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The Overlapping Coefficient: A Decision-Theoretic Study

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SUMMARY

The overlapping coefficient (OVL) is a widely used measure for quantifying the similarity between probability distributions. Although its relationship with classification error has been studied in previous literature, its role within an overlap-augmented decision criterion merits further examination. This paper examines the connection between distributional overlap and classification error under a decision-theoretic framework. For two probability density functions, $f(x)$ and $g(x)$, the overlapping coefficient is defined as the integral of their pointwise minimum. Under equal prior probabilities and equal misclassification costs, the minimum achievable probability of classification error is one-half of the overlapping coefficient. This is an established result, which serves as a foundation for the present study rather than a new theorem. We also discuss the relationship between the overlapping coefficient and total variation distance, and introduce an overlap-augmented loss function that combines false-positive and false-negative costs with a penalty for distributional overlap. The proposed formulation provides an additional measure for evaluating the difficulty of distinguishing two populations, although the overlap penalty is constant when the underlying distributions are fixed and therefore does not change the optimal classification rule. A numerical example illustrates the relationship among distributional overlap, classification error, and the proposed loss function. Applications to business risk assessment, medical diagnosis, and reliability analysis are discussed.

Key words: overlapping coefficient; statistical decision theory; Bayes error; total variation distance; classification; loss function.

1. Introduction

Statistical decision-making concerns the selection of optimal actions under uncertainty. In many practical problems, decisions require distinguishing between competing populations based on observed data. Examples include separating diseased and healthy individuals in medical diagnosis, identifying

high- and low-risk customers in finance, detecting defective and non-defective products in manufacturing, and classifying different risk categories in business analytics. In all of these applications, the difficulty of making correct decisions depends fundamentally on the degree of similarity between the underlying probability distributions.

The overlapping coefficient (OVL), introduced by Weitzman (1970), provides a natural measure of the common area shared by two probability distributions. For two continuous distributions with density functions $f(x)$ and $g(x)$, the overlapping coefficient is defined as

$$\Delta(f,g) = \int_{-\infty}^{\infty} min\{f(x), g(x)\}dx,$$

where the integral is taken over the common support of the distributions. The value of the coefficient ranges between zero and one. A value close to zero indicates that the distributions are well separated, while a value close to one indicates strong similarity between the distributions. Due to this intuitive interpretation, the OVL has become an important tool for comparing distributions in many fields, including the biological sciences, reliability theory, medical research, and pattern recognition.

Recent developments enhance its applicability for skewed, censored, or multimodal biomedical data (Eidous and Al-Talafha, 2022). Because analytical solutions are often limited to simple distributions, the numerical methods developed by Eidous and Abu Al-Hayja'a (2023a, 2023b) for Weibull distributions, and by Eidous and Alshorman (2023) and Eidous and Daradkeh (2024) for normal distributions improve computational accuracy for biomarker and dose-response comparisons. A new technique based on expected value using Weibull distribution to estimate the various OVL coefficients has been suggested by Eidous and Maqableh (2023, 2024) and Maqableh and Eidous (2024).

Nonparametric kernel-based approaches (Eidous and Al-Talafha, 2022) estimate the OVL by integrating the minimum of two kernel density estimates. Eidous and Ananbeh (2024, 2025) improved the kernel-based approaches using numerical methods. Comparable asymptotic treatments exist for Matusita's

coefficient (Alodat et al., 2022). Finally, generalizations of OVL to multiple normal distributions (Eidous and Alsheyyab, 2025) enable comparison of multiple treatment groups, which is highly relevant to multi-arm clinical trials.

Previous studies have mainly focused on estimation methods, distributional properties, and applications of the overlapping coefficient. Inman and Bradley (1989) investigated estimation approaches, while Pastore and Calcagnì (2019) reviewed overlap measures and their statistical applications. Martínez-Camblor (2023) examined the use of the overlapping coefficient in binary classification and established its relationship with the Youden index under specified conditions. Building on this literature, the present study formulates and interprets an overlap-augmented criterion for evaluating decision problems. The added overlap penalty is independent of the decision rule when the underlying distributions and penalty coefficient are fixed; therefore, it does not change the optimal decision rule. The contribution lies in interpreting this criterion as a supplementary measure of decision difficulty, rather than in establishing a new relationship between the overlapping coefficient and classification error.

From a statistical decision perspective, overlap has a deeper interpretation. When two probability distributions share a large amount of probability mass, observations generated from these distributions become difficult to distinguish. Consequently, even an optimal decision rule must produce a non-negligible probability of incorrect decisions. In contrast, when the overlap is small, the distributions can be separated more effectively, resulting in lower classification error.

The objective of this paper is to examine the interpretation of the overlapping coefficient within a decision-theoretic framework. Building on established relationships between distributional overlap, Bayes classification error, and total variation distance, we formulate an overlap-augmented criterion that combines conventional misclassification costs with an explicit penalty for distributional overlap. The resulting formulation provides a supplementary measure of the difficulty of distinguishing competing populations, while leaving the optimal decision rule unchanged when the underlying distributions and penalty

coefficient are fixed. The contribution lies in formulating and interpreting this criterion, rather than deriving new identities linking OVL to Bayes error or total variation distance.

2. Background and preliminary concepts

2.1. Statistical decision theory

Statistical decision theory provides a unified mathematical framework for selecting optimal actions under uncertainty, formalizing the interaction between a decision-maker and an unknown environment (Wald, 1950). Formally, a decision problem consists of a parameter space Θ, an observation space $\mathcal{X}$, an action space $\mathcal{A}$, and a loss function measuring the cost associated with each decision (Berger, 1985).

Let X denote an observed random variable with probability density $f(x \mid \theta)$, where $\theta \in \Theta$ represents the unknown state of nature. A decision rule is a function

$$\delta(x): \mathcal{X} \to \mathcal{A},$$

which determines the action taken after observing x. The performance of a decision rule is evaluated through the risk function, $R(\theta, \delta)$, which represents the expected loss over the observation space (Berger, 1985; Diakonikolas and Kane, 2023); $R(\theta, \delta) = E_\theta[L(\theta, \delta(X))]$, where $L(\theta, \delta(X))$ is the loss incurred by taking action $\delta(X)$ when the true state is θ.

A central objective in decision theory is finding decision rules that minimize expected loss. Under Bayesian decision theory, the optimal rule minimizes the posterior expected loss, while under frequentist decision theory, criteria such as minimax risk and admissibility are commonly considered. A fundamental decision problem is binary classification, where observations originate from one of two populations:

$$H_0: f(x), \qquad H_1: g(x).$$

The objective is to classify an observation into the correct population. The difficulty of this problem depends strongly on the separation between $f(x)$ and $g(x)$ (Wheat et al., 2025).

2.2. Bayes classification error

Consider a binary classification problem with prior probabilities

$$P(H_0) = \pi_0, \qquad P(H_1) = \pi_1,$$

where $\pi_0 + \pi_1 = 1$. Assuming equal costs for the two types of misclassification, the Bayes classifier assigns an observation to H_0 when $\pi_0 f(x) > \pi_1 g(x)$, and assigns it to H_1 otherwise (Duda et al., 2001; Bishop, 2006; Hastie et al., 2009). The minimum probability of classification error, known as the Bayes error (P_C), is derived by integrating the minimum posterior error over the feature space,

$$P_C = \int_{-\infty}^{\infty} min\{\pi_0 f(x), \pi_1 g(x)\} dx.$$

For equal prior probabilities, $\pi_0 = \pi_1 = {}^1\!/_2$, the Bayes error becomes

$$P_C = \frac{1}{2} \int_{-\infty}^{\infty} min\{f(x), g(x)\} dx.$$

Therefore,

$$P_C = \frac{\Delta(f, g)}{2}.$$

This fundamental relationship shows that the overlapping coefficient directly determines the smallest possible classification error when the two populations have equal prior probabilities.

2.3. The overlapping coefficient

For two probability density functions $f(x)$ and $g(x)$, the overlapping coefficient is defined as

$$\Delta(f, g) = \int_{-\infty}^{\infty} min\{f(x), g(x)\} dx.$$

It satisfies $0 \leq \Delta(f,g) \leq 1$. Equivalent expressions can be obtained using the identity

$$min(a,b) = \frac{a+b-|a-b|}{2}.$$

Since

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} g(x)dx = 1.$$

we obtain

$$\Delta(f,g) = 1 - \frac{1}{2}\int_{-\infty}^{\infty} |f(x) - g(x)|dx.$$

The quantity

$$TV(f,g) = \frac{1}{2}\int_{-\infty}^{\infty} |f(x) - g(x)|dx,$$

is the total variation distance between the two distributions. Hence,

$$\Delta(f,g) = 1 - TV(f,g),$$

which provides an important connection between distributional overlap and statistical separability. A high overlap corresponds to a small total variation distance and therefore weak discrimination between the two populations. Conversely, low overlap corresponds to a large total variation distance and stronger statistical separation. This identity follows directly from the definition of total variation distance and is used here as an established relationship.

3. Overlap-based decision-theoretic framework

3.1 Motivation for an overlap-based loss function

In classical statistical decision theory, losses are usually defined according to the consequences of incorrect actions. For example, in a binary classification problem, two types of errors may occur: assigning an observation from

population $f(x)$ to population $g(x)$ (false negative error) or assigning an observation from population $g(x)$ to population $f(x)$ (false positive error).

However, the magnitude of these errors is not determined only by the decision rule. It is also affected by the intrinsic similarity between the competing populations. When two distributions overlap substantially, even the optimal decision rule cannot completely avoid misclassification. Therefore, a meaningful decision framework should incorporate the degree of overlap directly into the loss structure. Let

$$\Delta(f, g) = \int min\{f(x), g(x)\}dx,$$

denote the overlapping coefficient between two populations. Since $\Delta(f, g)$ measures the amount of unavoidable similarity between the distributions, it can be interpreted as an intrinsic component of decision uncertainty. Accordingly, we define an overlap-based decision loss function as

$$L_{\Delta} = C_{FN}\, P(FN) + C_{FP}\, P(FP) + C_{\Delta}\, \Delta(f, g),$$

where C_{FN} represents the cost of false negative decisions, C_{FP} represents the cost of false positive decisions, and C_{Δ} represents the cost associated with intrinsic distributional overlap.

The proposed criterion combines the conventional costs of false-negative and false-positive decisions with an additional penalty for distributional overlap. For fixed distributions (f and g), and a fixed coefficient (C_{Δ}), the overlap penalty ($C_{\Delta}\Delta(f, g)$) is constant with respect to the decision rule (δ). Consequently, this additional term does not change the decision rule that minimizes the conventional misclassification cost. Instead, it provides a supplementary measure of the difficulty of distinguishing the two populations.

The first two components represent conventional decision losses, whereas the third component represents the unavoidable uncertainty caused by the similarity of the underlying populations.

To define the two error probabilities formally, let $\delta(x)$ be a binary decision rule, where $\delta(x) = 0$ assigns an observation to H_0 and $\delta(x) = 1$ assigns it to H_1. Define the decision regions as

$$D_0 = \{x: \delta(x) = 0\}, \qquad D_1 = \{x: \delta(x) = 1\}.$$

If π_0 and π_1 are the prior probabilities of H_0 and H_1, respectively, then the unconditional false-positive and false-negative probabilities are

$$P(FP) = \pi_0 \int_{D_1} f(x)dx$$

$$P(FN) = \pi_1 \int_{D_0} g(x)dx$$

Throughout this paper, $P(FP)$ and $P(FN)$ denote prior-weighted contributions to the overall misclassification probability, rather than conditional error rates. Their sum equals the overall probability of classification error.

Thus, the false-positive probability corresponds to assigning an observation from H_0 to H_1, while the false-negative probability corresponds to assigning an observation from H_1 to H_0. These definitions formally connect the error probabilities in the proposed loss function to the decision rule.

3.2. Properties of the overlap-based loss function

The proposed loss function possesses several desirable mathematical properties required for stable risk evaluation (see, for example, Li et al., 2025, for a survey of loss functions in deep learning).

Proposition 1: The overlap-based loss function is non-negative and bounded.
Proof: Since $0 \le P(FN) \le 1$, $0 \le P(FP) \le 1$, and $0 \le \Delta(f,g) \le 1$, then for positive constants, $C_{FN}, C_{FP}, C_\Delta > 0$, we have $L_\Delta \ge 0$.

Furthermore, $L_\Delta \le C_{FN} + C_{FP} + C_\Delta$. Therefore, the loss is bounded.
Proposition 2: For fixed probability densities f and g, let $C_\Delta > 0$. The overlap component of the proposed criterion, $C_\Delta \Delta(f,g)$, is strictly increasing as a function of the overlap coefficient $\Delta(f,g)$.

Proof: Since $C_\Delta > 0$, the derivative of the overlap component with respect to Δ is

$$\frac{d}{\partial \Delta}(C_\Delta \Delta) = C_\Delta > 0.$$

Hence, the overlap penalty is strictly increasing in Δ. This result concerns the dependence of the penalty on the overlap coefficient; it does not assert monotonicity under every possible change in the underlying distributions.

This result indicates that the proposed loss function incorporates the difficulty of the statistical decision problem. Even if the classification procedure is optimal, large overlap imposes a minimum unavoidable decision cost (Memon et al., 2025). However, when $f(x)$ and $g(x)$ are fixed, the overlap term is constant with respect to the decision rule. Consequently, adding this term does not change the rule that minimizes the conventional expected misclassification cost – it adds an overlap penalty to the evaluation of the decision problem.

4. Theoretical relationships and numerical illustration

4.1. Relationship between OVL and Bayes error

As stated in subsection 2.2, for two continuous populations with density functions $f(x)$ and $g(x)$, and equal prior probabilities $P(H_0) = P(H_1) = 1/2$, with equal misclassification costs, the Bayes classification error satisfies

$$P_C = \frac{\Delta(f, g)}{2}.$$

This establishes the identity from the definitions; the identity itself is an established result. It shows that the overlapping coefficient is not merely a descriptive measure of distribution similarity. It directly determines the smallest possible probability of classification error. Two extreme cases illustrate this relationship:

- Complete separation: If $\Delta(f, g) = 0$, then $P_C = 0$, meaning perfect classification is theoretically possible.

- Identical distributions: If $\Delta(f,g)=1$, then $P_C=1/2$, which corresponds to a minimum error probability of one-half under equal prior probabilities and equal misclassification costs.

Therefore, the overlapping coefficient provides a direct measure of the difficulty of distinguishing two populations.

4.2. Relationship between OVL and total variation distance

The Bayes error can be expressed in terms of total variation distance as

$$P_C=\frac{1}{2}\big(1-TV(f,g)\big).$$

This can be illustrated directly by considering

$$P_C=\frac{\Delta(f,g)}{2},$$

and $\Delta(\mathrm{f},\mathrm{g})=1-\mathrm{TV}(\mathrm{f},\mathrm{g})$, which gives the above formula for $\mathrm{P_C}$. This result provides a unified interpretation of three fundamental concepts:

- Overlapping coefficient: measures common probability mass.
- Total variation distance: measures distributional separation.
- Bayes error: measures optimal decision uncertainty.

These quantities provide equivalent mathematical descriptions of distributional overlap and minimum classification error under the stated assumptions. The identities are well established; their purpose here is to connect the concepts within the paper's decision-theoretic framework.

4.3. Numerical example: two normal distributions

Consider two populations under the hypotheses

$$H_0: X\sim N(0,1), \qquad H_1: X\sim N(2,1).$$

Assume equal prior probabilities, $\pi_0=\pi_1=1/2$, and equal costs for false-positive and false-negative decisions. Since the two normal distributions have

equal variances, the Bayes decision threshold is the midpoint between their means, $x = 1$. The corresponding decision rule is

$$\delta(x) = \begin{cases} 0, & x \leq 1, \\ 1, & x > 1. \end{cases}$$

Let Φ denote the standard normal cumulative distribution function (CDF); then the prior-weighted false-positive probability is

$$\begin{aligned} P(FP) &= \pi_0 P(X > 1 \mid H_0) \\ &= \frac{1}{2}\Phi(-1) \\ &\approx 0.0793. \end{aligned}$$

Various approximations to Φ have been developed to facilitate its numerical evaluation (see, for example, Eidous and Ananbeh, 2022; Eidous and Al-Rawwash, 2025).

Similarly, the prior-weighted false-negative probability is

$$\begin{aligned} P(FN) &= \pi_1 P(X \leq 1 \mid H_1) \\ &= \frac{1}{2}\Phi(-1) \\ &\approx 0.0793. \end{aligned}$$

Consequently, the minimum overall probability of classification error under the assumed priors and costs is

$$\begin{aligned} P_B &= P(FP) + P(FN) \\ &= \Phi(-1) \\ &\approx 0.1587. \end{aligned}$$

The overlapping coefficient between the two distributions is therefore

$$\begin{aligned} \Delta &= \int_{-\infty}^{\infty} min\{f_0(x), f_1(x)\}\, dx \\ &= 2\Phi(-1) \\ &\approx 0.3173. \end{aligned}$$

Thus, for two normal distributions with equal variances and equal prior probabilities, the overlapping coefficient satisfies $\Delta = 2P_C$. For the proposed overlap-augmented loss, consider

$$L_\Delta(\delta) = C_{FN}P(FN) + C_{FP}P(FP) + C_\Delta \Delta,$$

where C_{FN}, C_{FP}, and C_{Δ} denote the costs assigned to false-negative decisions, false-positive decisions, and distributional overlap, respectively. Substituting the numerical values gives

$$L_{\Delta}(\delta) \approx 0.0793C_{FN} + 0.0793C_{FP} + 0.3173C_{\Delta}.$$

If the two misclassification costs are equal, so that $C_{FN} = C_{FP} = C$, the expression reduces to

$$L_{\Delta}(\delta) \approx 0.1587C + 0.3173C_{\Delta}.$$

This example illustrates how the proposed loss separates the expected misclassification cost from an additional penalty reflecting the overlap between the underlying distributions. Because the overlap coefficient depends on the distributions rather than on the particular decision rule, the overlap penalty is constant across decision rules when the distributions are fixed. Therefore, this example illustrates the interpretation of the proposed loss, but does not, by itself, establish improved classification performance or a different optimal decision rule relative to the conventional expected misclassification cost.

5. Implications for statistical decision analysis

The proposed framework changes the interpretation of the overlapping coefficient from a descriptive similarity index to a decision-oriented measure. This has several important implications.

5.1. Risk assessment and business analytics

In financial and business applications, decision-makers frequently classify individuals, firms, or transactions into competing categories. Examples include:

- high-risk versus low-risk customers,
- profitable versus non-profitable products,
- fraudulent versus legitimate transactions,
- successful versus unsuccessful investment opportunities.

In these situations, a high overlapping coefficient indicates that the populations are difficult to distinguish. Consequently, decision rules based on available information will have limited predictive accuracy.

The OVL therefore provides a useful measure of the intrinsic uncertainty before selecting a particular statistical model or classification algorithm. The overlap coefficient does not, by itself, measure financial risk or establish which strategy is preferable. Its interpretation depends on the modeled distributions, the decision being considered, and the costs associated with incorrect decisions.

5.2. Medical diagnosis

Medical diagnosis represents another important application. Suppose two populations represent healthy and diseased individuals. If the distributions of a diagnostic marker strongly overlap, then no classification procedure can achieve very low error rates.

The proposed framework shows that this limitation is not necessarily due to the weakness of the diagnostic method but may be caused by the inherent similarity between the two biological populations. Thus, the overlapping coefficient can serve as an objective measure of diagnostic difficulty.

5.3. Reliability and quality control

In reliability engineering and industrial quality control, decisions often involve distinguishing acceptable and defective components. A large overlap between lifetime distributions or quality measurements implies that classification errors are unavoidable.

The proposed overlap-based loss function provides a natural way to incorporate this uncertainty into reliability decision models.

5.4. Extension to unequal prior probabilities

Although the main theoretical results were presented for equal prior probabilities, the framework can be extended to unequal priors. The Bayes error becomes

$$P_C = \int min\{\pi_0 f(x), \pi_1 g(x)\}dx.$$

In this case, the weighted overlap quantity

$$\Delta_\pi(f,g) = \int min\{\pi_0 f(x), \pi_1 g(x)\}dx.$$

provides a generalized measure of decision uncertainty. Therefore, the proposed framework remains valid when populations have different prior probabilities or unequal importance. For clarity, $\Delta_\pi(f,g)$ is a prior-weighted overlap quantity and is not generally equal to one-half of the ordinary overlapping coefficient. By definition, the Bayes classification error satisfies $P_C = \Delta_\pi(f,g)$. Under equal prior probabilities, $\Delta_\pi(f,g) = \Delta(f,g)/2$, recovering the established identity $P_C = \Delta(f,g)/2$. Furthermore, unequal misclassification costs require the Bayes decision rule to account for those costs explicitly.

6. Discussion and conclusions

This paper has developed a unified decision-theoretic framework based on the overlapping coefficient. Unlike traditional interpretations that view the OVL only as a descriptive measure of similarity between probability distributions, it examines how the overlapping coefficient can be incorporated into a supplementary decision-theoretic evaluation criterion, extending its interpretation beyond a descriptive measure of similarity between probability distributions.

A general overlap-based loss function was introduced to quantify the uncertainty associated with competing populations. The established relationship between the overlapping coefficient and Bayes classification error was examined through the identity

$$P_C = \frac{\Delta(f,g)}{2},$$

under equal prior probabilities and equal misclassification costs. Furthermore, the formula

$$\Delta(f, g) = 1 - TV(f, g),$$

was used to link overlap, statistical separability, and decision uncertainty within a single framework.

The results demonstrate that distributional overlap represents an intrinsic limitation on decision accuracy. When overlap is large, classification errors cannot be eliminated even by optimal procedures. Conversely, small overlap indicates that accurate discrimination is theoretically achievable.

Because the proposed framework is not restricted to a particular parametric family, it can be considered in a wide range of statistical decision problems, including economics, finance, business analytics, medical diagnosis, reliability engineering, and quality control. In practical applications, its implementation requires the relevant distributions or suitable estimates of their densities.

The contribution of the paper is the formulation and interpretation of the overlap-augmented loss function within a decision-theoretic framework. The established relationships between OVL, Bayes error, and total variation distance provide its theoretical foundation. Because the overlap penalty is constant for fixed distributions, further empirical work would be required to determine whether the proposed formulation offers practical benefits beyond conventional decision-cost evaluation.

REFERENCES

Alodat M.T., Al Fayez M., Eidous O.M. (2022): On the asymptotic distribution of Matusita's overlapping measure. Communications in Statistics – Theory and Methods 51(20): 6963–6977.

Berger J.O. (1985): Statistical decision theory and Bayesian analysis (2nd ed.). Springer-Verlag.

Bishop C.M. (2006): Pattern recognition and machine learning. Springer.

Diakonikolas I., Kane D.M. (2023): Algorithmic high-dimensional robust statistics. Cambridge University Press.

Duda R.O., Hart P.E., Stork D.G. (2001): Pattern classification (2nd ed.). John Wiley & Sons.

Eidous O.M., Abu Al-Hayja'a M.M. (2023a): Estimation of overlapping measures using numerical approximations: Weibull distributions. Jordan Journal of Mathematics and Statistics (JJMS) 16(4): 741–761.

Eidous O.M., Abu Al-Hayja'a M.M. (2023b): Numerical integration approximations to estimate the Weitzman overlapping measure: Weibull distributions. Yugoslav Journal of Operations Research 33(4): 699–712.

Eidous O.M., Al-Rawwash M.Y. (2025): Approximations for standard normal distribution function and its invertible. Journal of Algorithms & Computational Technology 19, 1–5. doi:10.1177/17483026251322100

Eidous O.M., Alsheyyab M.M. (2025): Generalization of the overlapping coefficient for $k \geq 2$ normal distributions. Mathematical Modelling of Natural Phenomena 20, Article 9. https://doi.org/10.1051/mmnp/2025008

Eidous O.M., Alshorman A.J.A. (2023): Estimating the Weitzman overlapping coefficient using integral approximation method in the case of normal distributions. Advances in Transdisciplinary Engineering 42: 1011–1020.

Eidous O.M., Al-Talafha S.A. (2022): Kernel method for overlapping coefficients estimation. Communications in Statistics – Simulation and Computation 51(9): 5139–5156.

Eidous O.M., Ananbeh E. (2022): Approximations for cumulative distribution function of standard normal. Journal of Statistics and Management Systems 25(3): 541–547.

Eidous O.M., Ananbeh E.A. (2024): Kernel method for estimating overlapping coefficient using numerical integration methods. Applied Mathematics and Computation 462, 128339.

Eidous O.M., Ananbeh E.A. (2025): Kernel method for estimating Matusita overlapping coefficient using numerical approximations. Annals of Data Science 12(4): 1265–1283.

Eidous O.M., Daradkeh S.K. (2024): On inference of Weitzman overlapping coefficient $\Delta(X, Y)$ in the case of two normal distributions. International Journal of Theoretical and Applied Mathematics 10(2): 14–22.

Eidous O., Maqableh H. (2023): Estimation of Matusita overlapping coefficient ρ for two Weibull distributions. Journal of Statistics and Computer Science 2(1): 1–16.

Eidous O., Maqableh H. (2024): Estimation of Morisita overlapping measure λ for two Weibull distributions. World Journal of Mathematics and Statistics 3(2): 13–21.

Hastie T., Tibshirani R., Friedman J. (2009): The elements of statistical learning: data mining, inference, and prediction (2nd ed.). Springer.

Inman H.F., Bradley E.L. (1989): The overlapping coefficient as a measure of agreement between probability distributions and point estimation of the overlap of two normal densities. Communications in Statistics – Theory and Methods 18(10): 3851–3874.

Li C., Liu K., Liu S. (2025): A survey of loss functions in deep learning. Mathematics 13(15): 2417.

Maqableh H.Y., Eidous O.M. (2024): On inference of Weitzman overlapping coefficient in two Weibull distributions. Journal of Health Statistics Reports 3(3): 1–6.

Martínez-Camblor P. (2023): About the use of the overlap coefficient in the binary classification context. Communications in Statistics – Theory and Methods 52(19): 6767–6777.

Memon S.A., Ahmad I., Shoaibullah (2025): Mathematical analysis of loss functions in classification and regression. Physical Education Health and Social Sciences, 3(2): 96–103.

Pastore M., Calcagnì A. (2019): Measuring distribution similarities between samples: A distribution-free overlapping index. Frontiers in Psychology 10, 589.
Wald A. (1950): Statistical decision functions. New York: John Wiley & Sons.
Wheat L., Mohrenschildt M.V., Habibi S. (2025): Bayes error rate estimation in difficult situations. arXiv preprint arXiv:2506.03159.
Weitzman M.S. (1970): Measures of overlap of income distributions of white and negro families in the United States (Technical Paper No. 22). US Bureau of the Census.