

EVALUATION OF TIME DEPENDENT PRESTRESS LOSSES IN PRETENSIONED CONCRETE MEMBER WITH TOP AND BOTTOM TENDONS

Vadzim PARKHATS ^{a*}, Rafał KRZYWONÓ ^b

^a MSc; Silesian University of Technology, Faculty of Civil Engineering, Department of Structural Engineering,
Akademicka 5, 44-100 Gliwice, Poland

*E-mail address: vadzim.parkhats@polsl.pl

^b PhD; Silesian University of Technology, Faculty of Civil Engineering, Department of Structural Engineering,
Akademicka 5, 44-100 Gliwice, Poland

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Abstract

Prestress losses are usually calculated by bringing the tendons in the cross-section to the single resultant tendon. This solution is correct only for tendons concentrated in one part of the cross-section, while with their wider distribution, for example after the use of upper tendons, it may result in underestimation of losses in the lower tendons and overestimation in the upper tendons. The paper presents formulae to determine time dependent prestress losses separately for the top and bottom tendons of a pretensioned concrete member. Furthermore, variations of stress in the prestressing steel are analysed for two pretensioned concrete members to evaluate the possibility of using the equation (5.46) from the Eurocode 2 for the resultant tendon instead of the deduced formulae. The aim of this study is to show how great the error of estimated loss of prestressing force on the basis of several prestressed concrete sections can be.

Keywords: Creep; Prestress losses; Pretensioned concrete member; Relaxation; Shrinkage; Stress in prestressing reinforcement; Tendons.

1. INTRODUCTION

Prestress losses in a pretensioned concrete member are divided into two groups: immediate and time dependent. The immediate losses include losses due to elastic shortening of concrete, short term relaxation of prestressing tendons, increasing the temperature while curing concrete, friction at the deflecting devices, and seating of strand in the anchorage device. The time dependent losses are losses due to relaxation of prestressing steel, creep and shrinkage of concrete. Accurate evaluation of prestress losses in a pretensioned concrete member is highly important, since the incorrect results can lead to immoderate deflection and cracking of a prestressed structure [1, 2]. It should be noted that both underestimation and overestimation of prestress losses are almost equally harmful.

Current prediction methods to estimate prestress losses were developed in the seventies and presented in CEB-FIP Model Code 1978 [3], nevertheless there is still some uncertainty about this issue. The problem of evaluation of the time dependent prestress losses separately for top and bottom tendons of a pretensioned concrete member is omitted from the widely available literature. The widespread approach consisting in reduction of the cross-sections of the top and bottom tendons to the resultant tendon seems objectionable, since it does not solve the mentioned problem. In this paper, equations to estimate the time dependent prestress losses separately for the top and bottom tendons are deduced. Additionally, variations of stress in the prestressing steel are calculated for pretensioned concrete members with different depths of the cross-sections of the concrete, loads, and ratios of the top and

bottom tendons to analyse an error between the results according to the deduced equations and those obtained on the basis of the equation (5.46) from Eurocode 2 [4] for the common substitute cross-section of the prestressing reinforcement:

$$\Delta\sigma_{p,c+s+r}(t) = \frac{\varepsilon_{cs}(t, t_0)E_p + 0.8\Delta\sigma_{pr}(t) + \frac{E_p}{E_{cm}}\varphi(t, t_0)\sigma_c(t_0)}{1 + \frac{E_p}{E_{cm}}\frac{A_p}{A_{cs}}\left(1 + \frac{A_{cs}}{I_{cs}}z_{cp}^2\right)\left[1 + 0.8\varphi(t, t_0)\right]} \quad (1)$$

where:

$\Delta\sigma_{p,c+s+r}(t)$ (marked as $\Delta\sigma_p(t)$ in the rest of the paper) is the variation of stress in the prestressing steel at time t ;

A_p is the area of the cross-section of the prestressing steel;

$\varepsilon_{cs}(t, t_0)$ is the shrinkage strain at time t (the loading is applied at time t_0);

E_p is the modulus of elasticity for the prestressing steel;

$\Delta\sigma_{pr}(t)$ is the variation of stress in the tendons at time t due to relaxation of the prestressing steel (determined for the initial stress in the tendons due to initial prestress and quasi-permanent actions);

E_{cm} is the modulus of elasticity for the concrete;

$\varphi(t, t_0)$ is the creep coefficient at time t (the loading is applied at time t_0);

$\sigma_c(t_0)$ is the stress in the concrete at the time when the loading is applied t_0 ;

A_{cs} is the area of the transformed cross-section;

I_{cs} is the second moment of the area of the transformed cross-section;

z_{cp} is the distance between the centre of gravity of the transformed cross-section and the prestressing steel.

2. EVALUATION OF THE TIME DEPENDENT PRESTRESS LOSSES SEPARATELY FOR THE TOP AND BOTTOM TENDONS OF A PRETENSIONED CONCRETE MEMBER

The variation of strain of the concrete at time t is given by [5]

$$\begin{aligned} \Delta\varepsilon_c(t) = & \frac{\sigma_c(t_0)}{E_{cm}}\varphi(t, t_0) + \\ & + \frac{\Delta\sigma_c(t)}{E_{cm}}[1 + \chi\varphi(t, t_0)] + \varepsilon_{cs}(t, t_0) \end{aligned} \quad (2)$$

where:

$\Delta\sigma_c(t)$ is the variation of stress in the concrete at time t ;

χ is the aging coefficient [6, 7].

The variation of strain of the prestressing steel at time t is

$$\Delta\varepsilon_p(t) = \frac{\Delta\sigma_p(t) - \chi_r\Delta\sigma_{pr}(t)}{E_p} \quad (3)$$

where: χ_r is the reduced relaxation coefficient [8].

It is assumed that on the level of the centroid of the tendons the variation of strain of the concrete is equal to that of the prestressing steel, $\Delta\varepsilon_c(t) = \Delta\varepsilon_p(t)$, so the variation of stress in the prestressing steel at time t is given by

$$\begin{aligned} \Delta\sigma_p(t) = & \frac{E_p\sigma_c(t_0)}{E_{cm}}\varphi(t, t_0) + \\ & + \frac{E_p\Delta\sigma_c(t)}{E_{cm}}[1 + \chi\varphi(t, t_0)] + \varepsilon_{cs}(t, t_0)E_p + \chi_r\Delta\sigma_{pr} \end{aligned} \quad (4)$$

The equation (4) can be transformed to:

$$\begin{aligned} \Delta\sigma_p(t) \left[1 - \frac{E_p\Delta\sigma_c(t)}{E_{cm}\Delta\sigma_p(t)}[1 + \chi\varphi(t, t_0)] \right] = \\ = \frac{E_p\sigma_c(t_0)}{E_{cm}}\varphi(t, t_0) + \varepsilon_{cs}(t, t_0)E_p + \chi_r\Delta\sigma_{pr}(t) \end{aligned} \quad (5)$$

By the omitting the effect of relaxation of the prestressing steel, it might be assumed that a ratio of the variation of stress in the concrete to that in the prestressing steel is equal to a ratio of the stress in the concrete (on the level of the centroid of the tendons) $\Delta\sigma_{c,p}$ to that in the prestressing steel $\Delta\sigma_{p,p}$.

$$\frac{\Delta\sigma_c(t)}{\Delta\sigma_p(t)} \cong \frac{\sigma_{c,p}}{\sigma_{p,p}} \quad (6)$$

By using the equation (6), the equation (5) for the top tendons can be expressed as:

$$\begin{aligned} \Delta\sigma_{p,top}(t) \left[1 - \frac{E_p\sigma_{c,p,top}}{E_{cm}\sigma_{p,p,top}}[1 + \chi\varphi(t, t_0)] \right] = \\ = \frac{E_p\sigma_{c,top}(t_0)}{E_{cm}}\varphi(t, t_0) + \varepsilon_{cs}(t, t_0)E_p + \chi_r\Delta\sigma_{pr,top} \end{aligned} \quad (7)$$

For the bottom tendons:

$$\begin{aligned} \Delta\sigma_{p,bot}(t) \left[1 - \frac{E_p\sigma_{c,p,bot}}{E_{cm}\sigma_{p,p,bot}}[1 + \chi\varphi(t, t_0)] \right] = \\ = \frac{E_p\sigma_{c,bot}(t_0)}{E_{cm}}\varphi(t, t_0) + \varepsilon_{cs}(t, t_0)E_p + \chi_r\Delta\sigma_{pr,bot} \end{aligned} \quad (8)$$

It is known that the tensile stress in the prestressing steel causes the compressive stress in the concrete. The relationships between these stresses for the top and bottom tendons are given by

$$\sigma_{c,p,top} = - \left(\frac{A_{p,bot} \sigma_{p,p,bot} + \frac{A_{p,top} \sigma_{p,p,top}}{A_{cs}} + \frac{A_{p,bot} \sigma_{p,p,bot} z_{cp,bot} (-z_{cp,top})}{I_{cs}} + \frac{A_{p,top} \sigma_{p,p,top} (-z_{cp,top})^2}{I_{cs}} \right) \quad (9)$$

$$\sigma_{c,p,bot} = - \left(\frac{A_{p,bot} \sigma_{p,p,bot} + \frac{A_{p,top} \sigma_{p,p,top}}{A_{cs}} + \frac{A_{p,bot} \sigma_{p,p,bot} z_{cp,bot}^2}{I_{cs}} + \frac{A_{p,top} \sigma_{p,p,top} z_{cp,bot} (-z_{cp,top})}{I_{cs}} \right) \quad (10)$$

Let:

$$\vartheta_{bb} = \frac{A_{p,bot}}{A_{cs}} + \frac{A_{p,bot} z_{cp,bot}^2}{I_{cs}} \quad (11)$$

$$\vartheta_{bt} = \frac{A_{p,bot}}{A_{cs}} + \frac{A_{p,bot} z_{cp,bot} (-z_{cp,top})}{I_{cs}} \quad (12)$$

$$\vartheta_{tb} = \frac{A_{p,top}}{A_{cs}} + \frac{A_{p,top} z_{cp,bot} (-z_{cp,top})}{I_{cs}} \quad (13)$$

$$\vartheta_{tt} = \frac{A_{p,top}}{A_{cs}} + \frac{A_{p,top} (-z_{cp,top})^2}{I_{cs}} \quad (14)$$

Therefore, the equations (9) and (10) can be expressed as:

$$\frac{\sigma_{c,p,top}}{\sigma_{p,p,top}} = - \left(\vartheta_{tt} + \vartheta_{bt} \frac{\sigma_{p,p,bot}}{\sigma_{p,p,top}} \right) \quad (15)$$

$$\frac{\sigma_{c,p,bot}}{\sigma_{p,p,bot}} = - \left(\vartheta_{bb} + \vartheta_{tb} \frac{\sigma_{p,p,top}}{\sigma_{p,p,bot}} \right) \quad (16)$$

It is assumed that a ratio of the stresses in the top and bottom tendons stays the same after the time dependent prestress losses, $\sigma_{p,p,top} / \sigma_{p,p,bot} = \sigma_{pm0,top} / \sigma_{pm0,bot}$. By using the equations (7), (8), (15), and (16), the variations of stress in the top and bottom tendons at time t are given by

$$\Delta \sigma_{p,top}(t) = \frac{\frac{E_p}{E_{cm}} \sigma_{c,top}(t_0) \varphi(t, t_0) + \varepsilon_{cs}(t, t_0) E_p + \chi_r \Delta \sigma_{pr,top}(t)}{1 + \frac{E_p}{E_{cm}} \left(\vartheta_{tt} + \vartheta_{bt} \frac{\sigma_{pm0,bot}}{\sigma_{pm0,top}} \right) [1 + \chi \varphi(t, t_0)]} \quad (17)$$

$$\Delta \sigma_{p,bot}(t) = \frac{\frac{E_p}{E_{cm}} \sigma_{c,bot}(t_0) \varphi(t, t_0) + \varepsilon_{cs}(t, t_0) E_p + \chi_r \Delta \sigma_{pr,bot}(t)}{1 + \frac{E_p}{E_{cm}} \left(\vartheta_{bb} + \vartheta_{tb} \frac{\sigma_{pm0,top}}{\sigma_{pm0,bot}} \right) [1 + \chi \varphi(t, t_0)]} \quad (18)$$

According to the equation (5.46) from Eurocode 2 [4], it is assumed that the values of the aging coefficient χ and the relaxation reduction coefficient χ_r are 0.8. Thus, final equations to evaluate the time dependent prestress losses for the top and bottom tendons of a pretensioned concrete member can be expressed as

$$\begin{aligned} \Delta P_{top}(t) &= A_{p,top} \Delta \sigma_{p,top}(t) = \\ &= A_{p,top} \frac{\frac{E_p}{E_{cm}} \sigma_{c,top}(t_0) \varphi(t, t_0) + \varepsilon_{cs}(t, t_0) E_p + 0.8 \Delta \sigma_{pr,top}(t)}{1 + \frac{E_p}{E_{cm}} \left(\vartheta_{tt} + \vartheta_{bt} \frac{\sigma_{pm0,bot}}{\sigma_{pm0,top}} \right) [1 + 0.8 \varphi(t, t_0)]} \quad (19) \end{aligned}$$

$$\begin{aligned} \Delta P_{bot}(t) &= A_{p,bot} \Delta \sigma_{p,bot}(t) = \\ &= A_{p,bot} \frac{\frac{E_p}{E_{cm}} \sigma_{c,bot}(t_0) \varphi(t, t_0) + \varepsilon_{cs}(t, t_0) E_p + 0.8 \Delta \sigma_{pr,bot}(t)}{1 + \frac{E_p}{E_{cm}} \left(\vartheta_{bb} + \vartheta_{tb} \frac{\sigma_{pm0,top}}{\sigma_{pm0,bot}} \right) [1 + 0.8 \varphi(t, t_0)]} \quad (20) \end{aligned}$$

It should be noted that the stresses in the concrete, $\sigma_{c,top}(t_0)$ and $\sigma_{c,bot}(t_0)$, can be caused by initial prestress P_{m0} , quasi-permanent actions $\psi_2 Q$, and self-weight G . These actions are generally applied at different times, so proper values of the creep coefficient shall be used in the calculations of the prestress losses (19), (20):

$$\sigma_{c,top}(t_0) \varphi(t, t_0) = [\sigma_{c,G,top}(t_0) + \sigma_{c,pm0,top}(t_0)] \varphi(t, t_0) + \sigma_{c,Q,top}(t_1) \varphi(t, t_1) \quad (21)$$

$$\sigma_{c,bot}(t_0) \varphi(t, t_0) = [\sigma_{c,G,bot}(t_0) + \sigma_{c,pm0,bot}(t_0)] \varphi(t, t_0) + \sigma_{c,Q,bot}(t_1) \varphi(t, t_1) \quad (22)$$

where:

t_0 is the time when initial prestress is applied;

t_1 is the time when quasi-permanent actions are applied;

$\sigma_{c,G,top/bot}(t_0)$, $\sigma_{c,pm0,top/bot}(t_0)$, $\sigma_{c,Q,top/bot}(t_1)$ are the stresses in the concrete on the level of the centroid of the top/bottom tendons at times t_0 and t_1 due to self-weight, initial prestress, and quasi-permanent actions, respectively.

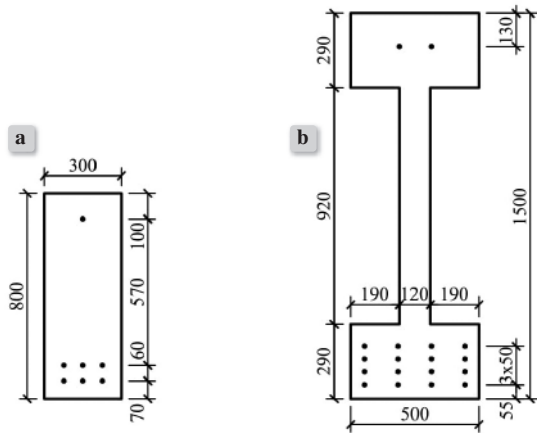


Figure 1.
Cross-sections of pretensioned concrete members: a) a rectangular beam, b) an I-shaped beam

3. ANALYSIS OF VARIATIONS OF STRESS IN THE PRESTRESSING STEEL OF PRETENSIONED CONCRETE MEMBERS

In this section, variations of stress in the prestressing steel of two pretensioned concrete members depending on their depths of the cross-sections of the concrete, loads, and ratios of the top and bottom tendons are presented. These variations found with the help of the equations (17) and (18) for the top and bottom tendons are compared with those calculated according to the equation (1) for the resultant tendon.

3.1. Parameters of analysed pretensioned concrete members

The cross-sections of two analysed pretensioned concrete members are shown in Fig. 1. The parameters of the members are presented in Table 1.

For both members:

- The prestressing strands Y1860S7-15.7, $E_p = 195$ GPa. The area of the cross-section of one tendon is 150 mm^2 . The class of relaxation – Class 2 (low relaxation). The stress in the prestressing steel immediately after transfer is 1395 MPa.
- The cement of Class N.
- The strength class of the concrete is C50/60.
- The density of the prestressed concrete is 25 kN/m^3 .
- The ambient relative humidity is 60%.
- The time when initial prestress is applied: $t_0 = 3$ days.

Table 1.
Parameters of the analysed members

	The rectangular beam	The I-shaped beam
The span L (m)	15	21
The load q (kN/m)	9	25
The perimeter of the member in contact with the atmosphere (m)	2.2	4.76

- The time when quasi-permanent actions are applied: $t_1 = 28$ days.

The presented cross-sections and parameters of the pretensioned concrete members are basic for the analysis. In the next section, the depth of the concrete cross-section, load, and ratios of the top and bottom tendons are changed to show differences in the predicted loss of stress in the prestressing steel according to the equations (1), (17), and (18).

3.2. Results and discussion

The results are presented in Fig. 2, 3, and 4. The vertical axes show the time dependent prestress losses ΔP in the per cent of initial stress P_{m0} , variations of stress in the prestressing steel $\Delta\sigma_p$, and ratios of variations of stress in the top and bottom tendons and the resultant tendon (the centre of gravity of the top and bottom tendons) to a variation of stress in the resultant tendon $\Delta\sigma_{p,resultant}$. It should be noted that creep nonlinearity is omitted from the calculations. Besides, the distance from the centroid of the top/bottom tendons to the top/bottom surface of a member remains constant.

The uniformly distributed load q is quasi-permanent and is applied to the top surface of a member. The effect of the factor ψ [9] on the value of the load is omitted from the analysis.

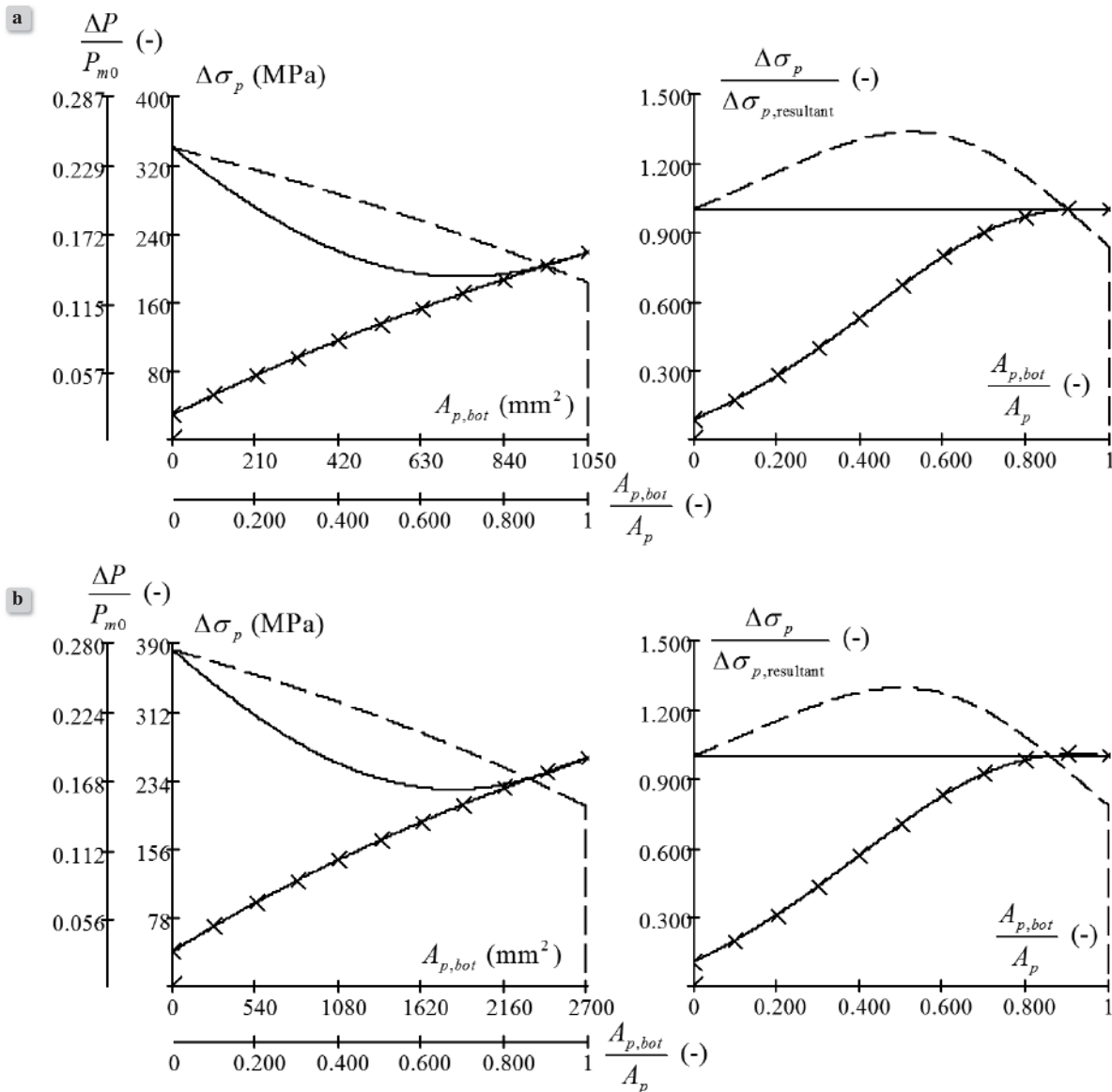


Figure 2.

Influence of the area of the cross-sections of the bottom tendons $A_{p,bot}$ on the variations of stress in the top tendons (the dashed line), the bottom tendons (the continuous line with crosses), and the resultant tendon (the continuous line) for: a) the rectangular beam, b) the I-shaped beam.

The most obvious is the influence of the distribution of the tendons in the cross-section. The greater the distance from the resultant tendon, the greater the error in the determined value of loss of prestressing could be expected. This is confirmed by the diagrams in Figure 2. The graphs are showing the full range of changes in the arrangement of the tendons. Limiting them to practical solutions, where usually the share of bottom tendons $A_{p,bot}$ exceeds 80%, a possible

error of the value of the deduced loss will be 8% for bottom tendons and 18% for top tendons. These values do not depend on the shape of the concrete cross-section.

The time dependent loss of the prestressing force is related not only to the value of the force with which the tendon has been tensioned, but also to the stress in the concrete surrounding the tendon. The top or bottom position of the tendon is very important here,

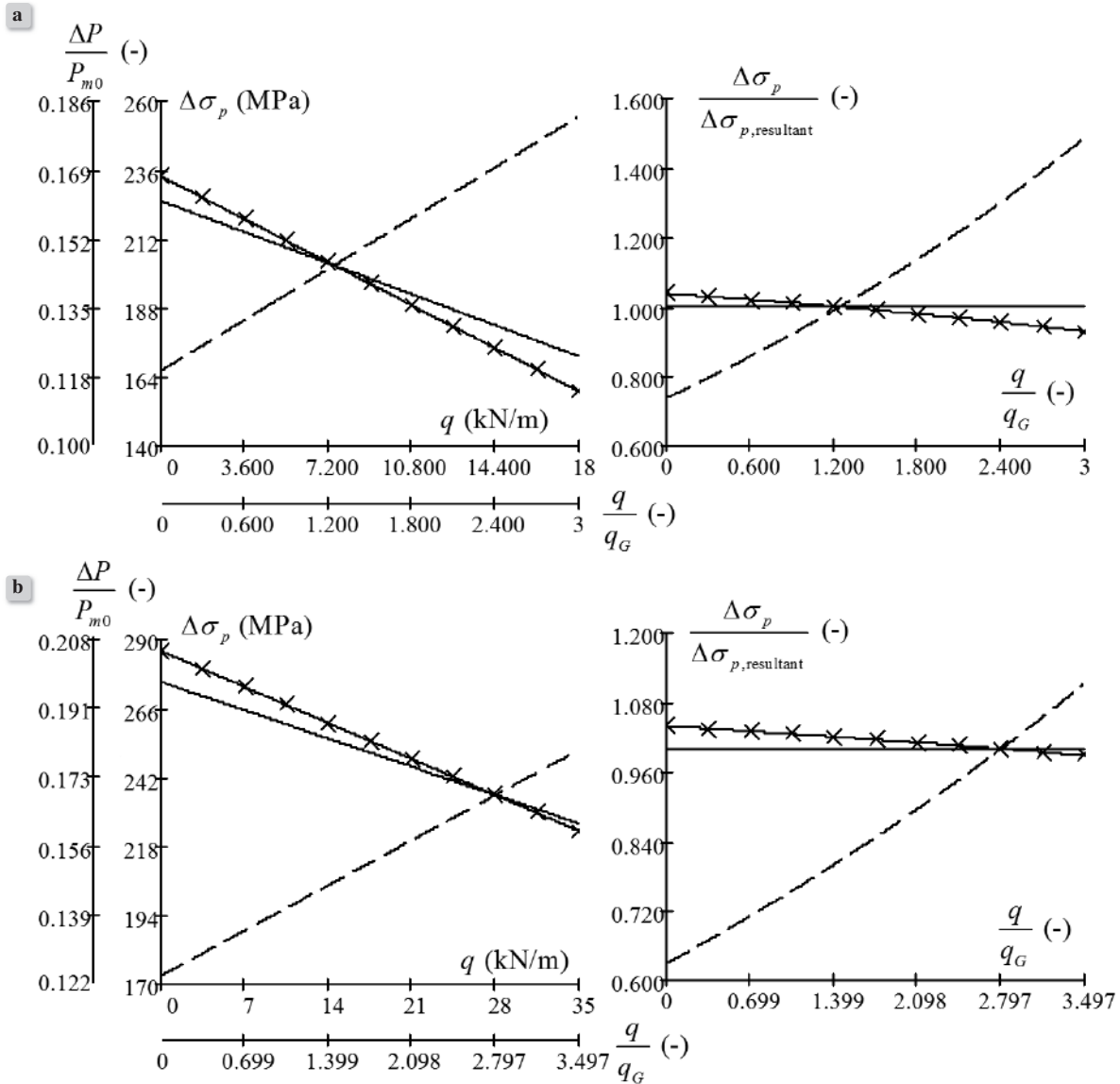


Figure 3.

Influence of the quasi-permanent load q on the variations of stress in the top tendons (the dashed line), the bottom tendons (the continuous line with crosses), and the resultant tendon (the continuous line) for: a) the rectangular beam, b) the I-shaped beam. q_G is the member weight.

as is the value of the external load bending the beam. This effect is shown in the graphs in Figure 3. It can be seen that a variation of stress in the top tendons is greater than that in the bottom ones.

This phenomenon is the effect of the type of stress around the tendons. The bottom tendons, similar to the resultant tendon, are present in the zone where tensile stresses from external loads occur, whereas the upper tendon usually appears in the compressive

stress zone. The area of the upper tendons is usually smaller, and therefore they are more susceptible to the impact of external loads.

Together with the span of a member, the bending moment caused by external load increases. The consequence is an error in calculated loss of prestressing force due to underestimation or overestimation of stresses in the concrete surrounding the tendon. Here also, due to the lower impact of top tendons on the

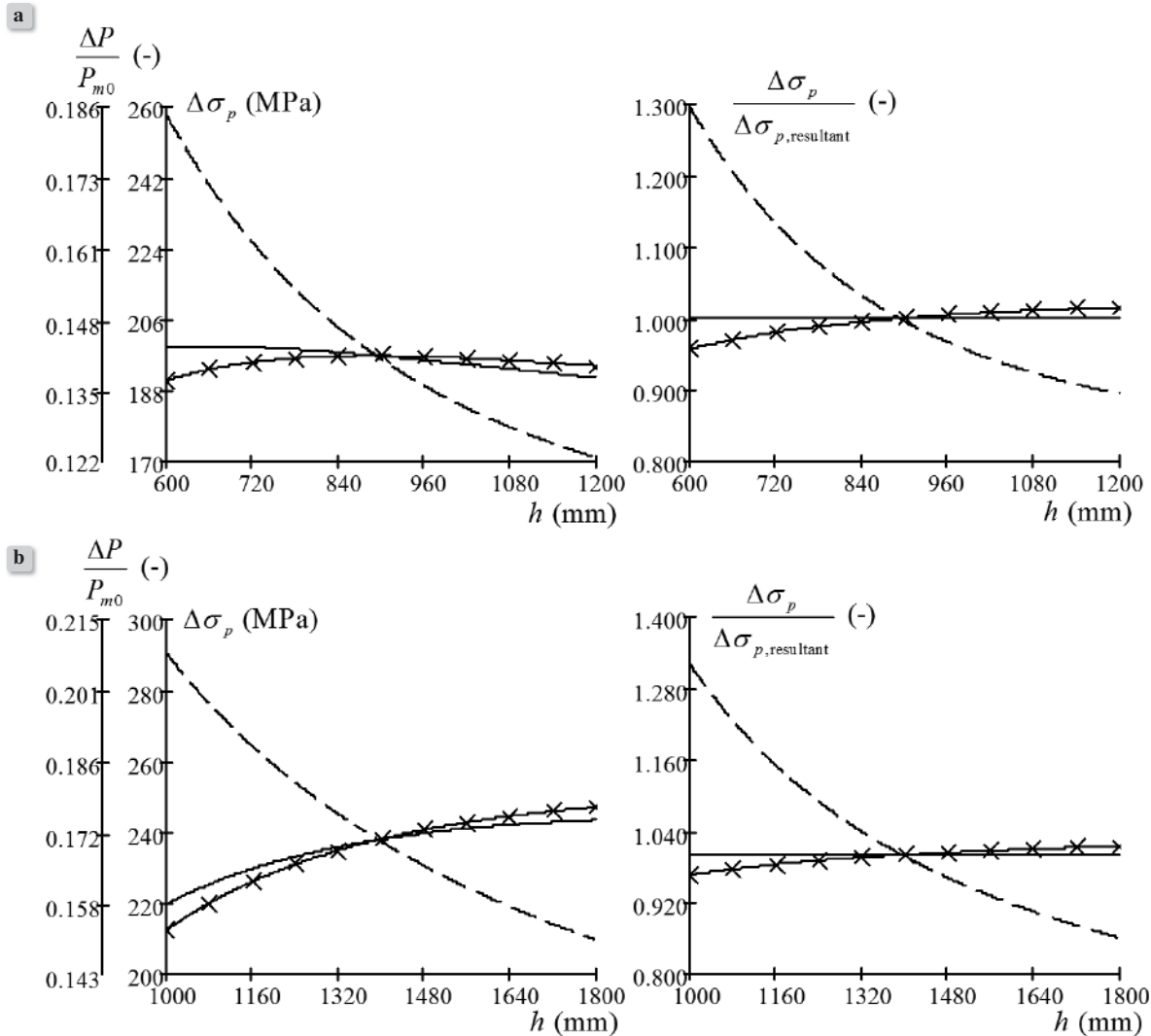


Figure 4.

Influence of the depth of the concrete cross-section h on the variations of stress in the top tendons (the dashed line), the bottom tendons (the continuous line with crosses), and the resultant tendon (the continuous line) for: a) the rectangular beam, b) the I-shaped beam

value of these stresses, the error of the calculated loss is greater.

The analysed pretensioned concrete members are similar to those that are used in the construction industry, therefore they are characterised by a great value of a ratio of the bottom tendons to the top ones. That is why the variations of stress in the bottom tendons are close to those in the resultant tendon in Fig. 3 and 4. If areas of the cross-sections of the top and bottom tendons are equal, it becomes clear that a value of a variation of stress in the resultant tendon found using the equation (1) is the average between variations of stress in the top and

bottom tendons calculated by the equations (17) and (18).

The presented in Fig. 2–4 results show that the error in estimating the value of the time dependent prestress losses, especially in the upper tendons, can reach up to several dozen percent in extreme situations. Thus, the equation (1) can't help to calculate variations of stress in the top and bottom tendons separately and the equations (17) and (18) should be used to do it.

4. CONCLUSIONS

In this paper, the equations (19) and (20) to evaluate the time dependent prestress losses separately for the top and bottom tendons are deduced. Moreover, the prestress losses are calculated for two pretensioned concrete members. The results are analysed and it is concluded that a variation of stress in the bottom tendons is greater than that in the top ones for pretensioned concrete members that are characterised by a great depth of the concrete cross-section, a small span, a great ratio of the bottom tendons to the top ones and are subjected to a small load. It should be noted that a shape of the concrete cross-section does not affect these patterns. It is established that the equation (5.46) from Eurocode 2 [4] for the resultant tendon can't be used to find the time dependent prestress losses separately for the top and bottom tendons. In the case of a change in a depth of the concrete cross-section, a span, and a load, the result of the equation (1) is the average between the values determined by the equations (17) and (18). In the case of a change in a ratio of the bottom tendons to the top ones, a variation of the stress found by the equation (1) changes parabolically.

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REFERENCES

- [1] Caro, L. A., Marti-Vargas, J. R., & Serna, P. (2013). Prestressed losses evaluation in prestressed concrete prismatic specimens. *Engineering Structures*, 48, 704–715.
- [2] Chandoga, M., Halvonik, J., & Pritula, A. (2009). Time depending deformation of pre-post tensioned beams. *Architecture Civil Engineering Environment*, 2(4), 51–60.
- [3] Comité Euro-International du Béton (CEB), Federation Internationale de la Précontrainte (FIP) (1978). Model Code for Concrete Structures. Paris: CEB, 348 pp.
- [4] EN 1992-1-1 (2004). Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings. Brussels: CEN.
- [5] Dyduch, K. (2015). Analiza opóźnionych strat sprężenia w konstrukcjach z betonu (Analysis of time-dependent prestress losses in concrete structures). In W. Derkowski, P. Gwoździwicz, M. Pańtak, W. Politalski, A. Seruga, & M. Surma (Eds.), *Konferencja Naukowo-Techniczna KS2015 Konstrukcje Sprężone*. Kraków.
- [6] Bazant Z. P. (1972). Prediction of Concrete Creep Effects Using Age-Adjusted Effective Modulus Method. *ACI Journal*, 69(4), 212–217.
- [7] Dilger W. H. (1982). Creep Analysis of Prestressed Concrete Structures Using Creep Transformed Section Properties. *PCI Journal*, 27(1), 89–117.
- [8] Ghali A., Trevino J. (1985). Relaxation of Steel in Prestressed Concrete. *PCI Journal*, 30(5), 82–94.
- [9] EN 1990:2002+A1 (2005). Eurocode – Basis of structural design. Brussels: CEN.