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SOLVING ECONOMIC DISPATCHING PROBLEM USING PATTERN SEARCH METHOD

RJEŠENJE PROBLEMA EKONOMSKOG DISPEČINGA KORIŠTENJEM METODE PRETRAŽIVANJA PO UZORKU

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Abstract: Methods of direct search are evolutionary algorithms used for constrained optimization problem solving. Information regarding objective function gradient is not required for finding an optimum solution. Pattern search is one such method. This paper gives an application of the method for non-convex economic dispatch problem solving. As a confirmation of its effectiveness, the method has been tested on the systems consisted of five, thirteen and forty thermal power units. Also, features of this method have been compared to other popular methods. The results are encouraging and show that the pattern search method is a very useful for non-convex economic dispatch problem solving.

Keywords: economic dispatch, convexity, pattern search method

Sažetak: Metode direktnog pretraživanja su evolucijski algoritmi korišteni za rješavanje optimizacionih problema sa ograničenjima. One ne zahtijevaju informacije o gradijentu funkcije cilja koja je predmet optimizacije prilikom pretrage za optimalnim rješenjem. Jedna od takvih metoda je metoda pretraživanja po uzorku. Ovaj rad predstavlja aplikaciju metode pretraživanja po uzorku za rješavanje problema nekonveksnog ekonomskog dispečinga. Korištena metoda je radi potvrde njene učinkovitosti testirana na proizvodnim sistemima sastavljenim od 5, 13 i 40 termojedinica. Pored toga, svojstva konvergencije i robusnosti korištene metode su ocijenjena kroz usporedbu s rezultatima predstavljenim u literaturi za sistem sa 40 termojedinica. Ishod je ohrabrujući i pokazuje da je metoda pretraživanja po uzorku primjenljiva za rješavanje problema nekonveksnog ekonomskog dispečinga.

Ključne riječi: ekonomski dispečing, konveksnost, metoda pretraživanja po uzorku

INTRODUCTION

Increase of the costs of production and needs for optimum use of production resources in power generation systems, impose the need for the use of economic dispatching, as an important planning function for compliance with the given demands [1]. The main goal of economic dispatching is the decrease of the overall costs of generation, by optimum allocation of output powers of the engaged thermal units, along with compliance with limitations formed as equalities and inequalities [2], [3]. The main operating characteristic of a thermal unit is the so-called input-output characteristic obtained by normative testing and data processing, or by calculations at the point of preparing the project and construction characteristics of a thermal unit. When presenting the input-output characteristics of a thermal unit, one can use continual form of characteristics in form of polynomial curve or a series of segmented straight lines fitting the experimental (or construction) data. In most cases, the approximation with quadratic characteristic is considered satisfactory. Real technical and technological execution of a thermal unit is such that the turbines are equipped with a number of regulatory vents, which open in sequences with the increase of output power of a thermal unit [4]. Therefore, input-output characteristic of a thermal unit shows discontinuities as a consequence of sudden increase of losses in jets of steam turbine during sequential intake of steam into the turbine. The precise form of input-output characteristic when intake of steam is crossing from one to another regulation vent is difficult to be determined through an experiment. In the majority of cases, as well as for relatively precise calculations, the approximated characteristic, which is convex, uninterrupted and differentiable function (mostly polynomially presented) is used, so that conventional optimisation techniques could be used in solving the problems of short-term optimisation of engagement and regime of a thermal unit [2], [5].

Different techniques of optimisation have been used to solve the problem of economic dispatching (ED). Some of these techniques are based on classic optimisation methods, while others use artificial intelligence, or heuristic algorithms. Many sources show application of classic optimisation methods in solving of the ED problems, such as linear programming or quadratic programming [6].

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Linear programming methods are fast and reliable, but their speed depends on the size of the problem. Non-linear programming methods are faced with problems of convexity and algorithm complexity. Algorithms based on Newton method have difficulties in working with a large number of inequality limitations [7]. Methods based on techniques of artificial intelligence, such as neural networks, are also being successfully applied [8], [9]. Recently, many heuristic searching techniques, such as optimisation with particle swarm [7], [9], [10], [11], genetic algorithms [9], [12], and bat algorithm [13], are being considered within the context of solving the ED problem. Additionally, hybrid methods have been developed [14], combining conventional approach of Lagrangian relaxation, gradient method and dynamic programming. Also, there is an alternative form of hybrid method for solving non-convex ED problem with differential evolution [15] as primary 'optimisation', where its advantages of global searching are being used. The best found solution is then determined by the sequential quadratic programming.

A lot of attention has recently been directed towards methods of direct searching. It is a family of methods of global optimisation, whose development ideas date from 1960. Methods of direct search are structured to explore a group of points around the current position, searching for the point which gives the value of objective function better than the current point. This family includes algorithms of searching by pattern, simplex methods (different from simplex algorithm used in linear programming), Powell optimisation and other methods [16]. Methods of direct search, unlike standard methods of optimisation, are frequently referred to as non-derivative, because they do not require any information on gradient or higher derivation of the objective function to search for the optimal solution. Therefore, the methods of direct search can be well used to solve non-continual, non-differential and multi-modal optimisation problems. Having in mind that the economic dispatching is one such problem, this method seems to be a good choice.

This work shows use of the pattern search method to solve problems of non-convex economic dispatching.

1. MODEL OF DYNAMIC ECONOMIC DISPATCHING

Problem of optimal distribution of the load to production units - economic dispatching, takes a special place in the optimum planning for power generation systems. Economic dispatching presents an operational function where the total generation is distributed between the engaged thermal units in a way to minimise total costs of production, while, at the same time, complying with all facility limitations and consumer requirements. For any specific conditions for system load, the economic dispatching defines output power of all thermal units, used to minimise the total cost of fuel and covering the needs of consumers [17]. Economic dispatching is used to manage production part of the power generation system to allocate the total generation of thermal units and to focus on coordination of their production costs.

Traditional presentation of the problem of economic dispatching, often approximates the power (cost) characteristics of thermal units with polynomial function (usually quadratic function), having the problem be solved with the use of optimisation technique, as a lambda-iterative method, gradient method, or similar [17]–[21]. To apply these methods, it is necessary to ensure differentiability of cost characteristics in order to find a global optimum solution. Real energy characteristics of thermal units are significantly non-linear due to presence of regulation valves on turbines, which open in sequences when increasing output power of a thermal unit. Therefore, energy characteristic of a thermal unit shows discontinuities as a consequence of sudden increase of losses in jets of steam turbine during sequential intake of steam into the turbine. The problem of economic dispatching becomes a non-convex problem to which classical optimisation techniques cannot be applied, ending, in principle, in finding any of the local optimums. Thence the practical problem of economic dispatching with non-convex energy (cost) characteristics still presents a challenging task.

1.1. Classical model of economic dispatching

Consider a simple configuration of a generation system, consisting of N thermal unit, settling the total load of the system P_p(t) in every time interval t. Consider for each of the generating thermal units to have a cost characteristic $F_i(P_i(t))$ and the output power of each unit to be $P_i(t)$ in every interval t. Figure 1 shows symbolic illustration of thermal units connected to the equivalent consumer node where the overall load of the system is concentrated. Such a scheme corresponds to the model of economic dispatching, where loses in transmission network are ignored.

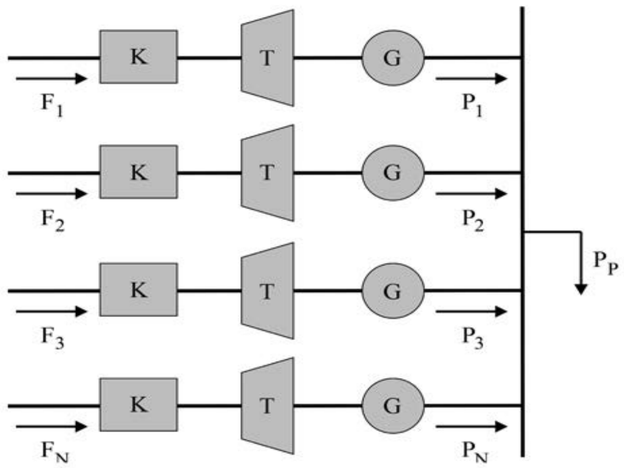

Figure 1: N thermal units connected to the equivalent node, covering the overall load in the system.

Figure 2 shows symbolic illustration of thermal units connected to the transmission network with distributed equivalent consumer nodes, whose power was reduced to the voltage level of transmission. Such a scheme corresponds to the model of economic dispatching, where loses in transmission network are taken into the account.

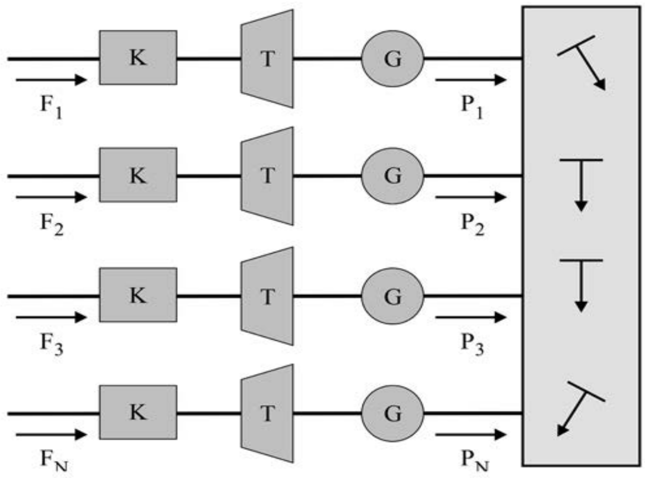

Figure 2: N thermal units connected to the transmission network with distributed consumer nodes, whose powers were reduced to the voltage level of transmission.

Mathematics formulation of the problem of economic dispatching, without considering the loses in the transmission network, is given as follows. Function of criteria is equal to the overall costs of fuel F_{uk} to cover the consumer demand for the power. The problem is reduced to minimisation of the function of criteria F_{uk}, satisfying the group of limitations:

$$\min F_{uk} = \sum_{t=1}^{T}\sum_{i=1}^{N} F_i(P_i(t)) \qquad \text{or}$$

$$\min F_{uk} = \sum_{t=1}^{T}\sum_{i=1}^{N} (a_i P_i^2(t) + b_i P_i(t) + c_i) \tag{1}$$

$$\sum_{i=1}^{N} P_i(t) = P_p(t) + P_l(t), \quad \forall t \tag{2}$$

$$P_i(t) - P_i(t-1) \le RU_i, \quad \forall i,\ \forall t \tag{3}$$

$$P_i(t-1) - P_i(t) \le RD_i, \quad \forall i,\ \forall t \tag{4}$$

$$P_i^{\min} \le P_i(t) \le P_i^{\max}, \quad \forall i,\ \forall t \tag{5}$$

Equation (1) shows the function of the objective (criteria), related to the overall costs of fuel for all N thermal units, during the optimisation period T. Equation (1) shows that the cost characteristics of thermal units are modelled with convex function - polynomial of second degree. Limitations in the type of equality (2) present balanced limitation towards which the overall generation of all engaged thermal units must cover the sum of the total load P_p and transmission losses in the system P_l for every time interval t. Limitations (3) and (4) present the so called ramp rate limits, or limitations to the allowed change of the output power in two consecutive time intervals. Limitation (3) is related to the so called *ramp-up* limit, or to the allowed change of output power of a thermal unit in two consecutive periods, when it increases its power. This means that the thermal unit i from the time period $t-1$, which corresponds to the output power $P_i(t-1)$, by crossing to the time period t can increase its power for a maximum amount of the allowed increase of RU_i, to which the power $P_i(t)$ will correspond. Limitation (4) is related to the so called ramp-down limit, or to the allowed change of output power of a thermal unit in two consecutive periods, when it decreases its production. This means that the thermal unit i from the time period $t-1$, which corresponds to the output power $P_i(t-1)$, by crossing to the time period t can decrease its power for a maximum amount of the allowed decrease of RD_i, to which the power $P_i(t)$ will correspond. Limitation (5) establishes limits to the allowed change of output power of a thermal unit, i.e. the output power of a thermal unit at any time interval must be within the technical limitations defined as minimum or maximum output power.

1.2. Model of economic dispatching with non-convex characteristics

Real technical and technological construction of a thermal unit is such that the turbines are equipped with a number of regulatory vents, which open in sequences with the increase of output power of a thermal unit. Therefore, input-output characteristic of a thermal unit shows discontinuities as a consequence of sudden increase of losses in jets of steam turbine during sequential intake of steam into the turbine. The precise form of input-output characteristic near to crossing of intake of steam from one to another regulation vent is difficult to be determined through an experiment. Therefore, the literature often uses the model of cost characteristic of thermal unit, containing a high level of non-linearity, and made of adding the "sinus" function to the fundamental characteristic modelled by the polynomial of the second degree. In this manner, the problem presented by relations (1) - (5) has the following form [22]-[25]:

$$\min F_{uk} = \sum_{t=1}^{T}\sum_{i=1}^{N} F_i(P_i(t)) \qquad \text{or}$$

$$\min \sum_{t=1}^{T}\sum_{i=1}^{N} \left\{ \begin{array}{l} a_i P_i^2(t) + b_i P_i(t) + c_i + \\ + \left| \xi_i \sin(\zeta_i (P_i(t) - P_i^{\min})) \right| \end{array} \right\} \tag{6}$$

$$\sum_{i=1}^{N} P_i(t) = P_p(t) + P_l(t), \quad \forall t \tag{7}$$

$$P_i(t) - P_i(t-1) \le RU_i, \quad \forall i,\ \forall t \tag{8}$$

$$P_i(t-1) - P_i(t) \le RD_i, \quad \forall i,\ \forall t \tag{9}$$

$$P_i^{\min} \le P_i(t) \le P_i^{\max}, \quad \forall i,\ \forall t \tag{10}$$

and it presents the full model of a non-convex economic dispatching. Coefficients ξ_i and ζ_i, for thermal unit *i*, reflect the effect of sequential opening of regulation vents at turbines of thermal units.

2. METHOD OF PATTERN SEARCH

Methods and techniques of direct search, unlike standard, classic methods of optimisation, are frequently referred to as non-derivation, because they do not require any information on gradient or higher derivation of the criteria function to search for the optimal solution. They can, therefore, be effectively used to solve non-continual and non-differential optimisation problems, such is the problem of non-convex economic dispatching [26]–[28]. Optimisation procedure for the pattern search algorithm represents an evolutionary technique, suitable for solving of different problems of global optimisation. Its main advantage is in simple concept, simple implementation and computer efficiency. Unlike other heuristic algorithms, such as genetic algorithms, the pattern search algorithm possesses a flexible and well balanced operator for improvement and adjustment of global and precision of local search.

Pattern search algorithm works by doing calculation of the function of criteria for series of points that can eventually be close to the optimum point. Algorithm starts with defining the group of points, i.e. a network around the specified point. This point can be specified by the beneficiary, or can be calculated in the previous step of the algorithm. The network is formed by adding the observed point to the series of vectors representing the pattern (Figure 3). If it becomes clear for any point within the network that it gives a better function of criteria in comparison to the current point of search, the said point becomes the observed point in the next iteration. Therefore, the pattern search algorithm starts in the initial point X_0 provided by the beneficiary as the starting point. In the first iteration, with the value of 1 scalar for the size of the network, vectors of pattern are [0,1], [1,0], [–1,0] and [0,–1]. Then, the algorithm adds these vectors to the initial point X_0 for the purpose of calculating the next points of the network [26], [28].

$$X_0 + [1,0]$$
$$X_0 + [0,1]$$
$$X_0 + [-1,0]$$
$$X_0 + [0,-1]$$

The algorithm examines the points of network by calculating the corresponding value of the function of criteria, until it finds the one for which this value is less than the value of point X_0. If there is such a point, than the research is successful and the algorithm defines another observed point X_1. After successful research, the algorithm moves to iteration 2 and multiplies the scalar for the size of network with 2 (expansion factor). In iteration 2, the network is construed from the following points:

$$2 \times [1,0] + X_1$$
$$2 \times [0,1] + X_1$$
$$2 \times [-1,0] + X_1$$
$$2 \times [0,-1] + X_1$$

The algorithm analyses the points of the network again, until it finds the one whose function of criteria is less than the one for point X_1. The first such point is named X_2 and the search is successful. Therefore, the scalar of the size of the network is multiplied by 2, and then for the iteration 3 it gains the value of 4. On the other hand, should the iteration 3 end as an unsuccessful search, i.e. should none of the points of the network have lesser value of the function of criteria for point X_2, the algorithm does not change its current observing point in the next iteration, i.e. $X_3 = X_2$. Additionally, the algorithm decreases the value of the network 2 times, i.e. multiplies it with the factor of shrinkage of 0.5. Examining of the point along a smaller network is then conducted.

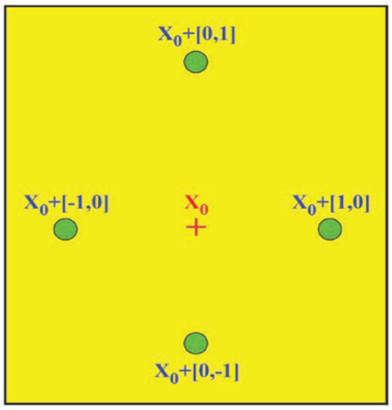

Figure 3: Illustration of position of different points in pattern search algorithm.

Pattern search algorithm will repeat the previously described patterns, until the optimum solution is find in the procedure of minimising the function of criteria. The algorithm is stopped in the following cases:

- the size of the network is smaller than the tolerant size;
- number of iterations reaches a certain, previously defined value;
- number of compared functions of criteria reaches previously defined value;
- distance between the point with successful examination and the next one with successful examination is lesser than the defined tolerance;
- difference between values of the function of criteria in one point with successful examination and the one for the next successful examination, is less than the defined tolerance.

The given steps and the form of evolution of the function criteria for the pattern search algorithm are further presented through the given pseudo-code. It needs to be mentioned that all parameters involved in this algorithm can be previously defined depending on the nature of the problem being examined. The beneficiary has a high possibility to control the direction of search. Illustration of the pseudo-code for the pattern search algorithm is given in the form [28]:

Initialising – defining the initial point x_0 and the length of steps Δ_0.

for $k = 0, 1, ...$

calculate $f(x_k)$

define the size of steps s_k by using the search of environment – create the network

calculate $\rho_k=f(x_k)-f(x_k+s_k)$

if $\rho_k \geq 0$, then $x_{k+1}=x_k+s_k$, otherwise $x_{k+1}=x_k$

update length of a step Δ_k

repeat until the criteria for stopping is complied with.

3. RESULT OF CALCULATION

In order to evaluate efficiency of the search by pattern method, we analysed three test cases of non-convex economic dispatching with 5, 13 and 40 thermal units. The first two cases are the examples showing efficiency, the method of work and the possibilities of the programme. All input parameters were taken into consideration, such as transmission loses, *ramp-up* and *ramp-down* limitations, and the consumer requests were given in a form of one-hour lasting load and multi-hour diagram.

On the other hand, the third case was more complex and efficiency and robustness of the method were seen during the calculation. Therefore, comparison with other methods was made through this example.

Non-linear problem of optimisation was solved with the previously defined function of pattern search, installed in the GA&DS group of tools of the Matlab programme package [29]. This function implements pattern search algorithm described in Chapter 2.

At the beginning, a few calculations were made with different values of key parameters of the method of search by pattern, such as initial size of the network and factors of shrinkage and expansion of the network. Based on those results, the size of the network was defined in further calculations for the value 1, factor of expansion was set to 2 and factor of shrinkage to 0.5. Furthermore, the vector of initial point X_0, was randomly generated. As for the criteria for stoping, all tolerances were set to 10^{-6}, maximum number of iterations to 1000, and maximum number of function compares to $2000 \cdot N$, with N being the number of thermal units.

All calculations were made on a computer of new generation, with Intel i7 processor and 8 GB RAM.

3.1. Test - example 1: case with 5 thermal units

The tested system contains 5 generation units whose details were given in Table I. The coefficient of transmission losses is given with *B*-matrix, and 24-hour diagram of load is given in Table II. All data were taken from [30].

Table I: Data on generation thermal units

unit	G_1	G_2	G_3	G_4	G_5
a (c.u*./h)	25	60	100	120	40
b (c.u./MWh)	2	1.8	2.1	2	1.8
c (c.u./MW2h)·10^{-3}	8	3	1.2	1	1.5
ξ (c.u./h)	100	140	160	180	200
ζ (rad/MW)·10^{-3}	42	40	38	37	35
P_{min} (MW)	10	20	30	40	50
P_{max} (MW)	75	125	175	250	300
RU (MW/h)	30	30	40	50	50
RD (MW/h)	30	30	40	50	50

*- currency unit

$$B = \begin{bmatrix} 0.000049 & 0.000014 & 0.000015 & 0.000015 & 0.000020 \\ 0.000014 & 0.000045 & 0.000016 & 0.000020 & 0.000018 \\ 0.000015 & 0.000016 & 0.000039 & 0.000010 & 0.000012 \\ 0.000015 & 0.000020 & 0.000010 & 0.000040 & 0.000014 \\ 0.000020 & 0.000018 & 0.000012 & 0.000014 & 0.000035 \end{bmatrix}$$

Table II: Daily diagram for the system load

hour	load (MW)	hour	load (MW)	hour	load (MW)
1	410	9	690	17	558
2	435	10	704	18	608
3	475	11	720	19	654
4	530	12	740	20	704
5	558	13	704	21	680
6	608	14	690	22	605
7	626	15	654	23	527
8	654	16	580	24	463

20 calculations were conducted for different initial points, and the best result (lowest costs) is 47,911 (c.u.). Results for optimum distribution of output powers to thermal units for this result are presented in Table III. The average duration of calculation was 68 seconds, or less than 3 seconds per hourly load.

Table III: Optimum distribution of output powers

hour	P_1 (MW)	P_2 (MW)	P_3 (MW)	P_4 (MW)	P_5 (MW)
1	16.8	36.0	107.2	102.7	150.8
2	41.5	20.0	112.7	124.9	139.8
3	58.8	43.5	112.7	124.9	139.8
4	75.0	73.5	122.6	124.9	139.8
5	75.0	98.6	126.2	124.9	139.8
6	75.0	98.5	166.2	136.1	139.8
7	75.0	98.6	175.0	145.8	139.8
8	75.0	98.5	175.0	174.5	139.8
9	75.0	100.3	175.0	209.8	139.8
10	75.0	114.8	175.0	209.8	139.8
11	75.0	125.0	175.0	209.8	146.0
12	75.0	125.0	175.0	209.8	166.6
13	75.0	114.8	175.0	209.8	139.7
14	75.0	100.2	175.0	209.9	139.8
15	75.0	102.5	135.9	209.8	139.8
16	62.0	86.5	168.9	177.8	91.8
17	46.0	88.3	168.9	177.8	83.5
18	16.0	98.5	158.0	209.8	133.5
19	39.8	98.6	175.0	209.8	139.7
20	69.9	119.9	175.0	209.8	139.7
21	46.3	120.0	142.0	209.8	171.8
22	33.2	103.9	110.0	177.8	187.8
23	33.5	88.0	78.0	193.8	139.7
24	12.5	65.8	105.5	143.9	139.7

3.2. Test - example 2: case with 13 thermal units

This test case we analysed optimum distribution of load with 13 thermal power generation units, whose main data is given in Table IV. The overall consumer demand amounts to 2,520 MW during the period of analysis of one hour [5].

The results of distribution of load to thermal units for this case are given in Table V. The overall cost of fuel for this distribution amount to 24,713 (c.u./h). A total of 100 calculations were made, of which the best result was selected. The total duration of calculation was 10 minutes and 4 seconds, i.e. 6 seconds by single calculation.

Table V: Optimum distribution of output powers

unit	power (MW)	unit	power (MW)
G_1	680	G_8	60
G_2	360	G_9	159.7
G_3	360	G_{10}	46.3
G_4	159.7	G_{11}	40
G_5	159.7	G_{12}	55
G_6	159.7	G_{13}	120
G_7	159.7		

Furthermore, optimum distribution of load was analysed for 13 thermal units, where the total consumer request was given at 12-hour time horizon, with the values presented in Table VI. The results of distribution of load to thermal units are given in Table VII, where the total cost of the facility, or the cost of fuel, amounts to 242,091 (c.u.). A total of 20 calculations were made and the average duration of a calculation was 79 seconds, i.e. less than 7 seconds per hourly load.

Table VI: Diagram of system load (MW)

period	$P_p(t)$	period	$P_p(t)$	period	$P_p(t)$
1	2,520	5	825	9	2,520
2	1,860	6	940	10	2,220
3	1,735	7	1310	11	2,875
4	1,305	8	2230	12	1,885

Table IV: Data on generating thermal units

unit	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}	G_{12}	G_{13}
a (c.u./h)	550	309	307	240	240	240	240	240	240	126	126	126	126
b (c.u./MWh)	8.10	8.10	8.10	7.74	7.74	7.74	7.74	7.74	7.74	8.60	8.60	8.60	8.60
c (c.u./MW2h)$\cdot 10^{-3}$	0.28	0.56	0.56	3.24	3.24	3.24	3.24	3.24	3.24	2.84	2.84	2.84	2.84
ξ (c.u./h)	300	200	150	150	150	150	150	150	150	100	100	100	100
ζ (rad/MW)$\cdot 10^{-3}$	35	42	42	63	63	63	63	63	63	84	84	84	84
P_{min} (MW)	0	0	0	60	60	60	60	60	60	40	40	55	55
P_{max} (MW)	680	360	360	180	180	180	180	180	180	120	120	120	120
RU (MW/h)	335	250	250	80	80	80	80	80	80	120	120	120	120
RD (MW/h)	360	290	290	130	130	130	130	130	130	120	120	120	120
P_0 (MW)	420	280	280	120	150	130	160	140	140	100	80	80	75

Table VII: Distribution of output powers of thermal unit at 12-hour timely horizon

hour	1	2	3	4	5	6	7	8	9	10	11	12
$P_1(t)$ (MW)	315.3	25.6	30.6	33.6	25.9	15.4	35.9	141.3	242.5	322.1	600.1	332.6
$P_2(t)$ (MW)	338.4	314.1	24.1	25.4	16.6	0.0	178.1	269.4	357.5	345.5	354.9	132.2
$P_3(t)$ (MW)	320.1	30.1	166.1	20.8	0.0	0.0	24.5	272.1	360.0	348.8	360.0	147.8
$P_4(t)$ (MW)	180.0	174.2	177.3	128.6	73.2	65.2	130.0	180.0	180.0	140.4	180.0	147.3
$P_5(t)$ (MW)	180.0	174.2	177.3	128.6	73.2	65.2	94.9	174.9	180.0	140.4	180.0	147.3
$P_6(t)$ (MW)	180.0	174.2	177.3	128.6	73.2	65.2	130.0	180.0	180.0	140.4	180.0	147.3
$P_7(t)$ (MW)	180.0	174.2	177.3	128.6	73.2	65.2	130.0	180.0	180.0	140.4	180.0	147.3
$P_8(t)$ (MW)	180.0	174.2	177.3	128.6	73.2	116.6	130.0	180.0	180.0	140.4	180.0	147.3
$P_9(t)$ (MW)	180.0	174.2	177.3	128.6	73.2	116.6	130.0	180.0	180.0	140.4	180.0	147.3
$P_{10}(t)$ (MW)	120.0	120.0	120.0	120.0	49.8	119.2	92.9	120.0	120.0	101.6	120.0	108.4
$P_{11}(t)$ (MW)	120.0	120.0	120.0	120.0	87.7	119.2	92.9	120.0	120.0	101.6	120.0	108.4
$P_{12}(t)$ (MW)	113.1	102.6	105.3	106.8	102.8	96.2	70.3	116.2	120.0	79.0	120.0	85.7
$P_{13}(t)$ (MW)	113.1	102.6	105.3	106.8	102.8	96.2	70.3	116.2	120.0	79.0	120.0	85.7

3.3. Test - example 3: case with 40 thermal units

This example analyses optimum distribution of load in IEEE test system with 40 units. The input data can be found in [31]. Transmission loses have been ignored, and the total consumption is 10,500 MW.
Results of distribution of load per thermal unit are presented in Table VIII.

Table VIII: Optimum distribution of output powers

unit	power (MW)	unit	power (MW)
G_1	110.8	G_{21}	523.3
G_2	110.8	G_{22}	523.3
G_3	112.0	G_{23}	523.3
G_4	179.7	G_{24}	523.3
G_5	87.8	G_{25}	523.4
G_6	140.0	G_{26}	523.3
G_7	259.6	G_{27}	10.0
G_8	284.6	G_{28}	10.0
G_9	284.6	G_{29}	10.0
G_{10}	130.0	G_{30}	90.0
G_{11}	168.8	G_{31}	190.0
G_{12}	168.7	G_{32}	190.0
G_{13}	214.8	G_{33}	190.0
G_{14}	394.3	G_{34}	164.8
G_{15}	304.5	G_{35}	199.6
G_{16}	304.5	G_{36}	199.8
G_{17}	489.3	G_{37}	109.9
G_{18}	489.3	G_{38}	110.0
G_{19}	511.3	G_{39}	110.0
G_{20}	511.3	G_{40}	511.3

The overall cost in this distribution amounts to 121,464 (c.u./h), which represents the best result of out a total of 100 calculations made with different initial points. All calculations were made for a total of 40 minutes, which gives an average of 24 seconds per calculation.

Table IX gives an extensive comparison of results reached through pattern search method with the results reached by other methods, such as particle swarm optimisation, evolution programming, genetic algorithm, plankton optimisation algorithm, bat algorithm, bio-geographic optimisation, as well as various options and modifications of those. Results were taken from [2], [5], [32], [33].

It is obvious that the method of search by pattern gives the best results.

Table IX: Comparison of results

Optimisation method	Costs of fuel (c.u.)
CEP	123,488
FEP	122,679
PSO	122,324
ESO	122,122
SPSO	122,050
APPSO	122,044
PSO-LRS	122,036
IGA	121,916
HPSOWM	121,915
PPSO	121,788
PC-PSO	121,768
NPSO	121,705
HDE	121,699
KH	121,695
BA	121,679
NPSO-LRS	121,664
MPSO	121,649
MBA	121,578
SOH-PSO	121,501
BBO	121,480
Search by pattern	121,464

Legend for abbreviations from Table IX:

CEP	- classical evolutionary programming
FEP	- fast evolutionary programming
PSO	- particle swarm optimization
ESO	- evolutionary structural optimization
SPSO	- sliced particle swarm optimization
APPSO	- agent based parallel particle swarm opti mization
PSO-LRS	- particle swarm optimization, local ran dom search
IGA	- interactive genetic algorithm
HPSOWM	- hybrid particle swarm optimization with wavelet mutation
PPSO	- perceptive particle swarm optimization
PC-PSO	- principal component particle swarm opti mization
NPSO	- novel particle swarm optimization
HDE	- hybrid differential evolution
KH	- krill herd
BA	- bat algorithm
NPSO-LRS	- novel particle swarm optimization, local random search
MPSO	- modified particle swarm optimization
MBA	- modified bat algorithm
SOH-PSO	- self-organizing hierarchical particle swarm optimization
BBO	- biogeography-based optimization

4. CONCLUSION

This work shows the use of the pattern search method to solve problems of non-convex economic dispatching. The method was applied to the test systems made of 5, 13 and 40 thermal units. Comparison made for the test system of 40 units, shows that this method has an advantage in comparison to other methods, for example evolution programming or particle swarm. On the other hand, the analysis of the calculations made showed that the pattern search is sensitive to initial parameters and it is based on the proximity of the initial point to the global solution. This makes this method very susceptible to falling into a trap of local minimum. Nonetheless, favourable speed of calculation provides room for additional calculations with the aim to increase the safety of the solution. It also needs to be mentioned that the other methods, to which this method is being compared, usually start with the population of initial points and not with one point as it is the case for the search by pattern method, which requires additional calculating resources.

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Damir Aganovic graduated the first study cycle at the Department for Electric Power Engineering of the Faculty of Electrical Engineering in Sarajevo in 2008, where he also graduated from the second cycle in 2010. He is now attending the third study cycle (doctoral studies) at the same school. He is employed at the JP Elektroprivreda BiH as an expert associate for operational management. The field of his interests include operational management of generation units and distributed sources of power.