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Numerical Modeling of Hole Under Opposite Biaxial Loadings

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Abstract

The exact concentration of the stress generated by the presence of a cavity is a problem of great significance in Mining and Civil Engineering. An interesting stress concentration problem is the biaxial one. A numerical analysis of stress around a cylindrical hole in an infinite elastic medium under opposite biaxial loading was investigated. This far-field loading is equivalent to a pure shear loading on planes rotated 45°. Analysis consisted of two-dimensional finite-difference computations carried out with the Fast Lagrangian of Continua (FLAC) code. The Stress Concentration Factor (SFC) is evaluated numerically and compared with the existing solution. Predicted results of stress distribution around the hole were found in good agreement with the analytic theory.

Key words: numerical modeling, hole, biaxial opposite loading, shear, Stress Concentration Factor

1 Introduction

Holes appear with various shapes in many types of engineering, such as : aerospace, mechanical, and tunneling [1]. The determination of displacements and stresses around the cavities, tunnels and mining excavations is a problem of great magnitudes in Mining and Civil engineering. The exact concentration of the stress generated by the presence of a cavity is a problem of great significance [1]. Several works studied the distribution of stress around the hole [2, 3]. An interesting stress concentration problem is the biaxial one. If the far-field stress is biaxial with a tensile stress in the horizontal direction and compressive stress in the vertical direction; the loading, in this case, corresponds to a pure shear loading on planes rotated 45° [4].
In this purpose, a numerical analysis of stress distribution around a cylindrical hole in an infinite elastic medium under a biaxial opposite loading using FLAC code [5] is investigated. The usefulness of proposed results for the stress distribution is highlighted, and then the Stress Concentration Factor (SCF) is evaluated numerically and compared with the analytic existing result.

2 Analytic solution

2.1 Kirsch solution

The determination of displacements and stresses around a cylindrical cavity with a radius ‘*a*’ belonging to an infinite medium, isotropic, homogeneous with a linear elastic behavior was discussed firstly by Kirsch in 1898 [6].
Considering a point in polar coordinate (r,θ) near an opening radius a (Fig. 1) has stresses given by expressions [7] (Eq.1):

$$\begin{cases} \sigma_r = \dfrac{P_1+P_2}{2}\left[1-\dfrac{a^2}{r^2}\right]+\dfrac{P_1-P_2}{2}\left[1-\dfrac{4a^2}{r^2}+\dfrac{3a^4}{r^4}\right]\cos 2\theta \\ \sigma_\theta = \dfrac{P_1+P_2}{2}\left[1+\dfrac{a^2}{r^2}\right]-\dfrac{P_1-P_2}{2}\left[1+\dfrac{3a^4}{r^4}\right]\cos 2\theta \\ \tau_{r\theta} = -\dfrac{P_1-P_2}{2}\left[1+\dfrac{2a^2}{r^2}-\dfrac{3a^4}{r^4}\right]\sin 2\theta \end{cases} \tag{1}$$

Where:
σ_r , σ_θ and $\tau_{r\theta}$ are respectively the radial, hoop and tangential stresses.
P_1, P_2 the applied stress field respectively in the horizontal and vertical directions.
a: radius of the hole.
r, θ: the polar coordinates.

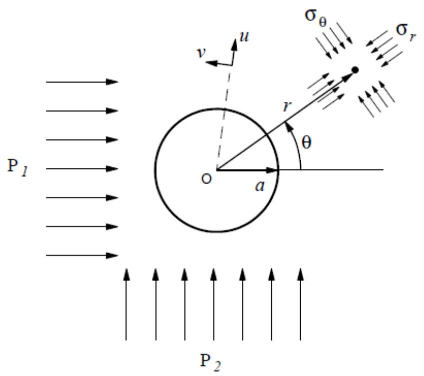


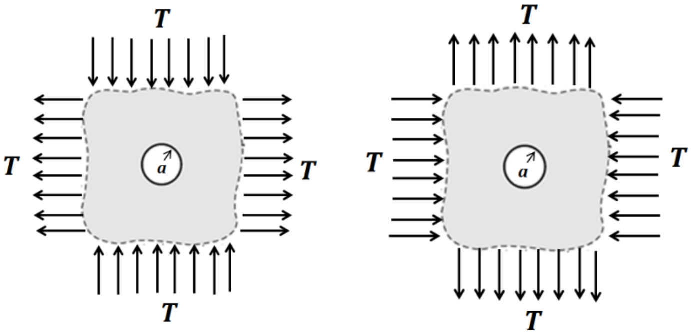
Figure 1: Stress around a cylindrical hole in an infinite elastic medium.

2.2 Hole under biaxial opposite loading

Considering a cylindrical hole subjected to a tensile and compressive stress in an infinite medium. Two cases of opposite loading can be obtained: the first one a tensile stress in the horizontal direction equal to T and compressive stress in the vertical direction equal to -T (Fig. 2a); and the second case corresponds to a compressive stress in the horizontal direction and a tensile stress in the vertical one (Fig. 2b). Stresses at the limits in both directions based on Eq. (1) are therefore equal to Eq. (2) and Eq. (3) for the first and the second cases respectively:

$$P_1 = +T \qquad P_2 = -T \tag{2}$$

$$P_1 = -T \qquad P_2 = +T \tag{3}$$



(a) Case 1 (b) Case 2

Figure 2: Cylindrical hole subjected to opposite biaxial loading

We obtain the stress field around the hole for the two cases by substituting the Eq. (2) and Eq. (3) into Eq. (1). So the expressions of stresses are given by Eq. (4) and Eq. (5) respectively:

$$\begin{cases} \sigma_r = +T \left[1 - \dfrac{4a^2}{r^2} + \dfrac{3a^4}{r^4}\right] \cos 2\theta \\ \sigma_\theta = -T \left[1 + \dfrac{3a^4}{r^4}\right] \cos 2\theta \\ \tau_{r\theta} = -T \left[1 + \dfrac{2a^2}{r^2} - \dfrac{3a^4}{r^4}\right] \sin 2\theta \end{cases} \tag{4}$$

$$\begin{cases} \sigma_r = -T \left[1 - \dfrac{4a^2}{r^2} + \dfrac{3a^4}{r^4}\right] \cos 2\theta \\ \sigma_\theta = +T \left[1 + \dfrac{3a^4}{r^4}\right] \cos 2\theta \\ \tau_{r\theta} = +T \left[1 + \dfrac{2a^2}{r^2} - \dfrac{3a^4}{r^4}\right] \sin 2\theta \end{cases} \tag{5}$$

The uniform field is disturbed by the presence of the hole. It is assumed that this disturbance is local, therefore the disturbing field will decay to zero as we move far away from the hole [4, 6].

2.3 Stress Concentration Factor

The influence of the presence of the hole is very local character, if the radius increases stress approaching very rapidly from the value imposed at the limits. The high concentration of stress observed near the hole is of great practical importance. As a practical example we can cite the case of holes in the decks of ships [7]. It is mentioned that deterioration of the resistance and premature failure of the structures due to fatigue cracking and plastic

deformation that occurs is caused by high concentration of stress at points having the highest Stress Concentration Factor [8].
The ratio between the maximum and the minimum stress is defined as Stress Concentration Factor. It only depends on the geometry, loading mode and the type of material chosen. It appears that the equal biaxial loadings enhance the local stress field, but the opposite biaxial one gives the highest concentration effect [4]. In order to confirm this high effect, the Stress Concentration Factor will estimated by the numerical model and the value will be compared with the analytical one.

3 Numerical analysis

The main objective of this part is a numerical modeling of the cases shown in Fig. 2 using the finite-difference element code FLAC^{2D} version 2007.
Fast **L**agrangian **A**nalysis of **C**ontinua FLAC^{2D} is a two dimensional explicit finite difference code for engineering mechanics computations which discretizes the behavior of structures built of soil, rock or other materials. A grid which is adjusted by the user is forming by elements or zones. The shape of the discretized material can be fitted by the user. Several stress/strain laws can be prescribed linear or nonlinear in response to the applied forces or boundary restraints [5].
Considering plane strain condition, this problem can be reduced to a bi-dimensional analysis. By reason for existing symmetry in the geometry view model and loading, only a quarter of the problem needs to be analyzed as shown in Fig. 3 which indicates the boundary conditions applied in our numerical model. The mesh must be refined near the hole for better precision in the area where the stresses and strains varied rapidly from one point to another [9].
A cylindrical hole with a radius a=1m exists in an infinite medium, isotropic, homogeneous and with elastic behavior. It is well noted as seen in Eq. (4) and Eq. (5) that the stresses around the hole does not depend on material's elastic properties. For this reason, the following material properties as example are assumed for the two cases: density ρ=2500kg/m^3 , shear modulus G=2.8GPa and Bulk modulus K=3.9GPa [5].

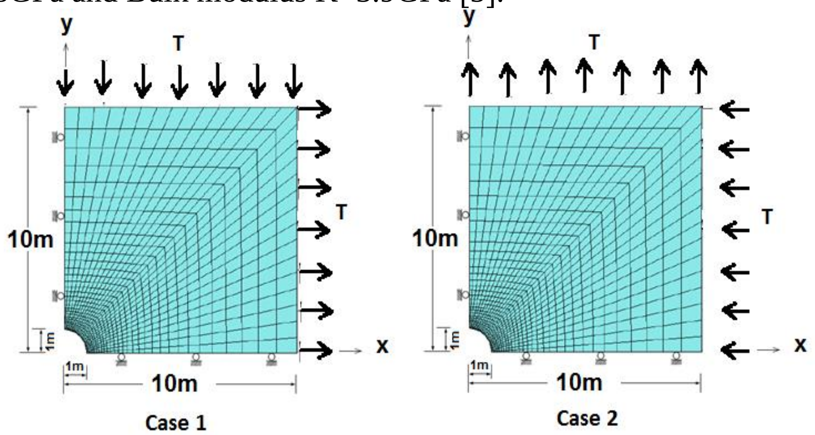


Figure 3: Cylindrical hole under opposite loading (boundary conditions)

The limits of this medium are subjected to a biaxial opposite loading, a tensile stress in the horizontal direction equal to T=30.10^6 MPa and a compressive stress in the vertical direction

equal to –T in the case 1.
But in the second case, we take a compressive stress in the horizontal direction equal to -T and tensile stress in the vertical direction equal to +T. The two cases of opposite biaxial loading considered in this analysis are shown in Fig. 4 and Fig. 5 respectively.

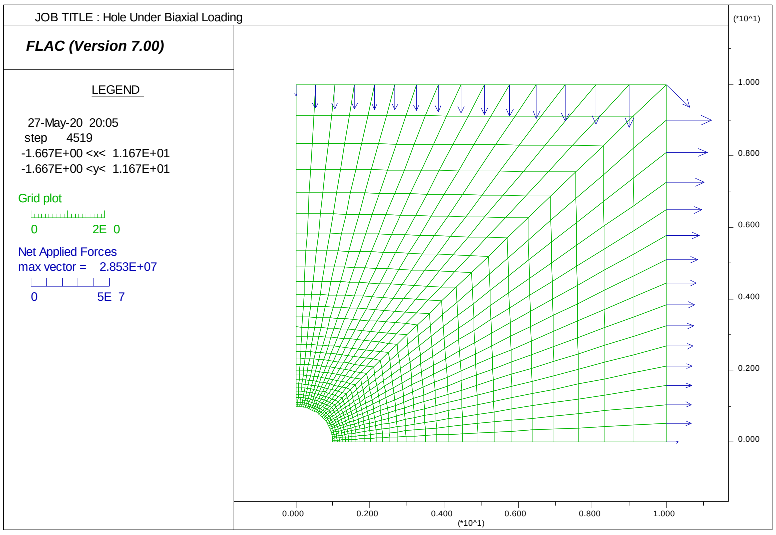


Figure 4: Applied loading, biaxial opposite loading: Case 1

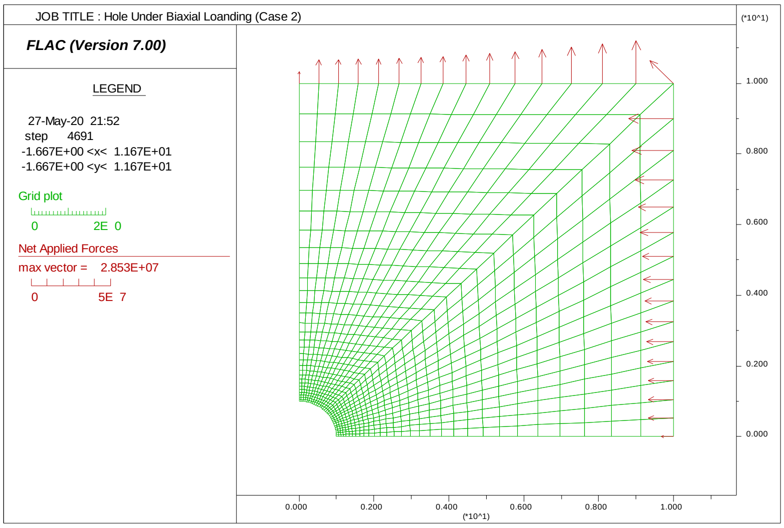
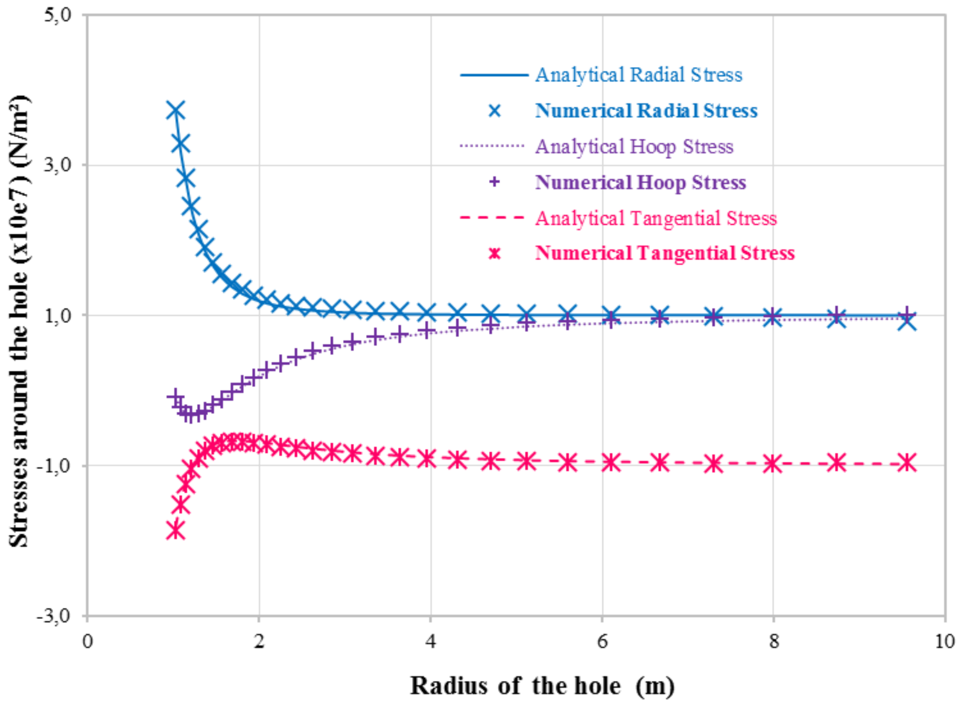


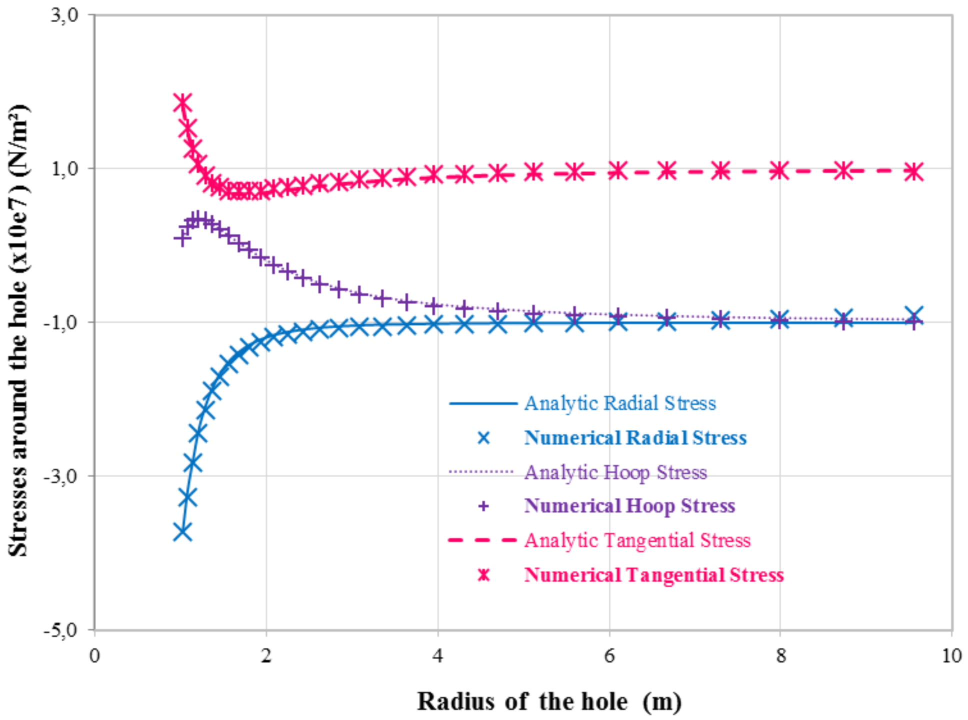
Figure 5: Applied loading, biaxial opposite loading: Case 2

4 Results and discussions

Stresses around the hole are evaluated numerically for the two cases in this study.
If $\theta=0$ and r varies from $a=1m$ to r equal to $10a$, the radial stress σ_r , Hoop stress σ_θ and maximum tangential stress $\tau_{r\theta}$ are calculated numerically for the two cases of opposite biaxial loading (Fig. 6) and then compared to those obtained analytically by the Eq. (4) and Eq. (5) respectively.



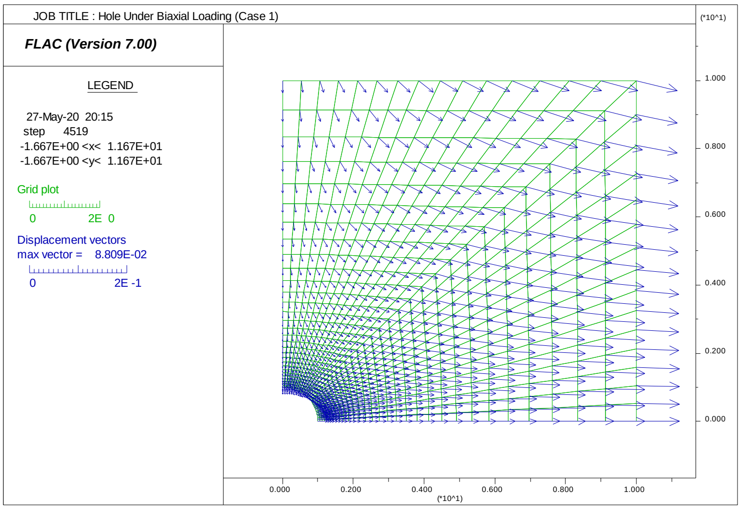
(a) Case 1



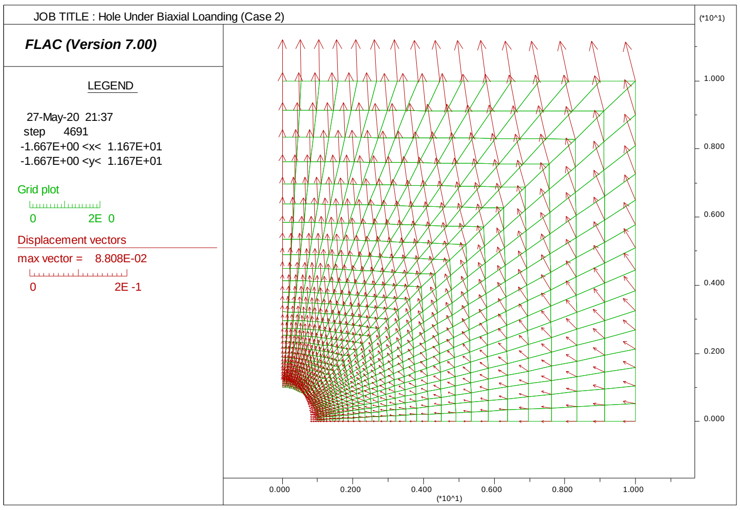
(b) Case 2

Figure 6: Numerical stresses around the cylindrical hole compared to analytic solution

As shown in Fig. 6, the stresses obtained numerically for the two cases (showed by cross points) are in very good agreement with those obtained analytically (showed by continuous line) by the formulas of Kirsch witch confirm the reliability of the numerical model.
The stresses to the inner edge of the hole (*r=a=1m*) are very large, they vary very quickly if we are moving away from the hole to stabilize at the end.
It is very clear from Fig. 6 that the two cases converge towards the same loading mode which will be confirmed and discussed in what follows.
The generated displacement field vectors during the application of the opposite biaxial loading imposed on the boundaries of the model obtained by FLAC for the two cases is shown in Fig. 7.



(a) Case 1

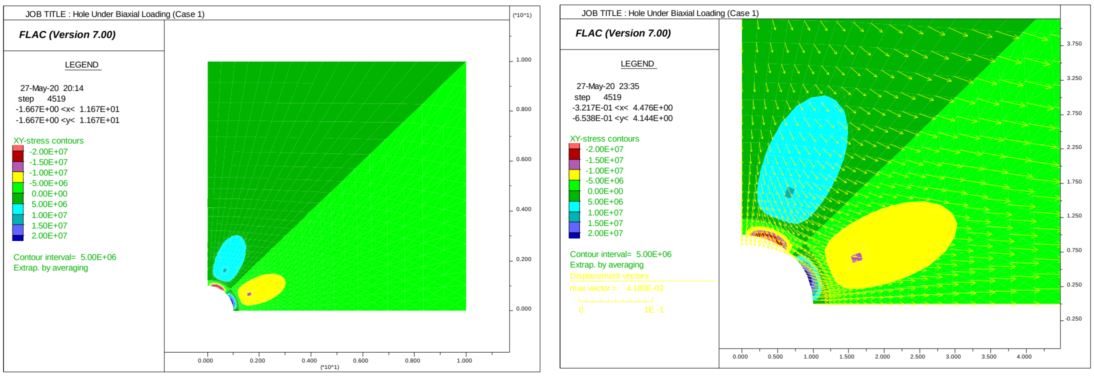


(b) Case 2

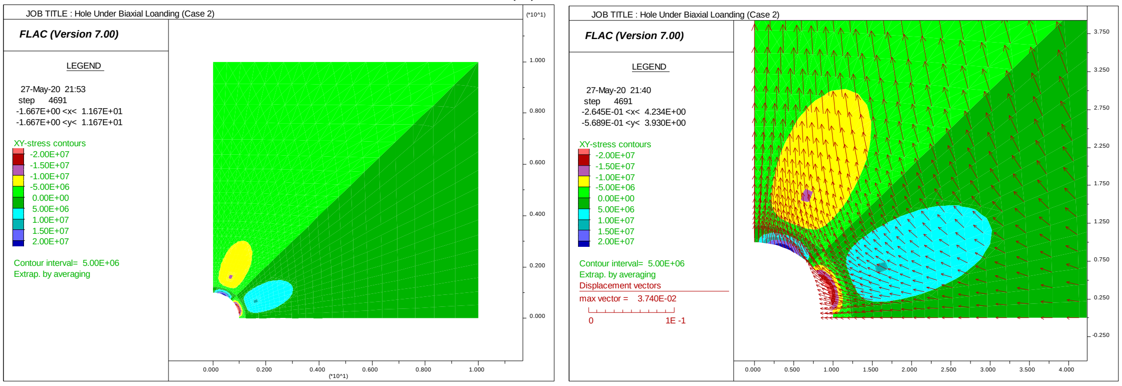
Figure 7: Displacement fields around the hole

As shown in Fig. 7 the medium tends to move from an angle equal to 45 degrees. This result obtained numerically confirms that these opposite biaxial loadings are equivalent to a pure

shear loading on planes rotated 45°.
The tangential stress and volumetric strain concentration are shown respectively in Fig. 8 and Fig. 9. We can observe easily the pure shear loading in the two cases.

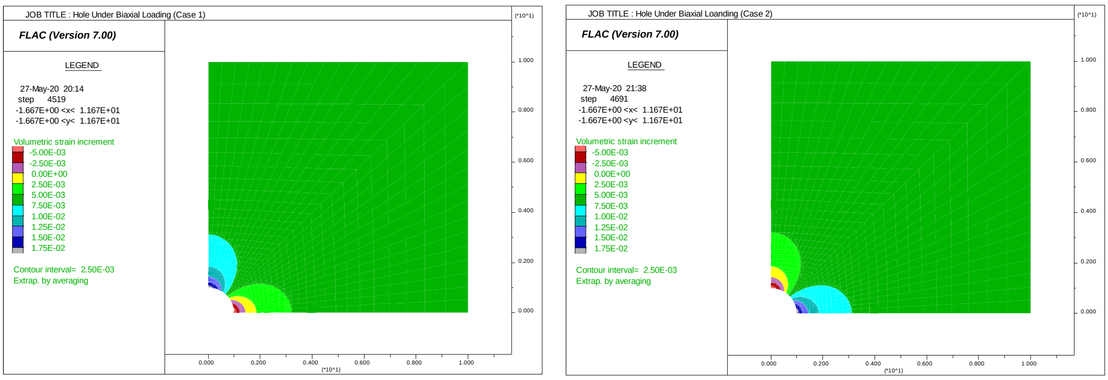


(a) Case 1



(b) Case 2

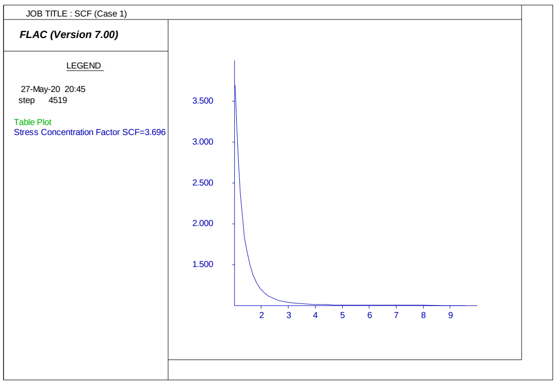
Figure 8: Tangential stress and displacement vectors around the hole



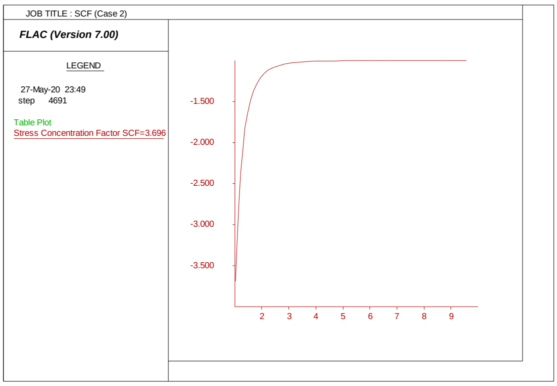
(a) Case 1 (b) Case 2

Figure 9: Volumetric strain around the hole

The Stress Concentration Factor (SCF) is calculated numerically for the two cases. This Stress Concentration Factor as a function of radius if the angle θ equal to 90° is shown in Fig. 10. The numerical result for Stress Concentration Factor (SCF) corresponding to this case is equal to 3.696.



(b) Case 1



(b) Case 2

Figure 10: Stress Concentration Factor (SCF) around the hole

Looking at Figs. 10a-b, we can observe if (*r/a=1*), the SCF is equal to 3.696 and then declines slowly with (*r/a*) to be stable at the end (equal to 1).
The analytic SCF is equal to 4 [4], so the error percentage between the numerical and analytical results is about 7.60%. This numerical result confirms the theory, thus giving the highest concentration effect (the analytic SCF is equal to 4).

5 Conclusion

A numerical evaluation of stresses, displacement and Stress Concentration Factor (SCF) has been confronted with the analytic solution.
Predicted numerical results of stress distribution and Stress Concentration Factor around the hole were found in good agreement with the analytic solution. From the obtained numerical predictions, the main findings are summarized below:
The biaxial opposite loading is equivalent to a pure shear around the hole.
The Stress Concentration Factor (SCF) for this case is equal to 3,696 which confirm the phenomenon of stress concentration around the hole.
This factor is higher than that obtained in the case of compression biaxial (1,948) and that obtained in the case of traction uniaxial traction (2,822) [10].
This numerical investigation is an intuitive appeal to the negative impact of the phenomena of stress concentration caused by discontinuities.

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