Wind-driven circulation in one- and two-layer oceans of variable depth

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ABSTRACT

The effect of bottom topography is studied for homogeneous and two-layer oceans with Ekman-type dynamics. In the case where friction is important only at a lateral boundary explicit solutions are obtained. Cases in which friction becomes important also in the interior are discussed qualitatively. Essential differences are found between the solutions for the homogeneous and the two-layer oceans. In the homogeneous ocean a western or an eastern boundary current occurs when $\partial/\partial y(f/H)$ is positive and negative, respectively (f is the Coriolis parameter, H the depth, y a northward coordinate), and there is a Sverdrup type interior circulation modified by bottom topography. This case has been discussed previously in the literature. In the two-layer model with the upper layer thin compared to the lower layer the motion in the lower layer becomes small. The upper layer solution is similar to the one-layer solution given by Stommel (1948) with a western boundary current, with some modification due to finite distortions of the interface. In the lower layer there is also a western boundary current. If $\partial/\partial y(f/H) > 0$ this current recirculates at the western boundary and no interior motion occurs. If $\partial/\partial y(f/H) < 0$ an eastern boundary current as well as an interior circulation is added. In both the homogeneous and the two-layer models free jets can cross over the ocean along a critical line, where $\partial/\partial y(f/H) = 0$. If a region with closed f/H-contours exists in the interior a corresponding gyre is added to the solution. The circulation in this gyre becomes very strong for the homogeneous ocean, weak for the two-layer ocean.

In the North Atlantic f/H increases northward, and closed contours exist only in a region at the Azores. According to the two-layer model the deep motion should be confined to a recirculating western boundary current. A deep anticyclonic gyre could possibly be found near the Azores with a jet connecting to the western boundary current along the 2 km depth contour.

I. The models

Consider an oceanic basin bounded by vertical walls in the zonal and meridional directions, and with variable depth. In the present analysis the curvature of the earth is neglected, except as it influences the vertical component of earth's rotation (the so-called β-plane). The basin then has a rectangular shape as shown in Fig. 1. The coordinates are x (eastward), y (northward), and z (upward). The depth H is a function of x and y. Over the ocean acts a zonal wind-stress that is a function of y. The horizontal dimensions of the basin and the scales of the topographic field $H(x, y)$ and the wind-stress field $\tau^w(y)$ are all assumed to be planetary, i.e. of the order of the earth's radius R. The depth H is small compared to this scale and also small compared to other horizontal scales that may occur in boundary layers, etc. This will allow the use of the hydrostatic approximation.

Two types of oceans are considered. The first is a homogeneous ocean, with a uniform density ϱ. The second is a two-layer ocean, with a shallow upper layer of depth h and uniform density ϱ, and a deep lower layer of depth $h' = H - h$ and uniform density $\varrho + \Delta\varrho$. $\Delta\varrho/\varrho$ is assumed small, permitting the Boussinesq approximation to be made. There is no mass exchange between the layers but stresses are assumed to occur at the interface when the layers have a relative motion. In fact, such stresses are found necessary in the two-layer model.

The dynamical equations are the ones introduced by Ekman (1905, 1923), balancing the horizontal pressure gradient by the Coriolis force due to horizontal motion and the friction force due to vertical shear. As mentioned, the β-effect (the variation of the Coriolis parameter with y) is considered. Non-linear acceleration

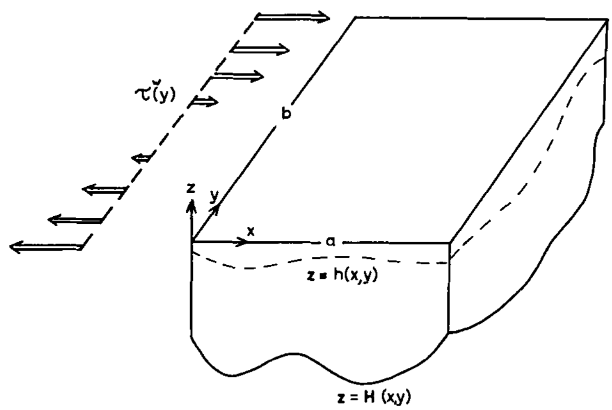

Fig. 1. The model ocean.

terms and lateral friction terms are neglected at present.

For a homogeneous ocean several theoretical studies of the topographic effects have been made in the past. Already Ekman (1923) discussed the "topographic term" in his equations. Some basic work on the topographic effects in rotating fluids including the non-linear acceleration terms has been given by Greenspan (1962, 1963, 1964) and Carrier (1965), and these studies have a direct bearing on the oceanic problem. Examples of wind-driven circulation in a closed β-plane ocean with bottom topography have been worked out by Holland (Diss., 1966). He also includes the effect of non-linear accelerations but has only friction due to horizontal shear.

For stratified oceans less theoretical work has been done. Warren (1963) has shown that the bottom topography probably controls part of the real Gulf Stream, but little is known about the general role of bottom topography in stratified oceans. In special, no solution has been given for the wind-driven circulation in a closed ocean with both stratification and topography included.

The novelty of this work lies mainly in the two-layer model. As seen, the one-layer model has already been extensively investigated by other authors, but a discussion of this case has been included to permit the reader to contrast the two models directly.

II. Equations for the homogeneous ocean

The Ekman type equations are integrated vertically from the top of the ocean $z=\zeta(x,y)$, where ζ is the small elevation from the level surface $z=0$, down to the bottom $z=-H(x,y)$. One finds

$$-fM_y = -(H+\zeta)\frac{\partial p}{\partial x} + \tau^w(y) - \tau^b_{xz}, \qquad (1)$$

$$fM_x = -(H+\zeta)\frac{\partial p}{\partial y} \qquad - \tau^b_{yz}. \qquad (2)$$

Here $M_x = \int_{-H}^{\zeta} \varrho u dz$, $M_y = \int_{-H}^{\zeta} \varrho v dz$ are the mass transport components, p the pressure, ϱ the density, and τ^b_{xz}, τ^b_{yz} the bottom stress components. f is the Coriolis parameter, that is a function of y. Note that the horizontal pressure gradient is independent of depth, as a consequence of the hydrostatic approximation.

By the Ekman theory the bottom stresses are related to the geostrophic velocity components u_g, v_g as follows:

$$\tau^b_{xz} = Df\varrho(u_g - v_g) \quad \tau^b_{yz} = Df\varrho(u_g + v_g), \quad (3\,\text{a, b})$$

where $D = (\mu/2f\varrho)^{\frac{1}{2}}$ is the Ekman depth (μ is a constant coefficient of viscosity), assumed small compared to H. D is a slowly varying function of y.

The geostrophic velocities can be expressed as

$$u_g = \frac{M_x}{\varrho H}, \quad v_g = \frac{M_y + \tau^w/f}{\varrho H}, \qquad (4\,\text{a, b})$$

with sufficient accuracy. As seen the transport term M_y has been corrected for the pure wind transport $-\tau^w/f$ (see Fig. 2).

Inserting these expressions for the bottom stresses in the equations (1, 2) and neglecting terms that are obviously small (of relative order ζ/H and D/H) the equations become

$$-fM_y = -H\frac{\partial p}{\partial x} + \tau^w(y) - \frac{D}{H}fM_x, \tag{5}$$

$$fM_x = -H\frac{\partial p}{\partial y} - \frac{D}{H}\tau^w(y) - \frac{D}{H}fM_y. \tag{6}$$

Dividing by H and eliminating p by cross-differentation gives

$$\frac{\partial}{\partial x}\left(\frac{fM_x}{H}\right) + \frac{\partial}{\partial y}\left(\frac{fM_y}{H}\right) = -\frac{\partial}{\partial y}\left(\frac{\tau^w}{H}\right) - \frac{\partial}{\partial x}\left(\frac{Df}{H^2}M_y\right) + \frac{\partial}{\partial y}\left(\frac{Df}{H^2}M_x\right), \tag{7}$$

where the small term $\partial/\partial x[(D/H^2)\tau^w]$ has been neglected. The equations (5–7) are obtained also for the more general case where $\tau^b_{xz} = Df\varrho(u_g - \varepsilon v_g)$, $\tau^b_{yz} = Df\varrho(\varepsilon u_g + v_g)$, allowing for an arbitrary angle between the geostrophic velocity and the bottom stress. Only the bottom stress component in the transport direction comes into play, and, effectively, one has the friction law used by Stommel (1948).

M_x and M_y satisfy the continuity equation

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} = 0 \tag{8}$$

that involves no approximation. Thus we can introduce a mass transport stream-function ψ through the equations

$$M_x = -\frac{\partial \psi}{\partial y}, \quad M_y = \frac{\partial \psi}{\partial x}. \tag{9a, b}$$

In terms of ψ the equation (7) becomes, after a change of sign:

$$J\left(\frac{f}{H}, \psi\right) = \frac{\partial}{\partial y}\left(\frac{\tau^w}{H}\right) + \frac{\partial}{\partial x}\left(\frac{Df}{H^2}\frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{Df}{H^2}\frac{\partial \psi}{dy}\right), \tag{10}$$

where J is the Jacobian with respect to x and y. This equation has to be solved subject to the boundary condition

$$\psi = 0 \quad \text{at} \quad x = 0, a \quad \text{and} \quad y = 0, b \tag{11}$$

III. A thermal analogy

The problem formulated in the previous section can be translated to an equivalent

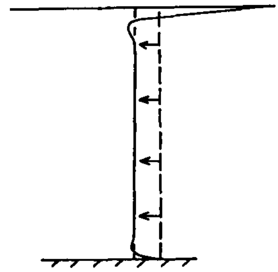

Fig. 2. Vertical velocity profile (one component) for the homogeneous ocean. The pure wind-current transport is $-\tau^w/f$ in the y-direction.

thermal problem, which may be found useful. Introduce a fictitious velocity field U, V with f/H as streamfunction:

$$U = -\frac{\partial}{\partial y}\left(\frac{f}{H}\right), \quad V = \frac{\partial}{\partial x}\left(\frac{f}{H}\right) \tag{12}$$

and write

$$\frac{Df}{H^2} = \varkappa(x, y), \quad \frac{\partial}{\partial y}\left(\frac{\tau^w}{H}\right) = Q(x, y). \tag{13}$$

The equation (10) can obviously be written

$$\frac{D\psi}{Dt} - \nabla\cdot(\varkappa\nabla\psi) = Q, \tag{14}$$

where $D/Dt = U(\partial/\partial x) + V(\partial/\partial y)$ is the rate of change following the fictitious velocity field, and $\nabla = \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y)$. This an equation describing the steady temperature distribution in a two-dimensional, incompressible fluid with the velocity field (U, V) that is heated proportional to Q. The heating function Q as well as the coefficient of heat conduction $\varkappa$ are functions of x and y. The boundary conditions are zero temperature along the lines $x = 0$, a and $y = 0$, b. Note that the fictitious motion in general penetrates the boundary.

IV. Solution for the homogeneous ocean: the case of meridional boundary layers

In the case where the f/H-contours cross over the basin from $x = 0$ to $x = a$, and $\partial/\partial y(f/\mathrm{H})$ everywhere has the same sign the problem can easily be solved. To simplify the problem we assume that $d\tau^w/dy = 0$ and $\mathrm{H} = $ constant at $y = 0$ and b, as will be seen this eliminates boundary layers at these walls.

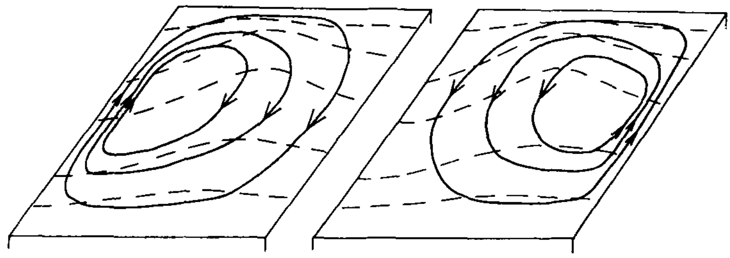

Fig. 3a–b. Circulation in the homogeneous ocean when $\partial/\partial y(f/H) > 0$ (case *a*), and when $\partial/\partial y(f/H) < 0$ (case *b*). In both cases is assumed that $\partial/\partial y/(\tau^w/H) > 0$. The circulation changes sign with $\partial/\partial y(\tau^w/H)$. Notation: ---, f/H contours; →, transport stream-lines.

We anticipate an interior region, where the bottom-stress terms are negligible, and a boundary layer region near $x = 0$ or $x = a$ where the bottom stresses grow large while the wind-stress can be neglected. The arguments for this approach are the same as have been given by several authors for the uniform-depth model. In the boundary layer we can further neglect the term $\partial/\partial y[(Df/H^2)(\partial\psi/\partial y)]$ against $\partial/\partial x[(Df/H^2)(\partial\psi/\partial x)]$, and the term $\partial/\partial x(f/H)\cdot\partial\psi/\partial y$ against $\partial/\partial y(f/H)\cdot\partial\psi/\partial x$. Finally the functions f/H, Df/H^2 etc. and derivatives of these occurring in the boundary layer equations can be evaluated at the boundary ($x = 0$ or a), since these functions have planetary scales. (The notations $(\ )_{x=0}$ and $(\ )_{x=a}$ are omitted in the following.)

Accordingly we write for the *interior region*

$$J\left(\frac{f}{H}, \psi\right) = \frac{\partial}{\partial y}\left(\frac{\tau^w}{H}\right) \tag{15}$$

and for the *boundary layer region:*

$$\frac{\partial}{\partial y}\left(\frac{f}{H}\right)\cdot\frac{\partial\psi}{\partial x} + \frac{\partial}{\partial x}\left(\frac{Df}{H^2}\frac{\partial\psi}{\partial x}\right) = 0. \tag{16}$$

Let us assume that $\partial/\partial y(f/H) > 0$. In this case the topographic effect is in the same direction as the β-effect, and we expect a boundary layer at the western boundary $x = 0$. We start the interior solution at the eastern wall, putting $\psi = 0$ there, and follow the f/H-contours westward with the velocity given by (12). The rate of change of ψ in this system is given by the right hand side of (15), and we can compute the values of ψ all the way to the western boundary $x = 0$, these final values there are called $\psi_1(y)$. (The method described corresponds to the integration of the partial differential equation (15) along the characteristics, that obviously are $f/H = \text{constant}$).

Integrating the boundary layer equation (16) once we have

$$\frac{\partial}{\partial y}\left(\frac{f}{H}\right)[\psi - \psi_0(y)] + \frac{Df}{H^2}\frac{\partial\psi}{\partial x} = 0, \tag{17}$$

where $\psi_0(y)$ is an arbitrary function. (As explained before $\partial/\partial y(f/H)$ and Df/H^2 are treated as constants.) Since the solution has to join the interior solution at the boundary layer edge we require $\psi_0(y) = \psi_1(y)$. The solution to (17) is

$$\psi = \psi_1(y) + A(y)\exp\left[(-H^2/Df)\partial/\partial y(f/H)\cdot x\right]. \tag{18}$$

Since we have assumed $\partial/\partial y(f/H) > 0$ the boundary layer solution decays in the positive x-direction. The condition $\psi = 0$ at $x = 0$ determines the remaining arbitrary function: $A(y) = -\psi_1(y)$.

In the case where $\partial/\partial y(f/H) < 0$ the roles of the western and eastern boundaries are simply interchanged. The interior solution extends now to the western boundary, while a boundary layer occurs near $x = a$. The two cases are exemplified in Figs. 3*a* and *b*.

The width of the boundary layer is $\delta = (Df/H^2)/[\partial/\partial y(f/H)]$, that is of order $(D/H)\cdot R$. It will vary along the boundary depending on the variations of H, f and D. The boundary layer approximations made can be shown to be justified in the case where δ is small compared to the planetary scale R, i.e. when $D/H \ll 1$, as has already been assumed. In special, it can be verified that the bottom stress terms are negligible away from the western (eastern) boundary and that the wind-stress is negligible near this boundary.

The above arguments break down, however, if $\partial/\partial y(f/H) = 0$ at some point. In the neighbourhood of this point the boundary layer width becomes large and some of the neglected terms in the equation must come into play. This case is discussed further in next section.

The solutions derived above have no boundary layers along the zonal walls $y = 0$ and b. The forcing function in the interior equation (15) vanishes at these walls, where is assumed that H = constant and $d\tau^w/dy = 0$, and the boundary condition $\psi = 0$ can be satisfied without any bottom-stress term. If we have arbitrary fields of H and τ^w, boundary layers do occur along the zonal walls. We do not discuss these layers since their nature is similar to that of the meridional boundary layers. As can be seen from the general equation (10), or even more clear from the thermal analogy equation (14), the differential equation itself makes no distinction between the x- and y-directions, such a distinction only comes through the assumed distributions of f/H, τ^w, and Df/H^2.[1]

V. Solution for the homogeneous ocean: cases with interior friction

In the case where $\partial/\partial y(f/H) = 0$ at some point along the boundary a new phenomenon occurs. As one exmple, consider again a basin with f/H-contours that run from $x = 0$ to $x = a$, but assume that $\partial/\partial y(f/H) > 0$ in the southern half and < 0 in the northern half of the basin, with $\partial/\partial y(f/H) = 0$ along a certain "critical contour" (all examples are worked out for a northern hemisphere case). The previous boundary layer solution is expected to hold at some distance from the critical contour, and one should find a western boundary current in the southern region and an eastern boundary current in the northern region of the basin, as shown in Fig. 4. As one follows the western boundary current north and approaches the critical contour a rapid increase in the boundary layer width δ occurs, and at the critical contour the theory predicts that the frictional effects

[1] If the depth is constant along a zonal boundary, and therefore also f/H is constant, $d\tau^w/dy$ will force a boundary layer that grows parabolically downstream. This is a special case that is not included in the discussion.

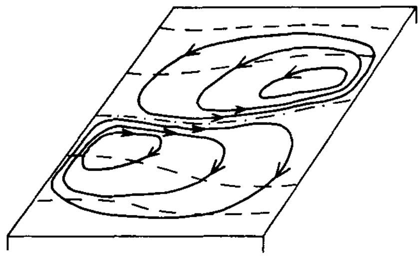

Fig. 4. Circulation in the homogeneous ocean when $\partial/\partial y(f/H) > 0$ in the southern half and < 0 in the northern half (northern hemisphere case). Notation as before. —·—· represents the critical contour.

penetrate all the way to the eastern boundary. However, the full description of the flow near the critical line cannot be obtained from the boundary layer equation. The y-scale is no longer large compared to the x-scale and the complete equation (10) has to be used. It is to be noted that the boundary current carries a certain net transport balancing the interior meridional transport. As we pass the critical contour this boundary layer transport is shifted from the western to the eastern side. By mass continuity a free jet must therefore occur. This jet will run along the critical contour that is itself a line of constant f/H, for the f/H-fields considered here.

One can get some feeling for the different solutions of the present problem from the thermal analogy mentioned earlier (Sect. III). The analogy predicts thermal boundary layers where there is a normal component U_n of the "fictitious velocity" towards a boundary, while the heat is swept into the interior where the normal velocity is away from the boundary. At a critical point where $U_n = 0$ the heat can be diffused into the interior, and a "thermal jet" is formed.[1]

[1] The magnitude of the width of this jet may be of interest to know. One finds that a particle is advected across the basin during a time $t \sim a/(\partial U/\partial y)_{\text{crit.}} \cdot \delta$, if it lies at a distance δ from the critical contour. The diffusion will in the same time spread the heat laterally over a width of order $(\varkappa t)^{\frac{1}{2}}$. The equation $\delta \sim (\varkappa t)^{\frac{1}{2}}$ gives the width of the jet. $\delta \sim (\{\varkappa a/(\partial U/\partial y)_{\text{crit.}}\}^{\frac{1}{3}}$. In terms of oceanic variables $U = -\partial/\partial y(f/H)$, $\varkappa = Df/H^2$, and the width is

$$\delta \sim \left\{ \frac{Dfa}{H^2 \partial^2/\partial y^2 (f/H)_{\text{crit.}}} \right\}^{\frac{1}{3}}. \tag{18}$$

The order of magnitude is $(D/H)^{\frac{1}{3}} \cdot R$, if R represents the basin dimensions as well as the scale of the

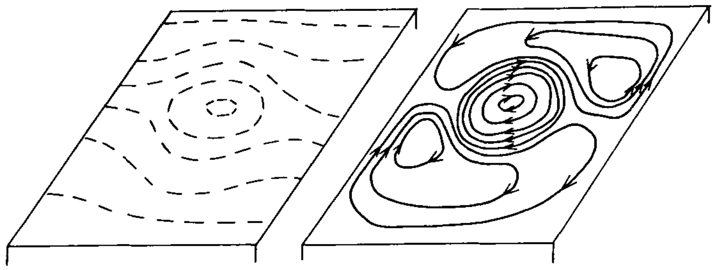

Fig. 5a–b. Circulation in the homogeneous ocean when closed f/H-contours exist. The case *a* shows the f/H-contours, the case *b* the transport streamlines. It is assumed that $\partial/\partial y(f/H) > 0$ in the southern half and < 0 in the northern half, and that $\partial/\partial y(\tau^w/H) > 0$. In the closed-contour region there is a strong circulation.

When f/H takes on a maximum or minimum value somewhere in the interior, another phenomenon of interest occurs. One now has a region with closed f/H-contours. A pure advection cannot hold there. Following a particle along one of the closed f/H contours one has a net input of $\partial/\partial y(\tau^w/H)$, if this keeps one sign. In the thermal analogy, it corresponds to a closed flow under continued heating (or cooling). Obviously diffusion of heat is needed in the entire region of closed contours. If the closed contour region does not extend to the boundary, the heat will be transported from this region to the boundary in a "thermal jet" along a critical contour. Example of such a solution is shown in Fig. 5.

The transports obtained in such a region is large when D/H is small. If the region has a radius r_1, the order of the total transport in the region by a uniform $\partial/\partial y(\tau^w/H)$ is of order $[\partial/\partial y(\tau^w/H)\cdot Hr_1^2]/4Df$. This transport can be compared to the total interior transport, integrated across the ocean, that is of order $(\tau^w/f)\,R$. The ratio of the two values is of order (1/4) $(H/D)(r_1/R)^2$. If the radius r_1 is comparable to R one gets an enormous transport in the closed region. If r_1 is 0.1 R, i.e. about 600 km, one finds that for $D = 20$ m and $H = 5000$ m the above ratio is about 0.6. If r_1 grows to 0.3 R, i.e. about 1900 km, the ratio is about 6, and the added closed-contour circulation would completely dominate the picture.

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f/H-field. This width is larger than the side boundary layer, to the ratio $(D/H)^{-\frac{1}{4}}$. As example, for $D = 20$ m, $H = 5000$ m the ratio is about 40.

VI. Equations for the two-layer ocean. Assumption of quasi-compensation

The dynamic equations are integrated vertically for the upper layer over a depth h (neglecting ζ against h), and for the lower layer over a depth $h' = H - h$:

$$-fM_y = -h\frac{\partial p}{\partial x} + \tau^w(y) - \tau^i_{xz}, \tag{19}$$

$$fM_x = -h\frac{\partial p}{\partial y} - \tau^i_{yz}, \tag{20}$$

$$-fM'_y = -h'\frac{\partial p'}{\partial x} + \tau^i_{xz} - \tau^b_{xz}, \tag{21}$$

$$fM'_x = -h'\frac{\partial p'}{\partial y} + \tau^i_{yz} - \tau^b_{yz}. \tag{22}$$

The transport components and the pressure for the lower layer are denoted by M_x', M_y' and p'. The upper layer is driven by the wind-stress and is retarded by the interface stresses τ^i_{xz}, τ^i_{yz}. The same interface stresses act as driving force for the lower layer. In this layer one has bottom-stresses τ^b_{xz}, τ^b_{yz} as for the one layer model.

Application of the Ekman theory to the interface gives for the interface stresses

$$\tau^i_{xz} = D_i f\varrho[u_g - u'_g - (v_g - v'_g)], \quad \tau^i_{yz} = D_i f\varrho \times [u_g - u'_g + v_g - v'_g], \tag{23 a, b}$$

where $u_g - u_g'$, $v - v_g'$ are the relative geostrophic velocities, and $D_i = (\mu_i/2\varrho f)^{\frac{1}{2}}$ is an Ekman depth related to the interface. The effective coefficient of viscosity μ_i is related to the coefficients μ and μ' of the two fluids according to the formula $\mu_i^{\frac{1}{2}} = 1/(\mu^{-\frac{1}{2}} + \mu'^{-\frac{1}{2}})$.[1] It will be assumed that D_i is of the same order as the Ekman depth D valid near the bottom.

It will further be assumed that the lower layer is much deeper than the upper one. The division of the real ocean in two layers is to a certain degree arbitrary. However, if we put the interface at the level where the average vertical density gradient is maximum, one finds that the ratio of the layer depths is about 0.1

When the assumption of a deep lower layer is introduced one finds that the lower layer motions becomes small compared to the upper layer motions (quasi-compensation). The quasi-compensation β, is physically explained as follows. The wind-stress curl acts as a source of vorticity for the upper layer. This vorticity can locally be changed by divergence-convergence due to a variation in layer depth, by the "geostrophic divergence", that is proportional to the northward velocity, and by an interface stress curl. Integrated over the entire layer no net effect is obtained from either of the two first terms. Therefore, the input of vorticity must be taken up by an interface stress curl. When the wind-stress curl and the interface stress curl are of the same order (in an integrated sense), the total transports in the upper and lower layers are also of the same order. Accordingly, the lower layer velocities must be small compared to the upper layer velocities, in proportion to the ratio h/h'.

These arguments are, of course, not rigorous, and at present the quasi-compensation should be considered as an independent assumption, the validity of which can be checked at the end of the calculation.

With this assumption the problem undergoes an important simplification. The upper layer solution can now be calculated as if the lower layer were at rest. The solution gives the shape of the interface and the values of the interface stresses, to the first approximation. The result is then applied to the lower layer. The interface stresses enter now as known forcing functions, and further the depth h' is given. The problem can therefore be treated in the same way as the one-layer case.

Since we are going to allow finite variations of the interface from its level position (no intersections of the interface with the free surface or the bottom are, however, allowed) the *upper layer* problem is non-linear. The non-linearities occur in the terms $h(\partial p/\partial x)$, $h(\partial p/\partial y)$. In the derivations it seems convenient to keep h rather than the transport stream-function ψ as the variable. The pressure gradient can be directly expressed in terms of h when the motion is assumed to vanish in the lower layer. One has $\zeta = \Delta\varrho h/\varrho$ + constant, and

$$\frac{\partial p}{\partial x} = g\varrho\frac{\partial \zeta}{\partial x} = g\Delta\varrho\frac{\partial h}{\partial x}, \quad \frac{\partial p}{\partial y} = g\varrho\frac{\partial \zeta}{\partial y} = g\Delta\varrho\frac{\partial h}{\partial y}. \tag{24 a, b}$$

The interface stresses, with u_g' and v_g' put equal to zero, become

$$\tau_{xz}^i = D_i f\varrho(u_g - v_g) = -D_i\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\right),$$

$$\tau_{yz}^i = D_i f\varrho(u_g + v_g) = D_i\left(\frac{\partial p}{\partial x} - \frac{\partial p}{\partial y}\right), \tag{25 a, b}$$

and are thus given in terms of $\partial h/\partial x$, $\partial h/\partial y$. The expressions for the transport components obtained from the equations (19, 20) are, neglecting obviously small stress terms:

$$M_x = -g\frac{\Delta\varrho}{f}\left(h\frac{\partial h}{\partial y} + D_i\frac{\partial h}{\partial x}\right), \tag{26}$$

$$M_y = g\frac{\Delta\varrho}{f}\left(h\frac{\partial h}{\partial x} - D_i\frac{\partial h}{\partial y}\right) - \frac{\tau^w}{f}(y). \tag{27}$$

Inserting these values in the continuity equation for the upper layer $\partial M_x/\partial x + \partial M_y/\partial y = 0$ gives a single equation for h:

$$-\frac{\beta}{f^2}h\frac{\partial h}{\partial x} + \frac{\partial}{\partial x}\left(\frac{D_i}{f}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{D_i}{f}\frac{\partial h}{\partial y}\right) = \frac{1}{g\Delta\varrho}\frac{d}{dy}\left(\frac{\tau^w}{f}\right), \tag{28}$$

[1] The result is easily derived from the complex Ekman solutions $w = w_0 \exp[-(i\varrho f/\mu)^{\frac{1}{2}}\cdot z] + w_g$, $w' = w_0' \exp[(i\varrho f/\mu')^{\frac{1}{2}}\cdot z] + w_g'$ where $w = u + iv$ is a complex velocity. The matching conditions are $w = w'$, $\mu(\partial w/\partial z) = \mu'(\partial w'/\partial z)$ at $z = 0$, that is taken to represent the interface. The Boussinesq approximation is assumed.

where $\beta = df/dy$. The boundary conditions are $M_x = 0$ at $x = 0$ and a, $M_y = 0$ at $y = 0$ and b, or

$$h\frac{\partial h}{\partial y} + D_i\frac{\partial h}{\partial x} = 0 \quad \text{at} \quad x = 0, a, \tag{29}$$

$$h\frac{\partial h}{\partial x} - D_i\frac{\partial h}{\partial y} = \frac{\tau^w}{g\Delta\varrho} \quad \text{at} \quad y = 0, b. \tag{30}$$

To avoid boundary layers at $y = 0$, b we assume that $d\tau^w/dy = 0$ at these boundaries.

For the *lower layer* the derivation follows the one given for the homogeneous ocean, with the interface stresses τ^i_{xz}, τ^i_{yz} and the layer depth h' given. Small deviations from the h'-values obtained from the upper layer solution must actually occur. The h'-values are computed under the assumption of perfect compensation, and one actually wants weak motions to occur in the lower layer. The deviations would come into the theory if one wants to evaluate the pressure gradients, but when working with a stream-function equation where pressure is eliminated they never show up explicitly. Their contributions to the h'-factor in the terms $h'(\partial p'/\partial x)$, $h'(\partial p'/\partial y)$ are certainly negligible.

For the bottom stresses we have the same formula (3a, b) as before. The geostrophic velocities are again expressed in terms of the transports:

$$u_g' = \frac{M_x' - \tau^i_{yz} f}{\varrho h'}, \quad v_g' = \frac{M_y' + \tau^i_{xz} f}{\varrho h'}. \tag{31 a, b}$$

In this case M_z' and M_y' are corrected for the pure interface stress transport (in the same way as M_y was corrected for the pure wind-transport in the one layer model). The equations corresponding to (5, 6) are

$$-fM_y' = -h'\frac{\partial p'}{\partial x} - \frac{D}{h'}fM_x' + \tau^i_{xz} + \frac{D}{h'}\tau^i_{yz}, \tag{32}$$

$$fM_x' = -h'\frac{\partial p'}{\partial y} - \frac{D}{h'}fM_y' + \tau^i_{yz} - \frac{D}{h'}\tau^i_{xz}, \tag{33}$$

with obviously small terms neglected. Eliminating the pressure and introducing a transport stream-function for the lower layer ψ' we get the equation

$$J\left(\frac{f}{h'}, \psi'\right) = \frac{\partial}{\partial y}\left(\frac{\tau^i_{xz}}{h'}\right) - \frac{\partial}{\partial x}\left(\frac{\tau^i_{yz}}{h'}\right) - \frac{\partial}{\partial x}\left(\frac{D}{h'^2}\tau^i_{xz}\right) - \frac{\partial}{\partial y}\left(\frac{D}{h'^2}\tau^i_{yz}\right) - \frac{\partial}{\partial x}\left(\frac{Df}{h'^2}\frac{\partial\psi'}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{Df}{h'^2}\frac{\partial\psi'}{\partial y}\right). \tag{34}$$

This corresponds to the equation (10) for the homogeneous ocean. The forcing function includes now a meridional stress that cannot be neglected. In fact, this will become the dominant term. The "correction terms"

$$\partial/\partial x[(D/h'^2)\tau^i_{xz}],\ \partial/\partial y[(D/h'^2)\tau^i_{yz}]$$

are expected to be small, but strictly we need the estimates of the interface stresses and the relative scales in the x- and y-directions before their contributions can be seen. The boundary conditions for the lower layer are finally

$$\psi' = 0 \quad \text{at} \quad x = 0, a \quad \text{and} \quad y = 0, b. \tag{35}$$

VII. Solution for the upper layer

The equation for the upper layer in the "compensated" approximation (28) is similar to the equation for a homogeneous ocean of uniform depth discussed by Stommel (1948). Thus we try a solution of the same general type, i.e. an interior region valid up to the eastern wall $x = a$, and a boundary layer at the western wall $x = 0$. The boundary layer simplifications are the same as given earlier for the homogeneous ocean. Boundary layers at $y = 0, b$ are avoided by the assumption that $d\tau^w/dy = 0$ at these walls. The *interior form* of the equation is

$$-\frac{\beta}{f^2}h\frac{\partial h}{\partial x} = -\frac{1}{g\Delta\varrho}\frac{d}{dy}\left(\frac{\tau^w}{f}\right) \tag{35}$$

and the *boundary layer form* applicable at the western boundary is

$$-\frac{\beta}{f^2}h\frac{\partial h}{\partial x} + \frac{\partial}{\partial x}\left(\frac{D_i}{f}\frac{\partial h}{\partial x}\right) = 0. \tag{36}$$

The boundary condition (29), in the interior approximation, states that h should be constant along $x = a$, we call this value h_0. It has to be prescribed in the problem (it can be related to the mean depth of the layer). Integrating the *interior equation* (35) with this boundary value we get the solution

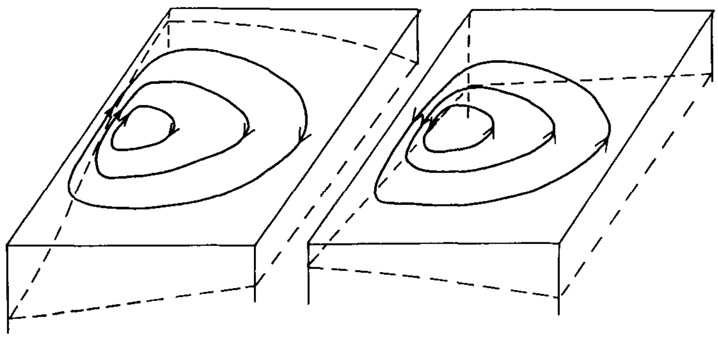

Fig. 6a–b. The upper layer solution for the two-layer ocean when $\partial/\partial y(\tau^w/f) > 0$ (case a) and < 0 (case b). The interface and the transport streamlines are shown.

$$h^2_{\text{int.}} = h_0^2 + \frac{2f^2}{\beta}\frac{d}{dy}\left(\frac{\tau^w}{f}\right)(a-x). \tag{37}$$

The value of $h_{\text{int.}}$ at $x = 0$ is denoted by $h_1(y)$, this is needed in matching the boundary layer. Integrating the *boundary layer equation* (36) once we have

$$\frac{\beta}{2f}[h_2^2(y) - h^2] + D_i\frac{\partial h}{\partial x} = 0, \tag{38}$$

where $h_2(y)$ is an arbitrary function. In order to joint the interior solution it is required that $h_2(y) = h_1(y)$. The solution to equation (38) is

$$h = h_1 \tanh\left[\frac{\beta h_1(y)}{2fD_i}\cdot x + C(y)\right] \quad \text{(this applies for}$$

$$h \leqslant h_1, \text{ for } h > h_1 \text{ change to coth}), \tag{39}$$

where $C(y)$ is another arbitrary function. It can be obtained as follows. From the boundary condition (29), applying at the wall $x = 0$, we get by use of equation (38)

$$\frac{\partial}{\partial y}h^2 = \frac{\beta}{f}[h_1^2(y) - h^2]. \tag{40}$$

This can be integrated in the y-direction to give h along the wall $x = 0$. The value of h at one point on the boundary needs to be known for this integration. As example, one can take the value at $y = 0$, where it agrees with the interior value. (This follows because M_y is geostrophic and moreover $M_y = 0$ along $y = 0$.) When $h_{x=0}$ is found $C(y)$ is given, $C(y) = \tanh^{-1}[h_{x=0}/h_1(y)]$, and the solution is completely determined. It can be shown that only a western boundary layer can exist (this is not immediately clear from (39), as $\tanh x \to 1$ both for large positive and large negative x). Numerical examples of solutions can be found in an earlier paper (Welander, 1966). A general sketch of the solution is found in Figs. 6a and b, in the cases of positive and negative $d/dy(\tau^w/f)$. The circulation looks much the same as for the uniform depth case treated by Stommel (1948). The width of the boundary current varies, however, along the boundary. If h_1 becomes very small and the boundary layer thus very wide, the boundary layer approximation breaks down and other terms come in. If $h_1 = 0$ at some point, i.e. if the interface intersects the free surface, a completely new problem arises, since the wind-stresses can now directly drive the lower layer. This case is not considered here.

The distribution of the interface stresses is of special interest, since these stresses drive the lower layer motion. The magnitude of the zonal and meridional stress can easily be estimated from the previous equations. One finds that the zonal interface stress is always small, of order $(D_i/h)\tau^w$. The meridional stress is of the same order in the interior, but in the boundary layer it grows large, to the order of τ^w. The forcing term in the lower layer equation (34) has its only essential contribution at the western boundary. The only possible exception is when the f/h'-contours are closed in the interior (see Sect. IX). The distribution of the interface stresses is examplified in Fig. 7, this case corresponds to the solution shown in Fig. 6a.

VIII. Solution for the lower layer: the case of meridional boundary layers

As for the homogeneous ocean we consider first the case where the f/h'-contours cross over the basin from $x=0$ to $x=a$, and $\partial/\partial y(f/h')$ has one sign. An interior region where the bottom stresses are negligible, and a boundary layer region at $x=0$ or $x=a$ where the bottom stresses are important is then anticipated. In contrast with the homogeneous ocean the driving stresses must be retained at the western boundary layer but can be neglected in the interior.

In this case we have a free geostrophic flow in the interior, governed by the equation

$$J\left(\frac{f}{h'}, \psi'\right) = 0 \qquad (41)$$

The transport is accordingly along the f/h'-contours.

To avoid boundary layers at $y=0$ and b the depth h' should be taken constant along these boundaries. This is a somewhat artificial condition, since we would have to change the total depth H according to the upper layer solution. However, when H (and its variation) is large compared to the upper layer depth h, the approximate condition is $H=$constant along $y=0$ and b, as in the homogeneous ocean model.

The derivation of the boundary layer equation valid at the western boundary $x=0$ requires certain care. To begin with the equation (34) can be simplified by dropping all the interface stress terms except the dominant term $-\partial/\partial x(\tau^i_{yz}/h')$. The neglection of $\partial/\partial y[(Df/h'^2)\,(\partial\psi'/\partial y)]$ against $\partial/\partial x[(Df/h'^2)\,(\partial\psi'/\partial x)]$ is also safe. In the Jacobian it is not obvious that the term $\partial/\partial y(f/h')\,\partial\psi'/\partial x$ dominates over $\partial/\partial x(f/h')\,\partial\psi'/\partial y$, similarly to the earlier boundary layer calculations, since h' itself has a boundary layer variation. The relative magnitude of the two terms in the Jacobian is obviously related to the boundary layer scales for h' and ψ'. Now the boundary layer scale for the upper layer, that applies also to h', is of order $(D_i/h)\,R$, and the boundary layer scale for ψ', determined from the internal balance in (34), is of order $(D/h')\,R$. As the lower layer depth is assumed large compared to the upper layer depth while D and D_i are assumed to be of the same order, it follows that the second scale is of smaller order. In this case the term $\partial/\partial x(f/h')\,\partial\psi'/\partial y$ can be neglected, and we can evaluate the coefficients of the remaining ψ'-equation at $x=0$. The larger boundary layer scale applies also for the forcing function, and accordingly this function can be neglected. However, this conclusion is valid only in the layer where the bottom stress terms are important. Outside this layer there will be a region where the forcing is important but the bottom stress term negligible. This is an "interior boundary layer". In this layer the scales for h' and ψ' are the same, and both the terms in the Jacobian have to be retained.

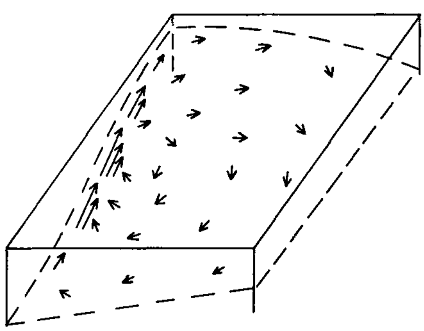

Fig. 7. Distribution of interface stresses in the solution shown in Fig. 6*a*.

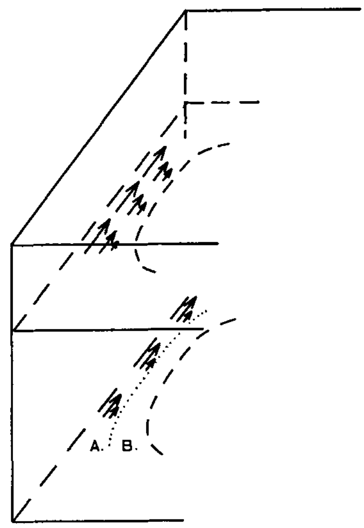

Fig. 8. Interface and bottom stresses at the western boundary in the two-layer ocean. At the bottom A represents the frictional boundary layer and region B the "interior boundary layer", that is directly forced by the interface stresses.

According to these arguments we have one *frictional boundary* layer next to the wall

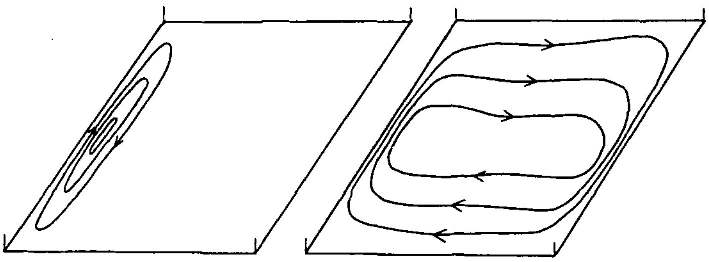

Fig. 9a–b. Circulation in the lower layer of the two-layer ocean when $\partial/\partial y(f/h') > 0$ (case a) and < 0 (case b). It is assumed that $d/dy(\tau^w/f) > 0$. If the sign of $d/dy(\tau^w/f)$ is changed the circulation reverses, there is only a small change in the stream-line pattern. Compare with Fig. 3a–b representing a corresponding case for the homogeneous ocean.

described by the equation

$$-\frac{\partial}{\partial y}\left(\frac{f}{h'}\right)\frac{\partial \psi'}{\partial x}+\frac{\partial}{\partial x}\left(\frac{Df}{h'^2}\frac{\partial \psi'}{\partial x}\right)=0 \quad (41\,\text{a})$$

and an *interior boundary layer* where the interface stresses force the solution according to the equation

$$J\left(\frac{f}{h'}, \psi'\right) = -\frac{\partial}{\partial x}\left(\frac{\tau^i_{yz}}{h'}\right). \quad (41\,\text{b})$$

The regions are shown schematically in Fig. 8.

At an eastern boundary the conditions are simpler. There is no driving interface stress, and h' has planetary scale. In this case an equation of the type (41a) holds throughout; there is no interior boundary layer.

The equation (41a) for the frictional boundary layer can be solved to give

$$\psi' = \psi_0'(y) + A(y)\exp\left[(h'^2/Df)(\partial/\partial y)(f/h')\cdot x\right] \quad (42)$$

where $\psi_0'(y)$ and $A(y)$ are arbitrary functions. These have to be determined from the matching to the interior boundary layer, for the western boundary layer solution, and to the interior, for the eastern boundary layer solution.

The solution obtained will have a character that depends critically on the sign of $\partial/\partial y(f/h')$. Look first at the case $\partial/\partial y(f/h') > 0$. Then a western boundary layer solution of the form (42) exists, while there is no boundary layer at the eastern boundary. Thus we start the interior solution from $x = a$ where $\psi' = 0$, and from (41) follows that $\psi' = 0$ over the interior. Going across the interior boundary layer at the western side ψ' changes, however. This change is found from the equation (41b), following the contours of constant f/h'. The value of ψ' at the edge of the frictional boundary layer is then known. Call this $\psi_1'(y)$, then the boundary condition at $x = 0$, and the joining condition determines $\psi_0'(y)$ and $A(y)$ in (42): $\psi_0'(y) = -A(y) = \psi_1'(y)$. The solution of this type looks as shown in Fig. 9a. One has a recirculating western boundary current and no motion in the interior.

Next look at the case where $\partial/\partial y(f/h') < 0$. For the homogeneous ocean a change in sign of $\partial/\partial y(f/H)$ simply meant that the western and eastern boundaries changed their role. In the present case this is not true. When $\partial/\partial y(f/h') < 0$ no frictional boundary layer exists at the western wall, as seen from (42), but the interior boundary layer does exist. Starting from $x = 0$ with $\psi = 0$, (41b) determines now non-zero values of the streamfunction at the interior side of this boundary layer, that are denoted $\psi_1'(y)$. These values are advected across the interior region along the f/h'-contours, according to (41). At the eastern boundary there is a frictional boundary layer described by (42), or, more conveniently by $\psi' = \psi_0'(y) + A(y)\exp\left[(h'^2/Df)\partial/\partial y(f/h')(x-a)\right]$.

The boundary condition and the joining condition again give $\psi_0'(y) = -A(y) = \psi_1'(y)$. The solution is represented in Fig. 9b. One has a forced, non-frictional boundary layer at the western wall that also induces an interior motion. At the eastern wall this motion is taken up by a frictional boundary layer.

It seems possible to generalize these results to cases where D_i is of different order than D. If D_i is made larger than D nothing is changed in the equations, it only means that the width of the interior boundary layer increases (we must, of course, all the time assume that the Ekman depth is small compared to the layer depth). If D_i is smaller than D, of relative order h/h', the interior boundary and the frictional boundary layer interact, and it is no longer possible to neglect bottom friction at the western side in the case when $\partial/\partial y(f/h') < 0$. Still the solution does not need to blow up. Although the free solutions for the frictional boundary layer blow up at the western side one may have forced solutions that behaves well and decay towards the interior. As example, look at the equation $-a(\partial\psi/\partial x) + \partial/\partial x[b(\partial\psi/\partial x)] = F(x)$, that is the forced form of the boundary layer equation (41a). For simplicity we treat the coefficients as constants. The solution can then be written $\psi = \psi_0(y) + A(y)$ exp $(ax/b) + \int_{t=0}^{x} A(t)$ exp $[a/b(x-t)]dk$. If $F(t)$ decays rapidly enough one can determine ψ_0 and A such that ψ vanishes at $x=0$ and remains finite for a ux. This is easily seen in the special case where F decays exponentially, $F(t) = e^{-ct}$. The solution then defines a ψ at the edge of the boundary layer that can be joined with the interior as before. If D_i is still smaller the frictional boundary layer is wider than the interior boundary layer. In this case the frictional part can be neglected altogether and one goes directly from an interior boundary layer at the wall to the interior solution.

IX. Solution for the lower layer: cases with internal friction

In the case where $\partial/\partial y(f/h')$ changes sign in the basin, assuming still that the f/h'-contours cross over from $x=0$ to $x=a$, one finds a transition from one type of boundary layer to the other, as the critical point is passed. An example is shown in Fig. 10, assuming $\partial/\partial y(f/h') > 0$ in the southern half and <0 in the northern half.

In the southern half one has a closed boundary layer circulation and no interior motion, in the northern half a forced western boundary layer and a frictional eastern boundary layer connecting by an interior flow as well as by a free jet along the critical contour.

The second case of interest is when the f/h'-contours are closed. In this case, violent effects were found for the homogeneous model. Here the situation is different since there is no interior forcing function. Accordingly there is no need for friction in the interior. However, the interior solution is then undetermined. Looking at small order terms one sees that interface stresses do occur also in the interior, and they may be critical here. The effect can easily be estimated. The order of magnitude of the interior interface stresses is $(D_i/h)\tau^w$. Assuming the region of closed contours to have a radius r_1 the total transport obtained in the closed contour region is of order

$$\tfrac{1}{4}(D_i/D)(h'/h)(r_1^2/R)(\tau^w/f).$$

We can compare this with the total lower layer transport, that is of order $(\tau^w/f)R$. The ratio of the "closed contour transport" to the total transport is therefore of order

$$\tfrac{1}{4}(D_i/D)(h'/h)(r_1/R)^2.$$

One finds that the "closed-contour" transport can be noticeable also in this model. If r_1 is 0.1 R, i.e. about 600 km, $D_i/D=1$, and $h/h'=0.1$, it would only be a few per cent of the total transport. If r_1 is increased to 0.3 R, i.e. about 1900 km, its contribution would have increased to over 20%. An example of a possible stream-line pattern is shown in Fig. 11.

X. General comparison of the two models. Effects of neglected terms

The present study shows that the homogeneous and the two-layer oceans respond quite differently to the bottom topography. The fact that the homogeneous model feels the topography throughout the depth, the stratified mainly in the lower layer is, of course, expected. More interesting is the result that the circulation in the lower layer of the stratified ocean differs in its general pattern from that of the homogeneous ocean. In the homogeneous ocean we find a western or an eastern boundary current depending on the sign of $\partial/\partial y(f/H)$. Where $\partial/\partial y(f/H)=0$ the whole boundary current separates and crosses over the ocean. In the two-layer case there is always a western boundary current in the lower layer. When $\partial/\partial y(f/H)<0$ there is an eastern boundary current as well (we do not distinguish between

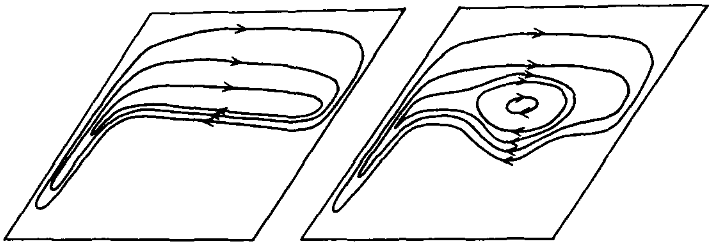

Fig. 10 *Fig. 11*

Fig. 10. Circulation in the lower layer of a two-layer ocean when $\partial/\partial y(f/h') > 0$ in the southern half and < 0 in the northern half. It is assumed that $d/dy(\tau^w/f) > 0$. Compare with Fig. 4 representing a corresponding case for the homogeneous ocean.

Fig. 11. Circulation in the lower layer of a two-layer ocean when a region of closed f/h'-contours is added. Compare with Fig. 5*b* that represents a corresponding case for the homogeneous ocean.

h' and H here, assuming the lower layer depth being mainly controlled by bottom topography). The separation of the western boundary current where $\partial/\partial y(f/H) = 0$ is only partial.

An essential difference occurs also in interior regions where the contours of f/H are closed. In both cases frictional effects must become important. In the homogeneous ocean these effects are, however, much stronger and associated with a gyre of geostrophic currents that may dominate the entire circulation picture. In the two-layer ocean the effects that occur in the lower layer are weaker, although not necessarily negligible.

These differences between the two models are explained by the different ways that the motion is forced. In the homogeneous ocean there is a windstress driving the circulation over the entire area, while in the lower layer of the stratified ocean the important driving comes from interface stresses set up in the western boundary layer region.

To obtain explicit solutions it has been necessary to assume the simplest possible dynamics, i.e. the Ekman-Sverdrup equations. One may ask about the influence on the solution of a closed wind-driven circulation by terms that have been neglected.

Studies including non-linear accelerations have been carried out by a number of authors for a homogeneous ocean. For a stratified case no complete solutions have been worked out. It is, however, expected that the lower layer in the present model will be affected by non-linear accelerations in much the same way as the homogeneous model. The boundary currents will probably overshoot the point where $\partial/\partial y(f/H) = 0$ and when (partially) leaving the wall meander around the controlling f/H-contour, as described by Robinson & Niiler (1966). When the non-linear effect is large the boundary currents may penetrate to the zonal boundaries and return in a quite sluggish way, as demonstrated in the numerical study by Veronis (1966). With regard to the "closed-contour regions", it is expected that the non-linear acceleration weakens the gyre predicted by the present frictional theory. If inertia allows a flow across these regions very strong frictional effects will not be needed.

With regard to lateral friction it can be easily seen that the homogeneous and the two-layer oceans will be differently affected. In the homogeneous ocean the addition of lateral friction merely changes details in the boundary currents, as can be seen by comparing the models by Stommel (1948) and Munk (1950). In the two-layer model, on the other hand, the addition of a strong lateral friction kills the motion in the lower layer, as the upper layer can balance internally without need of interface stresses. Then the effect of the bottom topography is completely lost. At the present it is not possible to estimate the relative effect of the two types of friction. There is further doubt whether the lateral friction can at all be represented by laminar-type terms. In the real ocean this friction is created by a turbulence in which the earth's rotation is very important, and it seems established that in certain cases this produces negative "Austausch"-coefficients.

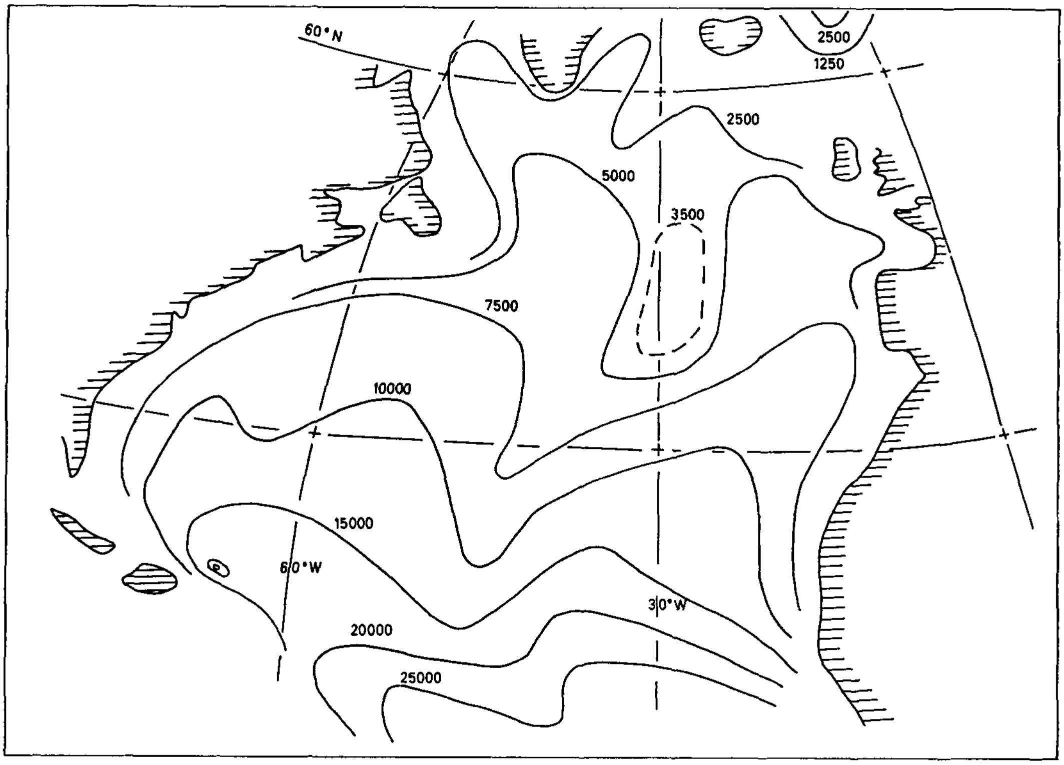

Fig. 12. Contours of $H^* = H/\sin\varphi$ for the North Atlantic. f/H is constant along these lines.

There is another essential approximation made in the present study that needs some comments, namely the two-layer representation of the stratification. In the real ocean there is a thermocline produced by a balance of vertical advective-diffusive processes. To replace this by a material surface neglecting all mass exchange between the surface layer and the deep sea is a drastic simplification. It is possible to include the thermohaline processes in a two-layer approximation, using results of recent theoretical studies by Stommel & Webster (1962). Some results from such a modelling will be presented in a later paper.

The two-layer representation leads to the existence of important interface stresses, that seem difficult to accept for the real ocean. This does not necessarily mean that the results derived are incorrect. In a three-dimensional model with continuous stratification there occurs a twisting of the vortex tubes due to vertical shear, and this process is of importance to keep the over-all vorticity balance. In the two-layer model, or any layered model, this twisting is excluded since the fluid layers have no shear. At the interface there is, however, a concentrated shear. Through the interface stresses these sheares formally enter in the vorticity balance. If one thinks of the two-layer model as representing a real three-dimensional ocean one should therefore look at the interface stresses as representing concentrated "twisting effects" rather than real forces. It seems important to have this question investigated in more detail starting from the full hydrodynamic equations.

XI. Application to the North Atlantic

It is of interest to see which general type of circulation is predicted for a real oceanic basin. As example we look at the North Atlantic. As seen from the theory the controlling feature is f/H. In Fig. 12 is shown a bottom topography chart for the North Atlantic in which the real depths are replaced by $H^* = H/\sin\varphi$ where φ is the latitude. These contours are therefore also contours of constant f/H.

Generally there is a decrease in the depth H northward, i.e. increase in f/H northward. There are no important closed contours in the interior except for a region of about 1000 km extension near the Azores. Applying the homogeneous model to this basin one would find a western boundary current and a topographically distorted "Sverdrup transport" in the interior. The closed-contour region would be associated with a quite strong anticyclonic gyre, with velocities comparable to those in the Gulf Stream. The homogeneous model will in this case give a prediction that looks less realistic than a constant-depth model, such as the one by Stommel (1948).

Applying the two-layer model, that is assumed to be a better representation of the real ocean, the upper layer solution looks essentially as the Stommel solution. In the lower layer one would find a recirculating western boundary current and negligible interior circulation. The boundary current would have an anticyclonic circulation in the main part of the basin, from about 15° N to 50°. As an isolated feature one would find an interior, deep gyre of anticyclonic circulation in the closed contour region at the Azores. The velocities in this gyre could be of a magnitude comparable to the upper layer velocities. The gyre would connect to the western boundary with a jet following about the 2000 m depth curve.

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ВЕТРОВАЯ ЦИРКУЛЯЦИЯ В ОДНО- И ДВУХСЛОЙНЫХ ОКЕАНАХ ПЕРЕМЕННОЙ ГЛУБИНЫ

Изучается эффект топографии дня для однородного и двухслойного океанов с динамикой экмановского типа. Для случая, когда трение важно только на боковой границе, получены явные решения. Случаи, при которых становится важным также и трение в открытом океане, обсуждаются качественно. Обнаружены существенные различия между решениями для однородного и для двухслойного океанов. В однородном океане пограничное течение у западного или восточного берегов имеет место, когда величина $\partial/\partial y(f/H)$ положительна или отрицательна, соответственно (f — параметр Кориолиса, H — глубина океана, y — координата, положительное направление на Север), имеется

также и циркуляция типа Свердрупа в открытом океане, модифицированная топографией дна. Этот случай уже обсуждался в литературе. В двухслойной модели с верхним слоем, более тонким по сравнению с нижним слоем, движение в нижнем слое становится малым. Решение для верхнего слоя подобно решению для одного слоя, полученному Стоммелом (1948) для пограничного течения у западного берега, с некоторой модификацией, обусловленной конечными возмущениями границы раздела. В нижнем слое также имеется пограничное течение у западного берега. Если $\partial/\partial y(f/H) > 0$, это течение циркулирует у западного берега, а в открытом океане движения нет. Если $\partial/\partial y(f/H) < 0$, то добавляется пограничное течение у восточного берега, также как и циркуляция в открытом океане. Как в однородной, так и в двухслойной моделях, свободные струи могут пересекать океан вдоль критической линии, где $\partial/\partial y(f/H) = 0$. Если существует область с замкнутыми контурами f/H во внутренних частях океана, то к решению добавляется соответствующий круговорот. Циркуляция в этом круговороте становится очень сильной для однородного океана, и более слабой для двухслойного океана.

В Северной Атлантике величина f/H возрастает в северном направлении, и замкнутые контуры существуют только в области Азорских островов. Согласно двухслойной модели глубинное течение должно ограничиваться замкнутым пограничным течением у западного берега. Глубинный антициклонический круговорот вероятно может быть найден вблизи Азорских островов со струей, связывающей его с пограничным течением у западного берега вдоль контура глубины 2 км.