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FLUX WEAKENING OPTIMAL CONTROL OF THE THREE-PHASE INDUCTION MOTOR

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Abstract. ***This paper illustrates the importance of control on energy saving opportunity in electrical drives systems. The paper's objective is to develop an optimal control law for a variable flux drive systems with induction machines. The optimal control provides dynamic regimes with minimum input energy and performs the minimization of the copper losses from induction motor. The solution of the problem uses optimal control theory and the numerical integration of the matrix Riccati differential equation. The synthesis of the optimal energetic control law is accomplished by comparing, via simulation, the two control methods: conventional and optimal.***

Keywords: optimal control, matrix Riccati differential equation, electrical drive, induction motor, flux weakening

1. INTRODUCTION

Electromechanical conversion systems, electrical drives, are very common today, from tens of watts in power electronics applications, up to tens of megawatts of power in industrial applications.

These systems are characterized by yields of high conversion 0.9 ... 0.98, at the steady-state operation and loads around the nominal value, while in the dynamic regimes of start or stop this dramatic decrease in the values of either 0 4 .... 0.6. It results the necessity to investigate the electromechanical conversion systems by formulating of optimal energetic control problem, minimizing the energy absorbed for dynamical regimes of start, stop and reverse. One such problem was formulated and solved for engine D.C. with constant flow and rotor control [1] and three-phase induction machine operating at constant flux, [2]. In both cases they were declared significant cuts in energies entered in the system, by 6% to 26%, due to the reduction of energy losses components in the car and energy transferred to the working machines.

The latest optimal control approaches were based on differential evolution [3], the "sliding mode" of sensorless drives [4] on adjusting the position [5] on an adaptive system [6].

Next we developed a optimal control structure for the particular case of drives with three-phase AC machines operating at variable flow.

2. PROBLEM FORMULATION

By considering an electromechanical conversion systems with three-phase induction motor described by 4th order nonlinear model, equation (1)

$$\begin{aligned}
\frac{dx_1}{dt} &= -k_1x_1+k_1x_3 \\
\frac{dx_2}{dt} &= k_2x_1x_4-k_3x_2-k_4m_s \\
\frac{dx_3}{dt} &= -k_5x_3+k_6x_1+k_7x_2x_4+k_8\frac{x_4^2}{x_1}+k_9u_d \\
\frac{dx_4}{dt} &= -k_{10}x_4+k_{11}x_1x_2+k_{12}x_2x_3+k_{13}\frac{x_3x_4}{x_1}+k_{14}u_q
\end{aligned} \tag{1}$$

where:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} i_{m2} \\ \omega \\ i_{d1} \\ i_{q1} \end{bmatrix} \tag{2}$$

are the state quantities in the following order: magnetizing rotor current, angular velocity of the machine, d-q frame stator current components.

To use previous developments, including problem management solution synthesized optimal linear quadratic [1], the system (1) needs to be linearized. Linearization technique adopted by reaction exact linearization [7], was developed in [8], [9]. Achievement of the conditions of existence of the linearized system and adjustment needs require accomplishment of a MIMO system with two outputs.

$$H=\begin{bmatrix} h_1 \\ h_1 \end{bmatrix} = \begin{bmatrix} i_{m2} \\ \omega \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{3}$$

and two commands

$$v=\begin{bmatrix} v_d \\ v_q \end{bmatrix} \tag{4}$$

It is noted by

$$z=\begin{bmatrix} z_1\\ z_2\\ z_3\\ z_4\end{bmatrix}=\begin{bmatrix} x_1\\ k_1x_3-k_1x_1\\ x_2\\ k_2x_1x_4-k_3x_2-k_4m_s\end{bmatrix} \tag{5}$$

direct conversion of x-z coordinates, obtained through derivatives Lie, and by

$$x=\begin{bmatrix} z_1\\ z_2\\ z_3\\ z_4\end{bmatrix}=\begin{bmatrix} z_1\\ z_3\\ (z_2+k_1z_1)/k_1\\ (z_4+k_3z_3+k_4m_s)/(k_2z_1)\end{bmatrix} \tag{6}$$

the reverse transformation.

Linearized closed loop system has the form:

$$\begin{bmatrix} \dot z_1\\ \dot z_2\\ \dot z_3\\ \dot z_4\end{bmatrix}=\begin{bmatrix} z_2\\ v_d\\ z_4\\ v_q\end{bmatrix} \tag{7}$$

where v_d, v_q is the new linearized command and the equations 2 and 4 of (7) have the form

$$\begin{bmatrix} \dot z_2\\ \dot z_4\end{bmatrix}=\begin{bmatrix} L_1\\ L_2\end{bmatrix}+\begin{bmatrix} Lg_1 & Lg_2\\ Lg_3 & Lg_4\end{bmatrix}\begin{bmatrix} u_d\\ u_q\end{bmatrix} \tag{8}$$

With the above notations the system (8) becomes

$$\begin{gathered} \dot z_1=z_2\\ \dot z_2=az_1+bz_2+L_{12}+Lg_1u_d+Lg_2u_q\\ \dot z_3=z_4\\ \dot z_4=cz_3+dz_4+L_{22}+Lg_3u_d+Lg_4u_d\text{-}L_{23}m_s \end{gathered} \tag{9}$$

with the obvious notation

$$\begin{gathered} a=k_1k_6-k_1k_5\\ b=-k_1-k_5\\ c=-(k_1+k_{10})k_3\\ d=-(k_1+k_3+k_{10}) \end{gathered} \tag{10}$$

It is defined by so-called decoupling matrix by

$$L_d=\begin{bmatrix} Lg_1 & Lg_2\\ Lg_3 & Lg_4\end{bmatrix}=\begin{bmatrix} k_1k_9 & 0\\ 0 & k_2k_{14}x_1\end{bmatrix} \tag{11}$$

The decoupling matrix is non-singular in all cases where the magnetizing rotor current, x1, is zero, which is true for any operating regime.

Using the above notation system (9) is rewritten as

$$\begin{bmatrix} \dot z_1\\ \dot z_2\\ \dot z_3\\ \dot z_4\end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 & 0\\ a & b & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & c & d\end{bmatrix}\begin{bmatrix} z_1\\ z_2\\ z_3\\ z_4\end{bmatrix}+$$
$$+\begin{bmatrix} 0\\ L_{12}\\ 0\\ L_{21}\end{bmatrix}\begin{bmatrix} 0 & 0\\ 1 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix} Lg_1 & Lg_2\\ Lg_3 & Lg_4\end{bmatrix}\begin{bmatrix} u_d\\ u_q\end{bmatrix}+\begin{bmatrix} 0\\ 0\\ 0\\ L_{23}\end{bmatrix}[m_s] \tag{12}$$

System (12) has the compact form

$$\dot Z(t)=Az(t)+BL_du(t)+Gw(t)+L_f \tag{12}$$

Where:

- A is the system matrix,
- B is the control matrix,
- Ld is matrix who accumulate totaling nonlinear components
- G is the perturbation matrix
- U is the control vector
- W is the magnitude disturbing vector.

The system (12) need to be placed under the standard form

$$\dot Z(t)=Az(t)+Bv(t)+Gw(t) \tag{13}$$

where is the command of linearized system as a result of the control strategy adopted, while it is necessary control for the real system. Comparing equations (12) and (13) gives

$$L_du(t)+L_f=v(t) \tag{14}$$

and real command

$$u(t)=L_d^{-1}[-L_f+v(t)] \tag{15}$$

In view of equation (7) of the definition of transform x - z and the system (12), the state has the form

$$\begin{bmatrix} z_1(t)\\ z_2(t)\\ z_3(t)\\ z_4(t)\end{bmatrix}=\begin{bmatrix} i_m(t)\\ \frac{di_m(t)}{dt}\\ \omega(t)\\ \frac{d\omega(t)}{dt}\end{bmatrix} \tag{16}$$

It is inserted the function criterion, the functional by the minimized form

$$J=\frac{1}{2}\left[z(t_1)-z_f\right]^T S[z(t_1)\text{-}z_f]+\frac{1}{2}\int_{t_0}^{t_1}[z^T(t)Q_z(t)+$$
$$+v^T(t)R_v(t)]dt \tag{17}$$

where:

- is reached final state of the system;
- is the desired end state;
- are the constant matrices of appropriate size.

The first algebraic term from the criterion function, called terminal cost, penalizes the system for misconduct of achieving the desired end state during the final time. Thus, if the following matrix is adopted

$$S=\begin{bmatrix} s_1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & s_3 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \tag{18}$$

And

$$z_f^T = [i_{mf}\ \ 0\ \ \omega_f\ \ 0] \tag{19}$$

The terminal cost

$$\lambda = \frac{1}{2} s_1 [i_m(t_1) - i_{mf}]^2 + \frac{1}{2} s_3 [\omega(t_1) - \omega_f]^2 \tag{20}$$

minimizes the mean square error of achieving final speed and magnetizing current. At first glance it would be necessary only the second term, speed is the final condition imposed. In reality, since the two channels of adjustment are coupled it is necessary first term too.

Additionally will be necessary to correlate the final values impose for speed and the magnetization current.

The first integral term is considered in order to reduce the energy absorbed by the machine during the dynamic regime. Therefore, the weighting matrix is adopted by

$$Q = \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

By introducing the Q matrix in the functional cost the following equation is deducted

$$J_1 = \frac{1}{2} \int_{t_0}^{t_1} q_1 i_m^2(t) + \frac{1}{2} \int_{t_0}^{t_1} q_3 \omega^2(t) dt \tag{22}$$

where the first term, by form $q_1 i_m^2$, aims to reduce the energy dissipated in copper machine by magnetizing current component.

The second term has in view to minimize the loss proportional to the square of the angular speed (such as viscous friction and partly by the ferromagnetic cores). So this term of the criterion function has complex significance while minimizing energy absorbed, especially on account of variation form the trajectory of speed.

Finally, the last term of the functional index, by adopting the weighting matrix form as

$$R = \begin{bmatrix} r_1 & 0 \\ 0 & r_1 \end{bmatrix} \tag{23}$$

gain the following expression

$$J_2 = \frac{1}{2} \int_{t_0}^{t_1} r_1 v_d^2(t) dt + \frac{1}{2} \int_{t_0}^{t_1} r_3 v_q^2(t) dt \tag{24}$$

By minimizing the term (24) means in fact, keeping order

$$v = \begin{bmatrix} v_d \\ v_q \end{bmatrix} \tag{25}$$

respectively supply voltages, relationship (15), within reasonable limits.

3. THE SOLUTION OF THE OPTIMAL CONTROL PROBLEM

The solution of the problem exists and it is unique if the system (12) is controllable and completely observable, and the weighting matrix carry out the conditions Q ≥ 0, S ≥ 0 and R > 0 [10].

In [1], [2], [10], [11] was proposed a non-recursive solution that can be calculated at any time, if state and disturbance are known at the time of calculation [11].

The solution of canonical system, and therefore the optimal control problem, is given by

$$v^*(t) = -R^{-1}B^T P(t_1 - t) z(t) - R^{-1}B^T [P(t_1 - t)\Phi_{12}(t_1 - t) - \Phi_{22}(t_1 - t)] S z_f - R^{-1}B^T [P(t_1 - t)\Omega_1(t_1 - t) - \Omega_2(t_1 - t)] \tag{26}$$

where **P** is the solution of differential Riccati matrix equation; $\boldsymbol{\Phi}_{12}$, $\boldsymbol{\Phi}_{22}$ is partitioning transition matrix of the canonical system and $\boldsymbol{\Omega}_{12}$, $\boldsymbol{\Omega}_{22}$ are matrices determined from the transversality condition.

The solution (25) is non recursive and can be calculated at any time t, if state and disturbance are known. Obviously status is known, while the disturbance can be measured or estimated by various methods [12]. Certainly solution (26) will be implemented digital using a sampling period of low value, less than the time constants of the converter. As a result, the perturbation measured or estimated in the sampling period can be considered constant during one sample time. In this way, the next step of the integration can be done based on the final value.

The solution (26) consists of three components:
- the state feedback - z(t);
- the desired final state - z_f;
- the feedforward compensation of the load disturbance.

4. SIMULATION RESULTS

The solution was custom developed above for an electromechanical conversion systems with parameters of three-phase induction machine: $P_N = 2.2\ kW$; $U_N = 400\ V$; $\Omega_N = 148.67\ rad/s$; $T_N = 14.8\ Nm$; $I_N = 4.7\ A$; $p = 2$ and the coefficients of model, equation (1):

$k_1 = 3{,}5386$; $k_2 = 147{,}34$; $k_3 = 0{,}639$; $k_4 = 106{,}39$;
$k_5 = 125{,}787$; $k_6 = 33{,}867$; $k_7 = k_{12} = k_{15} = 2$;
$k_8 = k_{13} = k_{16} = 7{,}077$; $k_9 = k_{14} = 13{,}399$
$k_{10} = 159{,}65$; $k_{11} = 19{,}14$

It is considered that electromechanical conversion systems operates in steady regime being supplied to the rated voltage and frequency values, and at *t*=3s the drive is loaded with a static resistant torque m_s=3,0 Nm.

Obviously this state can be achieved either by direct starting, either by using an inverter with controlled startup through a traditional control method. For this case we chose the first option for the simulation optimal control and second for the control structure with two- quasi-independent cascade channels after *d* and *q*, and with to keep a constant supply voltage in the diminishing flux area, to simulate the conventional control. The MATLAB simulation results of both the optimal control and conventional systems are presented. In Figure 1 (a) the optimal direct starting is shown, and in Figure 1 (b) the conventional control simulation results are shown.

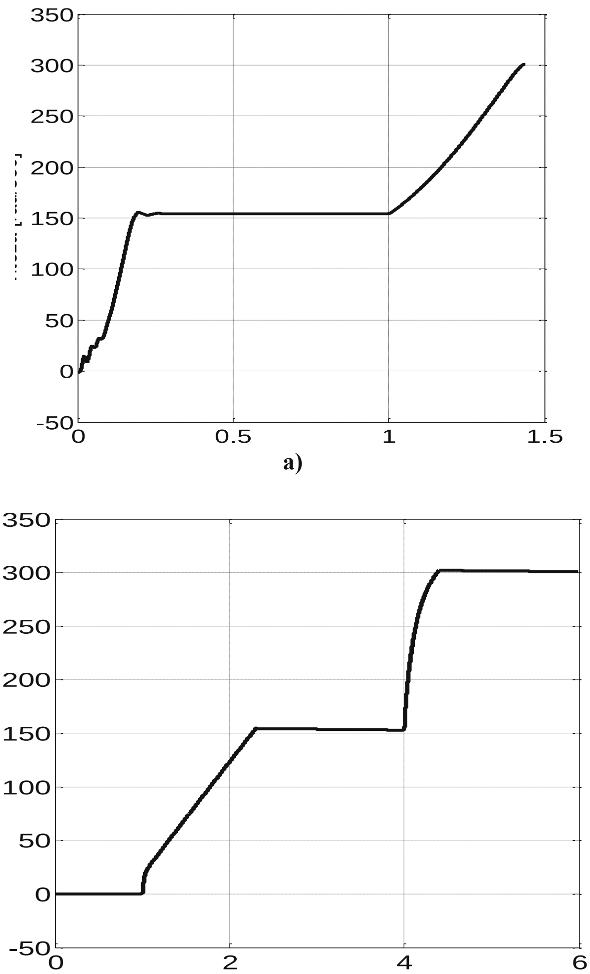


Figure 1. Angular speed [rad/sec] for optimal (a) and conventional (b) control

In the first case at initial time, *t*=0, the system of three-phase rated voltage is applied, taking place the uncontrolled direct starting with duration of approximately 0.2 seconds, after which get in the steady state regime required by the static resistant torque, imposed at shaft. In Figure 1 (a), by applying the optimal control after 1 sec (*t* > 1 sec), the angular speed variation is shown. During [0-1]sec the conventional control initiates the magnetization of the induction motor. At *t*=1 sec an acceleration command to the rated speed is applied, followed by the steady state regime. At *t* = 4 sec. the flux weakening control is approached, the high speed range limit being the double of rated speed. The main reason for this choice was determined by the aim pursued to create a system with greater generality as to allow comparison and assessment of new optimal control.

Other simulation results for optimal control are presented in Figures 2 – 6.

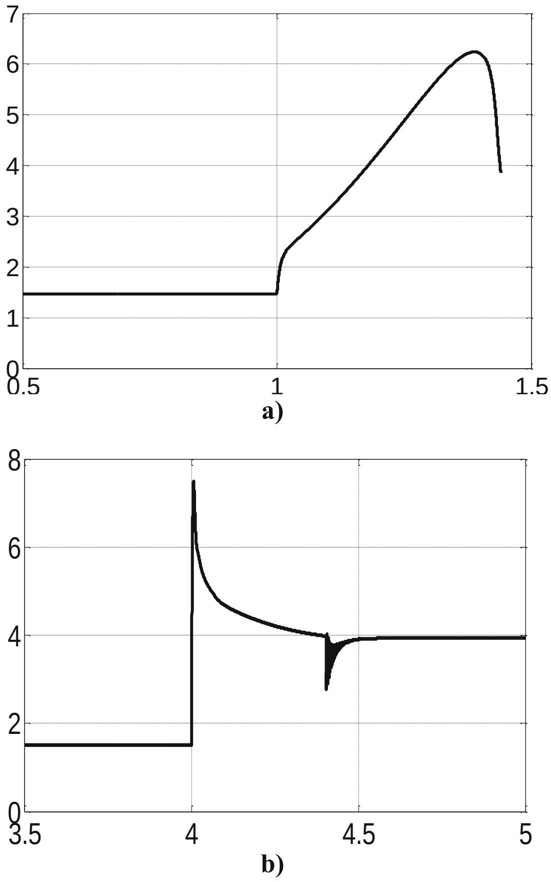
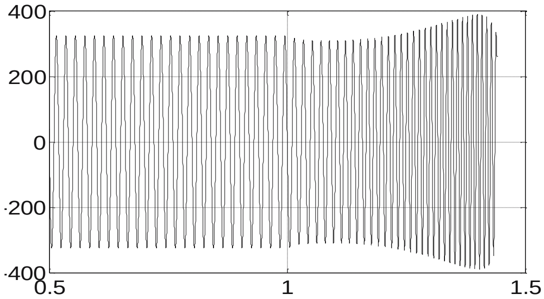
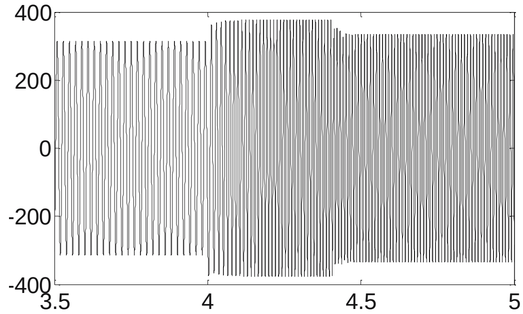


Figure 2. i_q current [A] for optimal (a) and conventional (b) control

- In **Figure 1(a)** good dynamic properties of optimal control are obtained. The angular speed increases linearly, the acceleration is constant, therefore, the electromagnetic torque shocks is avoided. The required speed is achieved with no steady state error;
- The transversal current component, **Figure 2(a)**, has a very interesting evolution. Thus the transversal current, has a low value at the start of dynamically regime, increasing slowly up to the end of the process and the maximum value reached is less than in the case of conventional control. Moreover, the current gradient is significantly lower. This advantage is exploited by the electric motor from cooling process point of view: for higher speed better self-ventilation is obtained.



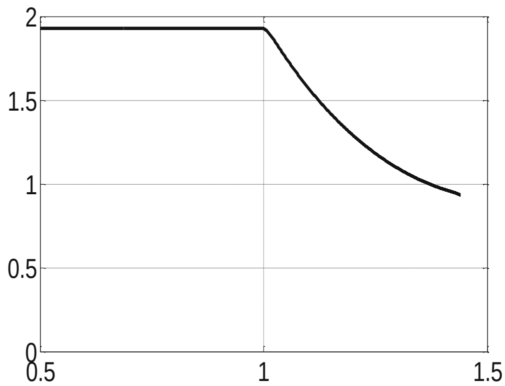
a)



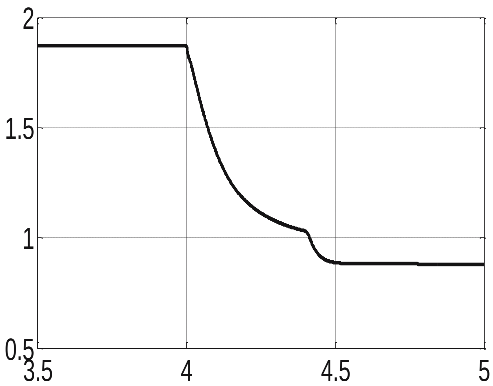
b)

Figure 3. Alternatively supply voltage [V] for optimal (a) and conventional (b) control

- In **Figure 3**, the supply voltage is presented, remarking changing the frequency and maintaining the magnitude value. There is a small increase in magnitude at the end of the dynamic regime allowable because it falls within the limits supported by the machine.

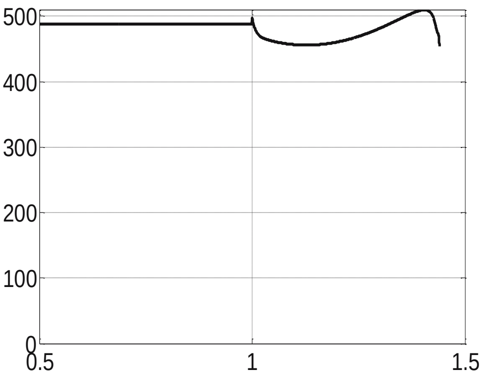


a)

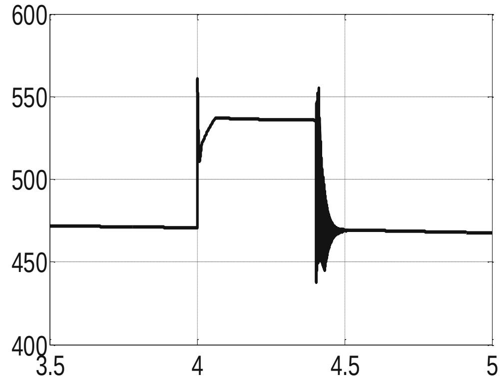


b)

Figure 4. Magnetizing rotor current [A] for optimal (a) and conventional (b) control

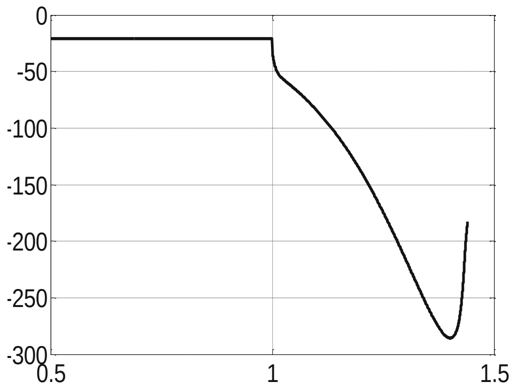


a)

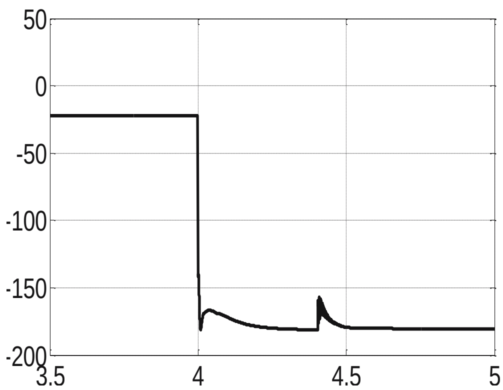


b)

Figure 5. V_q voltage component [V] **for optimal (a) and conventional (b) control**



a)



b)

Figure 6. V_d voltage component [V] for optimal (a) and conventional (b) control

- Magnetizing rotor current, i_m, **Figure 4** and symmetrical components of voltage, **Figure 5**, respectively **Figure 6**, poses no particular problem, it being admissible.

5. CONCLUSIONS

In this article an optimal control of dynamic variable flux induction machines has been investigated.

In order to achieve this goal an optimal drive system based on the energetic criteria has been proposed and an adequate Matlab-Simulink model has been performed.

The numerical simulation results have been obtained for two control structures: the optimal and the conventional control.

The simulation results for optimal control case (the rising of the angular speed is practically linear, constant acceleration, asymptotic entrance in the new steady state conditions, smoothly current and voltage variations by keeping the voltage within the allowable limits) confirms the realization of a control system with higher generality allowing comparison and assessment of the new optimal control structures.

6. AKNOWLEDGEMENTS

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