DOI: 10.2478/bhee-2017-0009

Review scientific paper/Pregledni naučni rad

REVIEW ON CATHODIC PROTECTION SYSTEM MODELLING

OSVRT NA MODELIRANJE SISTEMA KATODNE ZAŠTITE

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Abstract: Cathodic protection is most frequently used technique for protection of underground or underwater metallic infrastructures from corrosion. Design of the cathodic protection system implies determination of the electric potential and current density distribution on the electrode surfaces. These values must meet criterions defined by relevant standards. For the calculation of the electrical potential and current density distribution, different analytical and numerical methods can be used. In this paper, a review of the methods that are used for calculate of the cathodic protection system parameters is given. Also, due to frequent use, a mathematical model based on the boundary element method for the calculation of the cathodic protection system parameters is presented in more detail. Presented mathematical model was used for calculation of the cathodic protection system parameters of one geometry simple example.

Keywords: Cathodic protection system, current density distribution, electric potential, boundary conditions

Sažetak: Katodna zaštita je najčešće korištena tehnika za zaštitu podzemnih i podmorskih metalnih instalacija od korozije. Projektiranje sistema katodne zaštite podrazumjeva određivanje raspodjele električnog potencijala i gustoće struje na elektrodnim površinama. Ove vrijednosti moraju zadovoljiti kriterije definirane relevantnim standardima. Za proračun raspodjele električnog potencijala i gustoće zaštitne struje, različite analitičke i numerčke metode mogu biti korištene. U ovom radu je dat pregled metoda koje se koriste za proračun navedenih parametara sistema katodne zaštite. Također, zbog česte upotrebe, detaljnije je predstavljen matematički model baziran na metodi graničnih elemenata za proračun parametara sistema katodne zaštite. Predstavljeni matematički model je iskorišten za proračun parametara jednog geometrijski jednostavnog sistema katodne zaštite.

Ključne riječi: Sistem katodne zaštite, raspodjela gustoće zaštitne struje, električni potencijal, granični uslovi

INTRODUCTION

With the constant increase in energy demand as a general civilization trend, the number of metal installations such as metallic pipelines for the transport and distribution of gas and oil is growing. Also, a large number of industrial production branches use different types of metallic installations such as of pipelines, reservoirs and other underground installations, enabling demanding industrial activities [1]. However, the application of underground or underwater metal installations has one problem. Namely, non-noble metals or their alloys are mainly used as material for the production of these installations. These types of materials are highly susceptible to corrosion destruction. The problem of corrosion of underground installations has to be solved because of technical, but also economic and ecological reasons [2].

The economic aspects of corrosion of underground or underwater metallic installations are certainly the dominant reason for the adequate application of protection against corrosion. Corrosion-induced costs affect industry, governments and individuals, and their share depends on the degree of development of society, the political system and the lifestyle [3]. Extensive researches conducted in the United States over the period from 1998 to 2001 showed that annual costs due to the devastating effects of metal corrosion for 1998 amounted to about 276 billion US $, which is about 3.1% of the gross national income. Studies conducted in Japan and the European Community countries show that costs caused by corrosion are at a level of 4.2 to 4.5% of gross national income [4], [5]. The presented data are incomplete because they do not take into the account indirect corrosion-induced consequences such as physical disasters, human health hazards, ecological catastrophes, production losses, etc. It is assumed that the costs of indirect effects can be estimated and their amount is approximately equal to the costs of direct consequence of corrosion [4]. Recent research by the US company G2MT Lab indicates that annual costs by corrosion in the United States, taking into the account direct and indirect consequences in 2013, amounted to more than 1,000 billion US $, or about 6.2% of gross national income, and thus represents the highest individual cost of the economy [5], [6]. Also, it is considered that almost

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Paper submitted: October 2017 Paper accepted: December 2017

20% of the world's metallurgical production (production of metal from their ore) is only used to compensate for loss of lost metal in the corrosion process [7].

Previous research has shown that the greatest percentage of damage caused by corrosion can be prevented by the use of modern protection techniques. The metal can be protected from corrosion by using different methods. However, technical practice has shown that one of the most effective methods of protecting metallic structures from corrosion is the application of the cathodic protection system in combination with passive protection (insulating coatings based on polymers or concrete) [2], [7] – [13]. The cost of installing the cathodic protection system is approx. 0.1 - 1% of the total investment value of the object being protected.

The application of the cathodic protection system to underground metal pipelines results in extending the protected object's life span for 10 to 25 years, depending on the chosen solution of the cathodic protection system [2].

Modelling of corrosive processes implies the definition of the simplest possible mathematical model with the invariability of the basic characteristics of the electrochemical system. Practical engineering calculations can simplify the mathematical model, depending on the level of the required accuracy of the calculation. In case of complex geometries of the system, calculation method can be simplified only in cases of objects with a high degree of symmetry, which can be adequately modelled as one-dimensional or two-dimensional problems [7].

The design of the complex geometry objects installed in different corrosive aggressive environments (electrolytes) results in the introduction of more sophisticated scientific methods of calculating the parameters of the cathodic protection system. The development of underground metal infrastructure increases the need for application of cathodic protection systems [14]. More intensive application of cathodic protection systems results in the development of numerical methods for solving the distribution of electric potential and current density of these systems.

Defining the spatial distribution of cathode protection currents density is a preliminary assumption for the optimal design of electrochemical protection of underground metallic structures. The cathodic protection system must be designed to ensure the highest possible homogenous distribution of the protection current density on the surface of the protected object [15].

1. LITERATURE REVIEW ON CATHODIC PROTECTION MODELLING METHODS

For the purpose of determining the relevant parameters of the cathodic protection system parameters, mathematical models based on calculation of distribution of electrical potential and current density on the electrode surfaces are used [14], [16], [17]. To date, a number of analytical models and models based on numerical methods used for calculations when designing and applying cathodic protection systems have been developed.

The purpose of cathodic protection system modelling is to determine the distribution of electrical potential and protection current density on the electrode surfaces as well as at the corresponding points in/on electrolyte (usually at the points where the reference electrode is set for the purpose of verifying the measured values) [7], [14], [16]. The calculation of electric potential and current density distribution comes down to solving the well-known Laplace partial differential equation. Also, the correct calculation of the parameters of the cathodic protection system implies the definition of the appropriate boundary conditions at the overall boundary of the calculation domain. Boundary conditions in the cathodic protection system are most commonly defined through analytical expressions for functional relationships between the protection current density and the electrical potential on the electrodes surfaces [18] – [20]. The establishment of boundary conditions on electrode surfaces in cathodic protection systems is based on the physical and analytical definitions of electrochemical mechanisms of electrode reactions. The total anodic and cathodic current density is the result of electrochemical reactions, which occur simultaneously on the electrodes surfaces. Partial electrochemical reactions occurring simultaneously on the electrode surfaces result in a nonlinear relationship between the electrical potential and the current density i.e. nonlinear polarization characteristics as boundary conditions.

1.1. Analytical models

Analytical models used in designing of the cathodic protection system can generally be divided into two types. The first type is based on basic principles of electrical engineering [9], [21] - Ohm's law and Kirchhoff's laws. In such models, anodes and protected objects are equivalented by a combination of appropriate DC voltages sources and resistors, while electrolytes resistance and resistance of the connecting wires are represented by resistors. These types of models are very simple for application, since only basic laws of electrical engineering are applied. On the other hand, this type of models has a number of disadvantages, consisting in the fact that by representing the electrodes simply by resistors, the current density at all points on the electrode surface is obtained as a constant. This is a rough approximation of most real cases of cathodically protected objects. Also, polarization effects of the cathodic protection system cannot be taken into account with simple elements such as linear resistors. By introducing nonlinear (variable) resistors, which represent the polarization resistance of the electrodes, the previous model can be improved [22], [23]. However, the determination of the value of these nonlinear resistors is only possible for linearized polarization characteristics, which make this model unacceptable for modelling realistic cathodic protection systems.

Second type of analytical models is based on calculating the stationary current field by solving Laplace partial differential equation (LPD) using analytical methods such as variable separation, obtaining solutions in the form of infinite series, superposition method, confident mapping method, etc. The application of this approach is limited to simpler geometries with high the degree of symmetry such as cathodic protection of the inner wall of the well, assuming that the working point of the cathodic protection system is on the linear part of polarization characteristics [14], [23].

1.2. Numerical models

1.2.1. Finite difference method (FDM)

In the early seventies of the last century, the finite different method was used to solve the problem of the distribution of electrical potential and the current density of the cathodic protection system [24]. This method requires discretization of the entire domain of interest with a network. Solutions are located in network nodes. Although by using this method results with satisfactory accuracy are obtained, it is very rarely used when modelling realistic cathodic protection systems because of the need for discretization of the entire domain. The disadvantages that have eliminated this method from this field are long time needed for calculations, especially in the case of complex underground structures that need to be modelled in 3D space. Long time needed for calculation is caused by a large number of nodes required for calculation.

1.2.2. Finite element method (FEM)

The finite element method is one of the most frequently used numerical methods in science and engineering. It is characterized by robustness, modularity and multidisciplinary. In the finite element method, the entire domain of interest is divided into the volumetric elements of the finite dimensions. For each finite element, a set of algebraic equations is written, by solving those yields the distribution of fields within each finite element. Unlike the finite difference method, the finite element method is algorithmic and relatively simple to program for general equations of electromagnetic fields [14], [16]. As with the modelling of most engineering problems, the finite element method has been found to be applied in the field of cathodic protection systems [25] – [29]. However, this method has one disadvantage when modelling the cathodic protection system of underground pipelines due to the need for discretization of the entire domain (electrolyte) considered as a semi-infinite domain and due to a large difference in the subdomain dimensions (electrolyte as a semi-infinite subdomain and, for example, the pipeline wall thickness). This results in a large number of finite elements and therefore a large number of algebraic equations, and consequently requires a long time for calculation.

1.2.3. Boundary element method (BEM)

One of the most efficient methods for solving problems in the field of application of cathodic protection systems is the boundary element method (BEM). The basic advantage of this method is that discretization is required only at the boundary of the domain of interest and there is no need for discretization of infinite boundaries [30] – [32]. For the modelling of the cathodic protection system of underground or underwater metallic installations using the boundary element method, Laplace's partial differential equation (LPD), which is valid for the considered domain (electrolyte), is transformed into the integral field equation that only treats the boundary surfaces [33]. The discretization of this equation as well as the application of boundary conditions leads to the formation of the system of algebraic equations that need to be solved. For nonlinear boundary conditions, such as the case of cathodic protection system modelling, the system of equations is nonlinear, and it is necessary to use iterative methods such as the Newton - Raphson method [3], [34] – [38].

As the most suitable method for calculating the parameters of the cathodic protection system of underground metallic installations, this method has been improved during the last three decades through hybridization with other mathematical methods. As such, it has been used for specific calculations in the application of the cathodic protection system. Below are some interesting applications of these types of methods for specific calculations in the application of the cathodic protection system:

- for the analysis of the impact of the horizontal layered soil on the level of polarization of the protected pipelines by the cathodic protection system as well as the wells. Two approaches based on the boundary element method were proposed. The first approach, called the MRBEM (Multi Region Boundary Element Method), where all boundaries between neighbouring soil layers are to be discretized by the network of boundary elements [19], [39], [40] as well as the MLBEM (Multi-Layer Boundary Element Method) in which the layered soil is treated by applying the appropriate Green's functions [19], [30], [35], [38], [40], [41];

- to determine the optimal anode location and output current of the cathodic protection systems of underground pipeline or the ship's hull, many authors used a hybrid method based on the boundary element method for the calculation of the distribution of electric potential and optimization methods such as gradient method, Gauss - Newton method, a simulated annealing method [42], a genetic algorithm [43], or a combination of optimization methods [44], [45] for optimization of location and output current of anode;

- for the measurement results interpretation of the ON potential (the measured pipeline potential in relation to the reference electrode mounted on the ground surface directly above the pipeline during the operation of the cathodic protection system), the OFF potential (the measured electrical potential of the pipeline relative to the reference electrode mounted on the ground surface directly above the pipeline, cathodic protection system excluded) and the current density, in order to determine the condition of the coating and the estimation of the existence and size of defects on isolation coating based on the boundary element method and the nonlinear regression analysis. Due to the robustness of the electrode surface model, they are discretized by cylindrical boundary elements, the anode polarization curve is assumed to be linear, while the linearized polarization characteristic is used for the cathodic surface [46] – [48];
- for the calculation of the distribution of the electric potential of the cathodic protection system with dynamic polarizing characteristics based on potentiometric measurements caused by deposition of carbonate deposits on the surface of the protected object, approaches based on the method of boundary elements [49] – [52] were proposed. In order to obtain electrical potential values on the surface of the protected object, two approaches were proposed: the fictitious time method and the fictitious potential method combined with the boundary element method.

1.2.4. Coupled boundary element/finite element method

Although the boundary element method is one of the most frequently used methods for the modelling of the cathodic protection system, it does not give satisfactory results in the case of very long electrode surfaces such as long underground pipelines protected by a cathodic protection system. The reason for this lies in the fact that this method treats the electrode systems as equipotential, i.e. does not take into account the potential drop caused by the resistance of the material from which the electrodes are made. To overcome this problem, hybrid boundary element/finite element method is proposed in literature [7], [14]. The boundary element method is suitable for solving stationary current field problem in infinite and semi-subsurface domains, so it is suitable for calculating distribution of electric potential in the electrolyte. On the other hand, the finite element method is suitable for solving stationary current field of bounded domains; therefore it can be applied for the field calculation within the electrodes. The potential difference on the electrode/electrolyte interface is calculated using the hybrid boundary element/finite element method, as described in literature [7], [14], [53], [55] – [59].

1.2.5. Meshless methods

In order to accelerate the calculation of the distribution of electromagnetic fields, more attention has been paid to meshless numerical methods. W. J. Santos et al. [60] – [62] applied the method of fundamental solutions (MFS) in combination with the genetic algorithm (GA) to solve the problem of distribution of the current density and electric potential on the electrode surfaces of the cathodic protection system. As well as the boundary elements method, the method of fundamental solutions is applicable when the fundamental solutions of the field partial differential equations are known. Advantage of method of fundamental solutions over the boundary element method is it does not require any special integration or any special treatment of singular points. By this method, the approximation of the solution is obtained as a linear superposition of fundamental solutions. The singularity points are positioned outside the domain of interest. These singular points are called virtual sources or a "pseudo-boundary", which physically has no touch points with the actual domain boundary points. The basic disadvantage of this method is to determine the position of virtual sources as well as their current intensity. Their position must be determined in such a way as to meet the boundary conditions at the observation points that are set at the domain of interest. Due to the nonlinearity of boundary conditions, it is necessary to apply iterative techniques in combination with the method of fundamental solutions.

2. BASIC EQUATIONS FOR CATHODIC PROTECTION SYSTEM MODELLING

The distribution of electric potential in electrolyte and on electrode surface of cathodic protection system, under assumption that electrolyte is homogeneous and isotropic, is defined by the Laplace partial differential (LPD) equation for the static current field [8]:

$$-\nabla(\gamma \cdot \nabla \varphi) = 0 \quad (1)$$

where: γ electric conductivity of electrolyte in which cathodic protection system is placed, φ electric potential, a symbol ∇ is Nabla operator.

By solving the previous partial differential equations (1), the value of the electric potential at any point within domain of interest can be determined. It is well known that the LPD equation (1) have infinite solutions. Therefore, to obtain a unique solution, it is necessary to solve the LPD along with the appropriate boundary conditions. The boundary that needs to be take into account when modelling cathodic protection systems are the anode/electrolyte boundary and cathode/electrolyte boundary [63]. The boundary conditions applied at these boundaries are called polarization characteristics and they take into account the electrochemical reactions that take place on the electrode (anode and cathode) surfaces. Polarization characteristics represent functional relationships between the current density and the electrical potential of the electrode surfaces [64]. Due to the nature of the electrochemical reactions, these functional relationships are nonlinear. The mathematical form of these functional relationships depends primarily on the number and type of partial electrochemical reactions occurring simultaneously on the electrode surfaces. Partial electrochemical reactions occurring at the same time on the cathode surface are metal dissolution, oxygen reduction and hydrogen separation. Therefore, the total value of the polarization current density of the cathode current can be represented by the following equation [31], [59]:

$$j_c = j_{0,Fe} \cdot 10^{\frac{\varphi-\varphi_{Fe}}{\beta_{Fe}}} - j_{\lim,O_2} - j_{0,H_2} \cdot 10^{\frac{-(\varphi-\varphi_{H_2})}{\beta_{H_2}}} \tag{2}$$

where j_c is total current densities for cathode surface, $j_{c,Fe}$ current density corresponding to the metal dissolution reaction, j_{lim,O_2} is the limiting current density of oxygen reduction j_{H_2} is current density corresponding to the reaction of hydrogen separation, φ is potential difference of interface metal/electrolyte φ_{Fe}, and φ_{H_2} are equilibrium potentials of corresponding electrochemical reactions and β_{Fe}, and β_{H_2} are Tafel's slopes (coefficient) of metal dissolution and hydrogen separation, respectively.

From the previous equation (2) it is noticeable that the various electrochemical reactions do not result in the same form of functional relationship between the electric potential and the partial current density. It can be noticed that the functional relationship between partial current density and electrical potential characteristic for reactions of the metal dissolution and hydrogen separation have the same form but the opposite sign. The reason is that the electrochemical reaction of metal dissolution is an anode reaction and according to the electrochemical convention there is a positive sign while the electrochemical reaction of hydrogen separation is a cathode reaction and to the electrochemical convention has a negative sign. It is also noticeable that the functional relationship that represents the electrochemical reaction of oxygen reduction in has significantly different form in comparison to the remaining two functional relationships. The reason for this lies in the fact that the electrochemical reaction of oxygen reduction is concentration controlled and subject to diffusion laws, while the remaining two electrochemical reactions are actively controlled and their functional dependencies can be described by the Butler-Volmer equation.

Different electrochemical reactions take place on different electrodes. For anodic surfaces, it is characteristic that an electrochemical reaction occurs simultaneously are anode metal dissolution and oxygen reduction. The total value of the polarization current density of galvanic anode surface is defined by following equation [65]:

$$j_A = j_{O_2} \cdot \left(10^{\frac{\varphi-\varphi_A}{\beta_A}} - 1 \right) \tag{3}$$

where φ_A is the corrosive potential of the galvanic anode, and β_A is the Tafel's coefficient (slope) of the galvanic anode.

Previous equation (3) is suitable for the galvanic anode cathodic protection system modelling. In case of the impressed current cathodic protection system boundary condition on the impressed current anode surface have the following form [14]:

$$j_A = j_{O_2} \cdot \left(10^{\frac{\varphi-\Delta U_r-\varphi_A}{\beta_A}} - 1 \right) \tag{4}$$

where ΔU_r is the voltage added by the rectifier.

3. DIRECT BOUNDARY ELEMENT METHOD

The direct boundary element method used for the calculation of the electrical potential and current density distribution is based on solving the integral equation of the static current field. The integral equation of the static field can be obtained by applying Green's symmetric identity to the Laplace partial differential equation (1). Integral equation of the static current field has the following form [59], [66], [67]:

$$c(q)\varphi(q) + \int_\Gamma \varphi(p) \cdot \left(\vec{n} \cdot \nabla G(p,q)\right) d\Gamma = \int_\Gamma G(p,q) \cdot \left(\vec{n} \cdot \nabla \varphi(p)\right) d\Gamma \tag{5}$$

where q is observation point, p is field source point, $c(q)$ is constant, $\varphi(q)$ is the electric potential of the observation point q, $\varphi(p)$ is the potential of the field source point p, $G(p,q)$ is the Green's function, Γ is the boundary of the domain and $\vec{n}$ is normal unit vector.

In order to calculate the normal component of the current density on the electrode surfaces, the Ohms law needs be introduced, as follows [54] ,[59], [68]:

$$\vec{n} \cdot \nabla \varphi(p) = -\frac{j(p)}{\gamma} \tag{6}$$

where $j(p)$ is normal component of the current density of field source point p.

By incorporating the Ohm's law (6) into the integral equation of the static current field (5), an integral field equation is obtained that is suitable for the calculation of current density and electrical potential distribution on the electrode surfaces of the cathodic protection system, as follows:

$$c(q)\varphi(q) + \int_{\Gamma} \varphi(p) \cdot (\vec{n} \cdot \nabla G(p,q))\, d\Gamma = -\frac{1}{\gamma} \int_{\Gamma} G(p,q) \cdot j(p)\, d\Gamma \tag{7}$$

The previous integral equation of the static current field (7) is suitable for solving bounded problems such as the calculation of the current density and electric potential distribution of the inner wall of the pipeline or tanks. In order to calculate the parameters of the cathodic protection systems of metallic installations that are placed in infinite and semi-infinite domains, such as soil and sea water, the Gaussian boundary condition for infinite boundaries must be satisfied [16], [25], [52], [59], [63], [69], [70]:

$$\int_{\Gamma} j(p)\, d\Gamma = 0 \tag{8}$$

This boundary condition can simply be met by imposing a constant potential at the infinite bound φ_{∞} which is added to the right side of the integral field equation (7). Therefore, integral field equation for the modelling the cathodic protection systems placed in infinite and semi-infinite domain can be written in the following form [14], [25], [49], [50], [53], [54], [59], [68]:

$$c(q)\varphi(q) + \int_{\Gamma} \varphi(p) \cdot (\vec{n} \cdot \nabla G(p,q))\, d\Gamma = -\frac{1}{\gamma} \int_{\Gamma} G(p,q) \cdot j(p)\, d\Gamma + \varphi_{\infty} \tag{9}$$

By introducing a constant potential at the infinite boundary φ_{∞} in equation (7) provides a conservative flow of current between anode and cathode surface [16].

In the procedure of calculating the current density and electrical potential distribution on the electrode surfaces of the cathodic protection system using a direct boundary element method, the boundary surfaces must be discretized to a certain number of boundary elements. After the discretization of the boundary surfaces, equations (8) and (9) are solved over each boundary element. Boundary surfaces are usually discretized to the boundary elements of the same geometric shape and thus the solution of the integral equations of each element has the same form. After the application of the collocation point method, the coordinate transformation from global coordinate system to the local coordinate system and the Gaussian - Legendre numerical integration on each boundary element, the current density and the electric potential at the each point of all the boundary elements can be obtained by solving the following matrix equation [14], [63], [68], [71]:

$$\begin{bmatrix} H_{cc} & H_{ca} & -1 \\ H_{ac} & H_{aa} & -1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} \varphi_c \\ \varphi_a \\ \varphi_{\infty} \end{Bmatrix} = \begin{bmatrix} G_{cc} & G_{ca} \\ G_{ac} & G_{aa} \\ A_c & A_a \end{bmatrix} \cdot \begin{Bmatrix} j_c \\ j_a \end{Bmatrix} \tag{10}$$

where $[H]$ and $[G]$ matrix coefficients, $\{\varphi_c, \varphi_a, \varphi_{\infty}\}^T$ vector of unknown electric potentials of electrode surfaces and $\{j_c, j_a\}^T$ vector of current density. Indexes in the previous equation c and a denote the cathode and anode surfaces, respectively.

Since the elements of the current density vector are given by functional relationships between the current density and the electrical potential, which are nonlinear, the matrix equation (10) is nonlinear. Therefore, to solve nonlinear matrix systems it is necessary to use iterative techniques such as Gauss's method, Gauss - Seidel method or Newton - Raphson method [21,72,73,74]. For the solution of the nonlinear matrix equation (10), Newton - Raphson 's method was used in this paper.

3.1. Iterative Newton - Raphson method

The Newton - Raphson's iterative method, proposed in the papers [49], [50], [51], [52], was used in this paper to solve the nonlinear matrix equation (10). The first step of the Newton - Raphson's iterative method is the expansion of the current density vector into Taylor series. Then the current density vector in each iteration can be determined by applying the following equation:

$$\{j\}_n = \{j\}_{n-1} + \left[\frac{\partial j}{\partial \varphi}\right]_{n-1} \{\Delta\varphi\}_n \tag{11}$$

where n denotes the current iteration, n - 1 previous iteration, $[\partial j/\partial \varphi]$ is Jacobian's matrix and $\{\Delta\varphi\}$ is vector of electric potential increments between the two adjacent iterations. The vector of electric potential increments between two adjacent iterations can be calculated by using the following equation [68,71]:

$$\{\Delta\varphi\}_n = \begin{Bmatrix} \varphi_c \\ \varphi_a \\ \varphi_{\infty} \end{Bmatrix}_n - \begin{Bmatrix} \varphi_c \\ \varphi_a \\ \varphi_{\infty} \end{Bmatrix}_{n-1} \tag{12}$$

By adding the equations (11) and (12) in to the matrix equation (10), the following matrix equation is obtained [49] – [52], [63]:

$$\left([H]-[G]\left[\frac{\partial j}{\partial \varphi_S}\right]_{n-1}\right)\cdot\{\Delta\varphi\}_n=[G]\{j\}_{n-1}-[H]\{\varphi\}_{n-1} \tag{13}$$

From the previous matrix equation (13), a vector of electric potential increments can be determined between n^{th} and $(n\text{-}1)^{th}$ iterations. Further, by using equations (11) and (12), a vector of the current density and the vector of electrical potentials of all the boundary elements in the n^{th} iteration can be determined. The iterative cycle is repeated until convergence is achieved i.e. until all elements of electric potential increments vector is less than the set value (sufficiently close or equal to zero) [49] – [52], [68], [71].

In order to start the iterative cycle, it is necessary to guess the initial value of electric potential on all points of the electrode surfaces. Since the performance of Newton - Raphson's iterative methods depends on the initial guess, it is necessary to consider the choice of initial point on the polarization characteristics. In this paper the initial value of the electrical potential of all the points on the electrode surfaces was calculated from the functional relationships defined by equations (2) and (3) under assumption that there is no current flow in the system.

Since matrix $[H]$ and $[G]$ are full matrix, calculation of the equation (13) can be time consuming. Therefore, to accelerate the calculation of equation (13), the GMRES iterative method was used [68], [71].

4. BENCHMARK EXAMPLE

A previously presented mathematical model was used for the parameter calculation of one benchmark example of galvanic anode cathode protection system. The geometry of considered problem is given in Figure 1.

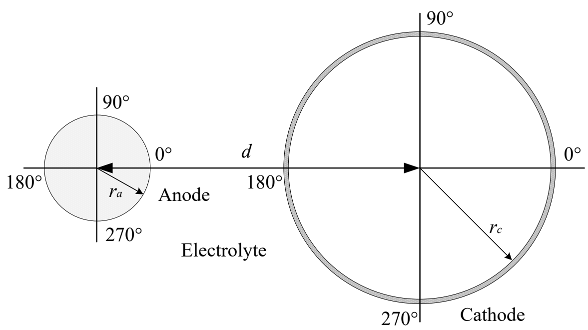

Figure 1: Geometry of analysed benchmark problem

The analysed cathodic protection system is placed in a homogeneous isotropic electrolyte with a electric conductivity of γ = 0.01 S/cm. The system consists of a cylindrical protected object (cathode) of radius r_c = 500 cm and a cylindrical galvanic anode radius r_a = 25 cm. Distance between centres of anode and cathode is d = 625 cm. Polarization characteristics parameters of anodic and cathodic surfaces are given in Table I.

Table I: Parameters of polarization characteristics [14], [31]

Parameter	Value
j_{0Fe}	1.0 (μA/cm²)
φ_{Fe}	-700 (mV) vs. CSE
β_{Fe}	60 (mV/dec)
j_{limO_2}	1.1 (μA/cm²)
j_{H_2}	1.0 (μA/cm²)
φ_{H_2}	-800 (mV) vs. CSE
β_{H_2}	120 (mV/dec)
j_{O_2}	1.0(μA/cm²)
φ_a	-1100 (mV) vs. CSE
β_a	60 (mV/dec)

Below are given results of the calculation of the electrical potential and current density distribution on the electrode surfaces of the analysed cathodic protection system. Due to the existence of symmetry on both analysed electrode surfaces, it is sufficient to give the distribution of the electrical potential and current density in the range 0° to 180°. Figures 2 and 3 show the distribution of electrical potential and current density on the surface of the protected object (cathodes).

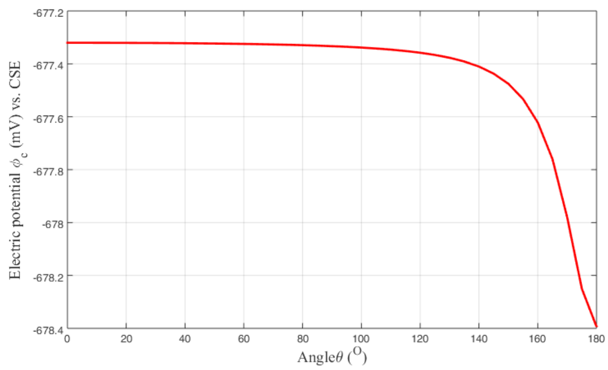

Figure 2: Distribution of electrical potential on the cathode surface

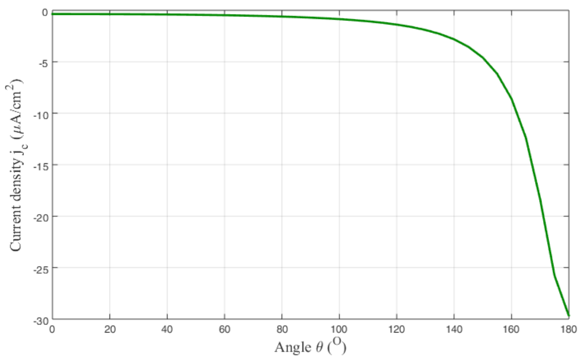

Figure 3: Distribution of current density on the cathode surface

The results shown in Figures 2 and 3 indicates that the highest negative value of the electrical potential on the cathode surface that is closest to the anode (the most negative value of the electric potential on the cathode surface is for α = 180°). Based on the above, it can be concluded that the cathode surface that is closest to the anodes will be more protected from the other parts of the cathode. Also, the cathode surface closest to the galvanic anode is the highest value of the protective current density.

The distribution of electrical potential on the galvanic anode surface is shown in Figure 4. From the results given in Figure 4, it can be noted that the highest value of the potentials is on the anode surface that is closest to the cathode (the highest value of the anode electrical potential is for α = 0°).

The results given in Figure 5 indicate that the value of the current density of the anode surface is highest on the surface closest to the cathode. Since the distribution of the protection current density is nonlinear on the surface of the galvanic anode, it will come to uneven dissolution of the galvanic anode over time. The previously presented results indicate that the galvanic anode surface that is closer to the cathode will faster dissolve.

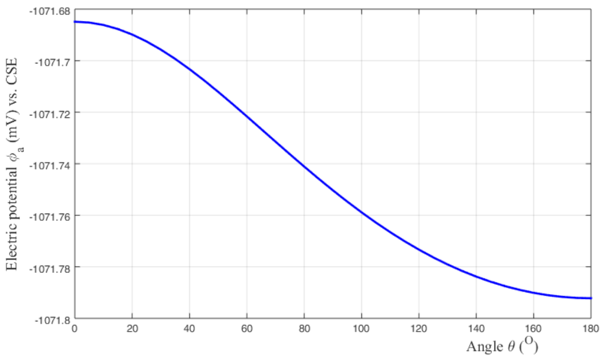

Figure 4: Distribution of electrical potential on the anode surface

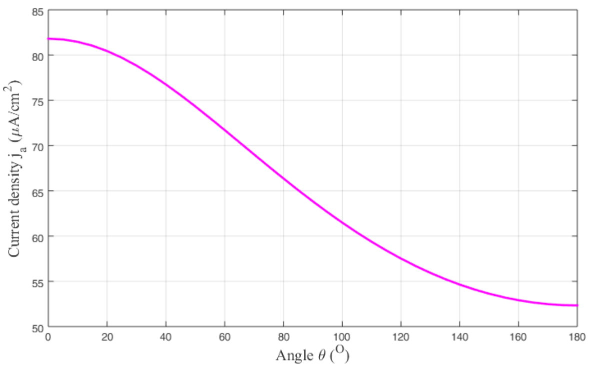

Figure 5: Distribution of the current density on the anode surface

5. CONCLUSION

When designing and verifying the efficiency of modern cathodic protection systems it is necessary to carry out very precise and accurate methods for the calculation of cathodic protection system parameters. The paper presents an overview of used mathematical models in literature for the cathodic protection system modelling. Also, a mathematical model based on the direct boundary element method for the calculation of parameters of the cathodic protection system is presented in this paper. The presented mathematical model takes into account the electrochemical reactions that take place on the electrode surfaces through the corresponding polarizing characteristics. An example of the application of the presented mathematical model is demonstrated in a geometrically simple example of the cathode protection system with galvanic anodes.

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