DOI: 10.2478/aucts-2025-0009

ACTA UNIVERSITATIS CIBINIENSIS – TECHNICAL SERIES
Vol. 77 2025

THE CALCULUS METHOD FOR COGGING FROM THE CONDITION OF TEETH BENDING STRAIN

FLOREA Radu
”Lucian Blaga” University of Sibiu, Romania
FLOREA Adriana
”Lucian Blaga” University of Sibiu, Romania
STOICA Augustin
”Lucian Blaga” University of Sibiu, Romania
ISARIE Claudiu
”Lucian Blaga” University of Sibiu, Romania
PRODEA Laurențiu
”Lucian Blaga” University of Sibiu, Romania

Corresponding author:viorel-radu.florea@ulbsibiu.ro

Abstract. *The paper presents a new toothing calculation procedure in the condition of the teeth bending deflections. Accepting the same hypothesis as in the case of the calculation at the tooth base bending, the method allows the toothing dimensioning by determining the distance between the axes, the module, or the checking, dependent on a maximum admissible arrow. Through the virtual or equivalent gear, the method can be extended to bevel or spur inclined gearings.*

Key words: *gear, toothing calculation procedure, teeth bending deflections, outflank.*

1. INTRODUCTION

The elastic strains intervene between the real geometry and the theoretical one, making engagement conditions worse. To link real conditions to the theoretical ones, it is recommended to outflank the cogging at parameters that are as close as possible to the value of maximum strains. A rational outflanking reduces dynamic effects, has favourable consequences on strains, vibrations, noises in functioning. In order to determine it, we will further on study the effects of teeth bending, through mathematical methods, and we will propose the following: an alternative to the teeth resistance calculus using bending rigidity; a scientific method of determining the outflanking parameters.

2. THE CALCULUS OF SPUR GEARINGS WITH STRAIGHT TEETH

The calculus of cogging from the rigidity condition allows the intervention of the designer in order to minimize the dynamic effects and to improve the gear noise level [1]. This favourable situation may be obtained by equaling the outflanking thickness with the feasible pointer of the tooth [9].
The calculus of tooth bending rigidity admits the same hypotheses as the calculus of tooth base bending resistance [2]. Decomposing the nominal force P according to tangential and normal direction on this, we get:

$P_{tx} = P_n \cos\alpha_F$, that strains tooth in bending (1)
$P_{rx} = P_{tx} tg\alpha_F$, that strains tooth in compression (2)

Neglecting compression, we have the P_{tx} component that may be expressed dependent on the tangential component of the normal force, through the relation:

DOI: 10.2478/aucts-2025-0009
ACTA UNIVERSITATIS CIBINIENSIS – TECHNICAL SERIES
Vol. 77 2025

$$P_{tx} = P_t \frac{\cos\alpha_F}{\cos\alpha} = \frac{2M_t}{d} \cdot \frac{\cos\alpha_F}{\cos\alpha} \tag{3}$$

The maximum pointer due to bending

$$f_{\max} = \frac{P_{tx} h_F^3}{3EI_z} = \frac{P_{tx} h_F^3}{3E\frac{bs_F^3}{12}} = \frac{4P_{tx} h_F^3}{bEs_F^3} \tag{4}$$

Replacing P_{tx} in the relation (3), we get:

$$f_{\max} = \frac{4P_t \frac{\cos\alpha_F}{\cos\alpha} h_F^3}{bEs_F^3} = \frac{P_t}{bE} \cdot \frac{4h_F^3}{s_F^3} \cdot \frac{\cos\alpha_F}{\cos\alpha} \tag{5}$$

Because the second part of the expression (5) is dimensionless, it can be replaced with a tooth parameter

$$Y_{Fa} = \frac{4h_F^3}{s_F^3} \cdot \frac{\cos\alpha_F}{\cos\alpha} \tag{6}$$

with which the expression (6) can be expressed as follows:

$$f_{i\max} = \frac{P_t}{bE} Y_{Fa} \tag{7}$$

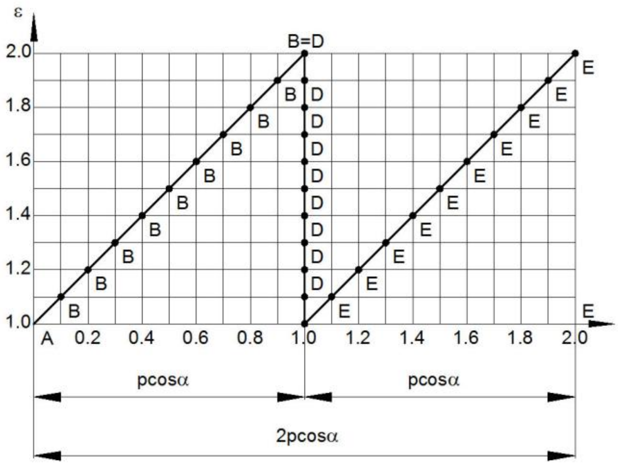

Fig. 1. [2]

In reality, strain of the entire nominal force P_n will take place only starting with point B (Figure 1). In order to determine the position of point B on the tooth flank, it must be restated that the gliding speed of flanks, in the cogging process varies linearly – from the extreme maximum values in A and E to zero in the pole. Correlating, in the cogging movement, tooth flank (A`C) with the cogging segment (AC), we get Figure 2.

From the AA`C and BB`C triangle, we obtain:

$$\frac{BC}{B'C} = \frac{AC}{A'C} \tag{8}$$

DOI: 10.2478/aucts-2025-0009
ACTA UNIVERSITATIS CIBINIENSIS – TECHNICAL SERIES
Vol. 77 2025

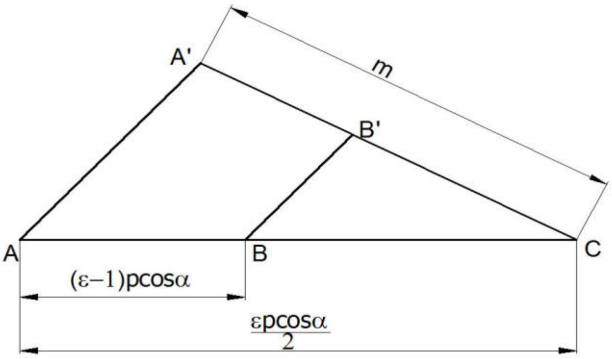

Fig. 2.

$$BC = \frac{\varepsilon p\cos\alpha}{2} - (\varepsilon - 1)p\cos\alpha \tag{9}$$

$$BC = \frac{(2-\varepsilon)p\cos\alpha}{2} \tag{10}$$

$$B'C = \frac{BC \cdot A'C}{AC} \tag{11}$$

$$B'C = \frac{(2-\varepsilon)p\cos\alpha \cdot m}{\varepsilon p\cos\alpha} = \frac{(2-\varepsilon)}{\varepsilon}m \tag{12}$$

$$A'B' = A'A - B'C = m - \frac{2-\varepsilon}{\varepsilon}m = \frac{2m(\varepsilon-1)}{\varepsilon} \tag{13}$$

Determining the position of the entrance point B in unitary cogging, the tooth bending strain scheme takes the shape of the one presented in Figure 3, where:

$$h_F = 2m - \frac{2m(\varepsilon-1)}{\varepsilon} = \frac{2m}{\varepsilon} \tag{14}$$

Under these circumstances, the shape factor Y_{Fa} can be presented through the shape factor Y_F, that has already been standardized [3]:

$$Y_{Fa} = \frac{4h_F^3}{s_F^3} \cdot \frac{\cos\alpha_F}{\cos\alpha} = \frac{6h_F m}{s_F^2} \cdot \frac{\cos\alpha_F}{\cos\alpha} \cdot \frac{2 \cdot 4m}{3s_F\varepsilon^2} \tag{15}$$

$$Y_{Fa} = Y_F\,\frac{2.66 \cdot m}{\varepsilon^2 s_F} \tag{16}$$

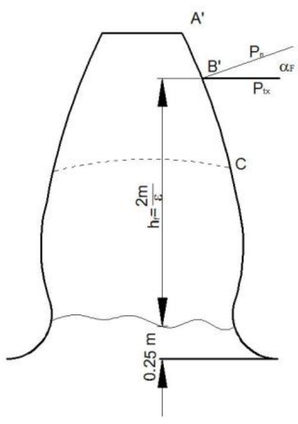

Fig. 3.

The last part of the expression is dimensionless and can be replaced with a bending factor:

DOI: 10.2478/aucts-2025-0009
ACTA UNIVERSITATIS CIBINIENSIS – TECHNICAL SERIES
Vol. 77 2025

$$Y_1 = \frac{2.66m}{\varepsilon^2 s_F} = \frac{2.66}{\varepsilon^2\left[\frac{\pi}{2} + 2x'tg\alpha - z\left(inv\alpha_y - inv\alpha\right)\right]} \cdot \frac{\cos\alpha_F}{\cos\alpha} \tag{17}$$

graphically represented in Figure 4.

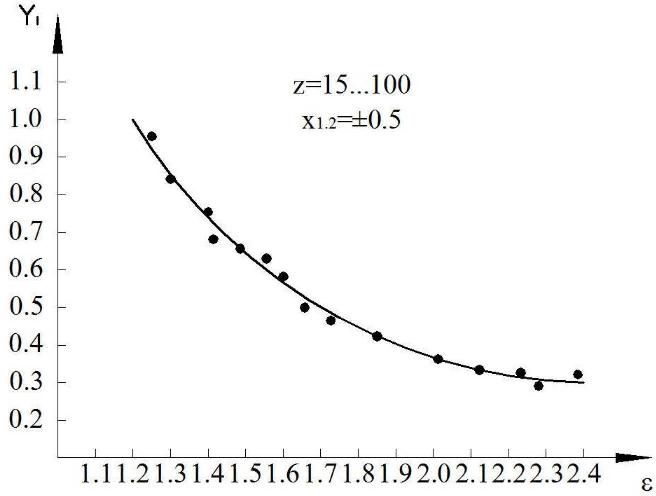

Fig. 4.

Under these circumstances, the expression (16) becomes:

$$Y_{Fa} = Y_F Y_1 \tag{18}$$

and the theoretical maximum pointer:

$$f_{\max} = \frac{P_t}{bE} Y_F Y_1 \tag{19}$$

Replacing the built-up factor of the strain, the relation (19) becomes:

$$f_{\max} = \frac{P_t}{bE} K_A K_V K_{F\beta} Y_F Y_1 \leq f_{ia} \tag{20}$$

allowing the checking of the tooth from the point of view of flexional strain [5].
In the relation (20):
P_t - is the tangential component of the nominal force;
b - cogging width;
E - coefficient of elasticity;
K_A - external dynamic factor – STAS 12268-84;
K_V - internal dynamic factor – STAS 12268-84;
K_β - factor of irregular repartition of strain on the width of the STAS 12268-84 cogging;
Y_F - shape factor, calculated by the definition relation or extracted from the STAS 12268-84 diagrams;
Y_1 - bending factor, calculated according to the definition relation or extracted from diagram (see Figure 4);
In order to dimension, the presented replacements are done further on.

- Dimensioning based on the distance between axes:

$$b = \psi_a a_w;\ d_1 = d_{w1}\frac{\cos\alpha_w}{\cos\alpha} = \frac{2a_w}{u \pm 1} \cdot \frac{\cos\alpha_w}{\cos\alpha} \tag{21}$$

$$f_{ia} = \frac{2M_{t1}(u \pm 1)}{\psi_a a_w^2 2E} \cdot \frac{\cos\alpha}{\cos\alpha_w} K_A K_V K_{F\beta} Y_F Y_1 \tag{22}$$

of which:

DOI: 10.2478/aucts-2025-0009
ACTA UNIVERSITATIS CIBINIENSIS – TECHNICAL SERIES
Vol. 77 2025

$$a_w = \sqrt{\frac{M_{t1}(u \pm 1)}{\psi_a E f_{ia}} K_A K_V K_{F\beta} Y_F Y_1 \frac{\cos\alpha}{\cos\alpha_w}} \tag{23}$$

the calculated a_w is brought to the standardised value.

- Dimensioning based on modulus:

$$b = \psi_m m;\ d_1 = \frac{2a_w}{u \pm 1} \cdot \frac{\cos\alpha_w}{\cos\alpha} \tag{24}$$

$$f_{ia} = \frac{2M_{t1}(u \pm 1)}{\psi_a m E\, 2a_w} \cdot \frac{\cos\alpha}{\cos\alpha_w} K_A K_V K_{F\beta} Y_F Y_1 \tag{25}$$

$$m = \frac{2M_{t1}(u \pm 1)}{\psi_m a_w f_{ia} E} K_A K_V K_{F\beta} Y_F Y_1 \frac{\cos\alpha}{\cos\alpha_w} \tag{26}$$

The choice of feasible value of the pointer is done according to the purpose of the gear. It must be lower than STAS 821-82 prescriptions. The experimental determinations allow us to recommend $f_{ia} \le \frac{\Delta a''_F}{5}$ where $\Delta a''_F$ is the depth of the outflanking, according to STAS 821-82.

3. THE CALCULUS OF SPUR GEARINGS WITH OBLIQUE TEETH

In order to keep the validity of admitted hypotheses for straight spur cogging, we replace the equivalent gear parameters in the relation (20). We obtain:

$$f_{i\max} = \frac{P_{tm}}{lE} K_A K_V K_{F\beta} Y_F Y_1 \tag{27}$$

but

$$P_{tm} = \frac{P_t}{\cos\beta} \tag{28}$$

$$l = \frac{b}{\cos\beta} \tag{29}$$

and introducing the cogging inclination factors Y_β and the coverage degree $Y\varepsilon$, the relation (27) takes the following shape:

$$f_{i\max} = \frac{P_t}{bE} K_A K_V K_{F\beta} K_{Fa} Y_F Y_1 Y_\varepsilon Y_\beta \le f_{ia} \tag{30}$$

In order to dimension, making the same replacements as in the straight spur cogging, we obtain:

$$a_w = \sqrt{\frac{M_{t1}(u \pm 1)}{\psi_a E f_{ia}}} K_A K_V K_{F\beta} K_{Fa} Y_F Y_1 Y_\varepsilon Y_\beta \frac{\cos\alpha_t}{\cos\alpha_{wt}} \tag{31}$$

and

$$m = \frac{M_{t1}(u \pm 1)}{\psi_m a_w f_{ia} E} K_A K_V K_{F\beta} K_{Fa} Y_F Y_1 Y_\varepsilon Y_\beta \tag{32}$$

4. THE CALCULUS OF STRAIGHT TEETH BEVEL GEAR

In order to maintain the validity of the hypotheses admitted for the straight spur cogging, the bevel gear is replaced with its virtual gear, at the level of the medium sections [7]. For the straight spur gearing wheels, this will be a gearing formed of spur wheels with straight teeth. Replacing the elements of the virtual gearing in the relation (20), we obtain:

DOI: 10.2478/aucts-2025-0009
ACTA UNIVERSITATIS CIBINIENSIS – TECHNICAL SERIES
Vol. 77 2025

$$f_{i\max} = \frac{P_{tm}}{bE} K_A K_V K_{F\beta} Y_F Y_1 \tag{33}$$

$$P_{tm} = \frac{2M_{t1}}{d_{mv1}} = \frac{2M_{t1}\cos\delta_1}{d_{m1}} \tag{34}$$

$$\cos\delta_1 = \sqrt{\frac{1}{1+tg^2\delta_1}} = \sqrt{\frac{1}{1+\left(\frac{1}{u}\right)^2}} = \frac{u}{\sqrt{u^2+1}} \tag{35}$$

$$f_{i\max} = \frac{2M_{t1}}{bd_{m1}E} \cdot \frac{u}{\sqrt{u^2+1}} K_A K_V K_{F\beta} Y_F Y_1 \le f_{ia} \tag{36}$$

5. THE CALCULUS OF MAXIMUM OUTFLANKING DEPTH

The STAS 821-82 recommendation present the maximum outflanking depth Δa_F, dependent on the mode and precision class, according to the operation smoothness criterion. Comparing the resulting values for a certain concrete case with those of the maximum strain caused by teeth bending in cogging, we find that they are approximately ten times bigger. As a result, outflanking according to the STAS recommendations avoids burden interferences, but includes dynamic effects that grow higher the higher the power and the rotative speed and that increase strains and noises that accompany the functioning of the gear.
In order to reduce dynamic effects and noises in functioning, we suggest the calculus of maximum outflanking depth Δa_F, starting from the relation 20. Replacing

$$P_t = \frac{2M_{t1}}{d_1} = \frac{2M_{t1}}{mz_1} \tag{37}$$

and

$$f_i = \Delta a_F$$

the relation (20) can be written as:

$$\Delta a_F = 1.5\frac{2M_{t1}}{bmz_1E} K_A K_V K_{F\beta} K_\alpha Y_\beta Y_\varepsilon Y_F Y_1 \text{ [mm]} \tag{38}$$

and it represents the maximum outflanking depth.
The 1,5 coefficient has the role of covering possible plus tolerances of dimensions [2].
The K_A, K_V, $K_{F\beta}$, $K_{F\alpha}$, Y_β, Y_ε, Y_F are according to STAS 12268-84 or DIN 3990-80.
- b is the width of the gear wheel [mm];
- m is the modulus [mm];
- M_t is the moment of rotation in [Nmm];
- z_1 is the number of teeth of the pinion wheel;
- E is the longitudinal elasticity modulus of the material [MPa].

6. CONCLUSIONS

The issue is whether the calculus at the tooth base bending is necessary anymore on fatigue conditions. The argument in favour has been the one referring to the flexional strains that might turn cogging worse because of the tendency towards burden interference. Outflanking avoids this danger, but at too much depth, it intensifies dynamic effects and noises [9]. It has been found out that rigidity varies cyclically and that it depends on the coverage degree.
A new method has been designed to calculate cogging from the condition of teeth bending strain.

DOI: 10.2478/aucts-2025-0009
ACTA UNIVERSITATIS CIBINIENSIS – TECHNICAL SERIES
Vol. 77 2025

Accepting the same hypotheses as in the case of the calculus for the tooth base bending, the method allows the dimensioning of cogging by determining the distance between axes or the modulus, or by checking dependent on a feasible maximum pointer.
By means of the equivalent ore virtual gear, the method can be extended to cover spur gearings and bevel gears al well.
Using the pointer produced by bending allows us to rigorously determine the outflanking depth.
The calculus and the experimental attempts have demonstrated that the recommendation for outflanking depth in STAS exceed the teeth pointer under stress more than ten times, which leads to the increase of dynamic effects and noises.

7. REFERENCES

1. Costopoulos TN., Spitas CA., Optimum gear tooth geometry for minimum fillet stress using BEM and experimental verification with photoelasticity., *Journal of Mechanical Design*, 128(5): 1159-1164, (2016).
2. Florea, R., Florea, A.: *Organe de maşini*., Editura Tehnică, Bucureşti, (2007).
3. ISO 6336 – Calculation of load capacity of spur and helical gears - Application for industrial gears, (2002).
4. Lisle TJ., Shaw BA. Frazer, RC. Frazer., External spur gear root bending stress: A comparison of ISO 6336: 2006, AGMA 2101-D04, ANSYS finite element analysis and strain gauge techniques. *Mechanism and Machine Theory*, 111: 1-9, (2017).
5. Matek, W., Muhs, D., *Maschinen-elemente*., Viewegs Fachbucher der Technik, (2000).
6. Niemann, G., Winter, H.: *Maschinen-elemente*., Springer-Verlag, (2003).
7. Pedersen NL., Improving bending stress in spur gears using asymmetric gears and shape optimization., *Mechanism and Machine Theory*, 45: 1707- 1720, (2010).
8. Thomas B., Sankaranarayanasamy K., Ramachandra S., Kumar SPS., Search method applied for gear tooth bending stress prediction in normal contact ratio asymmetric spur gears., Proceedings of the Institution of Mechanical Engineers, Part C: *Journal of Mechanical Engineering Science*, 232: 4647-4663, (2018).
9. Wen Q., Du Q., Zhai X. A new analytical model to calculate the maximum tooth root stress and critical section location of spur gear. *Mechanism and Machine Theory*, 128: 275-286, (2018).