

Basic operations in reasoning and conceptual exploration

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Abstract¹

Conceptualisation and reasoning involve intellectual operations that can and should be taught. This paper identifies pairs of basic operations in reasoning and conceptualisation that are comparable to the basic operations of arithmetic and just as important. Examples are provided to illustrate how these operations may be introduced in the classroom.

Key words

conceptualisation, operations, Piaget, reasoning

Introduction

This paper draws attention to some basic operations involved in thinking. They belong to the foundations of reasoning and conceptualisation. These operations should be of particular interest to those involved in philosophy in schools because philosophy specialises in logic, or the art of reasoning, as well as conceptual analysis and the construction of ideas, and therefore has a distinctive contribution to make to the development of students' abilities in these areas. Beyond that, however, they should not be neglected by anyone concerned with the teaching of thinking. In order to teach thinking, we need to be familiar with the operations involved, beginning with their most basic forms.

Before we get down to details, I should briefly say something about how I am employing the term 'operations'. I am using it to identify what Piaget long ago called 'operations,' or at least to refer to actions and processes that display the most important of the characteristics that Piaget used to define operations (see, for example, Piaget 1970). An operation, for Piaget, involves a transformation in action or in thought that is reversible. It may involve the manipulation of materials or may have become interiorised to the point of becoming a representational or mental process, but in either case the process involved

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can be carried out in one direction and then again in its reverse. Simple arithmetic provides obvious examples. Numerical addition has its reverse in numerical subtraction. We can add 3 to 5 to get 8; then subtract it again to return to 5. The same holds for multiplication and division. We can multiply 3 by 5 to get 15; then divide the result by 5 to come back to 3.

As the examples from arithmetic make clear, operations exist within *systems* of transformations. They also illustrate the fact that such systems and not merely incidental to the acquisition of human knowledge. Indeed, according to Piaget, human knowledge arises out of the assimilation of reality into such systems. That may be an overly restrictive account of human knowledge, but there is no denying that organised knowledge of the kind that underpins the school curriculum depends on systems of operations. It is therefore all the more important that we attend to them.

Reasoning: Logical justification and inference

Just as addition and subtraction are the basic operations of arithmetic, the operations of *logical justification* and *inference* form the basis of reasoning. I am going to use the words 'because' and 'therefore' to mark these operations, and suggest that students should be introduced to the use of these terms to carry them out. As they gain proficiency, students should become aware of the reversible nature of their relationship and able to move between them.

While we might not use terms like 'justification' and 'inference' with young children, we need to clearly distinguish between these operations. This includes clear instructions, such as 'give a reason' and 'draw a conclusion', as well as introducing and underlining the word 'because' to give a reason and 'therefore' to indicate that an inference is being drawn. (Some teachers of young children prefer to use the word 'so' instead of 'therefore' to introduce inference-making, but 'so' has *so* many other uses— isn't that *so*? Whereas 'therefore' is specific to the purpose.)

While I wish to focus on the reversibility of these operations, I should begin by saying precisely what these operations involve. Let us begin by placing the process of logical justification within the context of inquiry. Most simply put, logical justification is the giving of reasons in support of a suggestion. The examination of suggestions (ideas, opinions, hypotheses, propositions, etc.) through the give-and-take of reasons constitutes a large part of the inquiry process. From a cognitive point of view, we can say that logical justification deals with the relationship between *judgements*, where one or more judgements is used to justify another; or in terms of language use, we may say that it deals with logical relations between *statements*, where one or more statements are used to support another statement. Engaging students in logical justification therefore shifts their

attention from relations between the things about which they are judging to the relations between their judgements or between the statements that they make. In a word, it focuses their attention on *thinking*.

One way of introducing this operation is through exercises such as the following, which draws attention to logical justification as a relation between statements:

Exercise: Justifying beliefs and opinions

When we attempt to justify our beliefs and opinions, we give reasons to try to show that we are right—or, at least, that our beliefs or opinions have something to be said for them. The people below are attempting to justify some belief or opinion. In each case, state the belief or opinion and then connect it with the reason or reasons given for it using 'because'.

1. Angela insisted that a tomato is a fruit and not a vegetable, since it develops from a flower.
2. Robert wondered whether spiders are insects, but Naomi said that they aren't. She said that spiders have eight legs, while insects have only six legs.
3. Sam and Andy were talking about the difference between bees and flies. Andy said that bees' wings are different from flies' wings. He claimed that bees have two sets of wings, while flies have only one. Sam agreed, but added that bees also fold their wings into their bodies when they are at rest, whereas flies spread them out.

Questions such as 'Why is that?' or 'Why do you think so?' are the standard requests for logical justification when engaging in inquiry and should be asked whenever a suggestion is in need of support.

Let us turn to inference. Just as we *give reasons* to justify statements, we *reason* in order to draw conclusions or to infer one thing from another. The use of 'therefore' to draw inferences from statements is easily introduced by means of examples and exercises such as the following:

Exercise: Therefore

Below there are four pairs of statements where you can use the word 'therefore' to show that one statement follows from the other. Find the pairs and then put them together in the right order, using 'therefore' to connect them.

Robert is ten.

Jessica is taller than Jasmin, but shorter than Robert.

Jasmin was on top.

Robert is twice as old as Jasmin, who is only five.

Jasmin is the lightest of the three.

Jessica is lighter than Robert, but heavier than Jasmin.

Robert is the tallest of the three.

Jasmin sat on Jessica's shoulders, while Jessica sat on Robert's shoulders.

This brings us to the Piagetian property of reversibility. We have been using the words 'because' and 'therefore' to indicate the operations of justification and inference. Justification and inference are obviously different operations and yet they are related. They are the *inverse* of one another. Here is a simple example:

Justification: Scruffy's name is longer than Mutt's **because** it has three more letters.

Inference: Scruffy's name has three more letters than Mutt's. **Therefore** it is longer.

Once students are familiar with the two operations, it is time to make them aware of the connection between the two. With practice, they will be able to move backwards and forwards between justification and inference and this will be a great leap forward in their capacity to reason and inquire. The exercise below is designed to give students practice in moving between these operations. The first part is pitched at middle primary while the second is designed for junior secondary.² Notice that in some cases in the secondary exercise I have not used 'because' or 'therefore', requiring students to look for other clues to determine what Scruffy and Mutt are doing.

² The second part of the exercise comes from Cam (2013, p. 21).

Exercise: Giving reasons and drawing conclusions

In each case below can you say whether Scruffy and Mutt are giving a reason or drawing a conclusion? If they are giving a reason, turn around what they have said and make it into drawing a conclusion using 'therefore'. Similarly, if they are drawing a conclusion, then turn around what they have said and make it into giving a reason using 'because'.

1

Mutt: It is no fun being a stray dog because no one pats a stray dog.

Scruffy: I have to guard the house. Therefore I bark at people passing.

Mutt: I sit when I am told because I am an obedient dog.

Scruffy: I get to play with my friends at doggy day care. Therefore I like to go to doggy day care.

2

Mutt: There is no such thing as an ideal dog because no dog is perfect.

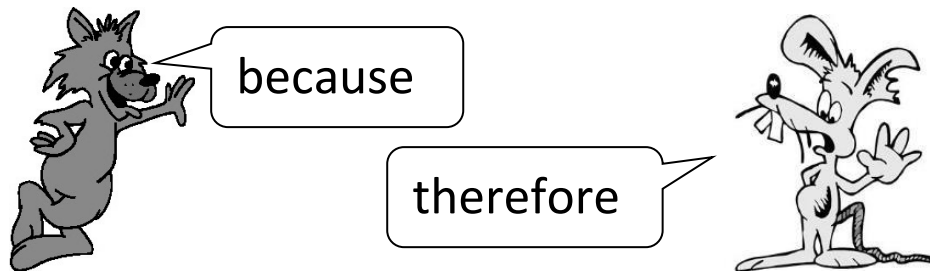
Scruffy: Given that dog-show judges measure actual dogs against the ideal, it follows that there must be an ideal dog.

Mutt: There is no such thing as a real circle because not one of the things we regard as a circle is perfectly circular.

Scruffy: There must be such a thing as a perfect circle. We use it to judge that something like a wheel isn't perfectly circular. (Be sure to fill out what 'it' means when you reverse what Scruffy is saying.)

Mutt: We can only see the perfect circle in our minds. Therefore the perfect circle cannot be a physical object.

It will assist primary students if you pin up a pair of cards relating to these operations when you teach them, as well as when you wish to emphasise their use or display associated written work.

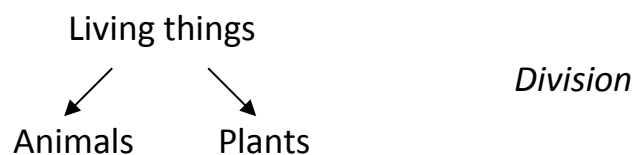


For secondary students, while $\therefore$ is the familiar shorthand for 'therefore', you should point out that $\because$ is the shorthand for 'because', making one symbol the inverse of the other. The folks who devised that arrangement knew what they were doing!

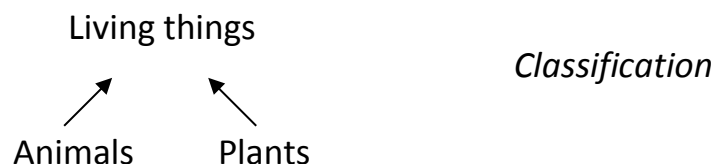
Conceptualisation: Classification and division

Just as it is common to talk in a vague kind of way about reasoning and reason-giving without clearly identifying the relevant operations, so too is failure to identify the operations that underpin conceptualisation. The most basic operations that we need to attend to here are division and classification.

Division involves dividing a kind into its various sub-kinds, as in dividing living things into animals and plants.



While this results in a classification scheme, the *operation* of classification is the inverse of division. To classify something is to assign it to a class of a more general kind. Thus, we classify animals and plants when we place them under the category of living things.



Note that the relationship between classification and division in conceptualisation is analogous to that between addition and subtraction in arithmetic. Adding things on and taking them away form a pair of operations in much the same way as grouping them together and separating them. When formalised, the first pair of these actions become

basic numeric operations while the second pair become the basic operations involved in conceptualisation. Learning to employ the elementary operations of classification and division is as indispensable for learning to think conceptually as is mastering addition and subtraction in learning to think mathematically.

The activity below involves the division of animals into a number of groups. It also involves the classification of the subgroups into animals of various kinds. The two operations work in tandem. The same activity can be carried out by replacing the animals with a variety of things of some other general kind with which your students are familiar.

Activity: Group³

Procedure:

1. Make sufficient copies of the set of animals below so that each small group of three or four children can be given a set. Then cut up the sheet to separate the animals and place each set in an envelope.
2. Divide the children into the groups and distribute one envelope to each group.
3. Tell the class that they are to divide their animals into groups. Explain that they can have as many groups as they like, except that one animal all by itself is not a group. Also tell the children that they will need to be able to name each group and be prepared to explain why the animals are in that group using the word 'because'.
4. Have one or more groups explain their scheme, naming each group.
5. Allow the other children to question or challenge any placement of an animal or grouping that they think does not work. In challenging or replying to a challenge, ensure that they give their reasons using the word 'because'.
6. After any improvements have been made through discussion, have the children glue their groups of animals onto sheets of paper and help them to label each group in writing.

³ From Cam (1995, pp. 84-86)



Recalling Piaget's claim that operations belong to systems of transformations, let us look at the way in which the conceptual operations of classification and division are the starting-point for both *distinction-making* and the classical method of *definition*. I will begin with distinctions. In teaching students to make a distinction, I like to begin by having them identify the general kind to which the things in question belong—the basic move in classification. Having identified that as precisely as possible, I ask them to work out how best to distinguish the particular things of the kind in question from one another. Here I stress that they are not looking for incidental differences, but for the most distinctive or elementary differences.

The combination of these two operations helps students to see that distinction-making is related to division and classification. It involves a division of a more general kind under which the things in question can be classified. It is important to make these connections clear to students. Understanding that they are building on what are by now familiar basic operations will strengthen their conceptual capability.

Exercise: Drawing distinctions

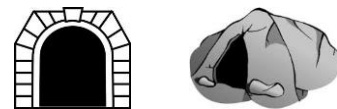
1. Begin by explaining what is required by way of an example. Suppose we were to ask our students to distinguish between mothers and fathers. First we ask: What are they both? The natural thing to say is that they are both parents. Then we ask: As parents, how do mothers and fathers differ? Here

we might point to all sorts of things, but the basic difference is that mothers are female parents, while fathers are male parents.

2. Now work with the class to make a distinction using one of the pairs below, beginning with the question 'What are they both?' before turning to the question 'How do they differ?' (It is best to begin with cases where we have a word for the general kind, as in *footwear* and *aircraft*.)
3. Have the class work in pairs to make a distinction using another pair. Remind them that they first need to identify what makes the two things the same kind of thing before they look for the difference between them.
4. Engage the class in discussion. Explore disagreements and encourage students to refine their suggestions. Use the opportunity to engage students in critical and creative thinking.
5. Continue in the same way for other examples, as time permits.



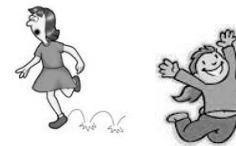
Slippers and shoes



A tunnel and a cave



A plane and a helicopter



Hopping and jumping

Finally, let us turn to the traditional way of constructing a definition that goes all the way back to the ancient Greek philosopher Aristotle. On Aristotle's way of doing things, definition involves identifying the general kind to which something belongs and the distinctive feature or features that mark it off from other things of that kind. Thus, a father can be defined as a male (distinctive feature) parent (general kind), while a mother is a female (distinctive feature) parent (general kind).

It is easy to see that this way of thinking about definition employs the operations of classification and division that we used in making distinctions. You classify whatever you are trying to define by first placing it under a more general kind and then using division to distinguish it from other things of that kind. So once students have satisfactorily made a distinction, you can have them construct a definition in the Aristotelian way, as in the following extension to the previous exercise:

6. Ask students to define any of the things they have adequately distinguished, such as *slippers*, *cave*, *helicopter* or *hopping*. They will need to combine the general kind with the distinctive feature, as in a cave is a *natural* (distinctive feature) *underground chamber* (general kind). You can have the class recall the two ingredients and then ask whether someone can put them together in a sentence, in this case beginning with 'A cave is ...'

I should not leave you with the impression that conceptual exploration is nothing more than a simple application of classification and division. Two and half thousand years ago, Socrates tried to discover criteria that are common to all cases of judging that something is good, just, brave or beautiful. In other words, he attempted to discover the nature of the general kind to which all things in such categories belong. Plato's *Dialogues* show Socrates striving but failing to reach this goal. This may be because there are no criteria that can be used to identify all the cases to which such complex and contestable concepts are applicable. As the 20th century philosopher Ludwig Wittgenstein has it, the criteria that govern the application of such concepts may vary from one case to another, the cases being related by nothing more than a family resemblance. The same caution applies to reasoning, which parallels mathematics in the complexity of its operations, as even an introductory textbook on formal logic will show. Nevertheless, complex reasoning and conceptualisation is complex in part because it involves more elaborate patterns of basic operations and not because it dispenses with them.

From a practical point of view, a great deal of work can be done by paying attention to these elementary logical and conceptual operations in the classroom. While I haven't the space to expand much beyond the things that I have already covered, even the briefest of further indications should suffice to make the point. As a simple illustration, consider comparisons exemplified by pairs such as before/after, younger/older, shorter/taller, heavier/lighter and hotter/colder. They are staples of the primary school classroom. So let us build on them. Any judgement employing one term allows a judgement in reverse using its opposite. If Jessica is taller than Jasmin, for example, then Jasmin is shorter than Jessica. If Jessica is lighter than Robert, then Robert is heavier than Jessica, and so on. The use of comparative terms provides ready opportunities for students to develop their powers of reasoning using basic operations.

Once you start looking at conceptualisation and reasoning in terms of basic operations, opportunities to employ them abound. Take the relationship between mutually exclusive categories in conceptualisation and reasoning with 'all' and 'none'. The conceptual claim that spiders are not insects, for instance, is equivalent to reasoning that if something is a spider, then it is not an insect. Coming back the other way, it follows that if something is an insect, then it is not a spider; or to put the matter conceptually, the class of insects does not include spiders. The logical point could equally well be put by saying that, as no

spiders are insects, so no insects are spiders—which has its conceptual equivalent in the claim that spiders and insects form mutually exclusive categories. There is no reason why students could not learn to move seamlessly between such logical and conceptual operations, which are certainly no more difficult than the mathematical operations they are expected to learn. Indeed, if we were as intent on teaching our students to be logically and conceptually literate as we are on ensuring that they are mathematically literate, then we would expend a commensurate effort in introducing students to logical and conceptual operations and in seeking to establish proficiency in their use.

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