B&H Electrical Engineering, Volume 20, Issue 2, 2026
ISSN:2566-3143, eISSN:2566-3151, DOI: 10.2478/bhee-2026-0028

DEVELOPMENT OF A TRANSFORMER MODEL USING OPTIMIZATION METHOD FOR PARAMETER ESTIMATION

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Abstract: The precise modeling of a transformer, including the estimation of its equivalent model parameters, is fundamental for power system analysis. Accurate determination of these parameters is required for both steady-state and transient studies, with particular attention to low-frequency transients such as transformer energisation. These phenomena are governed by the nonlinear characteristic of the transformer's iron core, with the nonlinear saturation curve playing a key role in their behaviour and representing a major challenge in transformer modeling. This paper proposes an algorithm for the transformer's nonlinear saturation characteristic estimation, based on measured inrush current waveforms. The method employs the Nelder–Mead optimization technique to minimize a defined objective function, ensuring minimal deviation between simulated and measured inrush currents with zero remanent flux. The validity of the proposed approach is demonstrated through a comparative analysis of measured and simulated inrush current waveforms.

Keywords: modeling, measurement, saturation curve, inrush current, estimation, Nelder-Mead method

INTRODUCTION

Transformers are key components of power systems, enabling efficient energy transfer across multiple voltage levels. Accurate and reliable transformer analysis is necessary for both normal steady-state operation and electromagnetic transients, such as inrush currents, ferroresonance, and overvoltages. Developing a precise mathematical transformer model is required to perform such analysis effectively.

One of the most important characteristics of a transformer is its nonlinear magnetization curve, which describes the relationship between magnetic flux and magnetizing current. This characteristic plays a key role in accurately representing transformer dynamic behaviour, and therefore its precise modeling is of particular importance. Special attention must be given to the curve shape in its deep saturation. In practice, the magnetization characteristic is most commonly determined indirectly by recalculating it from the experimentally obtained RMS voltage–current (U–I) characteristic using algorithms described in [1]. Two approaches are typically employed to obtain the RMS U–I curve; however, both suffer from inherent limitations. In the first approach, the nonlinear curve data are provided by the manufacturer, but usually only a limited number of points defining the saturation region are available, which are subsequently linearly interpolated in simulation models. This approach fails to capture the deep saturation, potentially resulting in significant inaccuracies in the simulation of nonlinear phenomena, particularly for high inrush currents or state conditions such as ferroresonance. Conversely, attempts to accurately measure the saturation characteristic under laboratory conditions are constrained by technical limitations. Power supply limitations may lead to insulation breakdown between the primary winding and the transformer tank, while prolonged operation in the deep saturation can cause excessive winding overheating. These practical challenges highlight the need for alternative methods capable of providing a more accurate and safer estimation of the nonlinear magnetization characteristic of the transformer iron core. Accordingly, the objective of this paper is to develop an efficient method for the reliable identification of the nonlinear parameters of a transformer model that overcomes the limitations of conventional measurements and standard testing procedures.

The proposed algorithm is based on minimizing a defined objective function using the Nelder–Mead optimization method, with the aim of minimizing the disagreement between simulated and measured transformer inrush currents with zero remanent flux. This procedure enables accurate estimation of the magnetization curve deep saturation [2], [3].

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To validate the accuracy and applicability of the proposed model, a comparative analysis of inrush current waveforms obtained from measurements and corresponding numerical simulations was performed. The results demonstrate the effectiveness of the proposed algorithm and confirm its reliability under real operating conditions.

1. TRANSFORMER MODELING

1.1. Equivalent Model of a Single-Phase Transformer

One of the most commonly used approaches for analyzing single-phase transformers under low-frequency electromagnetic transients and steady-state operating conditions is the standard T-equivalent model of a single-phase transformer, shown in Figure 1. The linear parameters of the equivalent model include the primary and secondary resistances and inductances, as well as the iron core resistance. These parameters are obtained from standard transformer tests, namely the short-circuit test and the open-circuit test. The nonlinear part of the model represents the magnetizing inductance of the transformer's iron core, which is determined from the flux–magnetizing current (Φ–i) characteristic. This characteristic reflects the nonlinear behavior of the core and enables accurate modeling of the transformer's magnetizing branch.

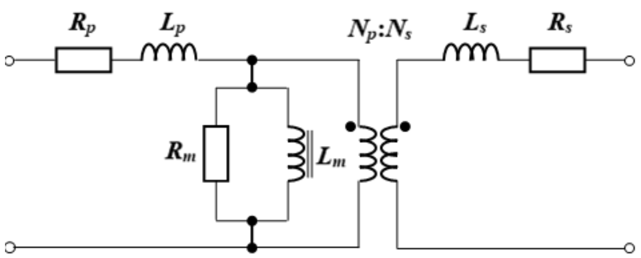

Figure 1: Standard T-equivalent model of a single-phase transformer

The labels in the figure represent the following model parameters:

R_p – primary winding resistance,
L_p – primary winding inductance,
R_s – secondary winding resistance,
L_s – secondary winding inductance,
R_m – resistance of the transformer's iron core,
L_m – nonlinear inductance of the transformer's iron core,
N_p/N_s – transformer turns ratio.

In the analysis of low-frequency transients, such as transformer inrush current, a constant value for the resistance of the transformer's iron core R_m , can be assumed, since the core losses are very small, which is taken into account in this paper.

1.2. Nonlinear core model

Modeling the nonlinear coil represents the greatest challenge and the most demanding part of the transformer modeling process. There are usually two approaches used for modeling a nonlinear coil, i.e. using:

- piecewise linearization
- analytical function.

A nonlinear coil is defined by a nonlinear function that establishes the relationship between the magnetizing current and the flux, i_m–Φ_m. At each point on the magnetization curve i_m–Φ_m, the inductance of the nonlinear coil L_m, is obtained as the corresponding gradient at that point on the curve. The magnetization curve is defined by a set of points (i_{mk}, Φ_{mk}), k = 1, 2, 3, ..., N, in the first quadrant. The parameter N represents the total number of known points on the magnetization curve. The given points can be approximated or interpolated using one or a set of known analytical functions.

Since the interpolation technique uses all measured points of the magnetization curve, it is evident that this approach provides higher accuracy compared to the use of approximation functions. Piecewise linearization of the nonlinear magnetization curve represents an efficient and simple method for modeling nonlinear elements, which can be easily implemented in a state-space equation form. Moreover, the simulation time remains within acceptable limits, as a linear state-space equation is solved at each time step. Therefore, in this paper, this particular approach to modeling of nonlinear coil will be implemented.

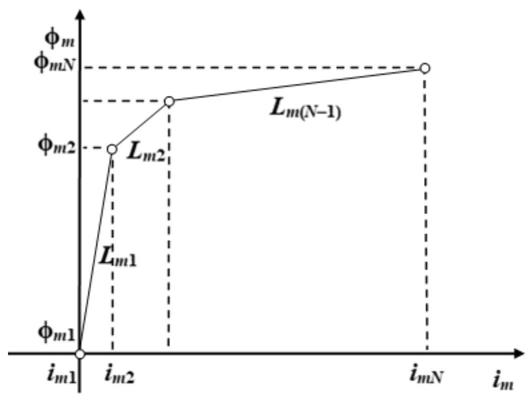

Figure 2: Piecewise linearization of the magnetization curve in the first quadrant

If the criterion for defining the magnetization curve with a total of N known discrete points in the first quadrant is satisfied (Figure 2), then the magnetizing current of the k- th linear segment, where k = 0, 1, 2, ..., N, can be calculated using the following relations [4]:

$$\begin{aligned}
&i_m = \frac{1}{L_{m_k}}\Phi_m + sign(\Phi_m)\cdot I_{mk} = \frac{1}{L_{m_k}}\Phi_m + S_{\Phi_k} \\
&i_{m_k} \le i_m \le i_{m_{k+1}},\ \Phi_{m_k} \le \Phi_m \le \Phi_{m_{k+1}} \\
&L_{m_k} = \frac{\Phi_{m_{k+1}} - \Phi_{m_k}}{i_{m_{k+1}} - i_{m_k}},\ I_{m_k} - \frac{1}{L_{m_k}}\Phi_{m_k}
\end{aligned} \tag{1}$$

ISSN:2566-3143, eISSN:2566-3151, DOI: 10.2478/bhee-2026-0028

In equation (1), S_{ϕ_k} represents the current source in the equivalent model of the nonlinear coil, which depends on the parameter k, i.e., on the position of the operating point on the magnetization curve. This means that, when modeling a nonlinear coil with a variable inductance L_m, it is equivalently represented by a parallel connection of a linear coil inductance L_{m_k} and current source S_{ϕ_k}, as shown in Figure 3.

Figure 3: a) Nonlinear core, b) equivalent model of a nonlinear coil

The parameters L_{m_k} and S_{ϕ_k} are functionally dependent on the position of the operating point on the magnetization curve, i.e., they depend on the parameter k.

The adjacent inductances L_{m_k} and $L_{m_{k-1}}$ are connected through the following relation, where k = 0, 1, 2, ..., N:

$$\frac{1}{L_{m_k}} = \frac{1}{L_{m_{k-1}}} + \frac{1}{\left(\Delta L_{m_k}\right)^2} \tag{2}$$

Accordingly, new variables related to the solution vector to be determined are defined as:

$$z_1 = \frac{1}{\left(\Delta L_{m_1}\right)^2}, z_2 = \frac{1}{\left(\Delta L_{m_2}\right)^2}, \cdots, z_{N-1} = \frac{1}{\left(\Delta L_{m_{N-1}}\right)^2} \tag{3}$$

The vector of unknown variables for optimization can finally be expressed as:

$$Z = \left[z_1 \quad z_2 \quad \cdots \quad z_{N-1}\right]^T \tag{4}$$

1.3. State-space equation

The equivalent model of a single-phase transformer energization by a voltage source $u(t)$ at the time $t=T_0$ is shown in Figure 4. The additional labels shown in the figure are:

$u(t)$ – sinusoidal voltage source,
Φ – nonlinear inductance flux,
i_p – transformer primary current,
i_R – core loss current,
i – magnetizing current.

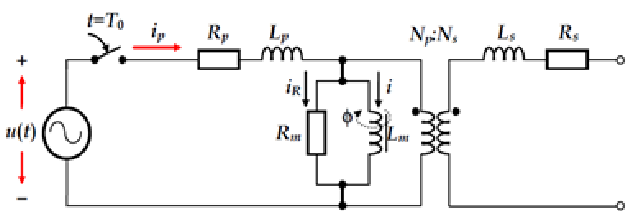

Figure 4: Equivalent model of a single-phase transfomer energization

The transformer model shown in the figure can be described using the standard state-space equation for transformer energization:

$$\frac{dX(t)}{dt} = A_k X(t) + B_k U(t) = F(X,t,Z) \tag{5}$$

The matrices are defined as follows: A_k - system matrix, B_k - control matrix, $X(t)$ - state space vector, and $U(t)$ - input (excitation) vector. The elements of both the state vector and the input vector are known in advance:

$$X(t) = \begin{bmatrix} i_p \\ \Phi \end{bmatrix}, U(t) = \left[u(t)\right]$$

The matrices A_k and B_k are functionally dependent on the operating point position k along the piecewise-linearized magnetization curve of the transformer core. Their elements can be determined using a coefficient generation procedure. Consequently, the final system and input matrices are obtained as follows:

$$A_k = \begin{bmatrix} -\frac{R_p + R_m}{L_p} & \frac{R_m}{L_p L_{m_k}(Z)} \\ R_m & -\frac{R_m}{L_{m_k}(Z)} \end{bmatrix}$$
$$B_k = \begin{bmatrix} \frac{1}{L_p} & \frac{R_m}{L_p} \\ 0 & -R_m \end{bmatrix} \tag{6}$$

For solving the state-space equation (6), the implicit Euler method has been chosen:

$$X_{n+1} = X_n + hF\left(X_{n+1}, t_{n+1}, Z\right) \tag{7}$$

The implicit Euler method was selected for its simplicity, absolute stability, and L-stability. The last one criterion makes it especially effective for solving stiff dynamic systems, such as the set of equations describing transformer energization. Using the known waveform of the measured inrush current in the time domain, together with the numerically obtained inrush current, the objective function can now be formulated as:

$$F(Z) = \frac{1}{N_{\max}} \sum_{n=1}^{N_{\max}} \left[i_{p_n}(Z) - i_{pmjerenje_n}\right]^2 \tag{8}$$

The aim is to determine the minimum of the defined objective function $F(Z)$, i.e., to find $F_{min} = F(Z)_{min}$ in n -dimensional space $Z \in R^n$. The Nelder–Mead method is a heuristic direct search algorithm widely used for solving unconstrained optimization problems in low-dimensional spaces. It operates without the need to compute the gradient of the objective function, making it suitable for optimizing discontinuous functions.

The algorithm is based on the iterative transformation of a simplex, a geometric figure with n+1 vertices in n -dimensional space, using reflection, expansion, contraction, and compression operations to improve the function value.

The advantages of the method include simple implementation, robustness in the presence of nonlinearity, efficiency for a small number of optimization parameters, and relatively low computational complexity, as it requires only one or two function evaluations per iteration. The choice of the initial optimization vector plays a significant role [5], [6].

2. SIMULATION RESULTS

The Nelder–Mead algorithm will be applied for the estimation of the nonlinear curve saturation of a laboratory single-phase transformer with a voltage ratio of 220/48 V, whose parameters are: R_p = 4.2 Ω, L_p = 2.8 mH and R_m = 3.4 kΩ. The supply voltage is represented by the sinusoidal function $e(t)$=310sin(ωt). In addition to the transformer parameters, the measured inrush current is also known, and its waveform in the time domain will be presented later in the text. By applying the optimization algorithm to the given objective function, the results for the optimal vector at three points corresponding to the saturation region are obtained and presented in Table I.

Table I: Optimization results

Number of points	Initial solution vector	Optimum solution vector
1	$Z_0 = [5]$	$Z_{opt.} = [6.0356]$
2	$Z_0 = \begin{bmatrix} 1.6591 \\ 2.0085 \end{bmatrix}$	$Z_{opt.} = \begin{bmatrix} 2.5912 \\ 7.5912 \end{bmatrix}$
3	$Z_0 = \begin{bmatrix} 0.0113 \\ 0.0161 \\ 0.0109 \end{bmatrix}$	$Z_{opt.} = \begin{bmatrix} 2.2013 \\ 4.8310 \\ 10.0007 \end{bmatrix}$

To illustrate the results of applying the Nelder–Mead optimization method, the initial transformer magnetization curve obtained from measurements (Figure 5), limited to the unsaturated region, is presented. This allows a graphical comparison between the initial measurements and the results obtained with the optimization method, clearly demonstrating its effectiveness in estimating the magnetization curve deep saturation.

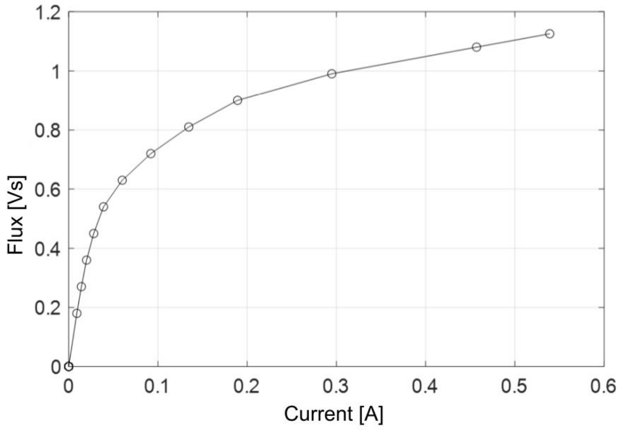

Figure 5: Initial magnetization curve

Figure 6 shows the initial magnetization curve used to start the optimization method, where the initial current values required for the algorithm are assumed. Three additional points were obtained by drawing a straight line through the last two measurement points, from which the additional points were taken.

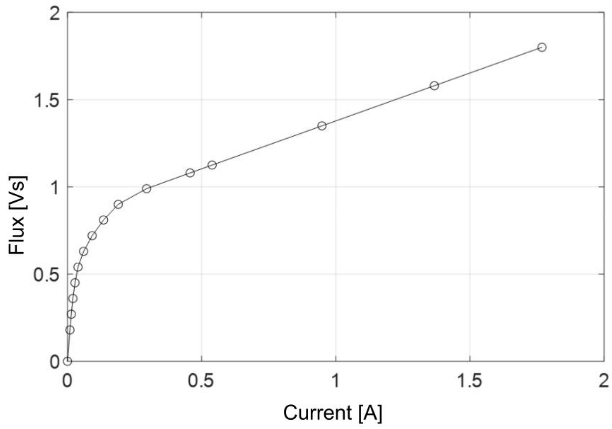

Figure 6: Initial magnetization curve: three points of the saturation

The resulting magnetization curve obtained using the Nelder–Mead optimization method is shown in Figure 7, where the estimated saturation is represented by three points corresponding to significantly higher magnetizing current values.

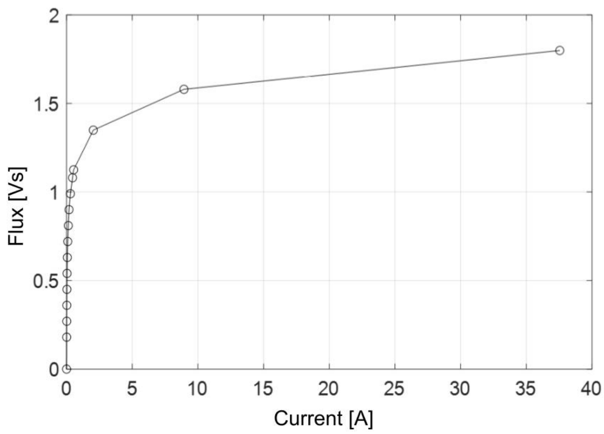

Figure 7: Optimum magnetization curve: three points of the saturation

The effect of estimating the magnetization curve in the saturation on the transformer inrush current is illustrated in Figure 8, showing a comparison between measured and simulated inrush currents. The saturation is represented by three points, determined by a straight line through the last two measurement points, as described earlier.

A diagram shows that this approach leads to significant deviations in the simulated inrush current compared to

ISSN:2566-3143, eISSN:2566-3151, DOI: 10.2478/bhee-2026-0028

measurements, resulting in considerable estimation errors. Applying the proposed optimization method minimizes these deviations, significantly improving the agreement between simulated and measured inrush currents.

Figure 9 shows this improvement, which compares the measured and simulated inrush currents based on the estimated nonlinear saturation curve. The simulated waveform closely matches the measured values, demonstrating the effectiveness and reliability of the proposed approach for modeling the transformer, particularly the nonlinear iron core.

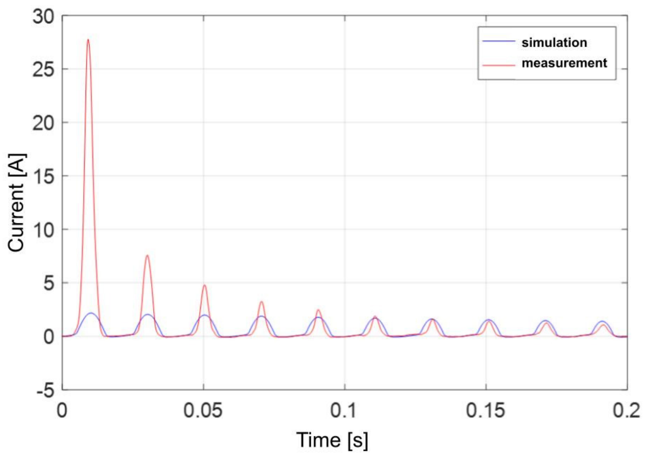

Figure 8: Transformer inrush current: measured and simulated without applying the optimization method

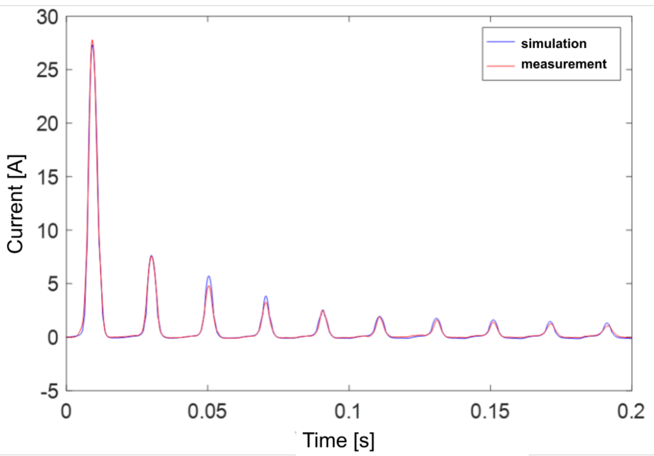

Figure 9: Transformer inrush current: measured and simulated using the optimization method

3. CONCLUSION

Precise transformer modeling, particularly the nonlinear magnetization curve, is very important for reliable analysis of electromagnetic transients in power systems. This paper presents an optimization method based on the Nelder–Mead algorithm to estimate the nonlinear saturation curve of a single-phase transformer using inrush current waveforms with zero remanent flux.

The analysis was performed on a single-phase transformer with a rated voltage ratio of 220/48 V. Laboratory measurements of the no-load U–I characteristic and the transformer inrush current were conducted. However, due to practical limitations of the experimental procedure, such as the risk of insulation breakdown between the primary winding and the transformer tank, as well as core overheating under deep saturation, accurate determination of the magnetization curve using conventional methods is hindered by practical experimental constraints.

Conventional approaches, which rely on a limited number of experimentally available data points, often result in significant errors in the simulation of inrush currents, particularly in magnetization curve saturation. Therefore, the proposed optimization method ensures very good agreement between simulated and measured waveforms, thereby improving model accuracy.

The obtained results confirm that the proposed algorithm represents an efficient and robust solution for identifying the nonlinear inductance of the transformer iron core, enabling more realistic transformer modeling over a wide range of operating conditions, including transient phenomena characterized by pronounced nonlinearities.

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BIOGRAPHY

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