BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 70 (74), Numărul 4, 2024
Secţia
CONSTRUCŢII DE MAŞINI
DOI:10.2478/bipcm-2024-0022

MODELING OF VELOCITIES AND ACCELERATIONS IN SLIDING PROCESSES

BY

CEZARA MĂRIUCA GAVRIL*

“Gheorghe Asachi” Technical University of Iași,
Faculty of Mechanical Engineering

Received: January 24, 2025
Accepted for publication: February 3, 2025

Abstract. This study aims to develop a mathematical model of the stick-slip phenomenon for various materials employed in sliding tribosystems. Specifically, it focuses on modeling the slip phase of a sliding cylinder, determining its velocity and acceleration. The methodology is illustrated through an example involving the first slip event in a steel-on-steel tribosystem. The modeling framework is based on extensive experimental investigations conducted in the Tribology Laboratory of the IMMR Department at the Faculty of Mechanics. Existing literature provides an overview of different tribosystems and experimental setups used to analyze stick-slip behavior. In this study, a simple sliding tribosystem was utilized, comprising a weight (oscillating mass) on a flat surface moving at a constant translational velocity. The weight was connected to a force sensor via a tensile helical spring. The system was integrated with a CETR UMT-2 Tribometer, enabling controlled variation of translational speed (0.02–8 mm/s) and real-time data acquisition of tangential force, time, velocity, and displacement.

Keywords: stick-slip, mathematical model, velocities, accelerations, cylinder.

---

*Corresponding author; *e-mail*: cezara-mariuca.gavril@academic.tuiasi.ro

1. Introduction

The stick-slip phenomenon occurs in numerous mechanical systems and is perceived through noise and vibrations, manifesting in various practical scenarios such as the sudden braking of vehicle or train wheels, vibrations generated in the wheel-road tribosystem during abrupt stops or rapid accelerations, and oscillatory motion in precision machinery. This dynamic instability arises due to alternating phases of static and kinetic friction, leading to undesirable mechanical effects, including wear, energy loss, and component degradation (Biswas *et al.*, 2019).

Given its detrimental impact on mechanical performance and durability, minimizing or eliminating the stick-slip phenomenon is a critical concern in engineering and tribology. Researchers have explored various mitigation strategies, including optimizing material pairings to reduce frictional inconsistencies, incorporating additive lubricants to smooth transitions between stick and slip phases, and increasing system rigidity to suppress unwanted oscillations. Additionally, modifying operating conditions, such as increasing sliding velocities or adjusting surface textures, has been shown to influence frictional behavior, reducing stick-slip effects (Fortes and Martins, 1990).

Beyond mechanical applications, stick-slip behavior is also relevant in fields such as robotics, where precise motion control is essential, and in geophysics, where it plays a fundamental role in earthquake dynamics. Consequently, understanding and controlling this phenomenon remain active areas of research, with implications across multiple scientific and industrial domains (Rice and Ruina, 1983).

2. The variation of speed and acceleration in the stick-slip process

The following methodology describes the systematic approach used to determine the temporal variations of velocity and acceleration for a specific practical case. This process follows a structured sequence of operations designed to capture, analyze, and interpret the dynamic behavior of the system during the sliding phase.

The experimental procedure begins with conducting the test and systematically recording the variation of the tangential force Fx over time. This step is crucial in understanding the dynamic interaction between the sliding components and quantifying the forces acting during the stick-slip phenomenon.

Figure 1 presents the time-dependent variation of the tangential force Fx, as recorded by the CETR-UMT 2 tribometer software. The graph provides a comprehensive depiction of the force fluctuations throughout the sliding process, effectively capturing both the stick and slip phases characteristic of this tribological phenomenon.

A careful analysis reveals two distinct mechanisms governing the interaction between the cylinder and the plate: adhesion, represented by the segments OA1, B1A2, B2A3, etc., and slipping, which occurs along the segments A1B1, A2B2, and so forth. The critical points A1, A2, A3, etc., correspond to the maximum values of the static friction coefficients. These coefficients are determined by the ratio of the tangential forces Fx associated with these peaks (FA1, FA2, etc.) to the weight of the cylinder, G. This analysis provides insight into the interplay between adhesion and frictional resistance, essential for understanding the tribological behavior of the system.

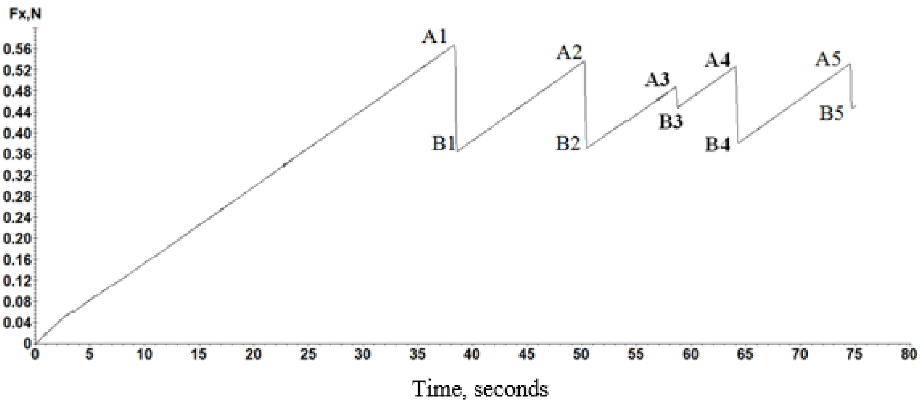

Fig. 1 – The variation of the tangential force Fx over time.

All data acquired from the force sensor and linear mass is continuously transmitted to the tribometer software, where it is processed and recorded in real time. This data includes variations in tangential force, displacement, and frictional behavior, providing a comprehensive dataset for further analysis. Once recorded, the information is exported to a separate Excel program, independent of the tribometer system, enabling additional post-processing, data visualization, and statistical analysis. The use of an external program allows for greater flexibility in handling the raw data, facilitating advanced computations, comparative assessments, and the generation of detailed reports for further investigation.

Figure 2 illustrates the following parameters: the tangential force Fx, the distance traveled by the linear mass Y, the displacement velocity of the platform V3, and the time sequences T at which the measurements are recorded. The variation in the distance traveled by the cylinder on the table surface, denoted as y(t), is determined by the ratio of the tangential force Fx to the elastic constant of the spring k.

	A	B	D	F	I	J	K	L
14	Data Cout-8/07/2023							
15								
16	Fx	Y	V3	T		T	y(t)=Fx/k	
17	N	m	mm/sec	sec		sec	m	
18								
19	0.001	0	0	0		0	1.35135E-05	
20	0.001	0	0	0.0103		0.0103	1.35135E-05	
21	0.001	0	0	0.02		0.02	1.35135E-05	
22	0.001	0	0	0.0303		0.0303	1.35135E-05	
23	0	0	0	0.041		0.041	0	
24	0.001	0	0	0.0513		0.0513	1.35135E-05	
25	0.001	0	0	0.061		0.061	1.35135E-05	
26	0.001	0	0	0.0713		0.0713	1.35135E-05	
27	0.001	0	0	0.0815		0.0815	1.35135E-05	
28	0	0	0.001	0.0923		0.0923	0	
29	0.001	0	0.002	0.1021		0.1021	1.35135E-05	
30	0.001	0	0.002	0.1123		0.1123	1.35135E-05	
31	0.001	0	0.002	0.1221		0.1221	1.35135E-05	
32	0.001	0	0.002	0.1323		0.1323	1.35135E-05	
33	0.001	0.002	0.011	0.1431		0.1431	1.35135E-05	
34	0.001	0.002	0.014	0.1533		0.1533	1.35135E-05	
35	0.001	0.002	0.014	0.1631		0.1631	1.35135E-05	
36	0.002	0.002	0.014	0.1733		0.1733	2.7027E-05	
37	0.002	0.002	0.015	0.1831		0.1831	2.7027E-05	
38	0.002	0.007	0.046	0.1934		0.1934	2.7027E-05	
39	0.002	0.009	0.058	0.2041		0.2041	2.7027E-05	
40	0.003	0.009	0.058	0.2144		0.2144	4.05405E-05	
41	0.002	0.009	0.058	0.2241		0.2241	2.7027E-05	
42	0.003	0.009	0.061	0.2344		0.2344	4.05405E-05	
43	0.003	0.016	0.092	0.2441		0.2441	4.05405E-05	

Fig. 2 – Data collected from the Tribometer program.

These values are essential for generating the graph that illustrates the temporal evolution of the distance traveled by the weight on the moving platform. By analyzing the recorded data, it is possible to assess the system's dynamic behaviour, particularly the variations in displacement over time.

Specifically, Fig. 3 shows the displacement pattern over a 75-second interval, revealing a characteristic stick-slip motion. The graph depicts an initial steady increase in displacement until reaching peak points (A1, A2, A3, etc.), followed by sudden drops to lower points (B1, B2, B3, etc.), indicating slip events. This cyclic pattern highlights the interplay between frictional forces and applied motion, where the system alternates between adhesion and slipping phases.

The repeated stick-slip cycles suggest a periodic behavior influenced by elastic deformation and frictional resistance. The displacement variations provide valuable insights into the mechanical response of the system, aiding in the understanding of tribological interactions and their impact on motion control and material performance.

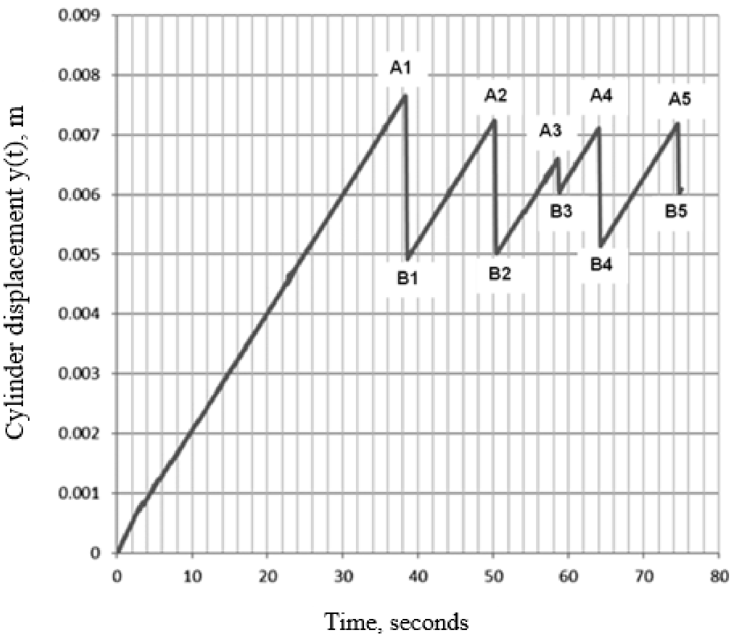

Fig. 3 − Variation of the displacement of the weight on the test stand table during the testing period.

In this stage, the variations in velocity and acceleration were analyzed for the first slip event. Since the sliding duration is very short, lasting only a few tenths of a second, the zero point was set at the moment when the weight begins its rapid sliding motion, transitioning from its maximum displacement to a lower position.

Figure 4 illustrates the displacement of the weight during this first slip event, showing a smooth but rapid decrease over approximately 0.2 seconds. This decline indicates the transition from static to kinetic friction, where the force acting on the weight overcomes static friction, initiating motion. The shape of the displacement curve provides insight into the velocity and acceleration behavior during the slip phase, which is crucial for understanding the system's kinematic response.

By examining this initial slip event, it is possible to assess the fundamental characteristics of the stick-slip phenomenon, including the rate of displacement change and the overall dynamic behaviour. This analysis contributes to a better understanding of frictional interactions and their influence on the motion of the weight, offering valuable data for tribological studies and mechanical system optimizations.

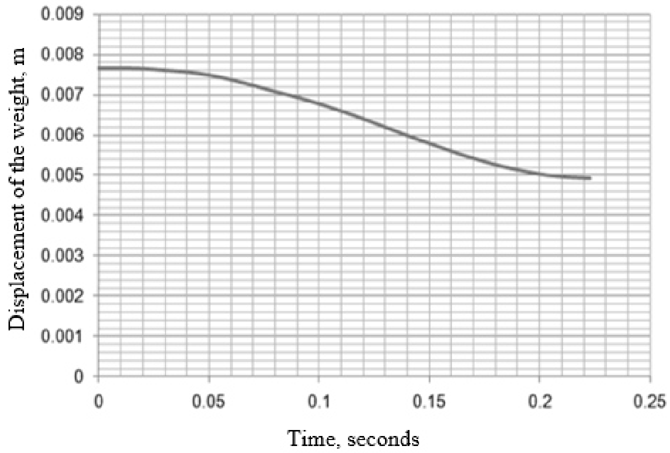

Fig. 4 − Variation of the weight displacement during the first slip event as a function of time, y(t).

Numerical differentiation was applied over the time interval in which the slip occurs, using the relation: $v_1(t) = \Delta y(t)/\Delta t$, where $\Delta y(t)$ represents the distance between two successive measurements, and $\Delta(t)$ is the time between two successive measurements. In Fig. 5, the dots represent the velocity values of the cylinder at each time interval, while the continuous curve represents the polynomial interpolation function.

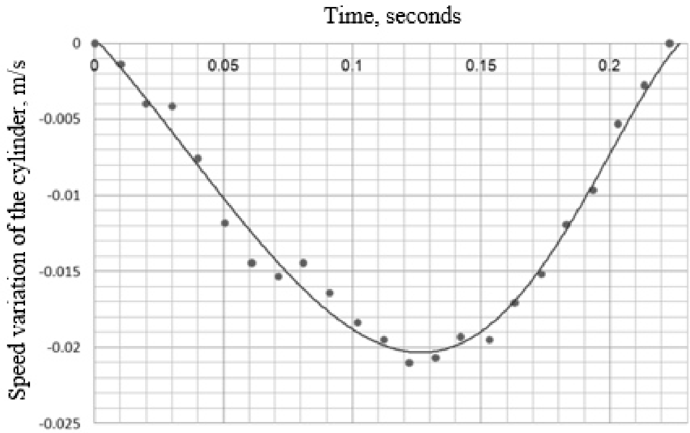

Fig. 5 − Variation of the cylinder velocity during the first slip event as a function of time.

The interpolation was performed using a 6th-degree polynomial function, with a deviation of $R^2 = 0.985$, resulting in the following equation:

$$v(t) = -1445.9t^6 + 652.16t^5 - 112.18t^4 + 17.913t^3 - 1.2877t^2 - 0.1802t + 0.0003 \quad (1)$$

where v(t) represents the variation of the cylinder's displacement velocity relative to a fixed system attached to the sensor pin.

To obtain the variation of acceleration during the slip event, Eq. (1) was differentiated, resulting in a 5th-degree polynomial. The graphical representation of this polynomial is shown in Fig. 6.

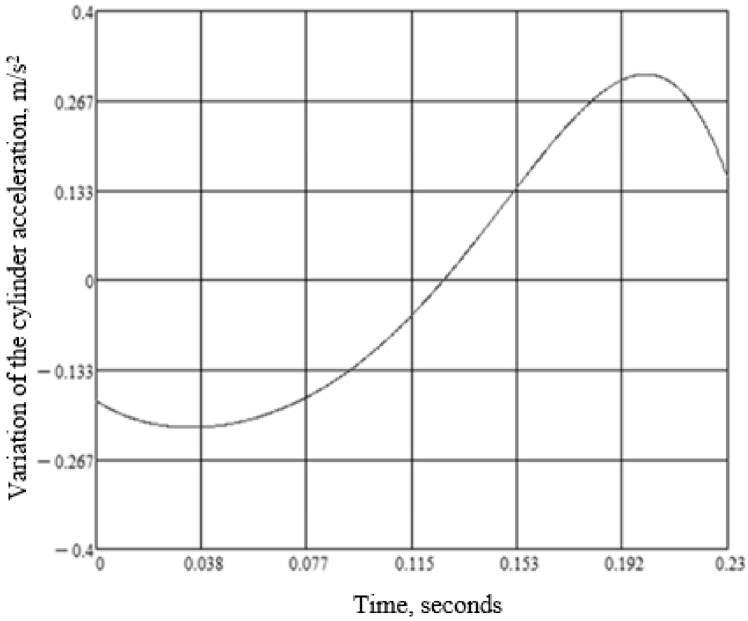

Fig. 6 − Variation of the weight acceleration during the first slip event as a function of time.

The acceleration variation curve in Fig. 6 crosses the zero value at 0.13 seconds from the moment the sliding begins, corresponding to the maximum point of the velocity v(t). Following the same methodology, the velocities and accelerations were determined for the next four slip events.

The next step involves comparing the analytically obtained accelerations with the measured values. Following the methodology for measuring accelerations during the cylinder's rapid sliding process, Fig. 7 presents the variations recorded throughout the test.

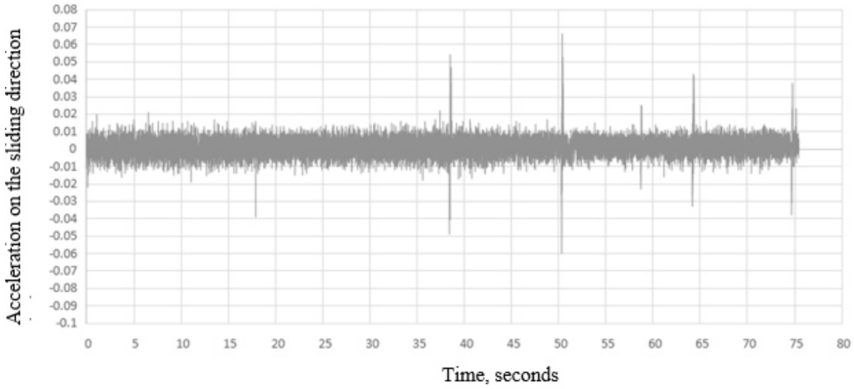

Fig. 7 – Variation of the experimentally determined acceleration of the weight as a function of time during the test.

In Fig.7, five acceleration variations can be observed, with limits within the range of (0.2–0.6) m/s², containing both negative and positive values. It is important to note that the acceleration values shown in Fig.7 are expressed relative to gravitational acceleration; therefore, the absolute values can be obtained by multiplying by g (9.81 m/s²). The diagram in Fig. 7 also shows background noise caused by vibrations produced by the tribometer's fan.

For the first slip event, which occurs 38.33 seconds after the start of the test, the measured acceleration variations during the slip are presented in Fig. 8, depicted as discrete points. The interpolation using a 5th-order polynomial is represented by the continuous curve, while the lines represent the average acceleration values calculated between two successive points. The acceleration values are given directly in m/s².

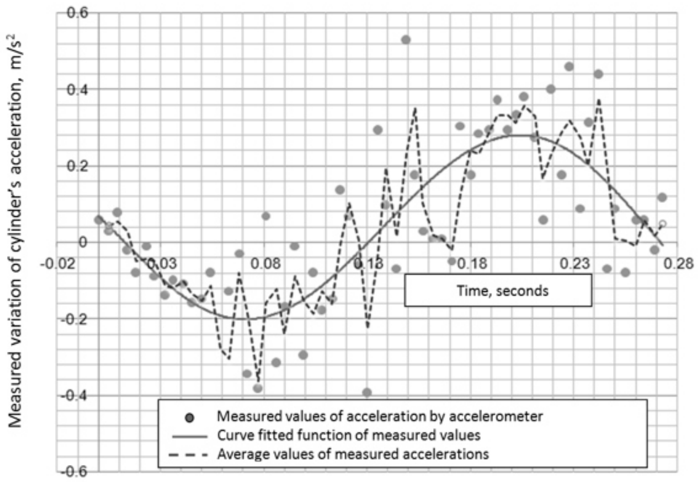

Fig. 8 – Variation of the measured accelerations during the first slip event, interpolation curve, and average acceleration values.

3. Conclusions

A Stick-Slip Dynamics and Frictional Behavior

The study confirms that the stick-slip phenomenon occurs over very short durations, typically between 0.15 and 0.3 seconds. During each slip event, the cylinder undergoes a rapid increase in velocity, reaching a peak before decelerating due to friction between the cylinder and the steel plate. This behavior is characteristic of sliding tribosystems and is influenced by the interaction of static and kinetic friction forces.

Experimental and Analytical Correlation

The experimentally obtained acceleration values align closely with the analytical results derived from polynomial modeling. The polynomial interpolation function provides an accurate representation of the variations in velocity and acceleration, validating the numerical differentiation approach. The consistency between theoretical and measured data highlights the effectiveness of the modeling methodology used in this research.

Impact of Tribosystem Parameters on Sliding Behavior

The results indicate that the magnitude of stick-slip effects depends on factors such as material properties, surface conditions, and applied forces. The study's setup, which included a weight connected to a force sensor via a tensile helical spring, successfully captured real-time variations in tangential force, displacement, and velocity, enabling precise analysis of the slip phase dynamics.

Influence of Sliding Speed on Stick-Slip Phenomenon

The tribometer tests, performed at varying translational speeds (0.02–8 mm/s), demonstrate that higher sliding velocities can influence the stick-slip behavior. Adjusting the speed can help mitigate the intensity of the stick-slip effect, which aligns with existing research suggesting that increasing velocity can reduce friction-induced oscillations.

Potential Applications and Further Research

The findings of this study have implications for engineering applications where minimizing stick-slip behavior is critical, such as precision machinery, transportation systems, and material testing. Further research could explore the effects of different material pairings, lubrication conditions, and external load variations on stick-slip characteristics. Additionally, refining the mathematical model to incorporate non-linear friction effects could enhance its predictive accuracy.

REFERENCES

Biswas S.K., Ghosh A., Sinha A.K., *An Experimental Study on Adhesion, Friction and Stick-Slip Phenomena*, Proceedings of the International Conference on Industrial Tribology (ICIT 2019), (2019).

Fortes M.A., Martins J.A.C., *Non-Linear Dynamics of Simple Elastic Systems Undergoing Stick-Slip Motion*, Journal of Applied Mechanics, vol. 57, no. 1 (1990).

Rice J.R., Ruina A.L., *The Occurrence of Stick-Slip Phenomenon*, Journal of Applied Mechanics, vol. 50, no. 2 (1983).

MODELAREA VITEZELOR ȘI A ACCELERAȚIILOR ÎN PROCESELE DE ALUNECARE

(Rezumat)

Acest studiu a avut ca scop să modeleze matematic fenomenul de stick-slip pentru mai multe tipuri de materiale utilizate în tribosistemele de alunecare. Această lucrare abordează modelarea alunecării unui cilindru în timpul fazei de slip, cu determinarea vitezei și a accelerației. S-a prezentat metodologia și un exemplu pentru primul salt dintr-un tribosistem de alunecare oțel pe oțel.

Modelarea s-a bazat pe numeroase experimente realizate în Laboratorul de Tribologie al Departamentului IMMR de la Facultatea de Mecanică.

Literatura de specialitate prezintă diferite tribosisteme și echipamente utilizate pentru studierea fenomenului stick-slip, astfel s-a utilizat un tribosistem simplu de alunecare, format dintr-o greutate (numită și masă oscilantă) plasată pe o suprafață plană care se mișca cu viteză constantă de translație. Greutatea a fost cuplată la un senzor de forțe printr-un arc elicoidal de tracțiune. Sistemul conceput a fost atașat la Tribometrul CETR UMT-2, care a permis varierea vitezei de translație a mesei tribometrului în intervalul 0,02 mm/s - 8 mm/s și colectarea unei serii de parametri în timp real: forța tangențială (Fx), timpul, viteza și deplasarea.