Radiative transfer of infrared radiation in model clouds

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ABSTRACT

The complete radiative transfer equation in its scalar form is evaluated to obtain the emissivity, transmissivity and reflectivity of model clouds at selected infrared wavelengths. The cloud models assume liquid water concentrations of 0.1 and 0.3 g/m^3 which are thought to be representative of layer clouds and fogs of moderately low and high water contents. It is the purpose of this study to determine relative errors in radiative transfer calculations in the infrared which are introduced by choices of model cloud droplet size distributions. This task is accomplished by comparing differences in infrared cloud characteristics resulting from the use of the well-known droplet distribution functions of Best (1951), Diem (1948), Deirmendjian (1964) and occasionally that of Warner (1969) for the same liquid water content. The water vapor window, subdivided into 5 spectral intervals, and a spectral region of the strong 6.5 micron water vapor band are investigated in detail for this purpose. It is found that the exponential and unimodal distributions of Best, Diem and Deirmendjian yield results which differ non-critically in practically all cases. The bimodal distribution of Warner, which is used only in conjunction with the lower liquid water content, shows appreciable differences with respect to the other three distribution functions. Of the many computational results which are available, only a few representative samples are displayed.

1. Introduction and statement of the problem

In order to determine the heat budget of the atmosphere in an accurate manner, it is imperative to study the infrared characteristics of clouds. In the past, for simplicity, the assumption was often made that clouds behave as perfect black bodies at all wavelengths in the infrared regions of the spectrum. Such assumptions are not always justified as follows, for example, from the recent papers of Yamamoto, Tanaka & Kamitani (1966), Zdunkowski & Choronenko (1969), Yamamoto, Tanaka & Asano (1970), and Korb & Zdunkowski (1970) which are based upon the exact formulation of the radiative transfer equation.

From the mathematical point of view, the problem of radiative transfer within and through horizontally homogeneous layer clouds of large horizontal extent, in principle, can be solved with any desired accuracy although practical calculations are very involved and difficult. An accurate mathematical solution of the radiative transfer problem, however, does not guarantee a close approximation to real atmospheric conditions since the physical properties of clouds are not known for any given situation and are only approximately known for average conditions. In order to carry out radiative transfer calculations pertaining to clouds, it is necessary to describe the cloud in terms of the cloud droplet size distribution function. The information on the distribution function provided in the literature, however, is not unique and it is difficult to decide which particular droplet distribution function should be used for a particular physical situation. Moreover, the size distribution functions are strongly at variance in their appearance and representation. Diem (1948), for example, describes the size distribution function in terms of graphs and tables while other

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authors such as Best (1951) and Deirmendjian (1964) provide interpolation formulas for the same purpose. It is, therefore, entirely possible that the radiative transfer computational results of the previously mentioned authors could have varied substantially had they chosen different size distribution functions for the same liquid water content.

It is the aim of the present investigation to compare radiative transfer calculations for selected spectral intervals due to the choice of various cloud droplet size distribution functions for given liquid water contents thought to be representative of radiation fog and layer clouds. This will enable one to estimate relative errors based on choices of size distributions. Since often calculations referring to the water vapor window are more sensitive to the choice of distribution functions than those in regions of strong gaseous absorption, the window spectral range is investigated in great detail. Nevertheless, a spectral region of a strong water vapor absorption band is studied also.

2. Liquid water content of layer clouds and cloud droplet size distribution models

The literature quotes numerous values of the liquid water content characterizing stratus clouds and fog. These values vary with the physical situation and, at least to some extent, are influenced by the type of measurement. Diem (1948), for example, quotes values of 0.09, 0.28, 0.29, and 0.40 g per cubic meter for strato-cumulus, alto-stratus, stratus and nimbostratus clouds, respectively. Values considerably larger and smaller for the quoted cloud types can also be found in the literature. For water contents of fog, Radford (1938), for example, obtains values ranging from 0.1 to 0.22 g per cubic meter. Interesting data collected by Russian authors, as reported by Feigelson (1964), quote average values for stratus clouds ranging from less than 0.1 to 0.3 g per cubic meter depending on the type of air mass. For the purpose of this study, two liquid water contents of 0.1 and 0.3 g per cubic meter are assumed which seem to cover the range of average values so frequently quoted. Four cloud size distribution functions are compared for their effect on the emissivity, transmissivity, and reflectivity of radiation relative to clouds of various thicknesses and base heights.

In terms of the droplet radius r and the liquid cloud water content W_L, the size distribution functions $n(r)$ due to Best (eq. 1) and Deirmendjian (eq. 2) are represented by the following interpolation formulas:

$$n(r) = \alpha W_L (2r)^{-0.7} \omega_L^{-3.3} \left[\exp\left(-\frac{2r}{\omega_L}\right)^{3.3}\right], \quad (\mathrm{cm}^{-3}\Delta\mu^{-1}) \qquad (1)$$

with $W_L = 1.1 \times 10^{-3} \omega_L^{1.79}$, and

$$n(r) = \beta r^6 \exp(-1.5r), \quad (cm^{-3}\Delta\mu^{-1}) \qquad (2)$$

where r and W_L are expressed in microns and g per cubic meter, respectively. The quantities α, β are constants which are fixed by the choice of the liquid cloud water content. ω_L is apparently independent of the cloud type. Inspection of eq. (1) indicates that the shape of Best's distribution function depends to some extent upon the liquid water content. This is not the case of Deirmendjian's function for which different choices of liquid water content result only in different particle numbers per radius interval.

The other two distributions due to Diem (1948) and Warner (1969) appear originally as graphs. For computer applications, points are extracted from the graphs and appropriate reductions are made to obtain the same liquid water content without disturbing the shapes of the distribution functions. For the liquid water content of 0.1 g per cubic meter, Diem's distribution curve (originally referring to $W_L = 0.09$) pertains to a strato-cumulus type cloud. A second distribution curve due to Diem, utilized in this study, refers to a stratus cloud having a liquid water content of 0.28 g per cubic meter. A slight reduction changes the value of W_L to 0.3 g per cubic meter. It should be noted that Deirmendjian refers to the referenced distribution function as Cumulus I rather than a layer cloud distribution function. This distinction in terminology is considered unimportant in this study. As a matter of interest, the fourth distribution function which is bimodal in character and not necessarily representative of layer clouds, is used for comparison purposes only.

For convenience of discussion, the Best distribution is considered to have modal radius less than one. For future reference, the distribution functions due to Best, Diem, Deirmendjian,

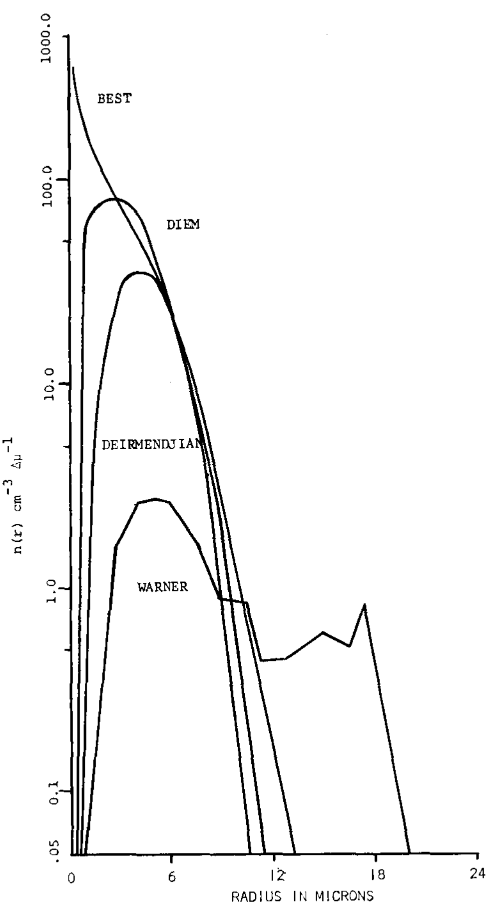

Fig. 1. Cloud droplet size distribution functions, $W_L = 0.1$ g/m³.

and Warner are also referred to numerically as 1, 2, 3, and 4, respectively. The number system is such that the larger numbers refer to distribution functions having greater modal radii. Fig. 1 displays the four distribution functions for $W_L = 0.1$ g per cubic meter.

3. Radiative input parameters

The droplet absorption and extinction coefficients for the previously quoted size distribution functions are tabulated in Tables 1, 2, and 3. Their computation is based on the complex index of refraction as compiled by McDonald (1960). The water vapor absorption constants used in this study are taken from Zdunkowski & Strand (1969).

Fig. 2 represents phase functions for each of

Table 1. *Droplet extinction coefficients* (km^{-1}), $W_L = 0.1$ g/m^3

	6.5 μ	9.0 μ	9.5 μ	10.0 μ	10.6 μ	11.5 μ
1	37.73	23.83	19.48	14.21	15.37	21.01
2	36.65	22.95	18.72	13.63	15.95	18.84
3	33.70	25.17	20.94	15.33	16.76	18.78
4	13.81		14.93		12.45	

Table 2. *Droplet absorption coefficients* (km^{-1}), $W_L = 0.1$ g/m^3.

	6.5 μ	9.0 μ	9.5 μ	10.0 μ	10.6 μ	11.5 μ
1	14.53	4.41	4.44	4.94	8.19	14.54
2	14.98	4.24	4.27	4.75	9.06	12.64
3	13.09	4.37	4.40	4.85	8.90	11.93
4	6.54		3.65		5.65	

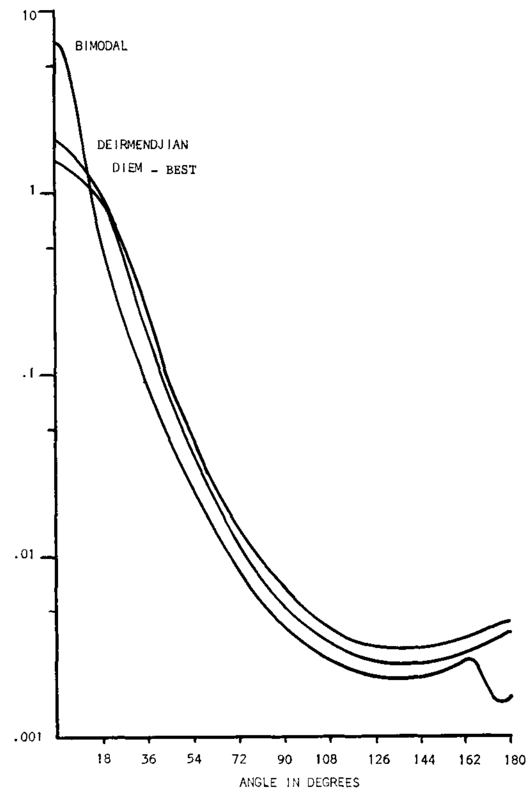

Fig. 2. 9.5. Micron phase function, $W_L = 0.1$ g/m³.

Table 3. *Droplet extinction and absorption coefficients* (km^{-1}), $W_L = 0.3\ g/m^3$.

	Extinction			Absorption		
	6.5 μ	9.5 μ	10.6 μ	6.5 μ	9.5 μ	10.6 μ
1	63.70	60.63	45.96	28.45	12.22	21.73
2	82.85	64.93	49.47	34.27	12.87	24.58
3	101.10	62.82	50.28	39.27	13.20	26.70

the size distributions ($W_L = 0.1$ g/m³) for the wavelength interval centered at 9.5 microns. Best & Diem distributions are nearly identical; differences do not show on the graph. As may be seen from this figure, distribution functions with greater modal radii generally exhibit greater forward scattering. Phase functions for other wavelength intervals and $W_L = 0.3$ g/m³ show similar characteristics and are not reproduced.

4. General remarks and mathematical procedure

All computations assume isothermal cloud temperatures while outside the cloud the temperature distribution follows the U.S. standard atmosphere. Saturation water vapor pressure is assumed to exist inside and a relative humidity of 0% or 70% outside the cloud depending upon the situation.

The mathematical analysis follows essentially the procedure of Korb & Zdunkowski (1970). Therefore, it is not necessary to restate the solution method of the radiative transfer equation. For details refer to their paper.

The analysis, taking into account multiple scattering by droplets and absorption and Planckian emission by droplets and gases, is based upon the expansion of the phase function into a series of Legendre polynomials. This results in an expression of the radiative transfer equation which is solved by iteration techniques. The iteration is discontinued if a certain convergence test is satisfied. Presently, the convergence condition that $\varepsilon < 0.001$ (eq. (30) of Korb & Zdunkowski) is used before the iteration is discontinued. This results in a very high precision of calculated quantities.

Let the optical thickness τ (droplets plus water vapor) of the cloud be counted vertically downward starting at the upper cloud boundary where $\tau = 0$. The total optical thickness of the cloud is $\tau = T$. For infrared radiation fields in horizontally homogeneous media of large horizontal extent, the radiative intensities depend only on the zenith angle ϑ of the radiation but not on the azimuthal angle. With $\mu = \cos\vartheta$, the upwardly and downwardly directed intensities at optical depth τ are generally denoted by $I_1(\tau, \mu)$ and $I_2(\tau, \mu)$, respectively.

Let $I_{1,e}(0, \mu)$ represent the radiative intensities due to the emission of an isothermal cloud which is not illuminated at its boundaries. For the purpose of computing cloud transmissivities and reflectivities, set the Planckian emission $B(\tau)$ of the source function (eq. 8 of Korb & Zdunkowski) inside the cloud equal to zero. For emissivity computations, however, the quantity $B(\tau)$ is not taken as zero but it is evaluated for the two cloud temperatures of 233°K and 283°K considered in this study. Furthermore, for transmission and reflection considerations, let the cloud be illuminated only at its lower boundary due to the presence of the water vapor below the cloud base and the surface of the earth. With these specifications, let $I_{1,t}(0, \mu)$ and $I_{2,r}(T, \mu)$ stand for the transmitted and reflected radiation at the upper and lower cloud boundaries, while $I_1(T, \mu)$ specifies the intensity illuminating the lower cloud boundary from direction μ. Then the flux emissivity (Em), transmissivity (Tr), and reflectivity (Re) of an isothermal cloud for a given spectral region characterized by the central wavelength are defined as

$$\mathrm{Em} = \frac{2\int_0^1 I_{1,e}(0,\mu)\,\mu d\mu}{B(T)} \tag{3}$$

$$\mathrm{Tr} = \frac{\int_0^1 I_{1,t}(0,\mu)\,\mu d\mu}{\int_0^1 I_1(T,\mu)\,\mu d\mu} \tag{4}$$

$$\mathrm{Re} = \frac{\int_0^1 I_{2,r}(T,\mu)\,\mu d\mu}{\int_0^1 I_1(T,\mu)\,\mu d\mu} \tag{5}$$

The intensities refer to spectral averages making use of the composite spectral line model for water vapor. For details refer to eqs. (19, 21) of Korb & Zdunkowski (1970). This composite spectral line model is chosen for reasons of simplicity. More advanced models are available but their application very probably does not alter the conclusions of this paper. From conservation of energy it follows that for an isothermal cloud

$$Em + Tr + Re = 1 \tag{6}$$

To have an additional check on the accuracy of the calculations, all three quantities are computed in each case. It is found that eq. (6) is satisfied with a very high degree of precision.

5. Discussion of results

To facilitate the discussion, results with respect to the liquid water contents of 0.1 and 0.3 g/m³ are examined, separately. First the water vapor window is treated; this is followed by a brief description of results pertaining to a central spectral region of the strong 6.5 microns water vapor band. In order to account for the non-negligible variations of attenuation coefficients and phase functions with wavelength, five spectral regions are used to represent the water vapor window in case of the smaller liquid water content. These intervals extend from *8.75–9.25, 9.25–9.75, 9.75–10.25, 10.25–11.00, 11.00–12.00* microns and are usually identified by their central wave-length. To keep computational efforts and computer costs within reasonable limits, only two spectral regions centered at 9.5 and 10.6 microns are used to represent the water vapor window in the case of the higher liquid water content of 0.3 g/m³.

(a) *Water vapor window: clouds with base heights at 500 m, $W_L = 0.1$ g/m³*. Figs. 3 and 4 represent graphically the variation of cloud emissivity, transmissivity, and reflectivity with cloud thickness for clouds of 500 m base height for two selected wave-lengths. The temperature within each cloud is 283°K. Only one curve, designated "Representative" is used to represent the three distribution functions due to Best, Diem, and Deirmendjian because of very slight differences among them. Similarly, small differences are found for the other window regions so that results need not be displayed separately. Maximum deviations are observed at 11.5 microns.

An even clearer picture of the differences in

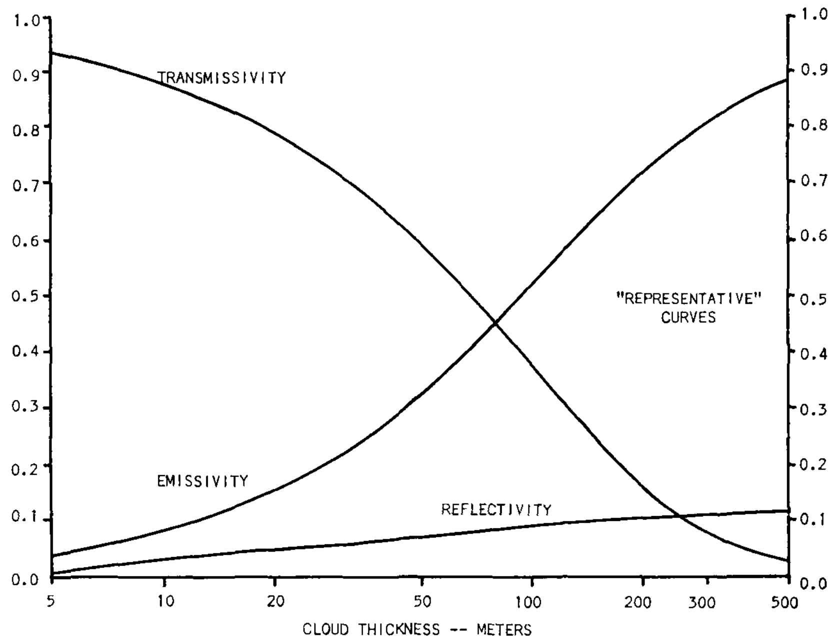

Fig. 3. Em, Tr, and Re of clouds of various thickness which are illuminated by radiation of wavelength 9.0 microns. Base height 500 m, $W_L = 0.1$ g/m³.

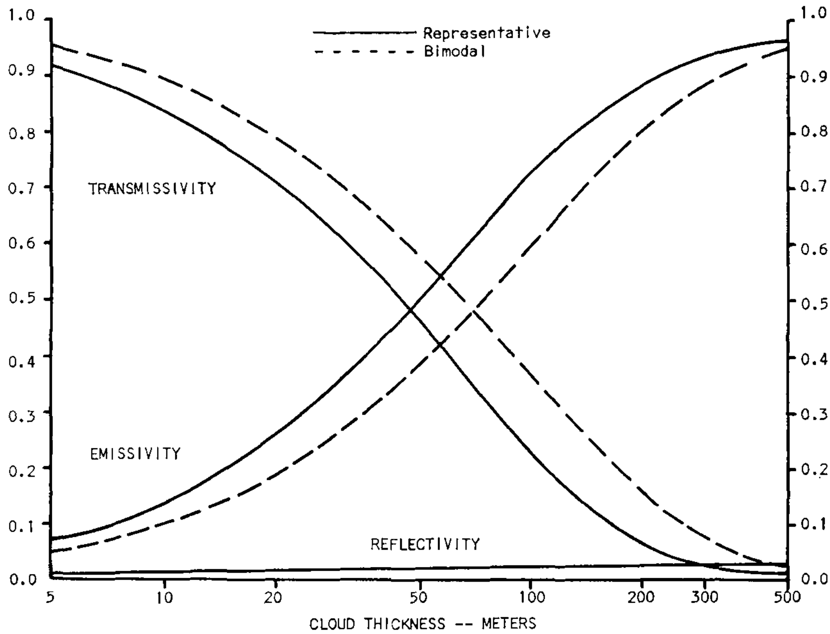

Fig. 4. Em, Tr, and Re of clouds of various thickness which are illuminated by radiation of wavelength 10.6 microns. Base height 500 m, $W_L = 0.1$ g/m³.

emissivity, transmissivity, and reflectivity among the distribution functions is seen by examination of Table 4. An investigation of this table reveals the following. At the wavelength of 9.0 microns, differences in Em, Tr, and Re are very small for distributions 1, 2, and 3. Differences greater than 1% between any two of the distributions are rare and no differences greater than 2% are found. Similarly small differences for distributions 1–3 are found for the other spectral regions of the window (not reported) except the 11.5 microns where differences up to 6.4% are found in clouds of moderate thickness (40 to 100 m). For very thin and for very thick clouds, differences among the distributions are negligible. Table 5 compares the distribution functions 2 and 4 after Diem and Warner, respectively. Results due to distribution functions 1 and 3 are not shown since they are approximated within 1% by distribution function 2. The bimodal distribution function of Warner shows rather significant deviations as compared with distribution function 2, as expected, since the bimodal size distribution is not necessarily representative of layer clouds due to the greater number of large drop sizes.

Fig. 5 represents the variation of Em with wave-length for clouds of various thicknesses with base heights at 500 m in terms of Best's and the bimodal distribution, respectively. Note well that the dashed line is drawn using only two data points.

(b) *Water vapor window: clouds with base heights at 8 200 m, $W_L = 0.1$ g/m³.* As a means of comparing radiative characteristics for a cloud base height other than 500 m, calculations for 9.5 and 10.6 microns are also made for model clouds based at 8 200 m. The temperature of the clouds is arbitrarily chosen at −40°C in order to get as great a temperature differential as possible. Although this specification is not real, it still serves the purpose of comparing the size distribution functions. It is assumed, for simplicity, that the cloud, now consisting of ice particles, is characterized by the same attenuation and scattering parameters of the lower cloud previously discussed.

A comparison indicates that emissivities, transmissivities and reflectivities for the high cloud agree within less than 2% with the corresponding results of the low clouds. Therefore, no separate comparison tables are given. The small differences are attributed to variations

Table 4. *Emissivity, transmissivity, and reflectivity by distribution and cloud thickness*

Cloud base 500 m, wavelength 9.0 microns, $W_L = 0.1$ g/m³

Depth (m)	Distr.	Emissivity	Transmissivity	Reflectivity
500	1	0.875	0.013	0.112
	2	0.872	0.014	0.114
	3	0.879	0.014	0.107
300	1	0.825	0.067	0.108
	2	0.817	0.072	0.111
	3	0.826	0.071	0.103
200	1	0.738	0.156	0.106
	2	0.727	0.165	0.108
	3	0.736	0.163	0.101
100	1	0.530	0.374	0.096
	2	0.517	0.385	0.098
	3	0.526	0.382	0.092
50	1	0.330	0.591	0.079
	2	0.319	0.601	0.080
	3	0.327	0.598	0.075
40	1	0.277	0.651	0.072
	2	0.269	0.659	0.072
	3	0.275	0.657	0.068
20	1	0.154	0.795	0.051
	2	0.149	0.800	0.051
	3	0.153	0.799	0.048
10	1	0.082	0.886	0.032
	2	0.079	0.889	0.032
	3	0.081	0.888	0.031
5	1	0.042	0.939	0.019
	2	0.041	0.941	0.018
	3	0.042	0.940	0.018

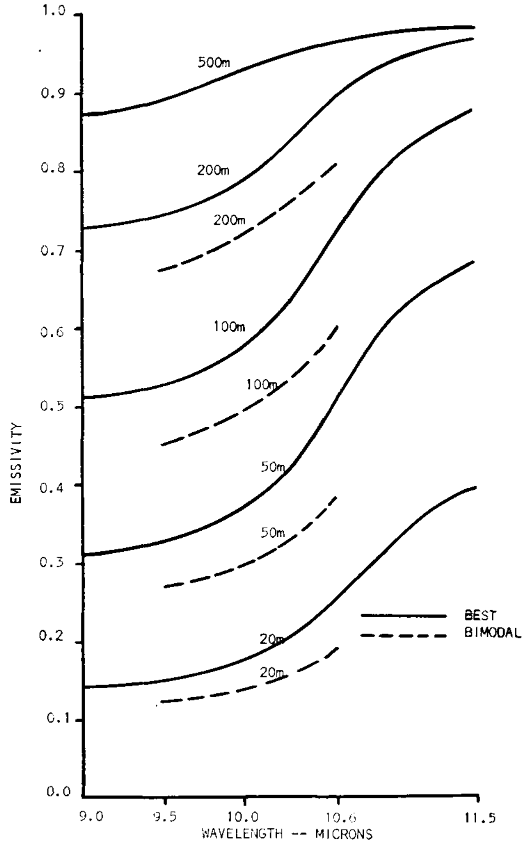

Fig. 5. Emissivity vs. wavelength for various cloud thicknesses. Cloud base heights 500 m. Cloud temperature 10°C, $W_L = 0.1$ g/m³.

in water vapor concentrations and pressure broadening effects of spectral lines.

(c) *Effect of ground albedo at a selected wavelength.* As a matter of interest, two sets of calculations are made to determine the effect of surface albedo on the total flux leaving the cloud top. Clouds of various thicknesses with base heights of 500 m are used for a wavelength of 10.6 microns. Results shown in Fig. 6 are representative of the size distribution models of Best, Diem, and Deirmendjian.

For very thin clouds most of the flux to space is due to radiation of the earth's surface since very little is absorbed. Hence, the greatest difference due to albedo is found with thin or no clouds. As the thickness of the cloud increases more of the ground radiation is abosorbed. The flux leaving the cloud now becomes nearly independent of the ground albedo but depends primarily on the cloud temperature.

(d) *6.5 Micron water vapor absorption band, $W_L = 0.1$ g/m³.* Selected calculations for the 6.25–6.75 micron spectral region are made for clouds with base heights at 500, 2 500, and 8 200 m. Table 6 is computed on the basis of particle size distribution function 2 after Diem. Results pertaining to the other three distribution functions agree within a few tenths of one per cent with the exception of distribution function 4 which varies from the other three by up to 1.4% for very thin clouds. Again, it is assumed for simplicity, that the radiative characteristics of cloud particles are the same as in low clouds.

Table 7 provides the comparison of Em, Tr, and Re by thickness and base height for the Deirmendjian distribution. Unlike the results

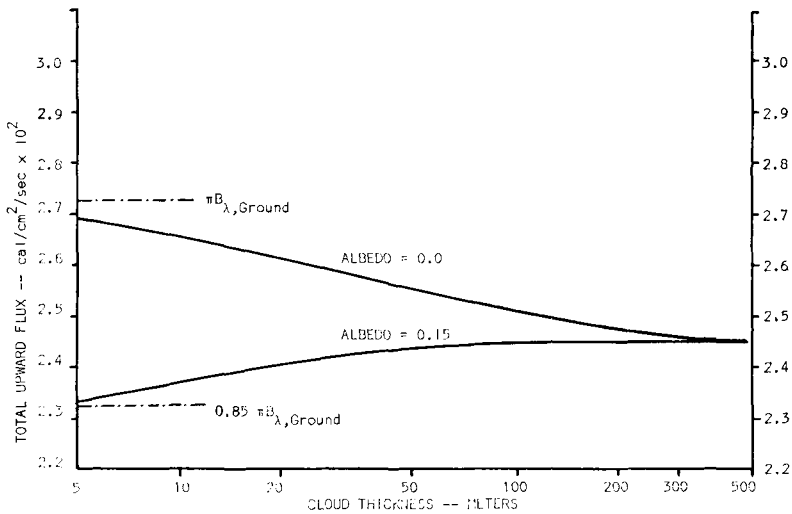

Fig. 6. Total upward flux (emitted and transmitted) from top of clouds of various thickness with base height 500 m for wavelength 10.6 microns and ground albedoes 0.0 and 0.15, $W_L = 0.1$ g/m³.

from the water vapor window, there is, at 6.5 microns, a great disparity in the three quantities with height of the cloud base. This is attributed to the quite noticeable height variation of water vapor density and pressure broadening of spectral lines. The spectral line intensity variation with temperature is less pronounced.

Table 5. *Emissivity, transmissivity, and reflectivity by distribution and cloud thickness*

Cloud base 500 m, wavelength 6.5 microns, $W_L = 0.1$ g/m³

Depth (m)	Distr.	Emissivity	Transmissivity	Reflectivity
500	2	0.889	0.018	0.093
	4	0.887	0.045	0.068
300	2	0.827	0.083	0.090
	4	0.790	0.144	0.066
200	2	0.732	0.180	0.088
	4	0.673	0.264	0.063
100	2	0.518	0.402	0.080
	4	0.454	0.491	0.055
50	2	0.321	0.615	0.064
	4	0.273	0.685	0.042
40	2	0.269	0.672	0.059
	4	0.228	0.734	0.038
20	2	0.150	0.809	0.041
	4	0.125	0.850	0.025
10	2	0.079	0.895	0.026
	4	0.065	0.920	0.015
5	2	0.041	0.944	0.015
	4	0.034	0.958	0.008

(e) *Water vapor window: clouds with base heights at 500 m, $W_L = 0.3$ g/m³.* For clouds ranging in vertical extensions from 10 to 200 m, emissivities, transmissivities and reflectivities are computed for a cloud base height of 500 m. Selected results are shown in Table 8 for the droplet size distribution 2 after Diem and a wave-length of 10.6 microns. No detailed discussion is necessary since results are similar to those pertaining to the smaller liquid water content of 0.1 g/m³.

Table 6. *Emissivity, transmissivity, and reflectivity for Diem's distribution and cloud thickness*

Cloud base 500 m, wavelength 6.5 microns, $W_L = 0.1$ g/m³

Depth (m)	Emissivity	Transmissivity	Reflectivity
500	0.996	0.0	0.004
300	0.996	0.0	0.004
200	0.996	0.0	0.004
100	0.996	0.0	0.004
50	0.996	0.001	0.003
40	0.995	0.002	0.003
20	0.979	0.018	0.003
10	0.919	0.078	0.003
5	0.806	0.191	0.003

Table 7. *Comparison of emissivity, transmissivity, and reflectivity for Deirmendjian distribution for various cloud thicknesses and base heights*

Wavelength 6.5 microns, $W_L = 0.1$ g/m³

Depth (m)	Base (m)	Emis-sivity	Trans-missivity	Reflec-tivity
200	500	0.994	0.0	0.006
	8 200	0.962	0.009	0.029
100	500	0.994	0.0	0.006
	2 500	0.989	0.001	0.010
	8 200	0.904	0.071	0.025
40	500	0.977	0.018	0.005
	2 500	0.964	0.027	0.009
	8 200	0.686	0.292	0.022

Differences (not shown) due to choices of various distribution functions and other spectral ranges again are found to be very small, ranging approximately from one to three per cent for cloud thicknesses of 100 m and greater. This conforms with previous results. For the very small cloud thicknesses of 40 m and less, however, relative errors can be as large as 7%, exceeding substantially previously detected differences.

(f) *6.5 Micron water vapor band,* $W_L = 0.3\ g/m^3$. Consider next the spectral region extending from 6.25–6.75 microns and the higher liquid water content of 0.3 g/m³. Results pertain to clouds with base heights of 500 and 8 200 m and thicknesses ranging from 10 to 200 m. Table 9 gives a comparison of emissivities, transmissivities and reflectivities on the basis of cloud droplet distribution function two.

Table 8. *Emissivity, transmissivity, and reflectivity for Diem's distribution and cloud thickness*

Cloud base 500 m, wavelength 10.6 microns, $W_L = 0.3$ g/m³

Depth (m)	Emis-sivity	Trans-missivity	Reflec-tivity
200	0.978	0.001	0.021
100	0.959	0.021	0.020
40	0.778	0.202	0.020
20	0.557	0.425	0.018
10	0.351	0.634	0.015

Inspection shows that a cloud of 40 m thickness, whose base is located at 500 m, for all practical purposes can be regarded a black body. This is in agreement with previous results of Zdunkowski & Choronenko (1969). For this low cloud and all heights considered in Table 9, all three distribution functions vary less than a few tenths of a per cent. Additional inspection shows that a cloud of 40 m thickness but of a base height of 8 200 m has black body deficiencies in emission of approximately 28 and 12% for the liquid water contents of 0.1 and 0.3 g/m³, respectively for the Diem model. This deficiency is mostly due to the smaller water vapor concentration at the higher level and the pressure broadening effect of spectral lines. In case of the high cloud, the choice of the model becomes somewhat critical for cloud thicknesses of 20 m and less where maximum differences in emissivities and transmissivities of 10% occur while reflectivity variances are less than 1 per cent.

Table 9. *Comparison of emissivities, transmissivities and reflectivities for two liquid water contents and cloud base heights*

Wavelength 6.5 microns

Depth (m)	Base (m)	Emissivity		Transmissivity		Reflectivity	
W_L (g/m³) ...		0.1	0.3	0.1	0.3	0.1	0.3
100	500	0.997	0.997	0.0	0.0	0.003	0.003
	8 200	0.920	0.968	0.053	0.004	0.027	0.028
40	500	0.995	0.996	0.002	0.000	0.003	0.004
	8 200	0.719	0.883	0.258	0.093	0.023	0.024
20	500	0.979	0.986	0.018	0.010	0.003	0.004
	8 200	0.503	0.728	0.476	0.251	0.021	0.021
10	500	0.919	0.938	0.078	0.058	0.003	0.004
	8 200	0.314	0.508	0.669	0.473	0.017	0.019

6. Summary and concluding remarks

Results due to the bimodal distribution function of Warner are not taken into account in the concluding remarks since this particular distribution function is not necessarily representative of layer clouds. Inspection of tables and graphs indicates that throughout the lower wavelength region of the water vapor window all results are nearly independent of the choice of the distribution function for $W_L = 0.1$ g/m^3; at the higher wavelength region of the window, small differences in modal radii of droplet size distributions cause not entirely negligible relative errors in transmission and emission values of moderately thick clouds (40 to 100 m in depth) while the reflectivity remains virtually unaffected.

In case of the higher liquid water content, the shorter as well as the higher wavelength regions of the window, as exemplified by the 9.5 and 10.6 micron intervals, are only little affected by the choice of distribution functions for cloud thicknesses of 100 m and larger. For cloud thicknesses less than 100 m for the 9.5 micron regions the choice of the size distribution is relatively unimportant while for the 10.6 micron region the choice of the distribution function becomes increasingly important. It may be summarized that in the water vapor window region for the lower as well as the higher liquid water content the choice of the distribution function causes only fairly small errors. The exception is found at the longer wavelength end of the window where relative errors become quite noticeable for moderately thick clouds. The 6.5 micron region is practically independent of the choice of any three of the distribution functions in case of $W_L = 0.1$ g/m^3 for all base heights and the 500 m base height for $W_L = 0.3$ g/m^3. Relative errors are significant for $W_L = 0.3$ g/m^3 and small cloud thicknesses if the base height is moved to 8 200 m.

In order to additionally check the accuracy the computations, an independent computational method (results still unpublished) is applied to a model case which reproduces accurately the previous results. This test, together with conservation of energy and the tests made by Korb & Zdunkowski (1970), establishes the accuracy of the computational procedure.

At the time that all the computational work was finished, a paper by Yamamoto et al. (1970) appeared in which emissivity, transmissivity and reflectivity values for various cloud thicknesses for the entire infrared spectrum were presented. Those authors used the Diem distribution function but the liquid water content of 0.28 g/m^3. In particular, their index of refraction differs substantially from the values used in this investigation so that a comparison of results is not directly possible. Professor Yamamoto also kindly informed us that they evaluated the transfer equation at points which differ from the wave-lengths used in this study. It appears for certain parts of the spectrum that the choice of the refractive index is as important or even more so that the selection of the cloud model.

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РАДИАЦИОННЫЙ ПЕРЕНОС ИНФРАКРНАСОЙ РАДИАЦИИ В ЬНЫХ ОБЛАКАХ

Для получения пропускания, излучательной и отражательной способности облаков в выбранных длинах инфракрасных волн рассчитывается полное уравнение переноса радиации в его скалярной форме. В моделях облаков предполагаются концентрации капельной воды в 0,1 и 0,3 г/м³, что соответствует слоистым облакам и туманам с онтосительно низким и высоким содержанием воды. Целью настоящего исследования является определение относительных ошибок в расчетах радиационного переноса в инфракрасной области, которые возникают при выборе различных моделей для распределений капель по размерам. Эта задача осуществляется путем сравнения различий в инфракрасных характерстиках облаков, возникающих при использовании хорошо известных функций распределения Беста (1951), Дьема (1948), Дейрменджана (1964) и, иногда, Уорнера (1969) для одного и того же содержания капельной влаги. С этой целью детально изучаются окно в спектре водяного пара, поделенное на 5 спектральных интервалов, и область спектра с сильной полосой водяного пара 6,5 мкм. Найдено, что экспоненциальные и унимодальные распределения Беста, Дьема и Дейрменджана приводят к результатам, некритически различающимся практически во всех случаях. Бимодальное распределение Уорнера, использовавшееся только для меньшего влагосодержания, приводит к заметным различиям по сравнению с остальными тремя функциями распределения. Из большого имеющегося объема результатов вычислений представлено лишь несколько репрезентативных примеров.