on movement than the isolated analysis of joint amplitude. It enables the simultaneous assessment of trajectory shape, local variability, global stability, and possible experimental artifacts. For this reason, the fractal model based on Mandelbrot dynamics can be considered a useful tool for the objective evaluation of recovery and for supporting physiotherapeutic decision-making in patients with rheumatoid arthritis.

4. Conclusions

The proposed mathematical model introduces a unified framework for the analysis of kinematic perturbations associated with rheumatoid arthritis, using Mandelbrot fractal dynamics to describe joint instability and the dispersion of movement trajectories. The integration of joint kinematics with fractal modeling enables the simultaneous representation of angular deviations, neuromuscular instability, and biomechanical variability experimentally observed in patients with rheumatoid involvement.

The model results highlight that the fractal cloud represents a geometric signature of functional joint degradation. The increase in fractal density and dispersion is directly correlated with the amplification of joint instability, kinematic stiffness, and loss of fine motor control. Thus, the fractal parameters introduced into the model allow the objective quantification of the severity of biomechanical impairment and the differentiation between physiological and pathological trajectories.

The biomechanical Mandelbrot coefficient introduced in the model controls both the lateral dispersion of the trajectory and the density of the fractal cloud, providing a direct link between the geometric behavior of the trajectory and the patient's biomechanical parameters. As the joint instability index and the dynamic variations of movement increase, the model generates dense and self-similar fractal structures, similar to the perturbations experimentally observed in patients with severe rheumatoid arthritis.

The model also allows the identification and separation of experimental errors from actual biomechanical perturbations through the use of fractal anomaly indices and the spatial distribution of the fractal cloud. This feature provides the possibility of developing robust computer-assisted kinematic analysis systems.

Overall, the comparative analysis performed before and after the implementation of the kinetic rehabilitation program demonstrates that the program produced a measurable improvement in kinematic behavior, reflected by the reduction of global instability, the increase in functional score, and the approximation of the real trajectory to the fractal domain associated with the healthy reference. However, the results indicate partial recovery rather than complete normalization of movement. This differentiation is essential in the assessment of patients with rheumatoid arthritis, as it allows the quantification of functional progress, the identification of persistent limitations, and the adjustment of therapeutic objectives.

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ANALIZA PERTURBAȚIILOR TRAIECTORIILOR CINEMATICE UTILIZÂND
DINAMICA FRACTALĂ PENTRU DETECTAREA ERORILOR EXPERIMENTALE
LA PACIENȚII CU ARTRITĂ REUMATOIDĂ

(Rezumat)

În contextul biomecanicii degetelor și al dezvoltării sistemelor biomecatronice, traiectoriile mișcărilor articulare prezintă variații neliniare și instabilități locale care nu pot fi descrise întotdeauna adecvat prin modele deterministe clasice; prin urmare, acest studiu propune un cadru matematic bazat pe introducerea unor perturbații fractale controlate, generate prin dinamica iterativă de tip Mandelbrot. Seriile temporale ale unghiurilor articulare MCP, PIP și DIP au fost procesate prin intermediul unei interfețe grafice dezvoltate în MATLAB, destinată analizei cinematice a degetului index, modelat ca lanț cinematic serial cu trei grade de libertate, permițând reconstruirea traiectoriei falangei distale și compararea acesteia cu o traiectorie de referință derivată din datele subiecților sănătoși. Prin calculul automat al erorilor carteziene, al deviațiilor normale și al discrepanțelor în spațiul articular, metoda propusă evidențiază variabilitatea mișcării și instabilitățile locale, contribuind la validarea datelor și la optimizarea evaluării biomecanice obiective în artrita reumatoidă.