

A DYNAMICAL STUDY OF AN INERTIAL TRANSPORT MECHANISM WITH RECTILINIAR MOTION

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Abstract. The paper deals with an application of a short transfer mechanism. Translating motion of an object on a horizontal support in a certain direction is due to the inertial force development by support plane motion. The support plane has rectilinear oscillatory movement given by a plane mechanism (shaping mechanism). The final scope of this study is to describe the object motion in horizontal support plane. The problem is about relative motion dynamics on a rectilinear trajectory.

Key words: shaping mechanism, relative motion, dynamics.

1. General considerations

The inertial short transfer mechanism with their elements shows as in Fig. 1. The driving element of the mechanism is the crank. Electric actuator (DC), rotates crank, by a wormed transmission.

In this paper, the movement of the object (W), has been studied. Dynamical study of this movement is about of relative motion, that is the

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sliding motion of the object (W) on the support plane. The curvilinear motion of the support plane (4) is the transportation movement (Corduneanu *et al.*, 2018).

It supposes, the constructive data of mechanism are known, and also the cranks angular velocity and friction coefficient of the object (W) on the support plane. The object (W) is considered of cuboid shape rotates crank, by a wormed transmission.

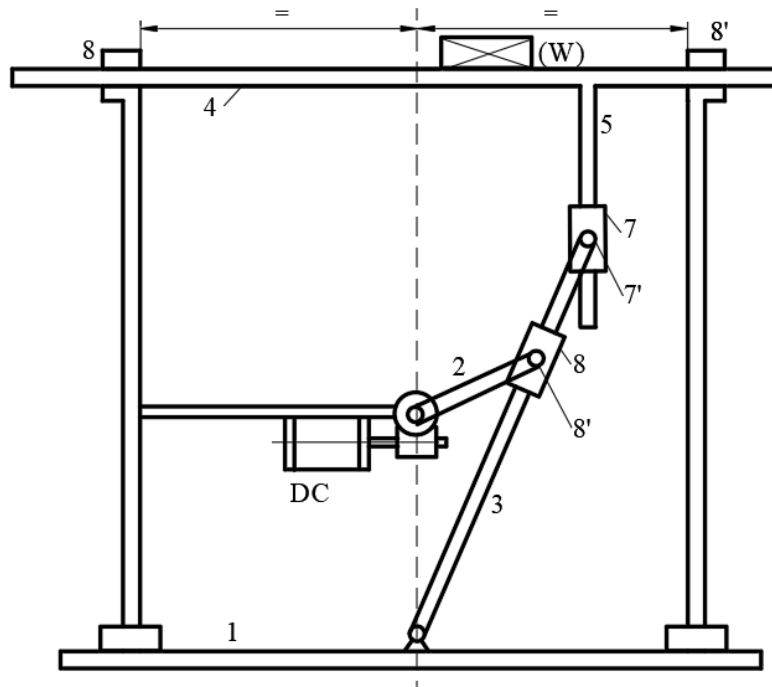


Fig. 1 – The short transfer mechanism, 1 – fixed support, 2 – driving crank, 3 – rocker, 4 – sitting plane, 5 – rod of the sitting plane, 6 – sliding joint between (2) and (3). 6' – revolute joint between (2) and (6), 7 – sliding joint between (3) and (5), 7' – revolute joint between (3) and (7), 8, 8' – sliding guides, 9 – rigid body (W) .

2. Initial data

A mechanical model of the mechanism has been considered (Fig. 2). It considers a mechanical model of the driving four bar linkage (Fig. 2). There: A – cranks revolute pair, O – rocker revolute pair, B – crank/rocker pair, C – rocker/sitting plane joint, Q – reference point of the sitting plane.

As hypothesis, the following parameters are established:

a – distance between fixed pairs, $a = OA$; r – length of the crank, $r = AB$, $r < a$;
 b – length of the rocker, $b = OC$, $b > a + r$.

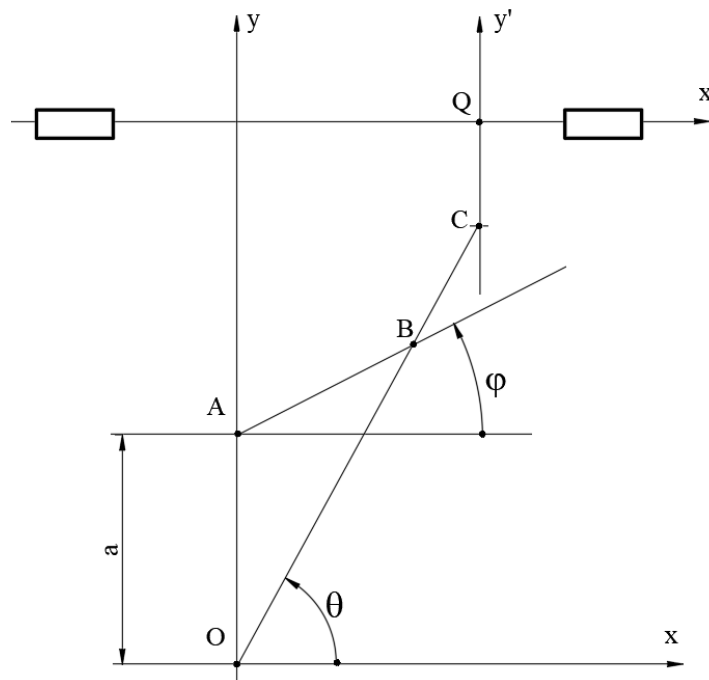


Fig. 2 – Mechanical model of the mechanism.

The geometrical model is related to a coordinate system, Oxy and to a mobile one $Qx'y'$ with horizontal axis Ox , Ox' and vertical axis Oy , Oy' .

It marks angle φ of the crank and θ the angle of the rocker, measured in direct sense from the Ox axis.

3. Statement of the problem

According to Fig. 2, the following relationship can be written:

$$\cot \theta = \frac{r \cdot \cos \varphi}{a + r \cdot \sin \varphi}. \quad (1)$$

For a full crank rotation, we have

$$\varphi \in [0, 2\pi), \quad (2)$$

respectively,

$$\theta \in \left[\frac{\pi}{2} - \arcsin \frac{r}{a}, \frac{\pi}{2} + \arcsin \frac{r}{a} \right]. \quad (3)$$

The cranks angular velocity is supposed to be constant, resulting the position angle as follows:

$$\varphi(t) = \omega_0(t), \quad t \geq 0, \quad (4)$$

where ω_0 is the constant angular velocity and t denotes the time.

Deriving expression (1) with respect the time, accordingly to relationship (4) it obtains

$$\frac{\dot{\theta}}{\sin^2 \theta} = \frac{\omega_0 r (a \cdot \sin \varphi + r)}{(a + r \cdot \sin \varphi)^2}, \quad (5)$$

or yet

$$\dot{\theta} = \frac{\omega_0 r (a \cdot \sin \varphi + r)}{(a + r \cdot \sin \varphi)^2} \cdot (1 + \cot^2 \theta)^{-1}. \quad (6)$$

Replacing (1) in (6) it obtains angular velocity of the rocker,

$$\dot{\theta} = \frac{\omega_0 r (a \cdot \sin \varphi + r)}{a^2 + r^2 + 2ar \cdot \sin \varphi}. \quad (7)$$

Deriving equation (7) with respect the time, according to relationship (4) it obtains angular acceleration of the rocker,

$$\ddot{\theta} = \frac{ar(a^2 - r^2) \cos \varphi}{(a^2 + r^2 + 2ar \cdot \sin \varphi)^2} \cdot \omega_0^2. \quad (8)$$

Taking in consideration (m) the mass of rigid body (W), the dynamic equation of its relative motion is (Mangeron *et al.*, 1978):

$$m\bar{a}_r = \bar{G} + \bar{N} + \bar{F}_f + \bar{F}_{in}, \quad (9)$$

with the following notations:

$\bar{a}_r$ - relative acceleration (horizontal);

$\bar{G}$ - gravitational force (vertical);

$\bar{N}$ - normal reaction (vertical);

$\bar{F}_f$ - friction force (horizontal);

$\bar{F}_{in}$ - inertial force (horizontal).

Projecting equation (9) on the axis of the mobile system, it obtains,

$$\begin{aligned} m\ddot{x}' &= -\mu N \cdot \text{sign}(\dot{x}') - m\ddot{x}_Q, \\ 0 &= -mg + N \end{aligned} \quad (10)$$

where, μ is the friction coefficient and g – gravitational acceleration, or yet

$$\ddot{x}' = -\mu g \cdot \text{sign}(\dot{x}') - \ddot{x}_Q, \quad (11)$$

i.e., equation of the relative motion, with

$$\dot{x}' = \frac{dx'}{dt}, \quad \ddot{x}' = \frac{d^2x'}{dt^2}, \quad \ddot{x}_Q = \frac{d^2x_Q}{dt^2}. \quad (12)$$

According to Fig. 2, we have

$$x_Q = b \cdot \cos \theta, \quad (13)$$

from where it obtains by deriving with respect the time

$$\ddot{x}' = -b\ddot{\theta} \sin \theta \quad (14)$$

$$\ddot{x}_Q = -b(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta). \quad (15)$$

Replacing (15) in (11) equation of relative motion can be written

$$\ddot{x}' = -\mu g \cdot \text{sign}(\dot{x}') + b(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta). \quad (16)$$

According to relationship (1) we have,

$$\sin \theta = \frac{a + r \cdot \sin \varphi}{\sqrt{a^2 + r^2 + 2ar \cdot \sin \varphi}}, \quad (17)$$

$$\cos \theta = \frac{r \cdot \cos \varphi}{\sqrt{a^2 + r^2 + 2ar \cdot \sin \varphi}}, \quad (18)$$

Replacing (7), (8), (17) and (18) in (16) it obtains the equation of the relative motion:

$$\ddot{x}' = -\mu g \cdot \text{sign}(\dot{x}') + \frac{abr(a^2 - b^2)(a + r \cdot \sin \varphi) \cos \varphi}{(a^2 + r^2 + 2ar \sin \varphi)^{\frac{5}{2}}} \cdot \omega_0^2 +$$

$$+ \frac{br^3(a \cdot \sin \varphi + r)^2}{(a^2 + r^2 + 2ar \sin \varphi)^{\frac{5}{2}}} \cdot \omega_0^2, \quad (19)$$

where φ is given by relationship (4).

4. Numerical results and discussions

To highlight the motion properties, we integrate the basic equation (10) using the software “odesolve” in a computational program. For integration, the equation of the relative motion (19) is written in the equivalent form

$$\frac{ds(t)}{dt} = v(t)$$

$$\frac{dv(t)}{dt} = -\mu g \frac{v(t)}{|v(t)|} + b \sin(\theta(t)) \left[\dot{\theta}^2 \cot(\theta(t)) + \ddot{\theta}(t) \right] \quad (20)$$

where $s(t) = x'(t)$ is the space, $v(t) = \frac{ds(t)}{dt} = \frac{dx'(t)}{dt}$ is the relative velocity of the body, the angular velocity $\dot{\theta}(t)$ and the angular acceleration $\ddot{\theta}(t)$ are given by the equations (7) and (8) respectively (Duca *et al.*, 2003).

The system (20) of the ordinary differential equations is considered along the initial conditions $s(0) = 0$, $v(0) = 0$. For numerical simulations, we used the following values of the parameters: $a = 0.25[m]$, $b = 0.40[m]$, $r = 0.10[m]$, $g = 9.81[m/s^2]$. For the rotation of the crank, we considered the cases $n_0 \in \{40, 60, 90\}[rpm]$. The angular velocity $\omega_0 = \frac{\pi n_0}{30}[s^{-1}]$, and the period is $T = \frac{2\pi}{\omega_0} = \frac{60}{n_0}[s]$.

Numerical values used for curves in Fig. 3 and Fig. 4 have been obtained for $n_0 = 40[rpm]$, therefore the cranks angular velocity $\omega_0 = 4.189[s^{-1}]$, and the motion period $T = 1.5[s]$. Fig. 3 and Fig. 4 present

the relative velocity and the space travelled by the body in the time interval corresponding to four rotation periods of the crank.

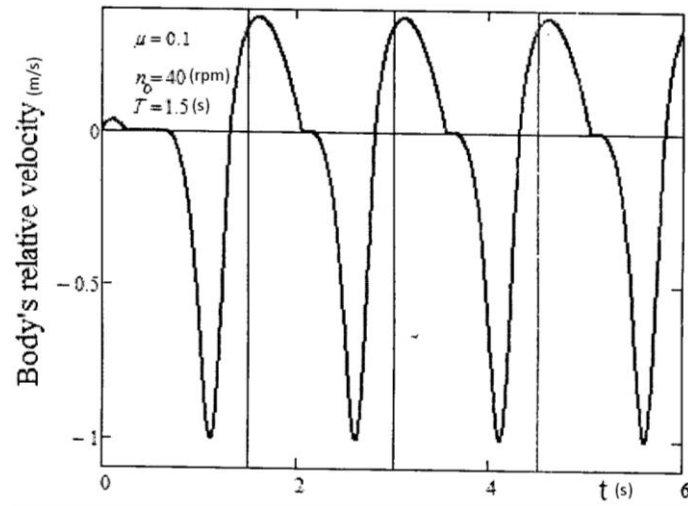


Fig. 3 – The profile of the relative velocity of the body for $n_0 = 40[rpm]$.

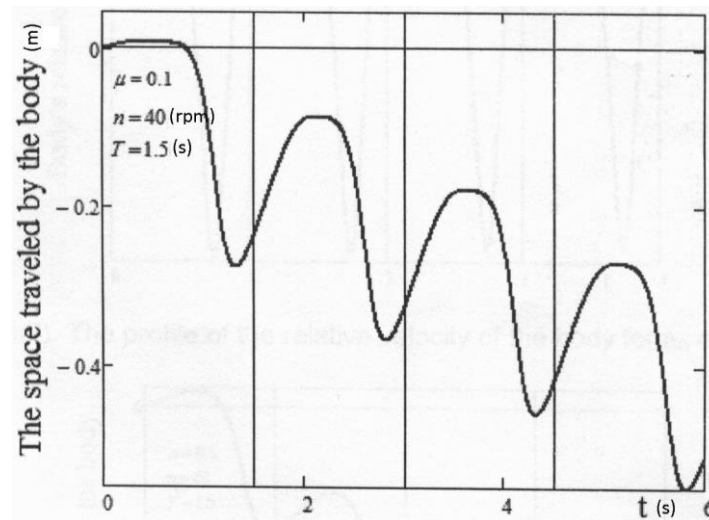


Fig. 4 – The profile of the space travelled by the body for $n_0 = 40[rpm]$.

As expected, except for the first period of the movement, the relative speed has a periodic evolution in time. In the first period of movement, there is a period of time in which the relative speed of the body equals zero. In the following periods of the movement, the time intervals in which the relative

speed equals zero is smaller than those of the first period. This evolution in time is due to the fact that the movement starts with zero relative velocity but, starting with the second period of the movement, the relative velocity of the beginning of the period is different from zero. For example, in the considered case we have $v(1.5) = v(3.0) = v(4.5) = v(6.0) = 0.34[m/s]$. The maximum value of the relative velocity is $0.375[m/s]$, and the minimum one is $-1.004[m/s]$.

Fig. 4 presents the time variation of the space traveled by the mobile body. Obviously, the spaces profile is in accordance with the relative velocity time variation given by the Fig. 3. The absolute value of the space increases after each time period T . Thus, in the analyzed here case, we have $|s(1.5)| = 0.237$, $|s(3.0)| = 0.330$, $|s(4.5)| = 0.422$, $|s(6.0)| = 0.515$.

Fig. 5 and Fig. 6 show the graphs of the relative speed and the space travelled by the transported body for the motor crank speed $n_0 = 60[rpm]$. Lets note that at this crank speed there is no longer a time interval in which the relative speed is equal to zero. Moreover, at the end of the period with order number $k, k > 1$, the relative speed has a different value, compared to the previous period. In the studied case we have $v(1) = 0.194$, $v(2) = 0.227$, $v(3) = 0.237$. The maximum and minimum values for each period are $t \in [0, 1]$, $v_{\max} = 0.194$, $v_{\min} = -1.947$; $t \in [1, 2]$, $v_{\max} = 0.388$, $v_{\min} = -1.898$; $t \in [2, 3]$, $v_{\max} = 0.421$, $v_{\min} = -1.885$; $t \in [3, 4]$, $v_{\max} = 0.431$, $v_{\min} = -1.881$.

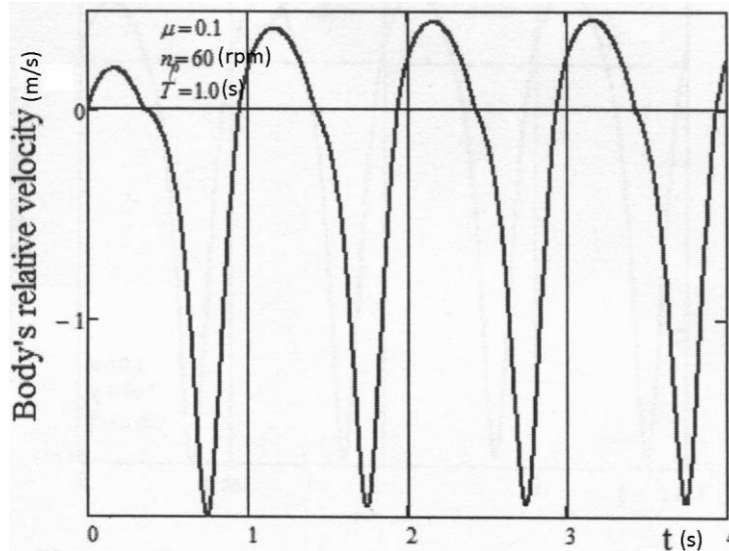


Fig. 5 – The profile of the relative velocity of the body for $n_0 = 60[rpm]$.

The time variation of the relative velocity leads to the increase in the space travelled by the moving body. Thus, the absolute values of the space at the end of each period are $|s(1)| = 0.399$, $|s(2)| = 0.695$, $|s(3)| = 0.969$, $|s(4)| = 1.257$.

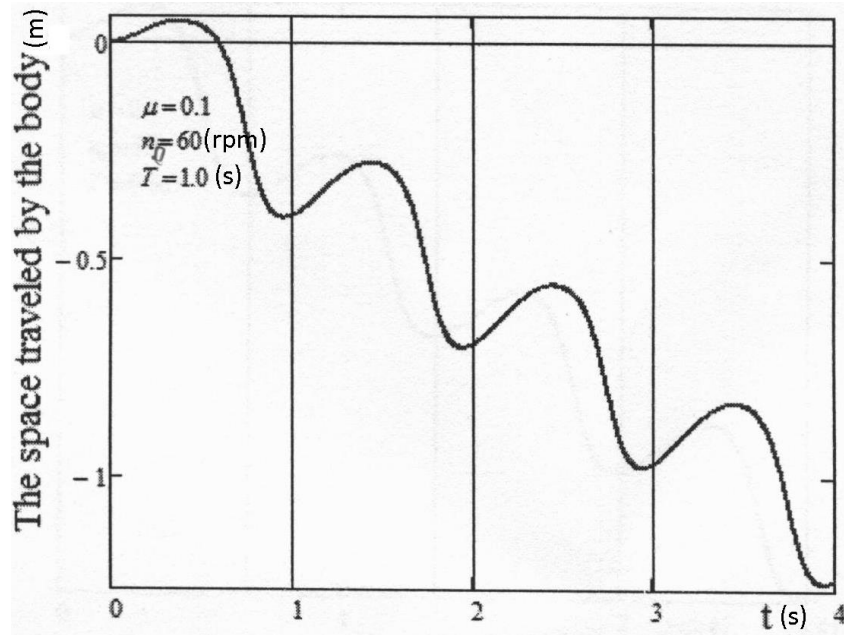


Fig. 6 – The profile of the space travelled by the body for $n_0 = 60[\text{rpm}]$.

The profiles of the relative velocity and the space traveled by the mobile body at the speed $n_0 = 90[\text{rpm}]$ are shown in Fig. 7 and Fig. 8 respectively. In this case, a significant increase in the absolute values of relative velocity and space is observed. The absolute values of the relative velocity are included in the range $0.57 \leq |v(t)| \leq 3.08$. The absolute values of space at the end of each period are given by $|s(0.667)| = 0.402$, $|s(1.334)| = 0.765$, $|s(2)| = 1.110$, $|s(2.667)| = 1.441$.

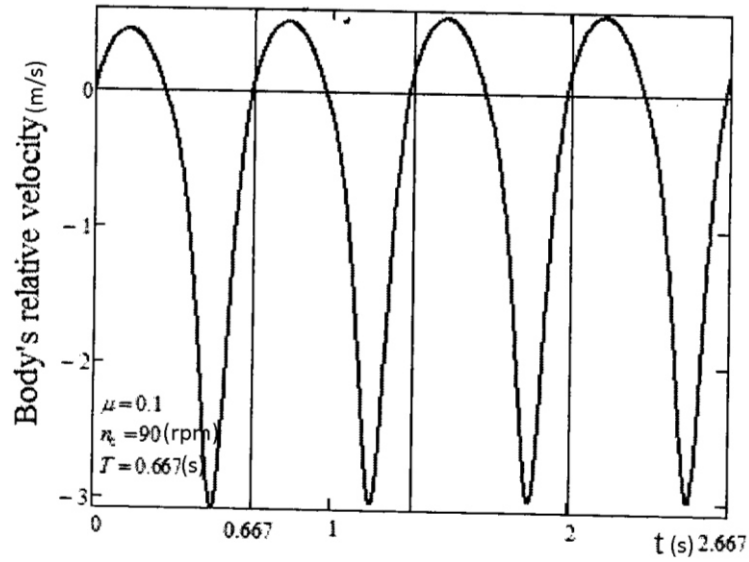


Fig. 7 – The profile of the relative velocity of the body for $n_0 = 90[rpm]$.

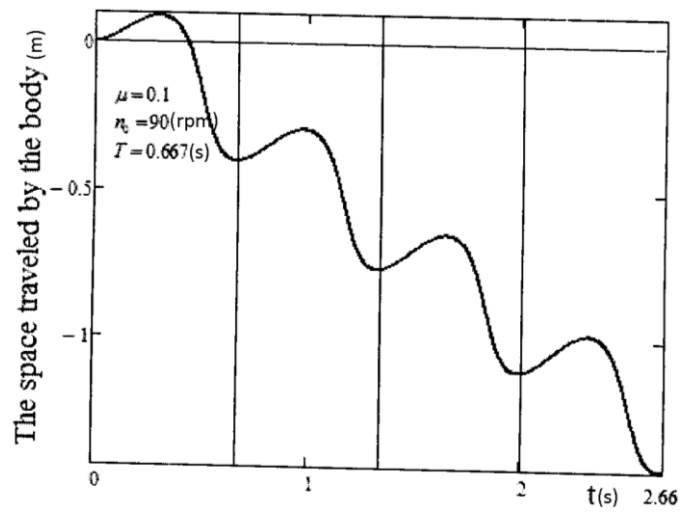


Fig. 8 – The profile of the space travelled by the body for $n_0 = 90[rpm]$.

The influence of the coefficient of friction on the movement of the body is presented in Fig. 9 and Fig. 10. We considered the values $\mu \in \{0.05; 0.10; 0.15\}$ for the coefficient of friction and for the rotation of the crank $n_0 = 30[rpm]$. As expected, at low values of the friction coefficient, the

relative velocity and the space travelled have significantly higher absolute values than for high values of the friction force. Also, it is important to point out that for small crank rotation values and high friction coefficient, the time intervals in which the mobile is at relative rest, are quite large.

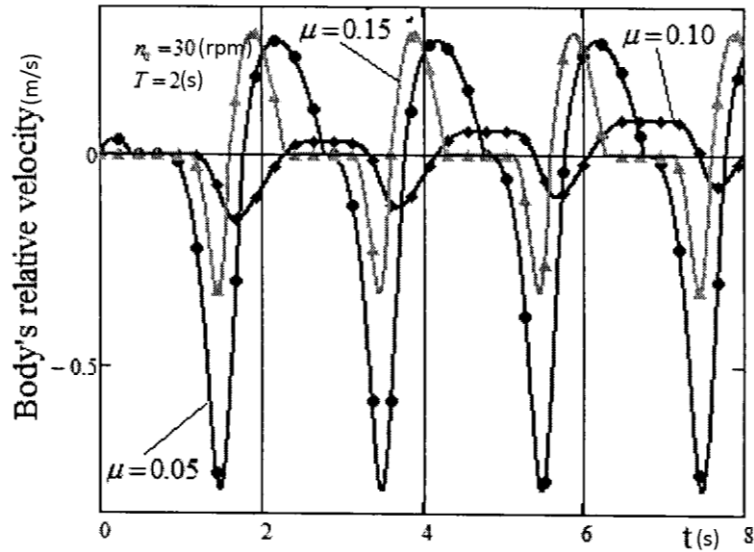


Fig. 9 – The profile of the relative velocity of the body for $n_0 = 30[rpm]$.

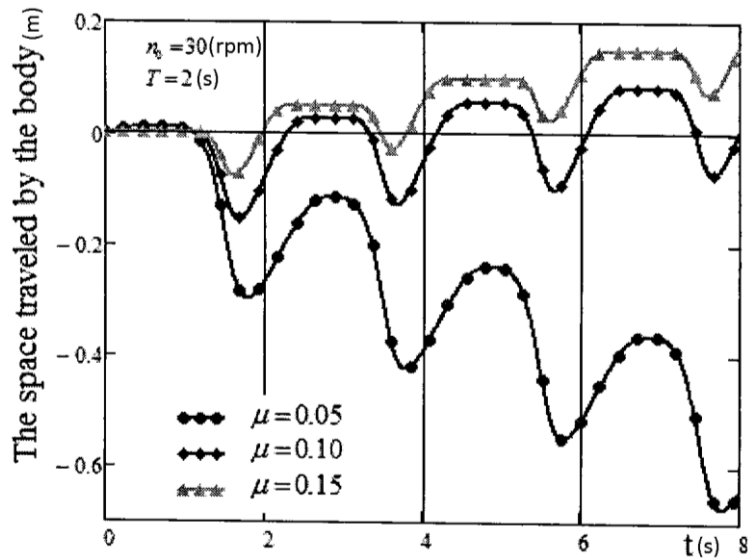


Fig. 10 – The profile of the space travelled by the body for $n_0 = 30[rpm]$.

5. Conclusions

The paper extends the methodology exposed in (Corduneanu *et al.*, 2019) to another transporting mechanism, with a bigger degree of complexity.

Different driving velocities were taken into account 30, 40, 60, 90 [rpm] and for each of them, the relative velocity and displacement of the mobile was studied, for different values of friction coefficients.

This study is valuable in designing of auxiliary systems, deserving robotics lines in different technical applications.

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STUDIUL DINAMIC AL UNUI MECANISM DE TRANSPORT INERȚIAL CU MIȘCARE RECTILINIE

(Rezumat)

Lucrarea prezintă o analiză cinematică și dinamică a unui mecanism de clasa 2, ordin 2 care poate fi tratat ca un mecanism complex format prin înserierea a două grupe structural 0/2/2 una de aspect 3 și alta de aspect 5. Pentru derularea studiului s-au folosit parametrii constructivi specificați în ultimul paragraph al lucrării. Ca rezultat al integrării numerice a ecuației de mișcare s-au obținut diferite diagrame cinematice specifice unor valori ale coeficientului de frecare dintre rigidul a cărui mișcare o studiem și suprafața de reazem.