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# CLASSIFICATION AND COMPERATION OF UNCERTANITY MODELLING METHODS IN POWER SYSTEMS

# *KLASIFIKACIJA I POREĐENJE METODA ZA MODELOVANJE NEODREĐENOSTI U ELEKTROENERGETSKIM SISTEMIMA*

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**Abstract:** Uncertainty is one of the most important factors contributing to the complexity of the power system operation and management. This paper presents some of the most important uncertainty modelling techniques and compares their advantages and disadvantages. In particular, this paper focuses on identification, classification and comparison of uncertainty modelling approaches used in power systems, highlighting the Distributed Generation (DG) allocation problem. The main objective of this paper is to identify the sources of uncertainty in DG allocation problem, review the most important uncertainty modelling methods and propose the appropriate matching approach between the sources of uncertainty and modelling methods

**Keywords:** allocation, distributed generation, fuzzy logic, power system, uncertainty

**Sažetak:** Neodređenost je jedan od glavnih faktora koji doprinosi kompleksnosti u procesu upravljanja elektroenergetskim sistemom. U radu su predstavljene i uporeñene metode koje se koriste za modeliranje elektroenergetskog sistema u uslovima neodređenosti. Rad se fokusira na prepoznavanje, klasifikaciju i usporedbu metoda za modeliranje neodređenosti, koje mogu biti korištene u problemu alokacije distribuirane proizvodnje. Osnovni ciljevi ovog rada su identifikacija izvora neodređenosti u elektroenergetskom sistemu sa posebnim osvrtom na problem alokacije distribuiranih izvora, pregled najvažnijih metoda za modeliranje neodređenosti i utvrđivanje načina na koji se određena metoda za modelovanje neodređenosti može izabrati za konkretnu primjenu, u zavisnosti od vrste problema.

**Ključne riječi:** alokacija, distribuirana proizvodnja, fuzzy logika, energetski sistem, neodređenost

## INTRODUCTION

Power systems are subject to a large number of uncertainties, which have traditionally been linked to the contingencies and risk management. In fact, the historical management of the uncertainties in power systems was focused on the assessment of security related issue using deterministic (e.g. N-1 criteria) and probabilistic (e.g. stochastic and robust) optimisation approaches, with the aim of achieving a trade-off between cost and security [1]. However, the application of these techniques is a part of an overall risk analysis and includes examination of unwelcome events which have a potential to cause hazards. More recently, however, sources of uncertainties have been linked to the increased penetration levels and intermittent nature of the renewable energy sources [1]. Renewable energy sources are of the random nature and for this reason uncertainties related to DG technologies are stochastic [2]. Further, power system planning and operation requires extensive simulation-based analysis such as load flow studies. In order to perform these simulations, adequate models are required. However, models are only approximations of the real systems and it is generally accepted that in power system studies model parameters are one of the major sources of uncertainty [3]. Finally, deregulation processes have increased decision making complexity and this fact is considered to be a direct cause of additional increase of power system uncertainty [4]. A formal definition of uncertainty is provided by [5] in a proposition which defines the uncertainty as a situation in which the crucial information required to make an appropriate description of the problem is not available.

## 1. CAUSES AND SOURCES OF UNCERTAINTY

The lack of information is the most common cause of uncertainty in a decision-making model [5]. In a broad sense, uncertainty parameters can be divided in two categories. The first group contains parameters which arise from uncertainty and imprecision related to measurements, observations and data collection complexity, conflicting evidence, ambiguity, and belief. The second group contains parameters that are inherently uncertain, due to their physical properties and relationship to external factors.

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Examples include power system load and DG generation curves. The traditional deterministic approaches are not entirely appropriate to address this group of parameters. Due to the modern technological challenges, the information available to describe the problem is available in various forms since the engineers design systems in which information can be qualitative, non-numerical and linguistic [6]. The most common types of information available to describe the problem with uncertainty are defined as numerical, interval, linguistic and symbolic information [5].

There are many sources of uncertainty in power systems such as the natural temporal variations, predictions and monitoring errors and physical parameter estimation [7]. The uncertainty is inherent in almost every part of the power system such as DG allocation problem [8], PV solar [9] and wind [10] production, system operation [11], system planning [12], reconfiguration [13], state estimation [14], electricity market [15], restoration [16], transmission expansion planning [17], control problems [18], fault location [19], risk based day ahead electrical vehicles aggregator scheduling [20], risk analysis [21], maximum loading point of electric power [22] and optimal load flow with HVDC Connected Wind Farms [23]. There are two fundamental forms of uncertainty: aleatory (which is random and irreducible) and epistemic (which is related to knowledge and validation and is reducible) [7]. In [24], power system uncertain parameters are classified in two major groups: technical and economical parameters. The most important sources of uncertainties which are relevant for the DG allocation problem are the load and generation output uncertainties. In the following section, load and generation output modelling are presented.

### 1.1. Load uncertainty

Power system loads are regarded as a major source of uncertainty in the power system models [25]. It is difficult to determine the accurate representation of the load, because it is variable, distributed and composed of various load types. However, single customer loads can be aggregated to provide one particular type of load which is predictable and can be represented by a typical load curve, such as residential, industrial and commercial. This approach is accepted as fair approximation for the purposes of power system steady state analysis. Load uncertainty can be reduced by structuring models according to their physical characteristic and by estimation of the parameter values from disturbance measurements [25]. A number of approaches are available for assessing and reducing the impact of uncertainty [25]. Probability and fuzzy approach can be used to model the load uncertainty [26]. Defining as the uncertain load, as the mean and as the standard deviation of the uncertain load, various probability distribution functions (PDF) can be used to represent the load uncertainty such as [26]:

- *Normal distribution*

$$f(P_D)=\frac{e^{-\frac{(P_D-\mu)^2}{\sigma^2}}}{\sigma\sqrt{2\pi}}$$
$$-\infty \le P_d \le \infty$$
$$\sigma > 0 \qquad (1)$$

- *Lognormal distribution*

$$f(P_D)=\frac{e^{\frac{\left(\ln\left(\frac{(P_D-\mu)}{m}\right)\right)^2}{2a^2}}}{\sigma(P_D-\mu)\sqrt{2\pi}}$$
$$P_D \ge \mu$$
$$\sigma > 0 \qquad (2)$$

where *m* is the scale parameter

$$f(P_D)=\frac{e^{-\frac{P_D-\mu}{b}}}{b}$$
$$P_D \ge \mu$$
$$b > 0 \qquad (3)$$

with *b* being the scale parameter.

- *Beta distribution*

$$f(P_D)=\frac{(P_D-d)^{a-1}(c-P_D)^{b-1}}{B(a,b)(c-d)^{a+b-1}}$$
$$=\frac{\Gamma(a+b)(P_D-d)^{a-1}(c-P_D)^{b-1}}{\Gamma(a)\Gamma(b)(c-d)^{a+b-1}}$$
$$d \le P_D \le c$$
$$a > 0$$
$$b > 0 \qquad (4)$$

where *a* and *b* are the shape parameters, *c* being the upper bound, *d* the lower bound, and *B (a, b)* the beta function.

- *Weibull distribution*

$$f(P_D)=\frac{a(P_D-\mu)^{a-1}}{b^a}e^{-\left(\frac{P_D-\mu}{b}\right)^a}$$
$$P_D \ge \mu$$
$$a > 0$$
$$b > 0 \qquad (5)$$

where *a* is the shape parameter, *μ* is the location parameter and *b* is the scale parameter.

### 1.2. Generation output uncertainty

The rapid increase in renewable energy introduces uncertainties related to the power generation output [27]. Ignoring the operational uncertainties, it can be stated that solar irradiation and wind speed are the greatest sources of uncertainty in solar and wind power systems, re-

spectively. Solar irradiation and wind speed are typically modelled by a probabilistic distributions, such as Beta and Weibull distributions in the cases in which historical data are available [28]. In the case in which historical data is not available, the fuzzy approach can be adopted to model the uncertainties associated with solar and wind power outputs. It is important to note that the prediction of power output from the irradiance introduces additional sources of uncertainty and it requires a sequence of models such as translation of irradiance to the plane-of-array, estimation of cell temperature and calculation of module electrical output [29]. The equivalent applies to the prediction of the wind generator output power based on the wind speed data.

## 2. METHODS FOR UNCERTAINTY MODELLING

Traditional uncertainty power system analysis uses two approaches to deal with uncertain parameters. The first one is the probabilistic analysis, usually used to determine generation reserve and the second one is contingency analysis used for transmission planning and operation [21]. The main purpose of various uncertainty modelling approaches is to determine the extent to which uncertainty of input parameter $X_i$ influences the output parameter $Y_i$ of the systems. There are various modelling methods which use different modelling techniques to represent uncertainty as shown in classification diagram in Figure 1.

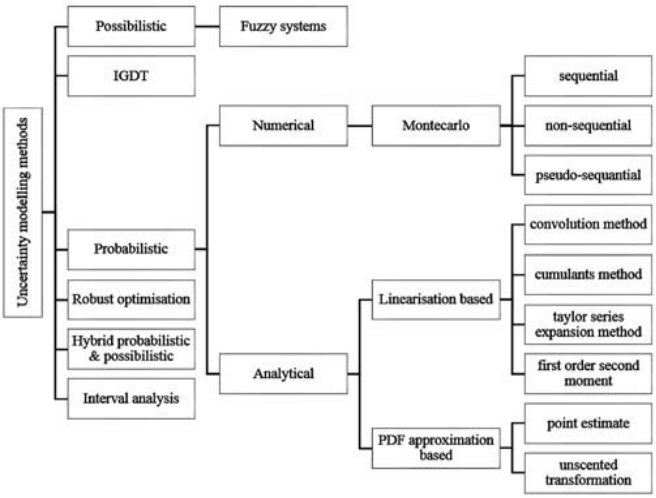



Figure 1: Classification of uncertainty modelling methods

### 2.1. Probabilistic methods

Probabilistic methods are used to model uncertainty in the case in which the uncertain parameters can be represented using the PDF. In probabilistic models, a multivariate function $y=f(Z)$ is available where $f$ describes the system, $Z=[z_1,...,z_m]$ is a vector of random parameter which can be represented with PDF and $y$ is to be determined [24]. Probabilistic analysis is used to design distributed energy systems [30] under conditions of uncertainty as shown in [31], where a parallel Monte Carlo method for optimum allocation of distributed generation is proposed [32]. In [30] an optimal distributed generation placement under uncertainties based on point estimate method using embedded genetic algorithm is developed. In [30], the interdependence between the random input variables is modelled by using the concepts of comonotonicity, countermonotonicity, while the independence between distributed generations and distributed consumptions is modelled using stochastic bounds which describe the statistical dependence between the input random variables. In [33], the probabilistic approach to solve the problem of DG allocation in isolated electrical service area is applied, considering the hourly load changes or the daily load cycle. In [32], a probabilistic power flow is adopted based on genetic algorithm approach to find a solution to the problem that is modelled mathematically under conditions of uncertainty and constraints, where the uncertainty parameters (wind generation, load growth, PV solar output and electricity price) are modelled using the point estimate method. In [34], the static voltage stability is evaluated using a two-point estimation method and continuation power flow (in distribution systems, considering DG which is represented using PDF). Finally, a method for solving a probabilistic three-phase power flow in radial distribution networks is proposed in [35], taking into account the technical constraints. The most common probabilistic methods are presented next.

#### 2.1.1. Monte-Carlo simulation

Assuming $z_i$ to be a represent uncertain parameter, the Monte Carlo simulation procedure is carried out according to the following steps [24]:

- Using the PDF, $z_i^e$ is generated for each $z_i$.
- Defining $Z=[z_1^e....,z_m^e]$, calculate the variable :

$$y^e=f(Z^e) \tag{6}$$

- Iterate the procedure for $N_{MC}$.
- Analyse the results using statistics, histograms etc.

Monte-Carlo simulation is widely used technique for probabilistic uncertainty modelling and considers uncertain parameters as random variables with probabilistic distribution and provides a random value as an output variable [3].

#### 2.1.2. Point estimate method

This method is carried out following these steps [24]:

1. Set $E(Y)=0$, $E(Y^2)=0$ and $k=1$.
2. Find $\varepsilon_{k,i}$ (the locations) and $P_{k,i}$ (probabilities of con-centration) using following relations:

$$\varepsilon_{k,i}=\frac{1}{2}\frac{M_3(z_k)}{\sigma_{z_k}^3}+(-1)^{i+1}\sqrt{n+\left(\frac{1}{2}\frac{M_3(z_k)}{\sigma_{z_k}^3}\right)^2} \tag{7}$$

$$P_{k,i} = (-1)^i \frac{\varepsilon_{k,3-i}}{2n\sqrt{n + \left(\frac{1}{2}\frac{M_3(z_k)}{\sigma^3_{z_k}}\right)^2}} \quad (8)$$

with $M_3(z_k)$ being the third moment of parameter $z_k$.

3. Find the concentration points $z_k$ according to the following equation:

$$z_{k,i} = \mu_{z_k} + \varepsilon_{k,i} \cdot \sigma_{z_i}, \quad i = 1,2 \quad (9)$$

where $\mu_{zk}$ is the mean and $\sigma_z$ the standard deviation of $Z_k$.

4. Calculate f for both $Z_{k,i}$ using the equation:

$$Z = \left[z_1, z_2, ..., z_{k,i}, ..., z_n\right], \quad i = 1,2 \quad (10)$$

5. Calculate $E(Y)$ and $E(Y^2)$ using following relations:

$$E(Y) = E(Y) + \sum_{i=1}^{2} P_{k,i} f\left(z_1, z_2, ..., z_{k,i}, ..., z_n\right) \quad (11)$$

$$E\left(Y^2\right) = E\left(Y^2\right) + \sum_{i=1}^{2} P_{k,i} f^2\left(z_1, z_2, ..., z_{k,i}, ..., z_n\right) \quad (12)$$

6. Put $k=k+1$ if $k < n$, then go back to the step 2, otherwise continue.

7. Find the mean and standard deviation using following formulas:

$$\mu_Y = E(Y) \quad (13)$$

$$\sigma_Y = \sqrt{E(Y) - E^2(Y)} \quad (14)$$

### 2.2. Possibilistic methods

Fuzzy models can be very useful in the process of electricity distribution network planning and management, particularly from the point of view of load forecasting [36] and representation of uncertainty [37] in the power system. The most frequently used fuzzy method for the inclusion of uncertainty parameters is α-cut method. The α–cut methods provide answer to the question: what is the membership function (MF) of if MFs of $X$ are known, where $y=f(x_1,...,x_n)$ is a function describing the system model and $X$ is a vector of uncertain input parameters to the system [24]. Let us consider a fuzzy set $\tilde{A}$ in $U$. Then, we define $A^\alpha$ as a crisp set called of the fuzzy set $\tilde{A}$, which contains all individuals of $U$ which have membership value greater than or equal to α [2, 38]:

$$A^\alpha = \{x \in U \mid \mu_A(x) \geq \alpha\} \quad (15)$$

$$A^\alpha = \left(\underline{A}^\alpha, \overline{A}^\alpha\right) \quad (16)$$

The α–cut for every variable $x_1^\alpha$ is calculated as shown in [2]. After that the α–cut of $y$, $y^\alpha$ is calculated as:

$$y^\alpha = \left(\underline{y}^\alpha, \overline{y}^\alpha\right) \quad (17)$$

$$\underline{y}^\alpha = \min f\left(x_1^\alpha, ..., x_n^\alpha\right) \quad (18)$$

$$\overline{y}^\alpha = \max\left(x_1^\alpha, ..., x_n^\alpha\right) \quad (19)$$

This applies that for every alpha cut two optimisations are done. The first is the maximisation which obtains the upper bound of y, whereas the second one is the minimisation which obtains the lower bound of y. The graphical representation of the set α–cut is shown in Figure 2. Possibilistic approach to uncertainty modelling is used in numerous applications. In [39], a computational approach for calculating the product of various fuzzy numbers is proposed using α-cuts and a regression model to obtain the membership function of the product of two fuzzy numbers. It also uses the computational approach based on α-cuts to compute solutions of fully fuzzy linear systems. Load flow analysis in this case becomes a fuzzy load flow analysis [40] and the final result is obtained as a fuzzy number, which requires defuzzification operation to move away from fuzzy domain and obtain a single valued, crisp output variable [8]. Reference [41] presents an improved method for modelling input parameter interactions in the possibilistic harmonic load flow. Possibilistic modelling of uncertainties is extensively used for system planning purposes. For example, [42] develops a robust possibilistic mixed-integer programming method for planning being applied to municipal electric power systems considering the uncertainty. Also, reference [43] presents a fuzzy model applied to the problem of multi objective planning of power distribution networks which determines the optimal site and size of the feeders and substations in distribution networks that optimises economic cost, reliability, and exposure using meta-heuristic algorithm based on Tabu Search. Further, reference [44] uses possibilistic uncertainty modelling of wind resources to solve optimal wind capacity integration problem, in a model which aims to minimise the total cost.

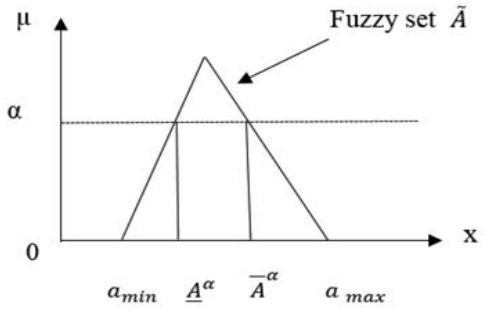



Figure 2: Graphical representation of the set α–cut

### 2.3. Hybrid possibilistic-probabilistic approach

Hybrid possibilistic-probabilistic approach is used in applications where both possibilistic and probabilistic parameters are present. This type of problems cannot be

solved with a single method and require the combined methods approach. There are two commonly used possibilistic-probabilistic approaches. The first one is a Mixed Possibilistic Monte-Carlo approach. Let us define the *f* as a function which describes the system model, *Z* as a vector which describes the uncertain input parameters using the PDF, and finally, *X* as a vector which describes the uncertain input parameters using the MF. The Mixed Possibilistic Monte-Carlo method defined as follows [24]:

- For every $z_i \in Z$ generate a value using its PDF $z_i^e$
- Calculate $(\overline{y}^{\alpha})^e$ and $(\underline{y}^{\alpha})^e$ in a following way:

$$(\underline{y}^{\alpha})^e = \min f(Z^e, X^{\alpha}) \tag{20}$$

$$(\overline{y}^{\alpha})^e = \min f(Z^e, X^{\alpha}) \tag{21}$$

$$X^{\alpha} = (\underline{X}^{\alpha}\,\overline{X}^{\alpha}) \tag{22}$$

The steps are repeated in order to obtain the output MF such as PDF.

The second hybrid possibilistic probabilistic approach is possibilistic scenario based approach and its steps are defined as follows [24]:

- Generate the scenario set which is used to describe behaviour of *Z* and $\Omega_J$
- Reduce the initial scenario to a smaller set $\Omega_s$
- Calculate $\overline{\mathbf{y}}^{\alpha}$ and $\underline{\mathbf{y}}^{\alpha}$ in a following way:

$$\underline{y}^{\alpha} = \min \sum\nolimits_{S \in \Omega_S} \pi_S \times f(Z_S, X^{\alpha}) \tag{23}$$

$$\overline{y}^{\alpha} = \max \sum\nolimits_{S \in \Omega_S} \pi_S \times f(Z_S, X^{\alpha}) \tag{24}$$

$$X^{\alpha} = (\underline{X}^{\alpha}\,\overline{X}^{\alpha}) \tag{25}$$

- Perform defuzzification operation of *y*.

There is an evidence of the possibilistic–probabilistic approach application in reliability analysis such as shown in [45]. In [46], a method for generation of a global quantification and characterization of the uncertainty in the output of a system which contains both probabilistic and possibilistic inputs is proposed. A novel analytical probabilistic-possibilistic tool for the power flow uncertainty assessment, based upon the evidence theory and joint propagation of possibilistic and probabilistic uncertainties is shown in [47]. Further, this approach is used in [48] to show the effects of wind speed probabilistic and possibilistic uncertainties on generation system adequacy using random fuzzy model to express the probabilistic and possibilistic uncertainties of wind speed simultaneously. In [49], a hybrid possibilistic-probabilistic method for assessment of DG influence under conditions of uncertainty is described. Another example of hybrid possibilistic–probabilistic approach to DG impact on energy losses is presented in [50].

### 2.4. Information gap decision theory

An information gap decision theory is different than classical approaches to uncertainty modelling and is defined as an unbounded family of nested sets which all share the same structure [6]. It was first proposed by Ben-Haim in 1980 and it is based on the difference of available and not available information [6]. It is used in applications in which, due to severe lack of data, PDFs or MFs are not available. It emerged, together with probability theory as a struggle between different schools of thought [6]. It is applied to make a robust decision making framework in the case of decision making under conditions of severe uncertainty. The Information Gap Decision Theory (IGDT) problem described by [4] where the general optimisation problem is represented as:

$$\min_x f(X, \gamma) \tag{26}$$

$$H_i(X, \gamma) \le 0, i \in \psi_{eq} \tag{27}$$

$$G_j(X, \gamma) \le 0, j \in \psi_{ineq}$$

$$\gamma \in \Gamma \tag{28}$$

where $\gamma$ is the vector of uncertain input parameter, $\Gamma$ is uncertainty set and *X* is decision variable set. Defining $\mu$ as the predicted value of uncertain parameter and $\xi$ as the maximum possible deviation of predicted from predicted value of the uncertainty parameter, the uncertainty set is written as [4]:

$$\forall \gamma \in \Gamma(\overline{\gamma}, \xi) = \left\{ \gamma : \left| \frac{\gamma - \overline{\gamma}}{\overline{\gamma}} \right| \not< \xi \right\} \tag{29}$$

It could be assumed that the uncertain parameter will not be different from the predicted value as follows:

$$f_b = \min_x f(X, \overline{\gamma}) \tag{30}$$

$$H_i(X, \gamma) \le 0, i \in \psi_{eq} \tag{31}$$

$$G_j(X, \gamma) \le 0, j \in \psi_{ineq} \tag{32}$$

In order to handle situations in which uncertain parameter differs from predicted value, two strategies might be used such as risk averse and risk seek as shown in [4]. Applications of IGDT in modelling uncertainty in the power system are numerous such as the Optimal Power Flow (OPF) With HVDC Connected Wind Farms [23] and congestion and voltage management in the presence of uncertain wind power output [51]. However, this type of uncertainty modelling is used in power systems to solve problems which require the risk management strategy [52]. IGDT is also used to address the issue of optimisation of strategic bidding in electricity markets. Some of the examples include risk-based bidding of large electric utilities considering demand response [53], a risk-constrained opti-

mal self-scheduling method for a wind power producer considering the uncertainty associated with market prices and wind generation [54], and risk-based optimal bidding strategy of generation company in day-ahead electricity market [15]. Further, IGDT is used in for restoration decision making in the distribution network [16], risk based day ahead scheduling of electric vehicle aggregator [42], energy trading [55], robust transmission and generation expansion [17], and decision making related to load procurement under conditions of severe uncertainty. There is, however, no evidence of IGDT applications to DG allocation problems.

### 2.5. Robust optimisation

Robust optimisation is a more recent technique used to solve optimisation problems under conditions of severe uncertainty such as lack of information, in which the model of uncertainty is not stochastic, but deterministic and set-based [56]. The robust optimisation was first proposed by [57] and is regarded as a tool which optimises a worst possible case of a problem under consideration [58]. The robust optimisation problem is defined as follows [4]: $z = f(X,y)$ Let be a liner function in $X$ (subject to uncertainty, with no PDF to describe it) and non-linear in $y$. The uncertainty of $X$ is modelled using the set $X \in U(X)$, where $X$ takes parameter from the set $U(X)$. The maximisation of the function $z$ is defined as [4]:

$$\max_{y} z = f(X,y)$$
$$X \in U(X) \tag{33}$$

Since the value of $z$ is linear with respect to $X$, and defining and $\tilde{X}$, $\overline{X}$ as $\hat{X}$ uncertain, predicted and maximum deviation of $X$ from $\overline{X}$, respectively, previous statement can be represented as:

$$\max_{y} z$$

$$\text{such that} \begin{cases} z \le f(\tilde{X},y) \\ H(\tilde{X},y) = A(y) * \tilde{X} + g(y) \\ \tilde{X} \in U(X) = \left\{ X \middle\| X - \tilde{X} \right| \le \tilde{X} \right\} \end{cases} \tag{34}$$

The robust optimisation is used to maximise $z$ and the robust counterpart version is defined as follows [4]:

$$\max_{y} z = f(X,y)$$
$$z \le f(X,y)$$

such that

$$\begin{cases} \sum_i w_i \\ 0 \le w_i \le 1 \\ f(X,y) = A(y) * \overline{X} + g(y) - \max \sum_i a_i(y) * \hat{X} * w_i \end{cases} \tag{35}$$

Now, according to equation (35) two nested optimisation problems need to be solved and the equation is linear with respect to wi and has dual form:

$$\min \left[ \Gamma\beta + \sum_i \xi_i \right] \tag{36}$$

$$\beta + \xi_i \ge a_i(y) * \hat{x}_i \tag{37}$$

and by substituting (37) in (35), we get:

$$\max_{y,\xi_i,\beta} z$$
$$z \le f(X,y)$$
$$\begin{cases} f(X,y) = A(y) * \overline{X} + g(y) - \Gamma\beta - \sum_i \xi_i \\ \beta + \xi_i \ge A(y_u) * \hat{X}_i \end{cases} \tag{38}$$

Robust optimisation method is widely used for uncertainty modelling in power systems applications. The application includes the area of distribution network reconfiguration as shown in [59] where two-stage robust optimisation model for the distribution network reconfiguration problem with load uncertainty is proposed. In addition, reference [60] proposes a two-stage robust reactive power optimisation considering uncertain wind power integration in active distribution networks. Robust optimisation is also applied to the reconfiguration of distribution systems with reliability constraints [61] which considers uncertainty of the reliability data by using a linear and adjustable robust approach. The robust optimisation is applied to model uncertainties in power dispatching applications such as the new adjustable robust optimisation formulation used to solve unit commitment (UC) problems under uncertainty method [62]. Novel active robust optimisation dispatch (AROD) model combines robust optimisation (RO) with dynamic optimization (DO) to optimise wind integrated power system economic dispatch considering hourly demand response [63]. In addition, flexible robust optimization dispatch for hybrid power system containing wind, PV, hydro and thermal energy sources with adjustable uncertainty budget dispatch model is presented in [64]. Additional examples of robust optimisation applications include optimization of distributed generation investment in buildings [65] and optimal bidding strategy of electricity retailers using a robust optimisation approach considering time-of-use rate demand response programs under market price uncertainties [66]. There is only limited evidence of robust optimisation applications to the DG allocation problem. For example, reference [67] proposes a fuzzy approach for assessment of propagation of the uncertainty of wind resources based on the wind speed distribution. An example of possibilistic approach to modelling uncertainties was shown in [68], which proposes a (DG) allocation strategy for radial distribution networks under

conditions of uncertainty related to load and generation parameters using adaptive genetic algorithm (GA).

### 2.6. Interval analysis

In interval analysis, the range of uncertain parameters is represented by an interval. If we assume that $f = (x_1,...,x_n)$ is the multivariate function and that $lb_1 \le x_i \le ub_i$, where $lb_1$ and $ub_i$ are the lower and upper limits of the objective function $f$, then the purpose of the interval analysis is to determine the lower and the upper limits of the objective function [24]:

$$Probability = \int_a^d A_1 \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \left[ \int_a^b \frac{x-a}{b-a} e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \int_b^c e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \int_c^d \frac{x-d}{c-d} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \right] \quad (39)$$

$$G(Probability) = \mu_{B_2}(Probability) \quad (40)$$

Research papers which use interval analysis to address uncertainties in power systems are not that numerous as possibilistic and probabilistic methods. Some examples are the application of interval analysis for determination of the maximum loading point of electric power systems considering load data uncertainties [22], optimal reconfiguration of distribution systems with under conditions of uncertainty [69], optimization of the electricity markets [70], control applications [18], and fault location [19]. Interval arithmetic, which takes into consideration uncertainty of the nodal information, is used to solve the power flow problem and to provide strict bounds for the solution of the problem [71]. There is no evidence of application of interval analysis to the DG allocation problem as this remains an unexplored research area [24].

## 3. MATCHING THE UNCERTAINTY MODELLING METHODS WITH UNCERTAIN PARAMETERS

Every type of uncertainty parameter can be related to a particular type of representation and quantification method, even if in some cases, the level of uncertainty can be unquantifiable [7]. Table I shows the classification of uncertainty properties, which can be used to represent uncertainty as a vector [5]. For example, a simple probabilistic uncertainty modelling profile can be described using the following vector:

$u = [a;a;c;a]$ (41)

In the case of possibilistic approach to uncertainty modelling, where lack of information is addressed using the description of input variables; the profile vector can be described as:

$u = [a;c;c;c]$ (42)

and after the output defuzzification

$u = [a;c;c;a]$ (43)

Renewable sources of energy are intermittent and are considered to be of stochastic energy flows which should be optimized by using hybrid stochastic-heuristic simulation methods. It is proposed that the DG allocation problem can be matched to the profile vectors described in the (42) and (43), depending on the approach adopted.

Table I: Uncertainty property classification

| | |
|---|---|
| A. CAUSES OF UNCERTAINTY<br>a) Lack of information<br>b) Abundance of information<br>c) Conflicting evidence<br>d) Complexity<br>e) Measurements<br>f) Beliefs | C. SCALE LEVEL OF NUMERICAL INFORMATION<br>a) Nominal<br>b) Ordinal<br>c) Cardinal |
| B. INPUT INFORMATION<br>a) Numerical<br>b) Set/interval<br>c) Linguistic<br>d) Symbolic | D. OUTPUT INFORMATION<br>a) Numerical<br>b) Set/interval<br>c) Linguistic<br>d) Symbolic |

## 4. CONCLUSION

This paper presents some of the most important uncertainty modelling techniques and compares their advantages and disadvantages. The most commonly used approached to uncertainty modelling in DG allocation problem are probabilistic, possibilistic and hybrid approaches. There is limited evidence of robust optimisation and IGDT methods application in DG allocation problem, while the interval analysis application in this area remains an unexplored research area. As the power system complexity continues to increase, new methods for uncertainty modelling continue to develop such as *Z*-number and two-time steps recalculation strategy. The development of new uncertainty modelling approaches and their matching with suitable applications is one of the promising future research directions.

## REFERENCES

[1] O.A. Mousavi, R. Cherkaoui: Discussion on uncertainty management in power systems, in Bulk Power System Dynamics and Control - IX Optimization, Security and Control of the Emerging Power Grid (IREP), 2013

[2] A. Soroudi, M. Ehsan, R. Caire, N. Hadjsaid: Possibilistic Evaluation of Distributed Generations Impacts on Distribution Networks, in IEEE Transactions on Power Systems, vol. 26, no. 4, pp. 2293-2301, Nov. 2011

[3] A.J. Abebe, V. Guinot, D.P. Solomatine: Fuzzy alpha-cut vs. Monte Carlo techniques in assessing uncertainty in model parameters, 4-th Int. Conf. on Hydroinformatics, Iowa City, USA, July 2000

[4] M. Aien, A. Hajebrahimi, M. Fotuhi-Firuzabad: A comprehensive review on uncertainty modeling techniques in power system studies, Renewable& Sustainable Energy Rev, Vol57, 2016, p. 1077-1089
[5] H.J. Zimmermann: Fuzzy Set Theory-and ItsApplications", 4th Edition, H.J. Zimmermann, "Fuzzy Set Theory and Its Applications,4th Edition, Springer Science+Business Media, 2001
[6] Ben-Haim: Chapter 2 - Uncertainty, In Info-Gap Decision Theory (Second Edition), in Chapter 2 – "Uncertainty In Info-Gap Decision Theory (Second Edition)", Academic Press, Oxford, ISBN 9780123735522, 2006, pp. 9-36.
[7] R. Preece, J. V. Milanović: Assessing the Applicability of Uncertainty Importance Measures for Power System Studies, IEEE Transactions on Power Systems, pp. 2076-2084, 2016
[8] M. Esmaeili, M. Sedighizadeh, M. Esmaili: Multi-objective optimal reconfiguration and DG power allocation in distribution networks using Big Bang-Big Crunch algorithm considering load uncertainty, Energy, Volume 103, 2016
[9] C.W. Hansen, A. Pohl: Which models matter: Uncertainty and sensitivity analysis for photovoltaic power systems, in IEEE 40th Photovoltaic Specialist Conference (PVSC), Denver, 2014
[10] A. Arabali, M. Ghofrani, Etezadi-Amoli, M.S. Fadali and M. Moeini-Aghtaie: A Multi-Objective Transmission Expansion Planning IEEE Trans. on Power Systems, vol. 29, no. 6, pp. 300, 2014
[11] C. Peng, S. Lei, Y. Hou, F. Wu: Uncertainty management in power system operation, in CSEE Journal of Power and Energy Systems, vol. 1, no. 1, pp. 28-35, March 2015
[12] V. Liebenau, J. Schwippe, S. Kuch, C. Rehtanz: Network extension planning considering the uncertainty of feed-in from renewable energies PowerTech, 2013 IEEE Grenoble, 2013, pp. 1-6
[13] S.M. Miri Larimi, M. R. Haghifam, A. Moradkhani: Risk-based reconfiguration of active electric distribution networks, in IET Generation, Transmission & Distribution, vol. 10, no. 4, pp. 1006-1015, 3 10 2016
[14] M. Negnevitsky, J. Terry, T. Nguyen: Using information entropy to quantify uncertainty in distribution networks, Power Engineering Conference, 2014 Australasian Universities, Perth, WA, 2014 pp.1-6
[15] S. Nojavan, K. Zare: Risk-based optimal bidding strategy of generation company in day-ahead electricity market using information gap decision theory, International Journal of Electrical Power & Energy Systems, pp. 83-92, 2013
[16] K. Chen, W. Wu, B. Zhang, H. Sun: Robust Restoration Decision-Making Model for Distribution Networks Based on Information Gap Decision Theory, in IEEE Trans. on Smart Grid, vol. 6, no. 2, pp. 587-597, March 2015
[17] S. Dehghan, A. Kazemi, N. Amjady: Multi-objective robust transmission expansion planning using information-gap decision theory and augmented $\epsilon$-constraint method, in IET Gen., Trans & Dist, vol. 8, no. 5, pp. 828-840, May 2014
[18] G. Shan, Y. Baoming, Y. Qiang: Educational and Information Technology (ICEIT), 2010 International Conference on, Chongqing, 2010, in Educational and Information Technology (ICEIT), 2010 International Conference on, Chongqing
[19] Z. Xu, M. Liu, G. Yang, N. Li: Application of interval analysis and evidence theory to fault location, IET Electric Power Applications, vol. 3, no. 1, pp. 77-84, 2009
[20] J. Zhao, C. Wan, Z. Xu, J. Wang: Risk-Based Day-Ahead Scheduling of Electric Vehicle Aggregator Using Information Gap Decision Theory, IEEE Tran on Smart Grid, vol. PP, no.99, , pp. 1-10, 2015
[21] H.M. Merrill, A.J. Allen: Risk and uncertainty in power system planning, in 10th Power Systems Computation Conference, Graz, 1990
[22] E.S. Pereira, V.M. da Costa: Interval analysis applied to the maximum loading point of electric power systems considering load data uncertainties, 84. Pereira E.S., and da Costa V.M., Interval analysis applied to the maximum loading point of electric power systems considering load data uncertainties, International Journal of Electrical Power & Energy Systems , pp. 334-340, 2014
[23] A. Rabiee Rabiee, A. Soroudi, A. Keane: Information Gap Decision Theory Based OPF With HVDC Connected Wind Farms, IEEE Transac on Power Systems, pp. 3396-3406, 2015
[24] A. Soroudi, A. Turaj: Decision making under uncertainty in energy systems: State of the art, Renew&-Sustai Energy Reviews, pp. 376-384, 2013
[25] I.A. Hiskens, B.C. Lesieutre, S. Roy: Uncertainty Evaluation and Mitigation in Electrical Power Systems, XI Symposium of Specialists in Electric Operational and Expansion Planning, BELÉM (PA) – BRASIL, 2009
[26] J. Zhu: Optimization of Power System Operation, 2nd Edition Wiley and Sons, Inc, 2015, 2015
[27] Z. Liu, F. Wen, G. Ledwich: Optimal Siting and Sizing of Distributed Generators in Distribution Systems Considering Uncertainties, in IEEE Transactions on Power Delivery, vol. 26, no. 4, pp. 2541-2551, Oct. 2011, pp. 2541-2551, 2011
[28] Y. Li, E. Zio: Uncertainty analysis of the adequacy assessment model of a distributed generation system, Renewable Energy, Vol 41, Pages 235-244, May 2012
[29] Y.C. Chen, X. Jiang, A.D. Domínguez-García: Impact of power generation uncertainty on power system static performance, in North American Power Symposium (NAPS), Boston, 2011
[30] G. Papaefthymiou, A. Tsanakas, P.H. Schavemaker, L. van der Sluis: Design of distributed energy systems based on probabilistic analysis, in Probabilistic Methods Applied to Power Systems, 2004 Int. Conference on, Ames, pp. 512-518,
[31] J.A. Martinez, G. Guerra: A Parallel Monte Carlo Method for Optimum Allocation of Distributed Generation, IEEE Transactions on Power Systems, pp. 2926-2933, 2014

[32] V.A. Evangelopoulos, P.D. Georgilakis: Optimal distributed generation placement under uncertainties based on point estimate method embedded genetic algorithm, IET Generation, Transmission & Distribution, pp. 389-400, 2014
[33] H.M. Khodr, M.R. Silva, Z. Vale, C. Ramos: A probabilistic methodology for distributed generation location in isolated electrical service area, Ele. Power Sys. Research, pp. 390-399, 2010
[34] K.Y. Liu, W. Sheng, L. Hu, Y. Liu, X. Meng, D. Jia: Simplified probabilistic voltage stability evaluation considering variable renewable distributed generation in distribution systems, IET Gen., Trans & Distribution, pp. 1464-1473, 2015
[35] M. Gómez-González, F.J. Ruiz-Rodriguez, F. Jurado: Metaheuristic and probabilistic techniques for optimal allocation and size of biomass distributed generation in unbalanced radial systems, IET Renewable Power Generation, pp. 653-659, 2015
[36] G. Cartina, G. Grigoras, E.C. Bobric, D. Comanescu: Improved fuzzy load models by clustering techniques in optimal planning of distribution networks, PowerTech,2009 IEEE Bucharest pp.1,6,
[37] S. Ganguly, N.C. Sahoo, D. Das: Multi-objective particle swarm optimization based on fuzzy-pareto-dominance for possibilistic planning of electrical distribution systems incorporating distributed generation, Fuzzy Sets and Systems, 2013
[38] T.J. Ross: Fuzzy logic with engineering applications 3rd edition, John Wiley& Sons, 2010
[39] R. Hassanzadeh, I. Mahdavi, N. Mahdavi-Amiri, A. Tajdin: Proceedings of the 2012 International Conference on Industrial Engineering and Opera, An $\alpha$ -Cut Approach for Fuzzy Product and Its Use in Computing Solutions of Fully Fuzzy Lin. Syst.
[40] M.A. Matos, E.M. Gouveia: The Fuzzy Power Flow Revisited, IEEE Transactions on Power Systems, vol. 23, no. 1, pp. 213-218, 2008
[41] A. Romero, H. C. Zini, G. Ratta: Modelling input parameter interactions in the possibilistic harmonic load flow, IET Generation, Transmission & Distribution, pp. 528-536, 2012
[42] Y. Zhou, Y.P. Li, G.H. Huang: A robust possibilistic mixed-integer programming method for planning municipal electric power systems, Int. Jour. of Electr. Power&Energy S.,pp.757-772, 2015
[43] J. Ramirez-Rosado, J.A. Dominguez-Navarro: Possibilistic model based on fuzzy sets for the multiobjective optimal planning of electric power distribution networks, IEEE Transactions on Power Systems, vol. 19, no. 4, pp. 1801-1810, 2004
[44] C. Sun, M. Xie, Z. Bie, J. Jiang, X. Song: Optimal wind capacity integration considering the possibilistic uncertainty of wind resources, in Smart Grid Technologies - Asia (ISGT ASIA), 2015 IEEE Innovative, Bankok, 2015
[45] J. M. Ling: A Possibility-Probability Based Model for Distribution System Reliability, in Computational Intelligence and Natural Computing, CINC '09,International Conference on, Wuhan 2009
[46] T. Whalen: Propagation of uncertainty in systems with both probabilistic and possibilistic inputs, in Norbert Wiener in the 21st Century (21CW), 2014 IEEE Conference on, Boston, 2014
[47] M. Aien, M. Rashidinejad, M. Fotuhi-Firuzabad: On possibilistic and probabilistic uncertainty assessment of power flow problem: A review and a new approach, Renewable and Sustainable Energy Reviews, Volume 37, pp. 883-895, 2014
[48] C. Sun, Z. Bie, M. Xie, G. Ning: Effects of wind speed probabilistic and possibilistic uncertainties on generation system adequacy, IET Generation, Transmission & Distribution, pp. 339-347, 2015
[49] A. Soroudi: Possibilistic-Scenario Model for DG Impact Assessment on Distribution Networks in an Uncertain Environment, IEEE Transactions on Power Systems, 2012
[50] A. Soroudi, M. Ehsan: A possibilistic–probabilistic tool for evaluating the impact of stochastic renewable and controllable power generation on energy losses in distribution networks—A case study, Renewable and Sustainable Energy Reviews, 2011
[51] C. Murphy, A. Soroudi, A. Keane: Information Gap Decision Theory-Based Congestion and Voltage Management in the Presence of Uncertain Wind Power, in IEEE Transactions on Sustainable Energy, vol. 7, no. 2, pp. 841-849, 2016
[52] X. Li, Z. He, S. Zhang: Robust optimization of risk for power system based on information gap decision theory, 2015 5th International Conference on Electric Utility Deregulation and Restructuring and Power Technologies (DRPT), Changsha, 2015
[53] M. Kazemi, B. Mohammadi-Ivatloo, M. Ehsan: Risk-based bidding of large electric utilities using Information Gap Decision Theory considering demand response, Electric Power Systems Research, pp. 86-92, 2014
[54] M. Moradi, B. Dalvand, Mohammadi-Ivatloo, N. Amjady, H. Zareipour, A. Mazhab-Jafari: Self-scheduling of a wind producer based on Info Gap Decision Theory, Energy, Vol 81,2015 pp. 588-600
[55] P. Mathuria, R. Bhakar: Info-Gap Approach to Manage Gen Co's Trading Portfolio With Uncertain Market Returns, IEEE Transactions on Power Systems, vol. 29, no. 6, pp. 2916-2925, Nov. 2014
[56] D. Bertsimas, D.B. Brown, C. Caramanis: Theory and Applications of Robust Optimization, Optimization SIAM REVIEW 2011, Society for Ind. and Applied Math Vol. 53, No. 3, pp. 464-501
[57] A.L. Soyster: Convex Programming with Set-Inclusive Constraints and Applications to Inexact Linear Programming, Operations Research, pp. 1154-1157, 1973
[58] A. Shapiro, W. Tekaya, M. P. Soares, J. P. da Costa: Worst-Case-Expectation Approach to Optimization Under Uncertainty, Operations Research 61 (6), 2013
[59] C. Lee, C. Liu, S. Mehrotra, Z. Bie: Robust Distribution Network Reconfiguration," IEEE Tran on Smart Grid, vol. 6, no. 2, pp. 836-842, 2015

[60] T. Ding, S. Liu, W. Yuan, Z. Bie, B. Zeng: A Two-Stage Robust Reactive Power Optimization Considering Uncertain Wind Power Integration in Active Distribution Networks, IEEE Transac. on Sustainable Energy, vol. 7, no. 1, pp. 301-311, 2016
[61] J. C. López, M. Lavorato, J. F. Franco, M.J. Rider: Robust optimisation applied to the reconfiguration of distribution systems with reliability constraints, in IET Gen., Transmission & Distribution, vol. 10, no. 4, pp. 917-927, 2016
[62] P. Xiong, P. Jirutitijaroen: Two-stage adjustable robust optimisation for unit commitment under uncertainty, IET Generation, Transmission & Distribution, vol. 8, no. 3, pp. 573-582, 2014
[63] W. Xu, J. Chuanwen, L. Bosong: Active robust optimization for wind integrated power system economic dispatch considering hourly demand response, Renewable Energy, v97, p798-808, 2016
[64] C. Peng, P. Xie, L. Pan, R. Yu: Flexible Robust Optimization Dispatch for Hybrid Wind /Photovoltaic/HydroThermal Power System, IEEE Tran on Smart Grid, vol. 7, no2, pp.751-762, 2016
[65] R. Taghipour, Shams Gharneh, G.B. Gharehpetian: Robust optimization of distributed generation investment in buildings, Energy, Volume 48, Issue 1, December 2012, Pages 455-463
[66] S. Nojavan, B. Mohammadi-Ivatloo, K. Zare: Optimal bidding strategy of electricity retailers using robust optimisation approach considering time-of-use rate demand response programs under market price uncertainties, IET Generation, Transmission & Distribution, 2015
[67] C. Sun, Z. Bie, M. Xie, J. Jiang: Assessing wind curtailment under different wind capacity considering the possibilistic uncertainty of wind resources, Electric Power Systems Research, Volume 132, pp. 39-46, 2016
[68] S. Ganguly, D. Samajpati: Distributed Generation Allocation on Radial Distribution Networks Under Uncertainties of Load and Generation Using Genetic Algorithm, IEEE Trans on Sust. Energy, vol. 6, no. 3, pp. 688-679, 2015
[69] L.W. de Oliveira, F. Seta, E.J. de Oliveira: Optimal reconfiguration of distribution systems with representation of uncertainties through interval analysis, International Journal of Electrical Power & Energy Systems, 2016
[70] A.T. Saric, A. M. Stankovic: An application of interval analysis and optimization to electric energy markets, IEEE Transactions on Power Systems, pp. 515-523, 2006
[71] Z. Wang, F. L. Alvarado: Interval arithmetic in power flow analysis, Power Industry Computer Application Conference, 1991. Conference Proceedings, Baltimore, MD, 1991, pp. 156-162

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