On the computation of the basic parameters of the interaction between the atmosphere and the ocean[1]

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ABSTRACT

Computations are discussed of turbulent friction stress and vertical turbulent heat and moisture fluxes in the near-water layer of the atmosphere by the "external" parameters the values of which can be determined from synoptic information. The analysis proceeds from considering the planetary boundary layer above the sea on the basis of the similarity theory. In so doing the specific features of the liquid underlying surface are taken into account and the processes in a viscous layer at the water-air interface are considered. The equations for computing the parameters of interest resulting from the similarity theory contain a number of universal functions. These functions are found by a special threatment of experimental data. It should be noted that the employed method of data systematization makes it possible to use the observations conducted above the land. Theoretical evaluation of the above-mentioned universal functions is also given. The obtained results may be of interest for further improvement of the hydrodynamical methods of weather prediction and the numerical experiments on the general circulation in the atmosphere and the ocean.

General character of the dependence of tanget stress and vertical turbulent heat and moisture fluxes on the "external" parameters

The basic physical parameters of the small-scale interaction between the atmosphere and the underlying surface are the near-ground values of the vertical turbulent heat H and moisture E fluxes as well as of the horizontal vector of turbulent friction stress which can be characterized by its magnitude τ and angle α which it forms with wind direction in the free atmosphere. The objective of the present paper is to investigate the "universal" relationship between the above-mentioned micrometerological parameters and the "external" ones which determine the structure of the planetary boundary layer of the atmosphere in the case of a horizontally homogeneous air flow. The requirement for stationarity of the regime can, apparently, be not imposed and a less limiting condition of diurnal periodicity in the changes of meteorological fields be used.

Instead of the values τ, H and E it would be convenient to use the following parameters uniquely related to these values: the friction velocity $u_* = \sqrt{\tau/\varrho}$ (ϱ is the air density), the temperature scale $T_* = -H/kc_p\varrho u_*$ (c_p is the heat capacity of the air at constant pressure, $k=0.4$ the von Karman constant) and the specific humidity scale $q_* = -E/k\varrho u_*$.

As the "external" parameters use can be made of the quantities which appear in the boundary conditions for the equations of the boundary layer theory—the roughness height of the underlying surface Z_0, wind velocity at the upper surface of the boundary layer (i.e. the geostrophic wind velocity) G, the differences in potential temperature $\delta\theta = \theta_h - \theta_0$ and in specific humidity $\delta q = q_h - q_0$ related to the entire layer under consideration (from the height Z_0 to the upper surface of the boundary layer h) as well as

[1] The paper is a somewhat enlarged version of the report at the XIV General Assembly IUGG 1967, IAMAP-D, Symposium on the Mechanism of Exchange Processes at the Earth-Atmosphere Interface.

the parameters entering into the coefficients of the equations—the Coriolis parameter f, the acceleration of gravity g and the buoyancy parameter $\beta = g/T$ (T is the characteristic value of the absolute air temperature). Following the paper by Zilitinkevich, Laikhtman & Monin (1967) and taking into account the humidity stratification effect (Zilitinkevich, 1966), it is not difficult to show that the relationship between the micrometeorological parameters u_*, α, T_* and q_* and the above-mentioned external parameters is expressed by the following equations:[1]

$$\ln \mathrm{Ro} = B(\mu) - \ln \frac{u_*}{G} + \sqrt{\frac{k^2}{(u_*/G)^2} - A^2(\mu)}, \quad (1)$$

$$\sin|\alpha| = \frac{A(\mu)}{k}\frac{u_*}{G}, \quad (2)$$

$$\frac{T_*}{\delta\theta} = \alpha_H^{(0)} \Big/ \left[\ln\left(\mathrm{Ro}\,\frac{u_*}{G}\right) - C(\mu)\right], \quad (3)$$

$$\frac{q_*}{\delta q} = \alpha_E^{(0)} \Big/ \left[\ln\left(\mathrm{Ro}\,\frac{u_*}{G}\right) - D(\mu)\right]. \quad (4)$$

Here $\alpha_H^{(0)}$ and $\alpha_E^{(0)}$ are reciprocals of the turbulent Prandtl number and the turbulent Schmidt number respectively, at neutral stratification (numerical constants close to unity);

$$\mathrm{Ro} = G/f Z_0 \quad (5)$$

is the non-dimensional surface Rossby number; A, B, C and D are universal functions of the non-dimensional argument

$$\mu = k^3\beta(T_* + 0.61\ Tq_*)/fu_*, \quad (6)$$

which represents a modified stratification parameter of Monin. The only additional assumption used besides rather general similarity hypothesis in deriving the equations (1)–(4) consists in that the wind, temperature and humidity profiles in the regions of small heights should satisfy asymptotically the Monin-Obukhov (1954) "log-linear" law (see Appendix).

To be able to make computations using the equations (1)–(6) it is necessary to indicate a method for determining the "external" parameters Z_0, G, $\delta\theta = \theta_h - \theta_0$ and $\delta q = q_h - q_0$, and to find the form of the functions A, B, C and D.

[1] The equation (1) was first derived for the case of neutral stratification in the paper by Kazansky & Monin (1961).

Height of the boundary layer

Let us consider the values G, θ_h and q_h representing wind velocity, potential temperature and specific humidity at the upper boundary of the boundary layer. The height h of the layer can be determined by the equation

$$h = \gamma L_* \quad \text{where} \quad L_* = ku_*/f \quad (7)$$

is the length scale introduced by Monin, γ is the non-dimensional coefficient. The dependence of h upon the "external" parameters can be found with the aid of the expression equivalent to the equation (7)

$$hf/G = \gamma k u_*/G, \quad (7a)$$

the right side of which contains the geostrophic drag coefficient u_*/G considered above. The choice of value for γ is roughly equivalent to the pre-assignment of one or another accuracy to the approximation of the asymptotic boundary layer by the finite thickness boundary layer. It is convenient to fix γ practically in such a way that, on the one hand, the accuracy of the above-mentioned approximation was not too low and, on the other hand, that the height h was small as compared to the effective thickness of the atmosphere (~ 10 km). The latter condition enables one to use the model of the horizontally homogeneous boundary layer in the presence of disturbances of the synoptic scale. Both requirements discussed are satisfied by the value $\gamma = 1$ corresponding to the height of the boundary layer of the order of 1 km.

It should be noted that in most cases the sufficient accuracy in determining the values of G, θ_h and q_h is achieved by replacing the variable height of the boundary layer by a constant value equal, say, to 1 km or to the height of the 850-mb surface.

Specific character of the sea surface. Regard to the processes in a viscous layer

We are interested in the possibility of applying the equations of the type (1)–(6) to the computation of the values u_*, α, T_* and q_* above the sea. In so doing it is necessary to take into account the peculiarities of the exchange processes near the waved and moving water surface.

The effect of waves results first of all in that the roughness parameter Z_0 appears to be essentially dependent on the external conditions. For the simplest case of statistically stationary waves the characteristics of which are fully determined by local wind the value of Z_0 can be written in the form (Monin & Zilitinkevich, 1967)

$$Z_0 = \frac{u_*^2}{g} F_0\left(\frac{u_*^3}{g\nu}\right), \tag{8}$$

where ν is the kinematic coefficient of air viscosity, F_0 is the non-dimensional universal function. The following considerations can be presented with respect to the form of this function.

At very small values of u_* (i.e. at weak wind) the sea surface becomes smooth and, hence, the acceleration of gravity g, governing the sizes of waves, should be excluded from among the determining parameters. This means that at small values of the argument the function $F_0(\zeta)$ should behave as ζ^{-1}. In this case the equation (8) becomes

$$Z_0 = m_0 \frac{\nu}{u_*} \quad (m_0 = \text{const.}). \tag{8a}$$

The expression (8) was discussed, in particular, in the papers of Deacon & Webb (1962), and Roll (1965). According to the latter, the empirical value of m_0 is approximately 0.48.

At great values of u_* more or less intensive waves are observed as a result of which the turbulent velocity pulsations prove to be considerable to the very sea surface. The viscous layer is practically of no importance in this case and, consequently, the viscosity coefficient ν falls out from among the parameters determining the value of Z_0. In other words, at large ζ the function, $F_0(\zeta)$ tends to a constant value, the equation (8) taking the form

$$Z_0 = m \frac{u_*^2}{g} \quad (m = \text{const.}). \tag{8b}$$

The obatined expression represents a well-known equation of Charnock (1955). The processing of extensive experimental material made in the paper by Kitaigorodsky & Volkov (1965*a*) shows, on the average, not bad justification of the above equation over a wide range of values of u_* at the constant m being equal to about 0.035.

The form of the function $F_0(\zeta)$ in the intermediate region can in principle be found empirically. As a simple interpolation formula with necessary asymptotic properties the following expression can be pointed out

$$F_0(\zeta) = m_0 \zeta^{-1} + m. \tag{9}$$

We further need to determine the magnitude of θ_0 representing air temperature at the lower boundary of the boundary layer, i.e. at Z_0 level. Since in most cases this quantity is not known it seems interesting to express it in terms of a comparatively easily determined temperature of the sea surface T_s. The temperatue difference $\theta_0 - T_s$, sometimes equal to several degrees, generally speaking, depends on wave characteristics and molecular processes[1]. With the same limitations at which the equation (8) is valid, the dependence under consideration can be written in the form (Monin & Zilitinkevich, 1967)

$$\theta_0 - T_S = T_* F_T\left(\frac{u_*^3}{g\nu}\right), \tag{10}$$

where F_T is the non-dimensional universal function depending, in addition to the indicated argument, on the Prandtl number. As in the case of the equation (8) we can give here asymptotic expressions corresponding to very small values of u_*. Thus at very weak wind when Z_0 is determined from the equation (8a) the non-dimensional relation $(\theta_0 - T_s)/T^*$ ought not to depend on g. This means that the function $F_T(\zeta)$ at $\zeta \to 0$ tends to a constant. No systematization of the experimental data was made on the basis of the equation of the type (10), however, some information of the dependence under study can be drawn from the empirical graphs contained in a number of publications which related the difference between water and air temperatures to temperature differences at two heights in the near-water layer (*vide*, for instance, Kitaigorodsky & Volkov, 1965*b*; Roll, 1965). In particular, it can be inferred from the graphs that the characteristic value of $F_T(\zeta)$ possibly has the order of several unities.

Let us consider now the quantity q_0 which, like θ_0, is the result of extrapolating the asymptotically logarithmic humidity profile downwards to $Z = Z_0$ level. Proceeding from the fact

[1] Strictly speaking, the magnitude of θ_0 represents the result of extrapolating the asymptotically logarithmic temperature profile downwards to $Z = Z_0$ level. Therefore the determination of $\theta_0 - T_s$ difference is equivalent to the determination of the heat transfer coefficient or the Stanton number.

that air humidity immediately near the water surface which is denoted by q_s is saturating and expressing the difference $q_0 - q_s$ in terms of the equation analogous to (10), we obtain

$$q_0 - \frac{R}{R_w}\frac{e(T_s)}{p_0} = q_* F_q\left(\frac{u_*^3}{g\nu}\right), \tag{11}$$

where R and R_w are universal constants of the air and water vapour, $e(T_s)$ is the maximum pressure of water vapour at temperature T_s, p_0 is the atmosphere pressure at sea level, F_q is one more non-dimensional universal function (depending, apart from the indicated argument, on the Schmidt number). The form of this function is probably fully analogous to the form of the function F_T. No processing of the experimental data by the equation (11), as far as the author knows, was made.

It should be noted that in determining the quantities $\delta\theta = \theta_h - \theta_0$ and $\delta q = q_h - q_0$ the use of the Reynolds analogy, i.e. direct substitution of water temperature T_s for θ_0 and the maximum specific humidity $q_s = Re(T_s)/R_w p_0$ corresponding to this temperature for q_0, is permissible only for rough estimations.

Thus the determination of the external parameters appearing in the equations (1)–(6) has been considered. Before proceeding to establish the form of the functions A, B, C and D, let us point out another effect specific for the atmosphere above the sea. The thing is that when considering a velocity field in this case it is necessary to keep in mind that apart from participating in wave motions, the liquid particles at the surface move with a certain velocity. When turning to the moving coordinate system we may exclude the displacement of the underlying surface and instead of the velocity components we obtain the differences between the values of these components and of the corresponding surface current components. Strictly speaking, it is to these differences that the results valid for the atmospheric boundary layer above the land can be applied. In most cases the associated corrections are of no importance. The only appreciable effect lies apparently in the fact that in the logarithmic boundary layer over the length of which the direction of the vectorial difference of wind velocity and surface current is constant with height, the wind direction proves to be very variable. Immediately near the water surface this direction coincides with the current direction; with increasing height it approaches the tangent stress direction.[1] Thus when using the equation (2) it should be kept in mind that the angle α between the direction of tangent stress near the water surface and that of wind in the free atmosphere, unlike the case of the atmosphere above the land, does not coincide with the angle of full wind turning in the boundary layer.

Experimental data and results of the semi-empirical theory

Figs. 1–4 contain empirical values of the universal functions A, B, C and D obtained by D. V. Chalikov and the author from the observations made at O'Neill[2] (Lettau & Davidson, 1957). It should be noted that the empirical points in Figs. 1 and 2 are of a markedly less scatter than those in Fig. 3 and particularly in Fig. 4. It is due to great relative errors in the measurements of the parameters T_*, $\delta\theta$, q_* and δq as well as to the fact that the similarity hypotheses on which the equations (1)–(4) are based under real conditions are fulfilled with higher accuracy for the wind profile than for the temperature and humidity profiles. A more considerable scatter in Fig. 4 is explained, in addition to the above-mentioned causes, also by the fact that during the observations the data of which are used here the accuracy of the measurements of q_* and δq was especially low because of the low values of the specific humidity observed.

The curves shown in Figs. 1–4 have been computed by B. G. Vager and the author on the basis of the theoretical model of the non-stationary stratified boundary layer. A problem in the diurnal variations of wind, temperature and the major characteristics of turbulence (pulsation energy, the mixing-length and exchange coefficient) was solved here. In so doing, the same system of the semi-empirical hypotheses was used for closing the equations as in the paper by Bobileva, Zilitinkevich & Laikhtman (1965). Except for the temperature difference in the boundary layer $\delta\theta = \theta_h - \theta_0$, the changes of

[1] A theoretical model of this phenomenon for pure drift current in the ocean was suggested by Laikhtman (1966).

[2] It will be recalled that observations made both above the sea and above the land are equally suited to the determination of the functions under consideration.

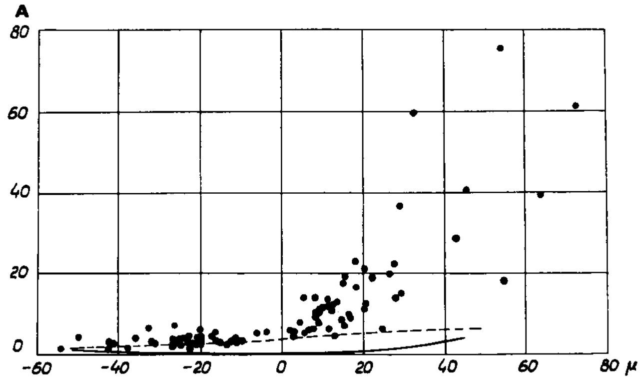

Fig. 1. "Universal" function $A(\mu)$.

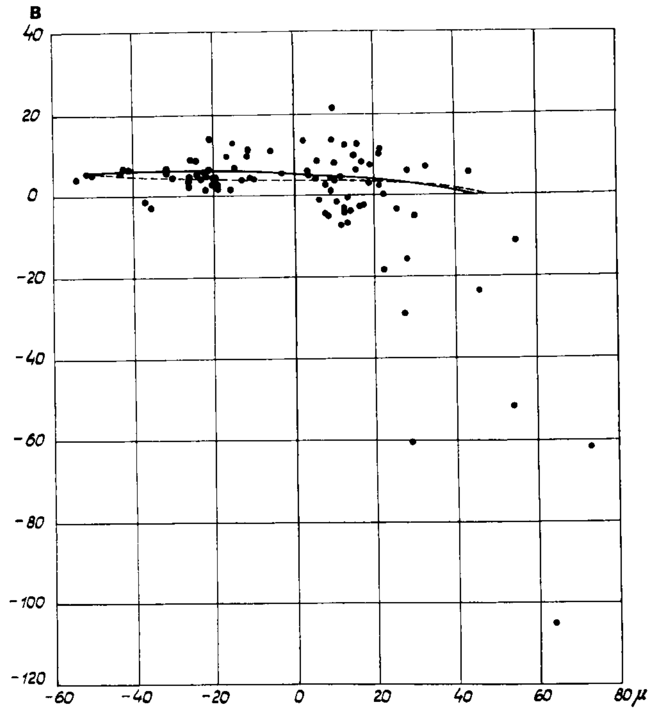

Fig. 2. "Universal" function $B(\mu)$.

which were assumed to be sinusoidal, all other "external" parameters were considered as fixed; the magnitudes of u_*, α, T_* as well as $\tilde{\mu} = k^3\beta T_*/fu_*$ were found as part of the solution. Thus it became possible to correlate each $\tilde{\mu}$ with u_*/G, α, $T_*/\delta\theta$ and, consequently (since the surface Rossby number R_0 is fixed), with the values of the functions A, B and C.[1] We can further take $D = C$ and substitute μ for $\tilde{\mu}$ (*vide* Zilitinkevich, 1967). In so doing, we obtain two, generally speaking, different estimates of the considered function for all μ, except for the extremal ones: one for the moment in the vicinity of which μ increases in the course of time and the other for the moment at which μ has the same value but decreases in the course of time. In the end, at each μ we have a pair of values of each function A, B and $C = D$.

The curves so constructed are given in Figs.

[1] Needless to say that when determining $T_*/\delta\theta$, and hence C, the vicinities of points at which $\delta\theta$ is turned into zero, should be excluded.

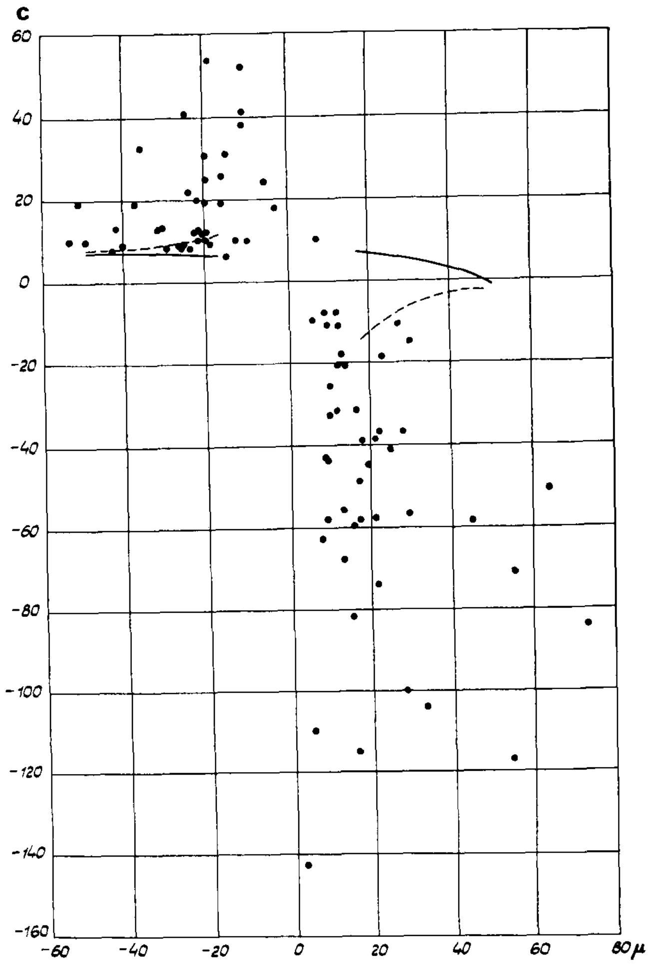

Fig. 3. "Universal" function $C(\mu)$.

1–4. The values corresponding to the moments of increasing μ (i.e. to the type of non-stationarity at which stability in the boundary layer grows) are plotted by solid lines, and those corresponding to the moments of decreasing μ (i.e. the growth of instability), by dashed lines. As one may see from the figures, the differences between curves in each of the pairs (in other words, the disturbances of the universal character of resistance and exchange laws caused by non-stationarity) are not too great. The validity of this conclusion is confirmed also by the results of the empirical determination of the universal functions under study, since the initial experimental material used here is relevant to the well-defined diurnal course.

When speaking about an agreement between the experimental data and the theoretical curves, it should be noted that under stabe conditions ($\mu > 0$) theory underestimates the role of stratification. Yet, qualitative agreement does exist.

The results presented herein make it possible in principle to compute tangent stress and vertical turbulent fluxes of heat and moisture from synoptic information. Such a promise is of interest in particular, for further development of the hydrodynamical methods of weather prediction (especially long-term ones) and the numerical experiments on the general circulation of the atmosphere and the ocean. The principal convenience of the approach suggested consists in that in computations based on the many-level models of the atmosphere no neces-

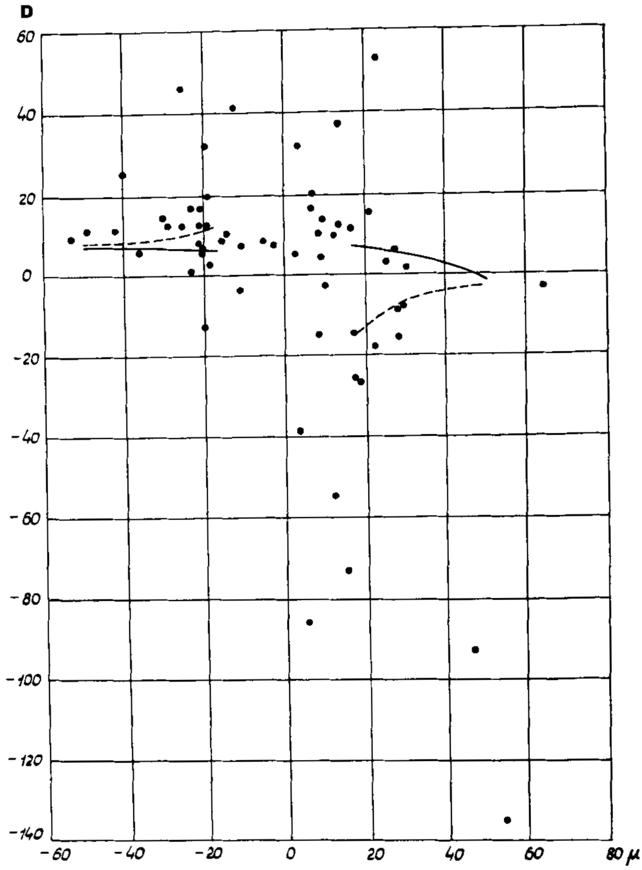

Fig. 4. "Universal" function $D(\mu)$.

sity exists for a detailed description of the vertical structure of the boundary layer. And the accuracy of the determination of the micrometeorological characteristics in question proves, undoubtedly, to be much higher than that which can be attained by numerical integrating the equations of the boundary layer theory if several computation levels are derived for the latter.

The author is indebted to Prof. A. S. Monin for useful discussions.

APPENDIX

A simple derivation of the equations of the type (1)–(4) is presented. Let us take the equation (3) as an example. Based on the similarity theory of Kazansky & Monin (1960) modified with regard to the humidity stratification effect for differences in potential temperatures θ at the heights Z_2 and Z_1, we shall write

$$\frac{1}{T_*}[\theta(Z_2)-\theta(Z_1)]=\varphi_\theta\left(\frac{Z_2}{L_*},\ \mu\right)-\varphi_\theta\left(\frac{Z_1}{L_*},\ \mu\right),$$

where L_* is the length scale of the equation (7), μ is the stratification parameter of the equation (6), φ_θ is the universal function of the two arguments. Let us take as Z_2 the height of the boundary layer assumed to be equal to L_* and put $Z_1 = L_*/n$, where n is a number which is considered to be large enough to have the Monin–Obukhov similarity equation (1954) fulfilled for

$$\theta(Z_1)=\theta_0+\frac{T_*}{\alpha_H^{(0)}}\left(\ln\frac{Z_1}{Z_0}+\beta_\theta\frac{Z_1}{L}\right),$$

where $L = u_*^2/k^2\beta(T_* + 0.61\,Tq_*)$, $\beta_\theta = \text{const}$. As a result of these substitutions we find

$$\frac{\alpha_H^{(0)}\delta\theta}{T_*} - \ln\frac{u_*}{fZ_0} = \ln\frac{k}{n} + \frac{\beta_\theta}{n}\frac{L_*}{L}$$

$$+\alpha_H^{(0)}[\varphi_\theta(1,\mu) - \varphi_\theta(1/n,\mu)].$$

Since the left side does not contain n, the right side should not depend on n either and thus proves to be the function only of μ. By denoting it as $-C(\mu)$ and using the identities $u_*/fZ_0 = \mathrm{Ro}\,u_*/G$ and $L_*/L = \mu$, we obtain the required relation.

The equations (1), (2) and (4) are derived in a similar way.

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О РАСЧЕТЕ ОСНОВНЫХ ХАРАКТЕРИСТИК ВЗАИМОДЕЙСТВИЯ АТМОСФЕРЫ И ОКЕАНА

Обсуждается вопрос о расчете напряжения турбулентного трения и вертикальных турбулентных потоков тепла и влаги в приводном слое атмосферы по „внешним“ параметрам, значения которых могут быть определены по синоптической информации. В основу анализа положено рассмотрение планетарного пограничного слоя атмосферы над морем на основе теории подобия. При этом принимаются во внимание специфические особенности жидкой подстилающей поверхности, а также учитываются процессы в вязком подслое на границе раздела вода–воздух.

Вытекающие из теории подобия формулы для расчета интересующих нас характеристик содержат ряд универсальных функций. Эти функции находятся путем специальной обработки экспериментальных данных. Отметим, что применяемый метод систематизации данных позволяет использовать материалы наблюдений, выполненных над сушей. Приводятся также теоретические оценки значений указанных универсальных функций.

Полученные результаты могут представить интерес для дальнейшего развития гидродинамических методов прогноза погоды, а также численных экспериментов по общей циркуляции атмосферы и океана.