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An analytical model for predicting the natural frequency of retaining structures

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Abstract

The importance of the retaining structures is crucial in geotechnical engineering and the accurate determination of static and seismic earth pressures and natural frequency is important for study the dynamic behavior of these structures. Usually analytical formulas which do not consider the earth pressures behind retaining structure are used. An analytical model for predicting the natural frequency of retaining structures including the earth pressures by failure wedges is proposed in the present analysis. The model considers the effect of Coulomb and Mononobe Okabe failure wedges. Backfill material is considered in the analysis as cohesionless. The failure wedge is an important factor which should be considered in determining the natural frequency of retaining structures. As the weight of failure wedge increases the natural frequency decreases significantly. The current model is validated using several analytical models reported in the literature of the earlier researcher.

Key words: Analytical model, dynamic behavior, retaining structures, natural frequency, failure wedge.

1 Introduction

An adequate determination of the natural frequency of retaining structures plays a primordial role in the study of its dynamic behavior in earthquake prone regions. Many researchers have developed several methods to determine the natural frequency of retaining structures. In these methods, the natural frequency is often calculated by the elastic wave theory and based on two parameters namely height of the backfill and the soil shear wave velocity. The effect of the properties of earth pressures didn't get any attention till today. In the last few years, there has been increased interest in the study of the effects of soil on the natural frequency of structures: for free spanning offshore pipelines [1], for soil-foundation system [2] and for foundations of offshore wind turbines [3].

In this paper, a complete analysis is carried out to determine the natural frequency of retaining structures analytically in a more general way by introducing the Coulomb and Mononobe

Okabe active failure wedge. An analytical model is proposed to determine the natural frequency of retaining structures. This model estimates the natural frequency of wall-backfill soil system by using the Coulomb [4] and Mononobe Okabe [5, 6] to evaluate soil wedge created by active earth pressure behind retaining structures in both static and seismic cases. The validity of the proposed model will demonstrate by using results within the Scott [7] and Ghanbari et al [8] models.

2 Model formulation

2.1 General model description

The importance of the retaining structures is crucial in geotechnical engineering and the accurate determination of seismic earth pressures and natural frequency is important for study the dynamic behavior of these structures. In the most previous investigations, the soil backfill was considered sufficient width as rectangular wedge whereas in the real, the backfill behind retaining structures repartitioned as triangular wedge. Rectifying this weakness, this study proposed an analytical model for calculating the natural frequency of retaining structures by considering the triangular active static and seismic failure wedges.
This work is first include the active failure wedges in determination of natural frequency of retaining structures and proposed a new model; a complete study has been carried out to determine the natural frequency in a more general way.
It is possible and common to obtain equivalent natural frequency for a backfill soil-retaining structure system from results of analytical analysis assuming the system has a single degree of freedom.
The normalized form of first vibration mode in a retaining wall without considering failure wedge has been used as the shape function for determining the natural frequency of retaining wall with considering failure wedges.

2.2 Mathematical Formulation of the Problem

The proposed retaining structure model is illustrated schematically in Fig. 1 for static case and under seismic pseudo static conditions, and it is defined by (a) structure’s characteristics: weight per unit length, modulus of elasticity, thickness and height (b) backfill soil characteristics: Poisson’s ratio, soil friction angle, Young modulus and weight per unit mass and (c) seismic excitation characteristics: horizontal and vertical seismic accelerations.

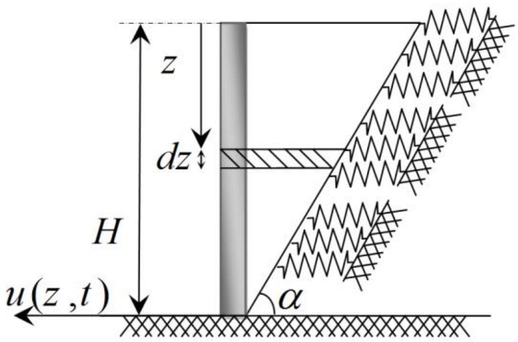

Figure 1: Description of model

In view of analytical studies, this model is designed to be as simple as possible, including Coulomb and Mononobe Okabe active failure wedges. More precisely, figure 2 represent the dynamic equilibrium and resultant forces acting on differential element of the modeled system. The following assumptions are adopted:

- the retaining wall behaves as a flexible structure [9];
- the static thrust is evaluated by Coulomb method [4];
- the seismic thrust is evaluated by Mononobe Okabe method [5,6];
- the inclined failure plane is modeled as a series of Winkler's springs [10];
- the natural frequency of wall-backfill soil system is estimated by Rayleigh method [11];and
- the backfill soil is assumed dry and cohesionless.

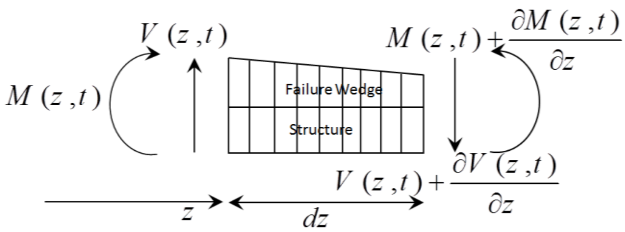

Figure 2: Dynamic equilibrium and resultant forces acting on differential element of the modeled system.

The kinetic energy can be calculated by integrating the mass through height of the retaining structure with failure wedge model and the potential energy by integrating the stiffness through the height of the model.
The kinetic energy in the system can be written as follows:

$$T_{\max} = \frac{1}{2}\int_0^H m(z)\left(\frac{\partial u}{\partial t}\right)^2 dz = \frac{\omega^2}{2}\int_0^H m(z)U^2(z)dz \tag{1}$$

The total potential energy, V, in the structure and backfill is written as follows:

$$V_{\max} = \frac{1}{2}\int_0^H E_m I(z)\left(\frac{\partial^2 U(z)}{\partial z}\right)^2 dz + \frac{1}{2}\int_0^H kU^2(z)dz \tag{2}$$

In this model, the mass of a thin element of thickness dz ‘soil & structure’ at depth z is given by:

$$m(z) = \left(\rho_{soil}.\frac{H-z}{\tan\alpha} + \rho_{str}.e\right)dz \tag{3}$$

The inertial moment of model can be expressed as,

$$I(z) = \frac{1}{12}\left(\frac{H-z}{\tan\alpha} + e\right)^3 \tag{4}$$

The equivalent elastic soil-structure modulus of model E_m may be calculated as follows:

$$E_m(z) = \frac{E_{soil}E_{str}(H-z+e)}{(H-z)E_{str} + e.E_{soil}} \tag{5}$$

The circular natural frequency, (ω) is defined as,

$$\omega^2 = \frac{\int_0^H E_m I(z)\left(\frac{d^2U(z)}{dz^2}\right)^2 dz + \int_0^H kU^2(z)dz}{\int_0^H m(z)U^2(z)dz} \tag{6}$$

In equation (6), the stiffness per unit length of Winkler’s spring (k) can be computed using subgrade reaction modulus (K) by Galin expression [8].

2.3 Solution for the retaining structure including soil failure wedge

Similar to the analysis of [8] using the Rayleigh’s quotient method, the natural circular frequency of retaining structures can be obtained in the following manner.
The extension of the method for including failure wedge is not presented. A detailed methodology of calcul is already reported in [8].
Combining Eqs. (3), (4), (5) and (6), and integrating over the height of the wall, it is straightforward to show that the first circular natural frequency (ω) of system is given by:

$$\omega^2 = \frac{\frac{E_m}{12}\left[3.11\left(\frac{e}{H}\right)^3 + \frac{7.54}{\tan(\alpha)}\left(\frac{e}{H}\right)^2 + \frac{6.31}{\tan^2(\alpha)}\frac{e}{H} + \frac{1.80}{\tan^3(\alpha)}\right] + \frac{Hk}{4}}{\frac{0.049}{\tan(\alpha)}\rho_{soil}.H^2 + 0.252\rho_{str}eH} \tag{7}$$

Equation (7) takes into account changes in soil wedge geometry and was used in the analysis given below.
From this equation (7), it is seen that ω is function of the retaining structure parameters, backfill soil parameters and failure wedge behind retaining structure.

3 Validation and comparison with available solutions

To verify the validity of the proposed model, the calculated natural frequencies were compared with those reported in literature ‘Scott’ [7] and ‘Ghanbari et al.’ [8].
For different values of wall height (Fig.3.a) and backfill friction angle (Fig.3.b), natural angular frequency associated with Coulomb and Mononobe Okabe failure wedges models computed in the present study is located between that of ‘Scott’ [7] and ‘Ghanbari et al’ [8] methods with υ_{soil}=0.2, E_{soil}=15 MPa, E_{str}=26 GPa, ρ_{soil}=1900 kg/m^3, ρ_{str}=2320 kg/m^3, H=7 m and a_h=0.3, a_v=0.5a_h.

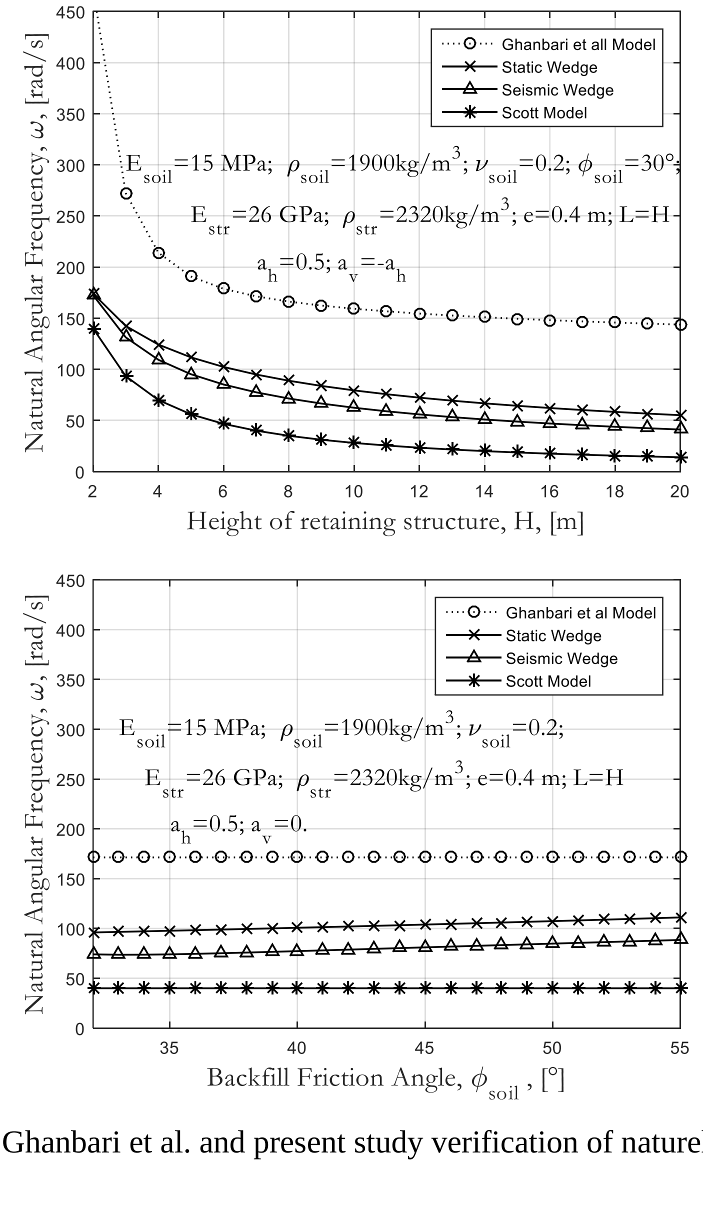

Figure 3: Scott, Ghanbari et al. and present study verification of naturel angular frequency

Scott (1973) [7] and Ghanbari et al. (2013) [8] methods do not consider the effect of backfill friction angle on the natural frequency of retaining structures.

4 Conclusion

In this research, by considering the earth pressures of the soil backfill behind retaining structures the natural frequency of retaining structures obtained. An analytical model using the Coulomb and Mononobe-Okabe failure wedges, to determine the natural frequency of retaining structures, is developed. In addition to the consideration of horizontal and vertical seismic acceleration coefficients, height and thickness of structure, Young modulus and density of the soil and the structure material and also Poisson ratio and friction angle of soil backfill are considered. Both static and seismic active thrusts are considered and it is found that values of natural frequency decreases significantly with the increase in the values of failure wedges.

The consideration of failure wedge plays an important role in determining the natural frequency of retaining structures, each variation of soil backfill and seismic accelerations changed the system and naturally the natural frequency.

The proposed model is reliable and is expected to give useful results for retaining structures.

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