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NUMERICAL ANALYSIS OF FRACTURE BEHAVIOR OF FUNCTIONALLY GRADED MATERIALS USING 3D-XFEM

ABSTRACT

This paper presents the numerical evaluation of mixed stress intensity factors (SIFs) and non-singular terms of William's series (T-stress) of functionally graded materials (FGMs) using three-dimensional extended finite element method (3D-XFEM). Four-point bending specimen with crack perpendicular to material gradation have been used in this investigation in order to study the effect of some parameters (crack position, crack size, specimen thickness) on the failure of FGMs materials. The fracture parameters (K_I , K_{II} , phase angle ψ and T-stress) obtained by the present simulation are compared with available experimental and numerical results. An excellent correlation was found of the 3D-XFEM simulations with those available in the literature. From the numerical results, a fitting procedure is performed in order to propose an analytical formulation and subsequently are validated against the 3D-XFEM results.

Keywords: *Functionally graded materials; Crack analysis; Maximum stress intensity factor; T-stress; Three-dimensional extended finite element method (3D-XFEM); Mixed mode; Phase angle*

INTRODUCTION

Materials have played a significant role in industries throughout the history. In engineering applications, the structures and machine components are subject to adverse thermo-mechanical environments in service. Practically, these mechanical parts are necessary to meet different requirements at different locations within the same component. So, the technological progress requires new materials that are multifunctional and durable under very severe service conditions. The composition of several different materials is often used in structural components in order to improve the responses of structures subjected to various loading condition simultaneously. To achieve this purpose, functionally graded materials (FGM) are a good choice. Functionally Graded Materials (FGMs) belongs to a class of advanced materials in which the volume fraction of the two or more materials is varied continuously across a particular direction in structure to achieve targeted performance with better reliability during applications. During manufacturing process of FGM structures appears some imperfections. These imperfections may be voids, inclusions, porosities, holes and cracks and so on. One of the most critical imperfections in FGM structures is the crack. The presence of defect (crack) in any structure reduces strength and reliability of structures considerably and even more reduction in reliability under the action of severe stress gradients induced by thermal and/or mechanical loadings (thermal cyclic loading). Several authors [1-3]

have been studied the fracture behavior of FGMs structures by analytical, experimental and numerical methods with various geometries and crack modes under various loading conditions. Carpinteri et al. [4] studied the crack propagation of brittle and fatigue crack growth in FGMs materials, they presented an analytical approach in order to investigate the crack growth in Mode I. Tilbrook et al. [5] presented an experimental study and numerical analysis to evaluate the mechanical behaviour and crack propagation in homogeneous and graded alumina–epoxy composite samples, under monotonic and cyclic loading. Also, Bhardwaj et al. [6-7] developed a new method to compute the fatigue life of interfacial edge cracked functionally graded materials (FGMs) based on the extended isogeometric analysis (XIGA). In the same context, the fracture and the mechanical behaviour of functionally graded materials (FGMs) has been reviewed by Ferreira et al. [8] and Swaminathan et al. [9]. The analytical methods can be successfully applied to cracked FGMs only for simple geometrical and loading conditions. The use of the finite element method in engineering projects has evolved a lot in recent years, mainly due to the use of numerical methods in commercial software. Finite Element Analysis (FEA) is an important tool to design a structural component in engineering field, such as the field in FGMs structures. It is known, that the microstructure of FGMs is generally heterogeneous and are characterized by gradual spatial variation in composition, microstructure and material properties of each component. So, the fracture behavior of FGMs becomes complicated due to the gradation in the material properties. For this purpose, finite element method (FEM), have been widely used to relieve the difficulties associated with the analytical solution to study the fracture behavior of FGMs [10-13]. Kim and Paulino [14-15] investigated the mixed mode stress intensity factor using the interaction integral approach in two-dimensional finite element method. Gu et al. [16] used an equivalent domain integral technique for calculating the SIF for FGMs using finite-element method (FEM). Dag et al. [17] analyzed the mixed-mode fracture problem of orthotropic functionally graded material under thermo-mechanical loading conditions. In the same context, a thorough examination of this subject can be found in Refs. [18–20]. However, FEM is not an appropriate method for the crack growth and fracture simulations as it requires conformal mesh to represent geometries and remeshing for the modeling of crack growth. The remeshing process used in FEM for the modeling of crack growth introduces error during the transfer of the field variables from an old mesh to the new mesh. Therefore, the researchers used advanced numerical methods such as extended finite element method [21, 22] in order to overcome the difficulties associated with FEM. On the other hand, most of the above mentioned papers have been devoted to two-dimensional (2D) crack analysis. In contrast, three-dimensional (3D) crack analysis in non-homogeneous materials and nonlinear elastic FGMs has been only rarely reported in literature [23-25]. Ayhan [26-27] numerically studied the surface crack in FGM plate under various loading, who used the three-dimensional enriched finite elements to compute mixed-mode stress intensity factors (SIFs). He concluded that the developed three-dimensional enriched finite elements are capable of accurately computing mixed-mode fracture parameters for cracks in FGMs. Sladek et al. [28] examined the fracture behaviour of functionally graded materials (FGM) by the Meshless method based on the Local Petrov-Galerkin approach (MLPG), in this method, no special integration techniques are required to evaluate the integrals. The analyzed domain is divided into small overlapping spherical subdomains on which the local boundary integral equations are applied. These methods are truly meshless methods, where in no elements or background cells are involved in either the interpolation or the integration. In the same subject, Zhang and his co-authors [29], presented interesting method to study the failure of FGMs materials, this method is called a boundary-domain-integral equation formulation, which uses the elastostatic fundamental solutions for homogeneous, isotropic and linear elastic materials and involves a

domain-integral due to the material's non-homogeneity. They found that the material gradation may have significant influences on the crack-opening-displacements and the stress intensity factors. Literature reviews indicate that there are only a few published results about T-stress calculation of cracked FGMs structures [30-32]. Therefore, the purpose of this paper is to introduce a Three-Dimensional Extended Finite Element Method 3D-XFEM approach for assessment of the mixed mode stress intensity factor, phase angle and T-stress of the surface cracks in four-point bending specimen with crack perpendicular to material gradation. The effects of variety of geometrical parameters such as thickness of specimen (B) and crack length to width ratio (a/W). Crack position on the SIFs and T-stress are investigated. Moreover, the effects of variety of the mechanical properties on the SIFs (K_I and K_{II}) of the cracks are discussed. The maximum values of K_I , K_{II} and T-stress are located at mid plane of specimen. Lastly, the level of fracture parameters (K_I , K_{II} and T-stress) and it's critical location along the graded region are analyzed for this loading condition.

EXTENDED FINITE ELEMENT METHOD

Finite element method (FEM) has been extensively used to treated and solved fracture behavior of mechanical components. In FEM, the modeling of discontinuity need a conformal mesh and required also a special element to represent correctly the cracks geometries. However, the generation of good quality meshes for complex geometries is an expensive and time consuming task. So, the Extended Finite Element Method XFEM was introduced to overcome the demerits of conventional FEM. The extended finite element method (XFEM), introduced by Belytschko and Black [33], overcomes the requirements of updating the mesh as the crack grows (Figure 1). XFEM employs local region enrichment using the concept of unit partition [34]. The displacement functions in the FEM formulation are enhanced by introducing additional enrichment functions that capture the jump in displacement across the crack surface and also the crack-tip singularities.

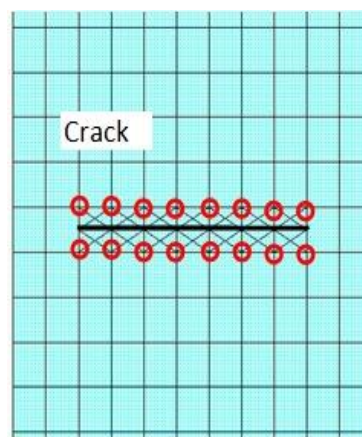


Fig. 1. XFEM Crack representation in a finite element model at the enriched zone

As previously mentioned, the concept of unit partition was introduced by Melenk and Babuska [34]. They developed the PUFEM method (Partition Unity Finite Element Method) which has been used in various computational disciplines and is the basis of the XFEM method. This concept establishes that the sum of the interpolation functions (shape functions) of all the nodes of an element, evaluated at any point of the same, provides as a result the unit:

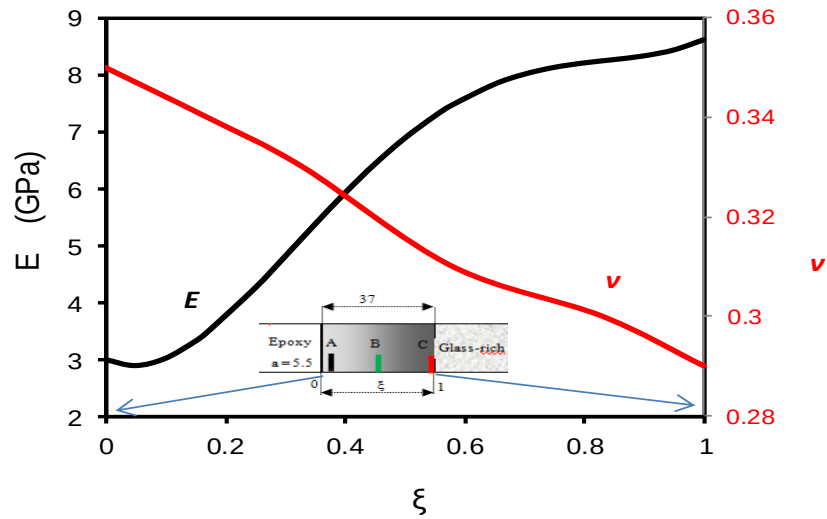


Fig. 3. Variations of Young's modulus E and Poisson's ratio ν along the graded region ($0 \leq \xi \leq 1$)

Let us consider a notched functionally graded beam under four-point bending treated experimentally by Rousseau and Tippur [3]. A crack of length $a = 5.5$ mm is assumed to exist perpendicular to the material gradation located in various position characterized by dimensionless parameters $\xi = 0.17, 0.58$ and 1 along the gradually zone, Figure 2. The mechanical properties at room temperature of FGMs plate are selected according to several references [3, 15] and are given in Figure 3.

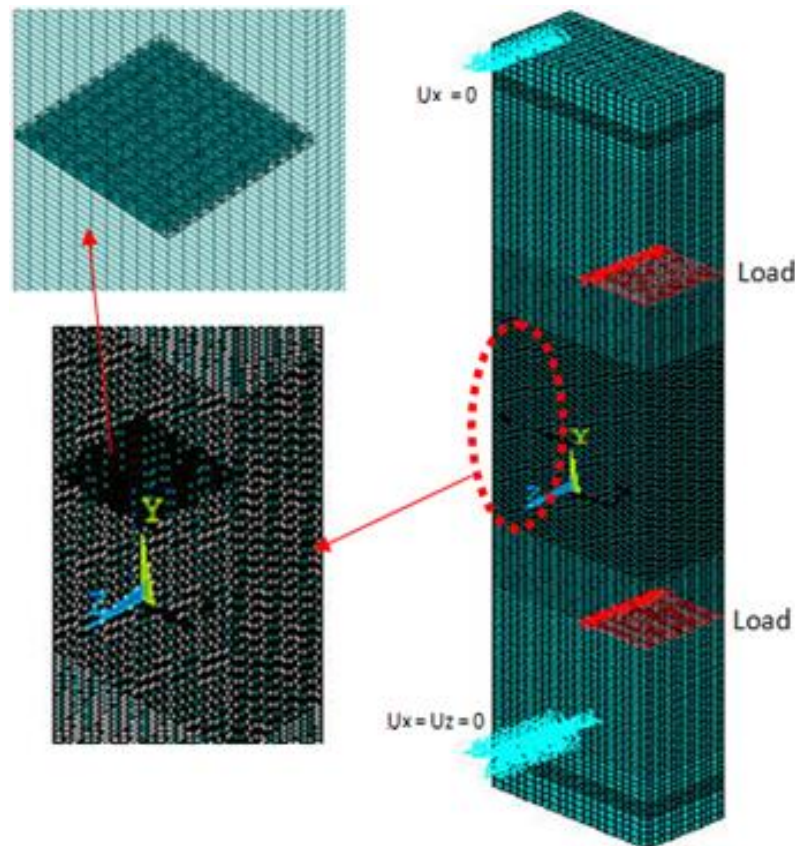


Fig. 4. Meshing model, loading condition and zoom at the enriched zone

Finite element simulation was done using the commercial software ANSYS® (ANSYS 19 [35]). The 08-nodes iso-parametric elements SOLID185 with quadratic base-function were used for mesh generation, whereas a very fine mesh was used in the region around the crack front to achieve precise results. Nonlinear behavior of the geometry and materials properties for the FGMs specimen was considered in this study. The mesh is designed to be highly refined close to the crack-tip in order to obtain more accurate results (typical element dimension in this zone is 0.05 mm). Various mesh schemes are tried to achieve convergence. The optimized model has 61340 nodes, 53872 elements. The mesh model with the boundary and loading conditions are given in Figure 4. The theory of incremental plasticity is introduced to modeling the material nonlinearity. The iterative method of Newton–Raphson is used as an approach to solve nonlinear equations by finite elements.

RESULTS AND DISCUSSION

Validation of the finite element model

To validate the numerical model developed in this work, a notched functionally graded beam under four-point bending was analyzed. In order to make a comparison, the same dimensions and material properties as those used by Rousseau and Tippur [3] and Kim and Paulino [15] were considered. The first task comprised an evaluation of the mixed stress intensity factor K_I and K_{II} along the crack tip for three position different along the gradient zone (crack A, crack B and crack C) and a crack size ratio $a/w = 0.25$. The crack location is characterized by dimensionless parameters $\xi = 0.17, 0.58$ and 1 along the material gradient.

Figure 5 shows the results obtained for K_I and K_{II} from the Three-Dimensional Extended Finite Element Method (3D-XFEM) of the SENB specimen. Also shown in Figure 5 are the values of K_I and K_{II} calculated from two-dimensional (2D) finite element method given by Kim and Paulino [15] for the same specimen. A first observation from the reading of Figure 5 is that whatever the crack position along the gradient, the K_I gradually increases along the crack-front and finally reaches the maximum value at the midspan of specimen thickness. However, Figure 5 shows that the K_{II} decreases along the crack-front and here maximum level is located at the centre of the specimen. This phenomenon can be explained by the fact that when a crack tends to propagate in pure mode I in FGMs materials, a mode mixity arises not only from geometrical and loading configurations but also from the material non-homogeneity parameter. On other hand, locally at the crack tip; there is exist mode mixity which is a critical parameter for interfacial fractures. The mode mixity is the relative proportions of tractions ahead of the crack tip in sliding mode (mode II) and opening mode (mode I) in the fracture. This behavior was confirmed by several authors [36–38]. We note a fairly good agreement is observed between the two sets of results can be considered as a validation for our numerical simulations using 3D-XFEM.

In order to better understand the influences, the location of the crack along the gradient zone, in Figure 6. one traces the evolution the maximum value of K_I and K_{II} along the gradients. We recall that the maximum value is located at mid-span of specimen thickness because fracture initiation occurs at this location. It can be found that the value of K_I increase with the increasing of non-dimensional variable ξ while the increasing of this latter decreases the value of K_{II} . In other words, the change position of crack from soft material to stiffer material leads to reduce the sliding mode and aggrandize the opening mode, that is to say a risk of brittle fracture of a material with high resistance.

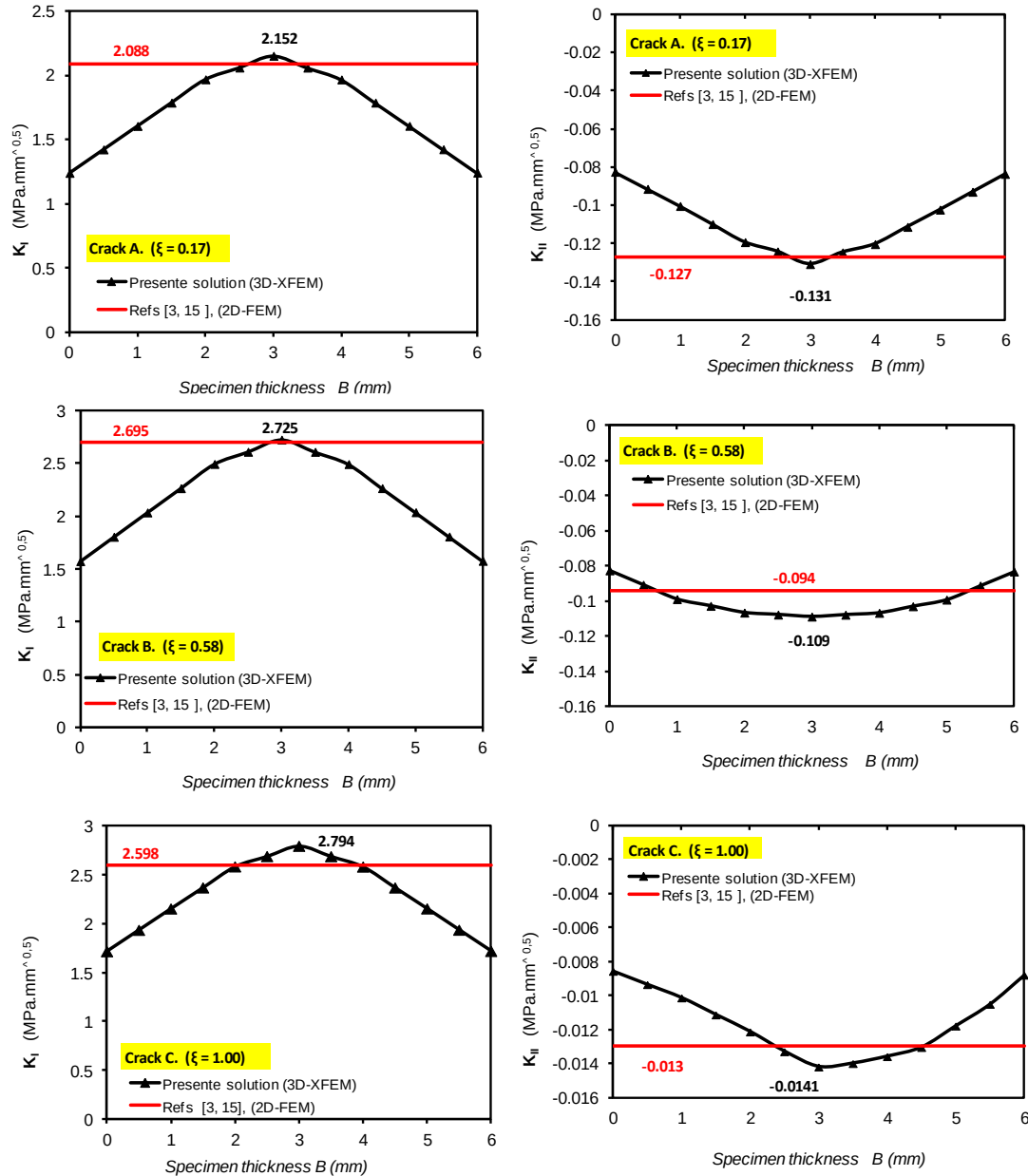


Fig. 5. Variations of K_I and K_{II} along the crack-front for $B=6$ mm and crack size $a/W = 0.25$

The phase angle $\psi = \tan^{-1} (K_{II}/K_I)$ plays an important role in the initiation and direction of crack growth of FGMs materials and hence is an important parameter in the characterization of interfacial fracture toughness. Figure 7 presents the phase angle versus crack position along the gradients. One can see that ψ increases when the crack is situated close to stiffer materials. This behavior can be explained probably by the fact that the change of mechanical properties of FGMs materials leads to arise a mode mixity loading, which lead a complexes stress distribution, on the other hand, the level of shear components (mode II) caused a rotation of the region close to the crack tip which leads to change of the phase angle ψ . We recall also, that the fracture of FGMs materials is variable and often break under mixed mode loading with complicated fracture patterns. One concludes, from the previous comparison, it can be noted that the 3D-XFEM method satisfactorily predicts the fracture parameters for four-point bending specimen and thus testifies the accuracy of the developed numerical modeling.

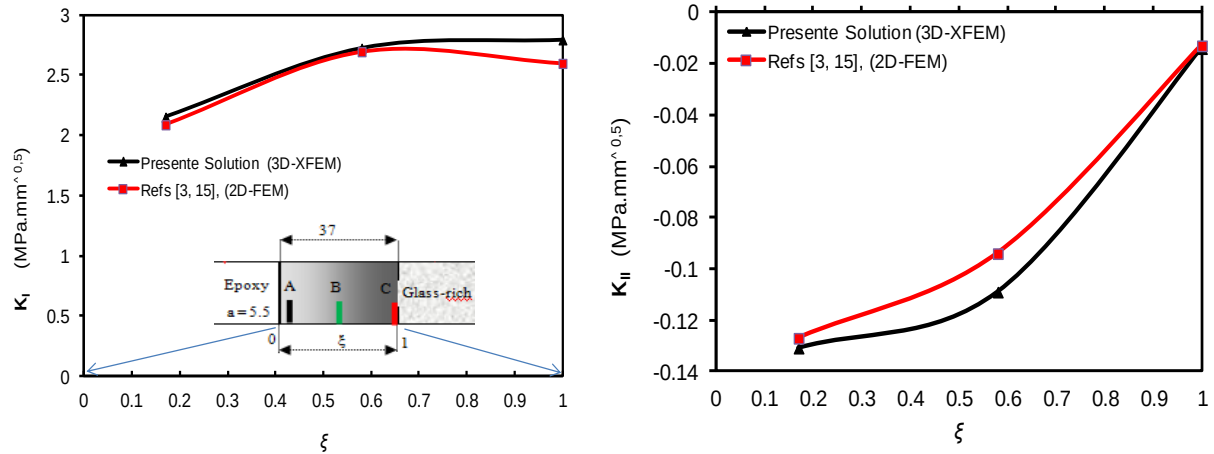


Fig. 6. Variations of maximum values of K_I and K_{II} vs. crack position (ξ) for $B=6$ mm and crack size $a/W = 0.25$

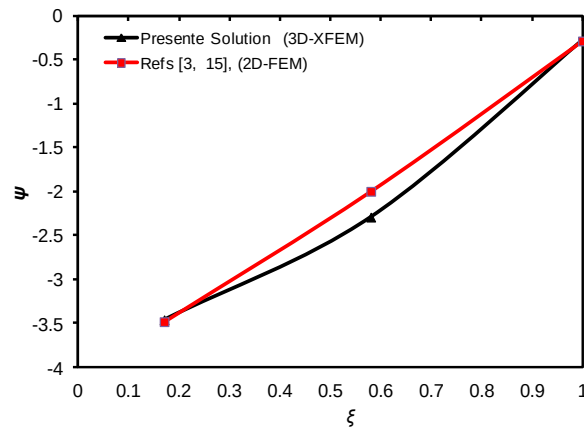


Fig. 7. Variations of phase angle vs. crack position (ξ) for $B=6$ mm and crack size $a/W = 0.25$

Figure 8 presents the evolution of T-stress along the crack tip for two positions of crack along the gradient zone characterized by the dimensionless parameters $\xi = 0.17$ and 1 . It can be observed that the T-stress is relatively instable through the thickness where the maximum value is located at mid-span of specimen. Whatever the dimensionless parameter ξ (crack location in gradient zone), the T-stress curves are of parabolic forms. Thereafter, we are interested in our study, the computation of failure parameters (K_I , K_{II} and T-stress) only at mid-span through the specimen thickness.

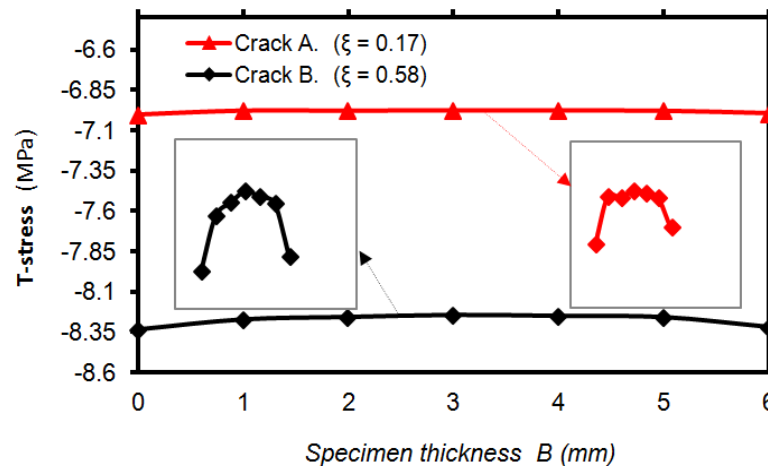


Fig. 8. Variation of T-stress along the crack-front for $B=6$ mm and crack size $a/W = 0.25$

Effect of thickness specimen

In order to study the effect of thickness specimen on the value of fracture parameters of a crack situated at different locations along the graded region, let consider the precedent geometrical model with two values of thickness equal to 6 and 11 mm. Figure 9 illustrates the variations of K_I , K_{II} and T-stress versus non-dimensional parameters (crack position along the graded region). It can be seen from these figures that thickness specimen and the crack position has a considerable influence on the fracture parameters (K_I , K_{II} and T-stress). Figure 9 also reveals a critical position of crack along the graded region characterized by the non-dimensional parameter $\xi = 0.26$, where the value of K_I and T-stress and absolute values of K_{II} seeks its maximum. On the other hand, the T-stress has an influence on crack growth and consequently its effects on the fracture behavior of FGMs materials. This behavior was confirmed by several authors [30-32].

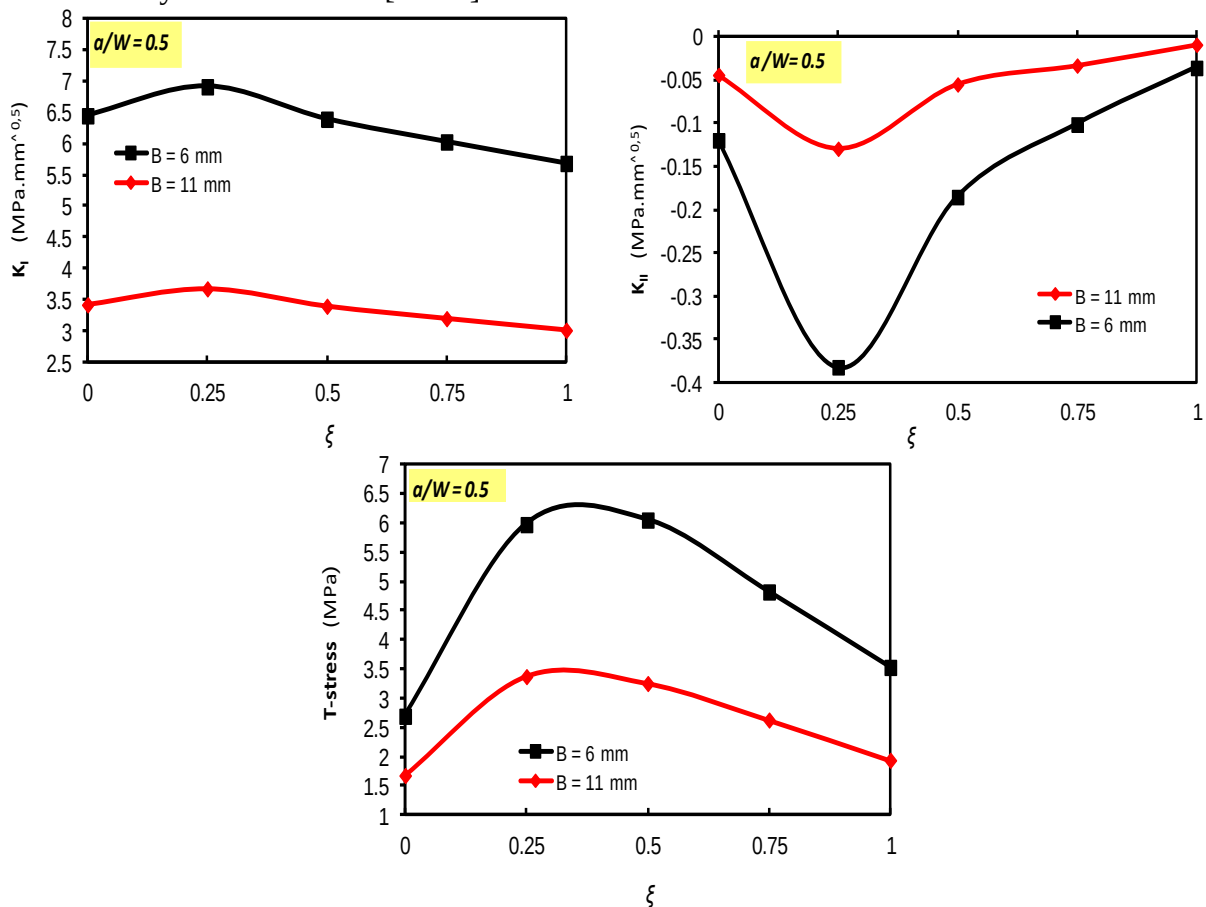


Fig. 9. Variation of maximum values of K_I , K_{II} and T-stress vs. crack position (ξ) for various specimen thicknesses and crack size $a/W = 0.5$

Effect of crack length

The effect of crack length is presented in Figure 10 in terms of stress intensity factor in (K_I , K_{II}) and T-stress versus crack locations along the graded region. Again, a critical position can be found for both crack length along the graded region ($\xi = 0.26$). Indeed, for crack length high, the value of fracture parameters is much more significant. However, Figure 10 shows also, that the crack sizes have a significant effect on the sign of the T-stress. This later (T-stress parameter) increases from high negative value for crack size lower to positive values when the crack length is higher. This behavior is probably related to the non-homogeneous distribution of the local stress and strain fields near to the crack due to the change of crack

position and mechanical properties of FGMs materials. Larsson and Carlsson [39] indicate that the sign and level of the T-stress substantially change the shape and size of plastic zone in case of plane strain.

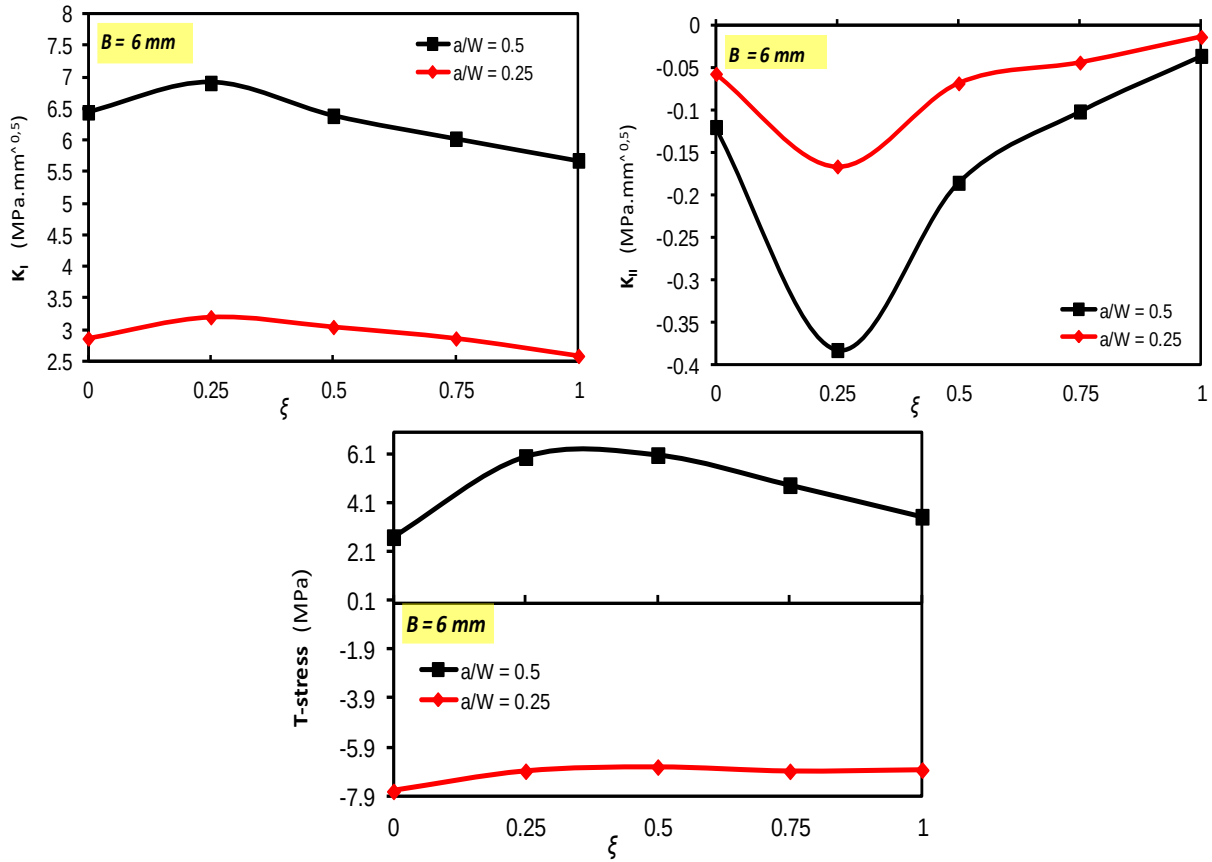


Fig. 10. Variation of maximum values of K_I , K_{II} and T-stress vs. crack position (ξ) for $B=6$ mm and various crack size a/W

Analytical formulation

The crack growth in FGMs materials is generally done via one of the three modes (Mode I, Mode II and Mode III) or through a combination of the three modes (mixed mode). It is known, that the opening mode (Mode I) are most dangerous and usually plays a dominate role in engineering application. From the previous analysis, the level of K_I is found to be maximum at mid-span of specimen. This demands 3D FE analysis of fracture specimen, which is difficult and takes a significant calculation time for any failure analysis. Hence, in this study, using the numerical results, an analytical expression is proposed to determine the SIF as a function of crack position, without any numerical computations, Full details of the fitting method from the numerical results are given elsewhere [40, 41]. We have tried to fit the data with a third order polynomial, the relation between K_I and ξ can be expressed as follows:

$$K_I = A_3 \left(\frac{a}{\xi}\right)^3 + A_2 \left(\frac{a}{\xi}\right)^2 + A_1 \left(\frac{a}{\xi}\right) + A_0 \quad (3)$$

where A_i : ($i = 0, 1, 2$ and 3) are polynomial factors. These factors are obtained through a systematic curve-fitting of the numerical calculations of the values of K_I presented in Figure 9. The amounts of A_i are defined in (Table 1). For this application, the maximum values of K_I

are collected for various crack size and then an empirical formula is proposed. The fitting formula for the maximum value of K_I is shown as follows:

$$K_I = \left[5.3008 \left(\frac{a}{W} \right)^2 + 8.8363 \left(\frac{a}{W} \right) - 0.4419 \right] \left(\frac{a}{\xi} \right)^3 + \left[21.966 \left(\frac{a}{W} \right)^2 + 3.3344 \left(\frac{a}{W} \right) + 2.3291 \right] \left(\frac{a}{\xi} \right)^2 + \left[16.306 \left(\frac{a}{W} \right)^2 - 7.166 \left(\frac{a}{W} \right) + 2.9341 \right] \left(\frac{a}{\xi} \right) + \left[11.461 \left(\frac{a}{W} \right)^2 + 5.4887 \left(\frac{a}{W} \right) + 0.7465 \right] \quad (4)$$

where a is the crack length, W is the specimen width and ξ is dimensionless parameter which define the crack position in graded region.

Table 1. Parameters of polynomial equation fitted for K_I

a/W	A3	A2	A1	A0
0.125	0.7455	-3.0891	2.2931	1.6117
0.375	3.5674	-6.4625	2.3869	4.3090
0.500	5.3511	-9.6938	3.5803	6.4635
0.750	9.1670	-17.1860	6.7317	11.3100

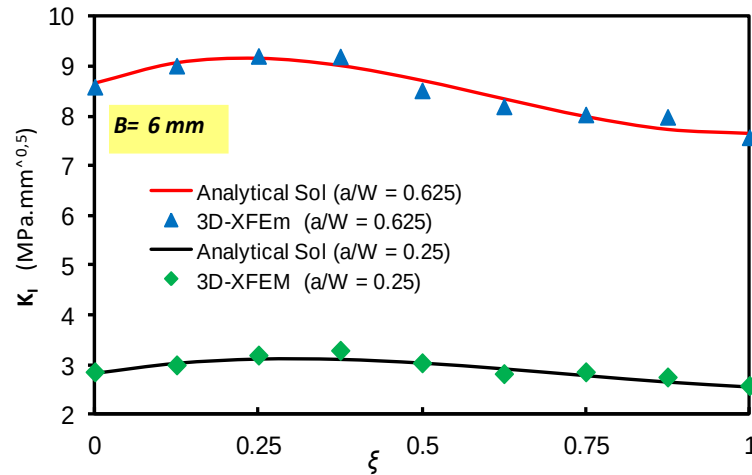


Fig. 11. Variation of maximum values of K_I vs. crack position (ξ) for $B=6$ mm and various crack size a/W , comparison analytical and 3D-XFEM solutions

To validate the proposed expressions in this work, additional three-dimensional finite element analyses were performed to evaluate the SIF for various crack position along the graded region. Figure 11 show an example of K_I calculated from the 3D-XFEM, together with the analytical solutions. Through these comparisons, one sees an excellent agreement was achieved and it can be concluded that the analytical solutions for K_I have been verified.

CONCLUSIONS

In this study, the effect of various parameters such as: material gradation, crack size, crack location, specimen thickness and loading condition on the fracture problems of Functional Graded Materials (FGM) under four point bending using three-dimensional extended finite element method (3D-XFEM). From these numerical results, we can deduce the following conclusions:

- The material gradation has a great effect on mixed mode SIFs.

- The level of SIFs along the crack front depends on the material gradation index.
- The level of T-stress and mixed mode SIFs varies along the crack-front and its variation is related to the specimen thickness and crack size.
- The fracture parameters at the specimen mid-span received special attention because fracture initiation occurs at this location regardless of crack position.
- Exists a critical position of crack along the graded region characterized by the non-dimensional parameter $\xi = 0.26$ when the crack is close to the soft side of the FGM
- Based on the 3D-XFEM results, an analytical expression is proposed for evaluation the K_I at the mid-span of FGMs specimen.
- Excellent agreement shown in this study provides sufficient confidence of the proposed analytical expression, and thus the application of this equation to cracked FGMs materials without any numerical computations

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