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Description of the sedimentation scheme used operationally in all Météo-France NWP models

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ABSTRACT

In this paper, a cost-efficient sedimentation algorithm is described. The numerical resolution of the sedimentation equation is replaced by three probabilities of transfer associated with the following three types of precipitation: (i) precipitation present in the layer at the beginning of the time step; (ii) precipitation coming from the layer above and crossing the layer under consideration and (iii) precipitation produced locally during the time step. This scheme can be considered as a special case of the more general statistical approach where a simple step function is chosen as the basis for these probabilities. The main advantage is a near-perfect reproduction of advective (Eulerian or Lagrangian) classical sedimentation schemes at low computation cost. The practical use of the scheme in the operational models of Météo-France is described. We believe that this approach can be used to compute sedimentation in any microphysical scheme. Moreover, the simplicity of the algorithm makes it possible to consider improvements which would have been complex to implement in the original schemes.

1. Introduction

The operational numerical weather prediction (NWP) models used at Météo-France are developed in cooperation with ECMWF (European Center for Medium-Range Weather Forecast) and the ALADIN consortium. Météo-France uses three models operationally. The first one, ARPEGE, is a global model with variable resolution. It uses a 4DVAR data assimilation system. At present, the highest horizontal resolution of this model is 15 km over France and its lowest resolution is 80 km at the antipode of France, few hundred kilometres east New Zealand. It has 60 vertical levels. The second model is the LAM (Limited Area Model) model ALADIN with a 10 km mesh, 60 vertical levels and 3DVAR data assimilation. The third one, AROME, is also a LAM with a 2.5 km mesh and 41 vertical levels. It uses the non-hydrostatic dynamical core which was developed in cooperation with the ALADIN consortium (Bénard et al., 2010).

The question of whether it is necessary to have an explicit treatment of the precipitating species sedimentation in NWP models remains open. In models where the precipitation is described through diagnostic equations it is assumed that the precipitation particles fall through the atmosphere within a time that is smaller than the characteristic time scale for horizontal transports, that is the particles will reach the ground before entering the adjacent grid-point column. In this case it is possible to prescribe stationarity and to neglect horizontal transport. This condition of vertical equilibrium for the precipitating species limits the validity of the approach to coarse resolutions. For scales of 10 km or larger, the numerical solution obtained by a diagnostic budget equation for rain and snow is nearly as accurate as a complete three-dimensional treatment (Ghan and Easter, 1992). When the model's spatial resolution is increased, the assumption of column equilibrium for the precipitating constituents becomes more and more unrealistic. If a characteristic value of 5 $m\,s^{-1}$ for the terminal fall velocity for rain and 1 $m\,s^{-1}$ for snow and a vertical fall distance of typically 2500 m for frontal precipitation are assumed, the time scales for sedimentation of rain and snow are estimated at 500 and 2500 s, respectively. At 15 $m\,s^{-1}$ wind speed, the corresponding horizontal displacements are about 8 km for rain and 40 km for snow. This effect should be taken into account for grid spacings of about 10 km or less. Horizontal transport is particularly important for the generation of lee-side precipitation. A diagnostic treatment tends to largely overestimate the precipitation amount at the mountain tops and to underestimate the amount on the downstream lee-side (Gassman, 2002; Baldauf and Schulz, 2004; Bouteloup et al., 2005).

For high-resolution models, a description of precipitating species through prognostics equations is then required, but the

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numerical treatment of sedimentation turns out to be quite difficult. In the case of precipitation falling through thin model layers near the surface, the sedimentation Courant number may become larger than one. In the ECMWF model, the choice has been made to compute sedimentation diagnostically. In the UK Meteorological Office's Unified Model, an intermediate way has been chosen (Wilson and Ballard, 1999): rain water is removed from the system during the time step but Eulerian scheme is used to compute the sedimentation of ice without distinguishing cloud ice from snow. To avoid numerical problems, it is assumed that the mixing ratio of ice at each level cannot decrease below a fixed value. This formulation is sensitive to the time step. In most other prognostic schemes, a process time-splitting method or a Lagrangian algorithm are used. For instance, in the Weather Research and Forecasting model (WRF), which is being developed as a collaborative effort among the NCAR Mesoscale and Microscale Meteorology (MMM) Division, the National Oceanic and Atmospheric Administration's (NOAA) and the National Centers for Environmental Prediction (NCEP), Eulerian sedimentation scheme using a small time step to ensure numerical stability is used (Hong et al., 2004; Skamarock et al., 2005). On the other hand, in the Regional Atmospheric Modeling System (RAMS), developed at Colorado State University, a Lagrangian scheme is used to transport precipitating species in the vertical direction (Walko et al., 1995). Finally, in the model used by the COSMO consortium, because the old time-splitting scheme was too time consuming, sedimentation is now computed with a quasi-implicit formulation with a predictor–corrector method allowing for microphysicals interactions during fallout (Gassman, 2003).

The microphysical scheme used in the operational models ARPEGE and ALADIN was first developed by Lopez (2002). It is based on the approach of Fowler and Randall (1996), where any time stepping is overcame by the use of a Lagrangian scheme for the fall of rain and snow. In contrast to its predecessor, the current microphysical scheme uses a prognostic treatment of precipitation to provide a finer description of the temporal evolution of the vertical distribution of precipitation and, thus, of the effects of latent-heat release associated with sublimation and evaporation. In this Lagrangian approach, the use of a double vertical loop is necessary to compute all processes acting on the precipitation fluxes. The numerical cost is then proportional to the square of the number of vertical levels. With the increase of the vertical resolution to 60 levels, it has then been decided to use the more efficient scheme which will be described in this paper.

In AROME, a detailed parameterization of cloud microphysics is used (Pinty and Jabouille, 1998; Caniaux et al., 1994). Five classes of hydrometeors are considered (cloud liquid water and cloud ice, rain, snow and graupels). Rain, snow and graupel precipitate but it is also possible to activate the sedimentation of cloud ice and cloud liquid water to obtain a better simulation of cirrus and fog. The sedimentation scheme uses a Eulerian algorithm. To maintain stability, the sedimentation is computed with a time-splitting technique. The time substep used for this computation is determined from the Courant stability criterion based on a maximum rain fall velocity of 10 m s^{-1}. The thinnest layer in AROME has a thickness of 35 m, which leads to a time substep around 3.5 s. With a time step of 60 s, the sedimentation process is calculated 17 times during the time step. The falling speed of each species is computed by integration over the size spectrum. The falling speed is different at each time step because the speed is recomputed at each stage of the time-splitting process. As in ARPEGE and ALADIN, to save computing time, it has been decided to use the scheme described in this paper in operation.

In this paper, a simple and stable scheme is proposed to compute the sedimentation of precipitation. It is used in all Météo-France operational models. The main idea is to replace the numerical resolution of the sedimentation equation by three probabilities of transfer associated with the following three types of precipitation: (i) precipitation already present in the layer at the beginning of the time step; (ii) precipitation coming from the layer above and (iii) precipitation produced locally by processes such as autoconversion, collection and melting. Sinks such as evaporation or melting of snow are taken to be evenly spread over the three types of precipitation.

The scheme is described in Section 2. In Section 3, the sedimentation process is studied for two academic cases. The results of simulations over France with ALADIN and with AROME during September 2008 are shown in Section 4. Finally, some conclusions are drawn in Section 5. In this paper, all of the description of the sedimentation process refers to application to single-moment bulk microphysics schemes, where the pronostic variable is the mixing ratio. At the end of Section 2 a comment explains how the method would apply to two-moment schemes.

2. Description of the sedimentation scheme

Here, we apply the reasoning to rain but it could be readily extended to any precipitating species. It is shown in this section that it is possible to define a sedimentation algorithm taking into account the microphysics based on two equations, a diagnostic equation for the precipitation flux and a prognostic equation for the mixing ratio. The only new hypothesis is the assumption that the precipitation flux is constant during the time step. In Section 2.1, the scheme is presented as a generalization of the approach of Rotstayn (1997) (R97 hereafter). In Section 2.2, the basic proportions used by the scheme are computed.

2.1. *Equations of the scheme*

The budget equation for rain water mass can be written as

$$\frac{\partial q_\mathrm{r}}{\partial t} = -\vec{V}.\vec{\nabla} q_\mathrm{r} + \frac{1}{\rho}\frac{\partial}{\partial z}F_\mathrm{r} + s_\mathrm{r}^o - s_\mathrm{r}^i. \tag{1}$$

In this equation, q_r is the mixing ratio of rain, F_r is the precipitation flux, ρ is the density of air, $\vec{V}$ is the wind and s_r^o and s_r^i are the sources and sinks of rain, respectively. For rain, sources are auto-conversion, collection and melting of snow; sinks are evaporation and freezing. The advection term is computed by the dynamical core of the model. The equation to be solved by the microphysics scheme is

$$\frac{\partial q_r}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}F_r + s_r^o - s_r^i. \tag{2}$$

In this paper, we assume that levels are numbered from top to bottom. Full-levels represent layers where prognostic variables of the model such as q_r are defined; half-levels are layer interfaces where fluxes such as F_r are defined. A layer j is conventionally defined as the part of the atmosphere between two half-levels $j-1$ and j.

The vertical and temporal discretization of eq. (2) is

$$q_r^+(j) = q_r^-(j) + \frac{\Delta t}{\rho \Delta z}\left[F_r(j-1) - F_r(j)\right] + S_r^o - S_r^i, \tag{3}$$

where Δz is the thickness of the current layer j, Δt is the time step, $S_r = \Delta t s_r$, $F_r(j-1)$ is the incoming precipitation flux at the top of the layer, $F_r(j)$ the output flux, q_r^- is the value of the rain mixing ratio before microphysics (including sedimentation) and q_r^+ the value after microphysics.

There are two unknown quantities in eq. (2), q_r and F_r. To solve the problem, F_r is generally related to q_r by

$$F_r = \rho q_r V_r^t, \tag{4}$$

where V_r^t is the mass-weighted bulk terminal fall velocity of rain. This leads to the following equation

$$\frac{\partial q_r}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho q_r V_r^t\right) + s_r^o - s_r^i. \tag{5}$$

Equation (5) is not easy to solve because s_r^o, s_r^i and V_r^t are non-linear functions of q_r. Lopez (2002) gives a Lagrangian solution. In many other schemes, eq. (5) is split into two equations which are solved sequentially. The first one takes microphysical processes into account (without sedimentation):

$$\left(\frac{\partial q_r}{\partial t}\right)_{\text{micro}} = s_r^o - s_r^i, \tag{6}$$

while the second one takes the sedimentation process alone into account

$$\left(\frac{\partial q_r}{\partial t}\right)_{\text{sed}} = \frac{1}{\rho}\frac{\partial F_r}{\partial z}. \tag{7}$$

All microphysical processes are computed before the sedimentation; q_r is updated and the sedimentation process alone is applied to the new value of q_r. This sequential approach requires small time steps. Evaporation and collection are not computed during the fall but only in the layer where the drops are at the beginning of the time step.

Following R97, it is possible to define a vertical discretized form of eq. (7), where the output flux is written as a function of q_r and V_r^t

$$\left(\frac{\partial q_r}{\partial t}\right)_j = \frac{F_r(j-1)}{\rho \Delta z} - q_r(j)\frac{V_r^t(j)}{\Delta z}. \tag{8}$$

Assuming that the flux coming from the upper-level $F_r(j-1)$ is constant during the time step, it is possible to integrate eq. (8), which yields

$$q_r^+(j) = q_r^-(j)\exp(-C) + \frac{F_r(j-1)\Delta t}{\rho \Delta z}\{1 - \exp(-C)\}/C, \tag{9}$$

where $C = (V_r^t\,\Delta t)/\Delta z$ is the sedimentation Courant number. For future convenience, (9) can be written as

$$q_r^+(j) = \left(1 - P_1\right)q_r^-(j) + \left(1 - P_2\right)\frac{F_r(j-1)\Delta t}{\rho \Delta z} \tag{10}$$

with $P_1 = 1 - \exp(-C)$ and $P_2 = 1 - P_1/C$. P_1 and P_2 are plotted in Fig. 1 as a function of non-dimensional time $\tau = t(V_r^t/\Delta z)$.

In R97, eq. (9) is integrated from the top to the bottom of the atmosphere. $F_r(j-1)$ is calculated from $q_r^+(j-1)$ and $V_r^t(j-1)$ using eq. (4).

The approach of R97 has some limitations. As shown in Fig. 1, P_1 is still smaller than 1 with a Courant number greater than 1. With an advective algorithm it is expected that all the rain mass have left the layer when $t > \Delta z/V_r^t$. Similarly, with an advective algorithm, P_2 should be zero until $t > \Delta z/V_r^t$, the time necessary for rain to cross the layer from the level above. Finally, the scheme accounts for sedimentation alone and it does not seem simple to build a generalized equation directly from eq. (10) in the presence of sources and sinks.

To avoid these limitations, we propose a more direct and phenomenological approach to define sedimentation fluxes,

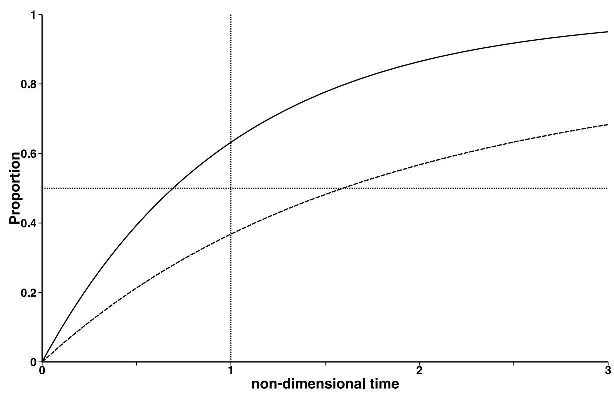

Fig. 1. Evolution of the two proportions P_1 (solid line) and P_2 (dashed line) following R97 (see text) as a function of the non-dimensional time $\tau = t(V_t/\Delta z)$.

allowing more flexibility while ensuring appropriate robustness. As a starting point, the vertical and temporal discretization of eq. (7) is written as

$$q_{\mathrm{r}}^{+}(j) = q_{\mathrm{r}}^{-}(j) + \frac{\Delta t}{\rho \Delta z}\big(F_{\mathrm{r}}(j-1) - F_{\mathrm{r}}(j)\big). \tag{11}$$

The expression of $q_{\mathrm{r}}^{+}(j)$ as a function of $q_{\mathrm{r}}^{-}(j)$ and $F_{\mathrm{r}}(j-1)$ is kept as in eq. (10) but P_1 and P_2 are left arbitrary. Then, combining eqs. (11) and (10), a diagnostic equation for F_{r} can be obtained,

$$F_{\mathrm{r}}(j) = P_1 \frac{\rho \Delta z}{\Delta t} q_{\mathrm{r}}^{-}(j) + P_2 F_{\mathrm{r}}(j-1). \tag{12}$$

The solution of the microphysics parameterization (microphysics and sedimentation) can be solved first by diagnostically computing F_{r} from the top to the bottom of the atmosphere without taking microphysical processes into account, using eq. (12), then by computing the temporal evolution of q_{r} through eq. (3). This formulation is equivalent to R97 but it is easier to generalize.

It is used operationally in AROME but with proportions P_i computed to behave as closely as possible to the Eulerian advective algorithm. They are described in Section 2.2.

When the time step is large, the error due to omitting source and sink terms in the computation of the precipitation flux F_{r} can be large. We propose a generalization of eq. (12) to solve this problem:

$$F_{\mathrm{r}}(j) = \left(1 - \frac{S_{\mathrm{r}}^{i}(j)}{q_{\mathrm{r}}^{-}(j) + (\Delta t/\rho \Delta z)\, F_{\mathrm{r}}(j-1) + S_{\mathrm{r}}^{o}(j)}\right) \times \left(P_1 \frac{\rho \Delta z}{\Delta t} q_{\mathrm{r}}^{-}(j) + P_2 F_{\mathrm{r}}(j-1) + P_3 \frac{\rho \Delta z}{\Delta t} S_{\mathrm{r}}^{o}(j)\right). \tag{13}$$

When sources and sinks are equal to zero, eq. (13) is equivalent to eq. (12). The contribution of source terms from the current layer j to the precipitation flux are taken into account by the proportion P_3, which is described and computed in Section 2.2. The precipitation flux is computed sequentially from the top to the bottom of the atmosphere, hence source terms from the levels above are contained in the incoming flux $F_{\mathrm{r}}(j-1)$. It is assumed that sinks are evenly spread over all the terms. This is taken into account by the first factor on the right-hand side of the equation. The microphysical computation ensures that the sinks cannot be larger than the sum of the other terms. Hence this factor is always positive and bounded by 0 and 1.

Terminal velocity, and source and sink terms are non-linear functions of q_{r}. They are computed using the known part of the precipitation flux. For instance, collection and evaporation processes are computed using $(\rho\Delta z/\Delta t)q_{\mathrm{r}}^{-} + F_{\mathrm{r}}(j-1)$. This means that rain drops that are created by the collection process do not evaporate and so collect, not in the current layer but in the layers below. Equation (13) is used operationally in ARPEGE and ALADIN.

However, it is necessary to compute the three basic proportions (P_1, P_2 and P_3). As will be discussed later, there is a wide choice of methods for computing these proportions. Météo-France's operational scheme has been built to mimic advective processes (Eulerian or Lagrangian). In the following section, the proportions are computed so as to behave as closely as possible to the advective algorithms.

2.2. *Computation of the three basic proportions*

The proportions P_1, P_2 and P_3 are now defined together with relevant assumptions. Although P_1 and P_2 could be computed through basic considerations, a more general formulation using an intermediate function P_0 is introduced here, to make the computation of P_3 clear and show the link with other schemes described in the literature.

Consider a set of raindrops with a terminal velocity V_{r}^{t}. From this set, the proportion $P_0(z, t)$ of drops able to pass at least a vertical distance z during a time t is

$$P_0(z,t) = \begin{cases} 0 & \text{if } \dfrac{V_{\mathrm{r}}^{t} t}{z} < 1, \\[2ex] 1 & \text{if } \dfrac{V_{\mathrm{r}}^{t} t}{z} \geq 1. \end{cases} \tag{14}$$

Now let us consider a layer of the model containing a certain quantity of rain. Assuming that the vertical distribution inside the layer is homogeneous, it is possible to compute the proportion P_1 of rain which leaves the layer during the time step Δt. All the raindrops fall during the time step Δt, but from a height which varies from 0 to the thickness Δz of the layer:

$$P_1 = \frac{1}{\Delta z} \int_0^{\Delta z} P_0(z, \Delta t)\mathrm{d}z. \tag{15}$$

Secondly, consider drops of rain which are not in the layer under consideration at the beginning of the time step but which come from the level above. Among all the drops that enter the layer under consideration (from above) during the time step, P_2 is the proportion of those which also leave the layer (by the bottom) during the time step. To compute P_2, it is necessary to make the hypothesis that the incoming flux of rain mass from the level above remains constant during the time step. This hypothesis is new and was not made in the original Lagrangian algorithm nor in the Eulerian one. Its impact will be shown later. In terms of P_0, P_2 can be written as

$$P_2 = \frac{1}{\Delta t} \int_0^{\Delta t} P_0(\Delta z, t)\mathrm{d}t. \tag{16}$$

Thirdly, consider the case of rain mass created by microphysical processes during the time step. It is assumed that the production of rain is uniform in time and in space. It is then possible to compute the proportion P_3 of rain mass created in the layer during the time step which also leave the layer during the time

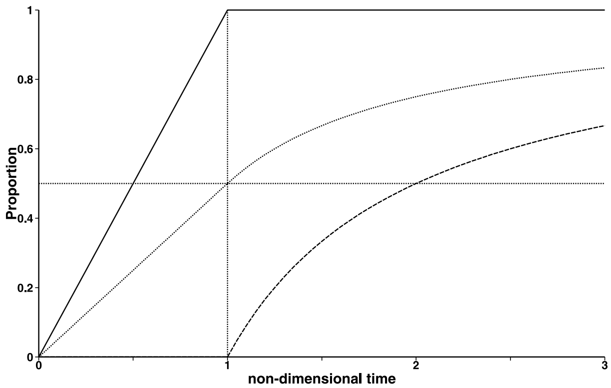

Fig. 2. Evolution of the three proportions P_1 (solid line), P_2 (dashed line) and P_3 (dotted line) defined in the text as a function of the non-dimensional time $\tau = t(V_t/\Delta z)$.

step. In terms of P_0, P_3 can be written as

$$P_3 = \frac{1}{\Delta t \Delta z} \int_0^{\Delta z} \int_0^{\Delta t} P_0(z, t) \mathrm{d}z \mathrm{d}t. \tag{17}$$

This double integral can be written

$$P_3 = \frac{1}{\Delta z} \int_0^{\Delta z} P_2(z, \Delta t) \mathrm{d}z = \frac{1}{\Delta t} \int_0^{\Delta t} P_1(\Delta z, t) \mathrm{d}t. \tag{18}$$

Using eq. (14) for P_0, it is found that

$$P_1 = \min(1, C), \tag{19}$$

where $C = (V_\mathrm{r}^t \, \Delta t)/\Delta z$ still denotes the sedimentation Courant number,

$$P_2 = \max\left(0, 1 - \frac{1}{C}\right), \tag{20}$$

$$P_3 = \begin{cases} C/2 & \text{if } C \leq 1 \\ 1 - 1/2C & \text{if } C \geq 1 \end{cases} \Rightarrow P_3 = \frac{P_1 + P_2}{2}. \tag{21}$$

The three proportions P_1, P_2 and P_3 are shown in Fig. 2 as a function of non-dimensional time $\tau = t(V_t/\Delta z)$.

2.3. Link with Geleyn et al. (2008)

Geleyn et al. (2008) (G08 hereafter) proposed a statistical approach for the sedimentation of prognostic precipitating species. They tried to find an alternative to the single fall speed hypothesis. The single fall speed is replaced by a spectrum of fall speeds going from zero to infinity. The single fall speed may be regarded as the mass-weighted average of this spectrum, that is the first moment of the associated probability density function (PDF). To ensure that the normalized probabilities are identical whatever the origin of the drops leaving a model layer through its bottom, they proposed that the PDF should be forced to always be a decreasing exponential. In G08, the proportion P_0 of drops able to fall at least a vertical distance z during a time t is defined as $P_0 = \exp\big(-z/\big(V_t t\big)\big)$. Proportions P_1, P_2 and P_3 are computed using eqs. (15)–(17). The scheme described in this paper can be considered as a special case of G08's PDF-based approach where a simple step function is used for probabilities.

Whatever the expression of the P_i, a new class of sedimentation scheme has been defined, where the precipitation flux is computed using eqs. (12) or (13) and where prognostic eq. (3) is used for q_r. These schemes are unconditionally stable. The proportions needed by the scheme can be built from the sedimentation equation as in R97, defined to mimic Eulerian or Lagrangian sedimentation algorithms as described here or based on a PDF of fall speeds as in G08.

2.4. Practical implementation in ARPEGE/ALADIN and AROME

The sedimentation algorithm presented in the previous section is implemented in both ARPEGE/ALADIN and AROME microphysics. The algorithm first computes the sedimentation flux using eq. (13) in ALADIN or eq. (12) in AROME, then solves the prognostic equation (3). In AROME, precipitating species are rain, snow, graupel and, optionally, fog liquid water and cirrus cloud ice. Terminal velocities are computed at each time step from the mixing ratio of the precipitating species, following Pinty and Jabouille (1998), as in the original time-splitting Eulerian scheme.

In ALADIN and ARPEGE, eq. (13) is used to diagnose the precipitation flux. Unlike the AROME case, ARPEGE/ALADIN simple microphysics has only two precipitating species, rain and snow. The terminal velocities are constant and equal to $0.9 \mathrm{~m\,s^{-1}}$ for snow and $5 \mathrm{~m\,s^{-1}}$ for rain.

In this paper, two applications are shown, the first one uses eq. (13) and constant terminal velocities, the second one uses eq. (12) but with variable terminal velocities. This choice has been made because the sedimentation scheme described in this paper has been introduced in the existing operational models without any other change in the microphysics parameterizations. Equation (13) is not used in AROME because its implementation would require extensive changes in the microphysics algorithmic and, as discussed in the conclusion, there is no evidence that it yields to any improvement of the model. It is easy to see that there is no problem to use eq. (13) with variables terminal velocities. Terminal velocities are used only to compute the proportion P_i, that is before the computation of the precipitation flux. Moreover, it is expected to use variable terminal velocities (and sedimentation of cloud) in ALADIN and ARPEGE during the year 2010. As shown in the next section, the main weakness of eq. (12) is a dependency of the results to the time step.

2.5. Generalization to two-moment bulk microphysics schemes

The previous description refers to application to single-moment bulk microphysics schemes. In eq. (5), q_r is the mixing ratio of rain and V_r^t the mass-weighted bulk terminal fall velocity. In a two-moment scheme, a very similar equation is introduced to describe the number concentration (Morisson et al., 2005)

$$\frac{\partial N_r}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho N_r V_{Nr}^t\right) + s_{Nr}^o - s_{Nr}^i \tag{22}$$

where N_r is the number concentration of raindrops, s_{Nr}^o and s_{Nr}^i the sources and sinks of N_r and V_{Nr}^t the number-weighted terminal velocity. The generalization of the method described here to number concentration can be easily done introducing a number concentration sedimentation flux F_{Nr} defined by

$$F_{Nr} = \rho N_r V_{Nr}^t. \tag{23}$$

This leads to a new formulation for the prognostic equation of N_r

$$\frac{\partial N_r}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z} F_{Nr} + s_{Nr}^o - s_{Nr}^i. \tag{24}$$

It is possible to compute the number concentration sedimentation flux using an equation similar to eq. (13), where the P_i are computed using the number-weighted terminal velocity and then to compute the evolution of the number concentration using eq. (24).

3. Study of the sedimentation process in two academic experiments

In this section, two academic experiments are described. The first one is a comparison of the proposed sedimentation scheme to the Lagrangian and Eulerian formulation in a one-dimensional study. In this experiment the sedimentation schemes are used alone, all other processes are switched off, that is eq. (13) is equivalent to eq. (12).

In the second experiment, we show the impact of the use of eq. (13) instead of eq. (12) when evaporation is switched on.

3.1. Academic study of sedimentation algorithms alone

A set of raindrops of a precipitating species is distributed uniformly in a layer around the height 10 km in an one-dimensional model. This means that the altitude of the corresponding full-level is 10 km. All processes except sedimentation are switched off. To test the algorithm used in ARPEGE/ALADIN, the falling speed is set to 5 m s^{-1}, which is the value used for rain in the operational model. AROME's algorithm is tested with the operational formulation to compute the terminal velocity. The vertical resolution is the same for all the academic experiments and is the 41-levels distribution used in operational AROME.

In the following figures, the evolution of the vertically integrated content of the precipitating species scaled by the initial content is plotted as a function of time. Before the first raindrop reaches the ground the value is 1. When the last raindrop has reached the ground, the value is 0. A value of 0.5 means that half the drops have reached the ground. It is then possible to estimate

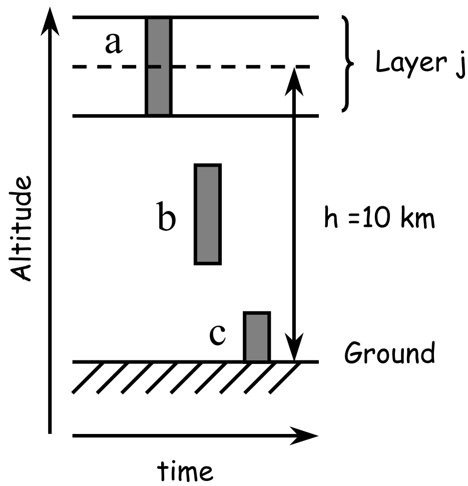

Fig. 3. Illustration of the academic experiment. (a) Initial state, a set of raindrops (grey rectangle) is distributed uniformly in a layer around the height $h = 10$ km. (b) The drops have run through a part of the atmosphere, but the integrated content is still equal to 1. (c) Half the drops have reach the ground, integrated content is equal to 0.5. In the case of a constant falling speed, the time between state (a) and state (c) is $t = h/V_t$.

the average velocity of the set of drops by dividing the height by the time corresponding to the value 0.5. In ARPEGE/ALADIN, the falling speed is constant, so it is expected that this value should be equal to the average velocity. A sketch illustrating the experiment is shown in Fig. 3 .

In Fig. 4 , the experiment is made with the old operational Lagrangian algorithm of ARPEGE/ALADIN with three different time steps: 300, 60 and 10 s. A small time step implies a dispersion of drops in the vertical direction. The smaller the time step, the greater the dispersion. With a small time step, the trajectory of the drops is shorter than the thickness of the level. A set of drops is then spread out over several levels.

In Fig. 5 , the experiment is made with the new operational algorithm of the ARPEGE and ALADIN models. The minimal dispersion is found with a time step of 60 s. With a time step of 10 s the cause of the dispersion is the same as for the Lagrangian algorithm. For larger time steps, a new phenomenon impacts the falling process. The hypothesis of a continuous incoming flux from the layer above becomes less valid. The result is a vertical dispersion of the drops.

Figure 6 shows the results with the original AROME sedimentation scheme. The Eulerian scheme uses a time-splitting technique which makes it time independent. The curve is superimposed with that of the new operational scheme which uses a 10 s time step. In addition to the vertical dispersion, there is a change in the average falling speed of the set of raindrops. In AROME, the terminal velocity is a function of the mixing ratio of the precipitating species. A more or less vertical dispersion of raindrops leads to a change in the mixing ratio and thus a change in the terminal velocity.

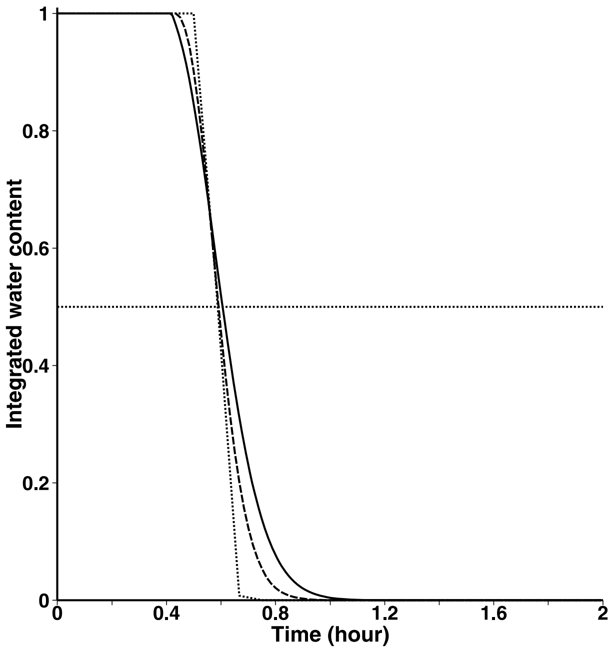

Fig. 4. Temporal evolution of the integrated content of a precipitating species normalized by the initial content (see text) for the old operational Lagrangien algorithm with three different time steps. Solid line: $\Delta t = 10$ s; dashed line: $\Delta t = 60$ s; dotted line: $\Delta t = 300$ s.

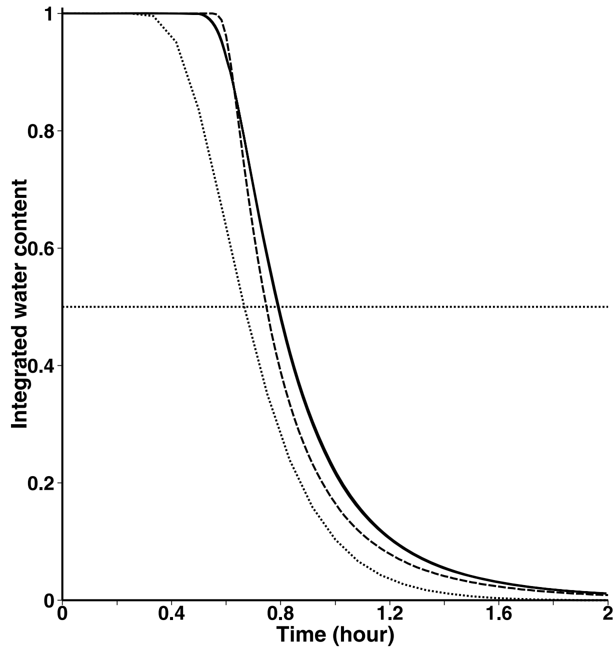

Fig. 6. Sensitivity to time step of the operational approach in the AROME sedimentation scheme. Solid line: $\Delta t = 10$ s; dashed line: $\Delta t = 60$ s and dotted line: $\Delta t = 300$ s. Eulerian scheme is plotted but the curve is superimposed with the 10s curve.

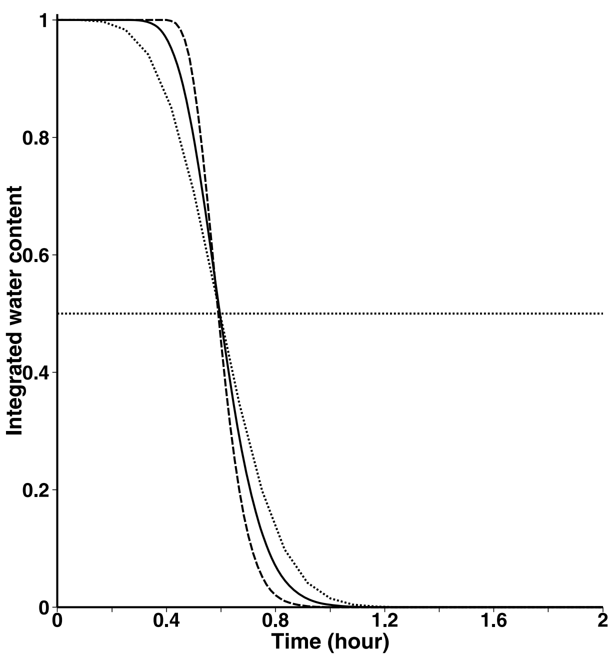

Fig. 5. Same as Fig. 4 but for the new operational algorithm used in ARPEGE/ALADIN. Solid line: $\Delta t = 10$ s; dashed line: $\Delta t = 60$ s and dotted line: $\Delta t = 300$ s.

3.2. Academic study of the evaporation process

In the previous experiment, all microphysical processes were switched off, implying that eq. (13) amounts to eq. (12). Another experiment was made, always in a simple academic framework, but with at least one microphysical process activated, to actually compare the behaviour of the two formulations. A very simple experiment is designed: a constant input flux of precipitation (1 mm h^{-1} with 5 m s^{-1} terminal velocity) comes from the upper level in a layer where the relative humidity is forced to be 60% during all the simulation. This means that the evaporation flux does not contribute to the evolution of the relative humidity. The thickness of the layer is chosen equal to 300 m, a value representative of the vertical resolution of the Météo-France operational models in the mid-troposphere. A sketch illustrating the experiment is shown in Fig. 7 . A stationary state is rapidly reached. The output precipitation flux converges towards a constant value and no evolution is observed. With the chosen settings, about 10 time steps are enough to reach stationarity and results are shown after 20 time steps. In the first case, the output flux is computed using eq. (12), in the second case, the output flux is computed using eq. (13).

The experiment is made with different values for the time step: 60, 120, 300, 600 and 1200 s. Results are shown in Fig. 8. With eq. (13) there is no dependency to the time step, whereas with eq. (12) there is an impact of the time step when the sedimentation Courant number is larger than 2 ($\Delta t > 120$ s). This

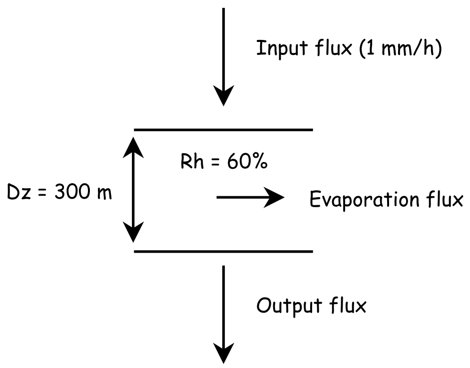

Fig. 7. Illustration of the second academic experiment. A precipitation flux of rain comes from the upper level in a layer with a relative humidity of 60%. The output flux is computed using eq. (12) or eq. (13).

experiment highlights the benefits to use eq. (13) in models with large time steps, and more specifically, large sedimentation Courant numbers.

Other experiments (not shown) testing other microphysical processes (accretion, melting, etc.) were made. They show that the dependence of eq. (12) to the time step is only for sink terms, not for sources. Further results, described in the next section, with the AROME model that uses eq. (12) show that it is not easy to highlight this weakness in three-dimensional real-cases.

4. Application to the ARPEGE/ALADIN/ AROME operational microphysical scheme

In this section, the practical implementation of the operational sedimentation scheme to the Météo-France operational models is described and forecast results for the period of September 2008 are shown.

4.1. *AROME case*

Due to the short time step (60 s) used in the operational model AROME and due to the algorithmic structure of the microphysical scheme, the precipitation flux was computed with eq. (12). The study was made for September 2008. A series of 30 forecasts was made. The 24 h precipitation accumulation for the most rainy day with the old Eulerian algorithm is presented in Fig. 9.

Results with the new operational sedimentation scheme are shown in Fig. 10. For this case, the precipitation pattern is very similar, both for low values and for the maximum observed. It is possible to look at these precipitation plots day by day but it is more convenient to condense the information into a representative score. This was done by interpolating raingauge observations and model predictions on a 0.1 grid over France. Then, Heicdke Skill Scores (HSS in the following) for six classes were computed. The classes intervals for 24 h precipitation accumulation (30 h forecast minus 6 h forecast) were [0.2, 1], [1, 2], [2, 5], [5, 10], [10, 20] and >20 mm. HSS was defined by

$$\mathrm{HSS} = \frac{\left(\frac{a+d}{n}\right)_{\mathrm{exp}} - \left(\frac{a+d}{n}\right)_{\mathrm{stand}}}{1 - \left(\frac{a+d}{n}\right)_{\mathrm{stand}}}, \tag{25}$$

where $n = a + b + c + d$ is the number of cases and a, b, c and d are defined in Table 1, subscript 'exp' is for experiment forecasts and subscript 'stand' is for standard forecasts. In this study, the previous day's accumulation of observed precipitation was taken as the standard forecast.

The HSS measures the fractional improvement of the forecast over the standard forecast. Like most skill scores, it is normalized by the total range of possible improvement over the standard, which means that HSS can safely be compared with different data sets. The range of the HSS is $-\infty$ to 1. Negative values indicate that the standard forecast is better, 0 means no skill, and a perfect forecast obtains an HSS of 1. The results presented in Fig. 11 are very similar, illustrating the innocuity of the more efficient proposed scheme for AROME.

4.2. *ARPEGE/ALADIN case*

The microphysical parameterization used in the Météo-France operational models ARPEGE and ALADIN arose from the work of Lopez (2002). To simulate the Lagrangian algorithm as closely as possible, eq. (13) is used to compute the precipitation flux. There are only two precipitation species, rain and snow.

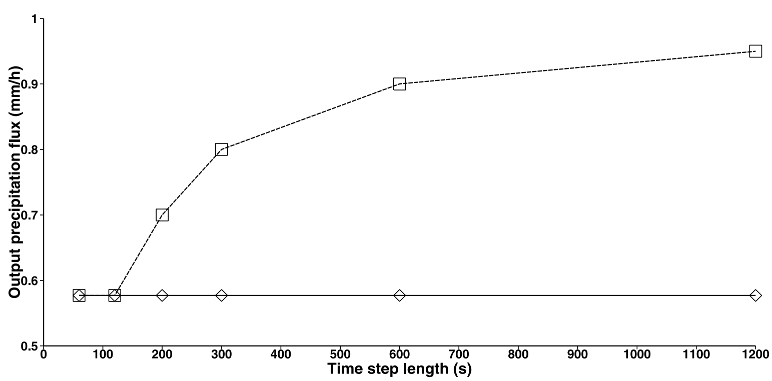

Fig. 8. Stationary output precipitation flux from a 300 m layer where the relative humidity is fixed to 60%. The output flux is plotted as a function of the time step length. With eq. (13) (solid line with diamond) there is no dependency to the time step. With eq. (12) (dashed line with square) there is an impact of the time step when the vertical CFL criterion is larger than 2.

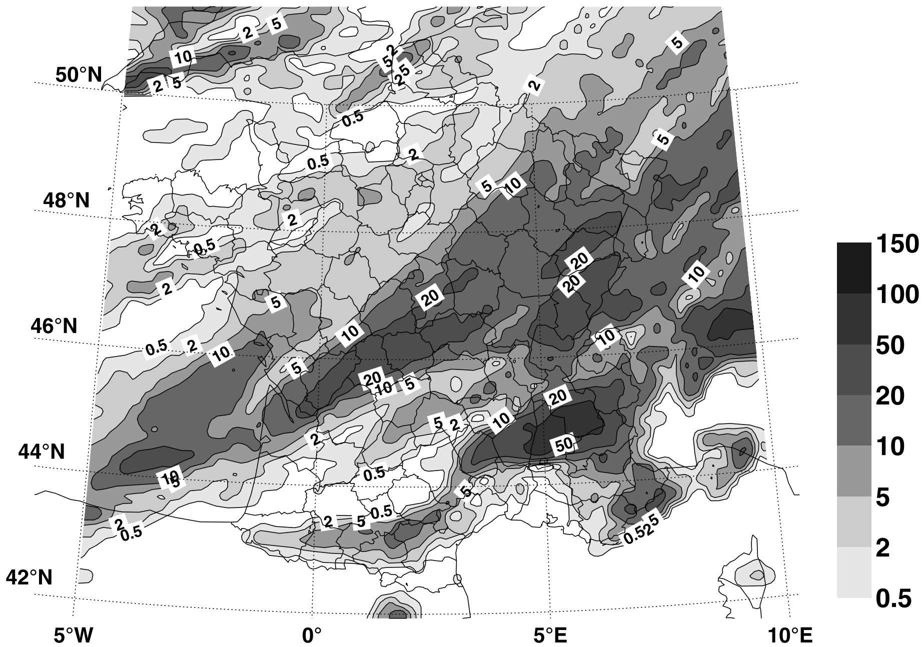

Fig. 9. Twenty-four hours precipitation accumulation over France on 3 September 2008 with AROME and the Eulerian sedimentation algorithm.

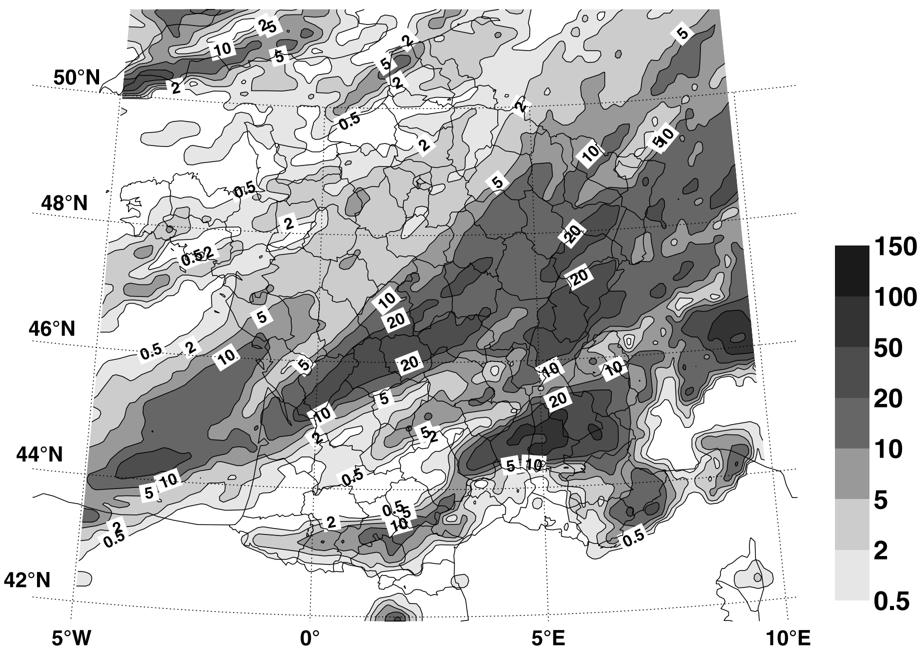

Fig. 10. Same as Fig. 9 but with the new operational sedimentation scheme.

Table 1. Definition of numbers a, b, c and d used to compute Heicdke skill score

	Obs yes	Obs no
Forecast yes	Hit (a)	False alarm (b)
Forecast no	Miss (c)	Correct rejection (d)

Equation (13) can be written for rain in this specific case

$$F_{\mathrm{r}}(j) = \left(P_{\mathrm{r}1}\frac{\rho\Delta z}{\Delta t}q_{\mathrm{r}}^{-} + P_{\mathrm{r}2}F_{\mathrm{r}}(j-1) + P_{\mathrm{r}3}\left(F_{\mathrm{r}}^{\mathrm{auto}} + F_{\mathrm{r}}^{\mathrm{coll}} + F_{\mathrm{r}}^{\mathrm{melt}}\right)\right) \times \left(1 - \frac{F_{\mathrm{r}}^{\mathrm{evap}}}{(\rho\Delta z/\Delta t)\,q_{\mathrm{r}}^{-} + F_{\mathrm{r}}(j-1) + F_{\mathrm{r}}^{\mathrm{auto}} + F_{\mathrm{r}}^{\mathrm{coll}}}\right). \quad (26)$$

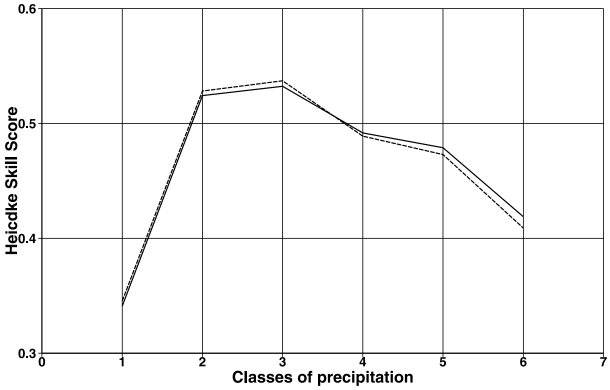

Fig. 11. Heicdke skill scores for six classes of precipitation, for two sets of AROME forecasts, during September 2008 over France. With the old Eulerian algorithm (solid line) and with the new operational approach (dashed line). [0.2, 1] mm: class 1; [1, 2] mm: class 2; [2, 5] mm: class 3; [5, 10] mm: class 4; [10, 20] mm: class 5; >20 mm: class 6.

Here $F_r(j)$ is the flux of rain drops at the bottom of the layer, $F_r(j-1)$ is the incoming flux of rain drops from the layer above. q_r^- is the content of rain in the layer at the beginning of the time step, ρ is the density of air. F_r^{auto}, F_r^{coll}, F_r^{melt} and F_r^{evap} are the production flux of rain by autoconversion, collection and melting, and the flux of evaporation respectively. They can be written as

$$F_r^{proc} = \frac{\rho \Delta z}{\Delta t} \Delta q_r^{proc}, \tag{27}$$

where proc is the process considered (auto, coll, melt or evap) and Δq_r^{proc} the quantity of rain produced or destroyed by the process considered during the time step.

For snow, eq. (13) can be written in a similar way,

$$\begin{aligned} &F_s(j) \\ &= \left(P_{s1} \frac{\rho \Delta z}{\Delta t} q_s^- + P_{s2} F_s(j-1) + \; P_{s3} \left(F_s^{auto} + F_s^{coll} \right) \right) \\ &\quad \times \left(1 - \frac{F_s^{evap} + F_r^{melt}}{(\rho \Delta z / \Delta t) \, q_s^- + \; F_s(j-1) + F_s^{auto} + F_s^{coll}} \right) \end{aligned} \tag{28}$$

Here, melting, the transformation of snow into rain, is now a sink term and appears with evaporation in the first term on the right-hand side of the equation. P_{ri} and P_{si} are the proportions described in the previous section but computed with the falling speeds of rain and snow, respectively. In ARPEGE/ALADIN, the falling speed is fixed: 0.9 m s^{-1} for snow and 5 m s^{-1} for rain drops. This was a limitation of the Lagrangian scheme caused by the search for the origin of the drops, but it is not mandatory in the new scheme for which all calculations are made locally. As shown previously with the AROME formulation, this is not a limitation. As for AROME, a comparison was made for September 2008. The results for 3 September are shown in Figs. 12 and 13. The differences between the two schemes are very small for the 24 h accumulated precipitation.

HSS was computed for the whole month (Fig. 14). The only, small, difference was for precipitation exceeding 20 mm. Nevertheless, the number of cases for this class is too low to allow any conclusions to be draw.

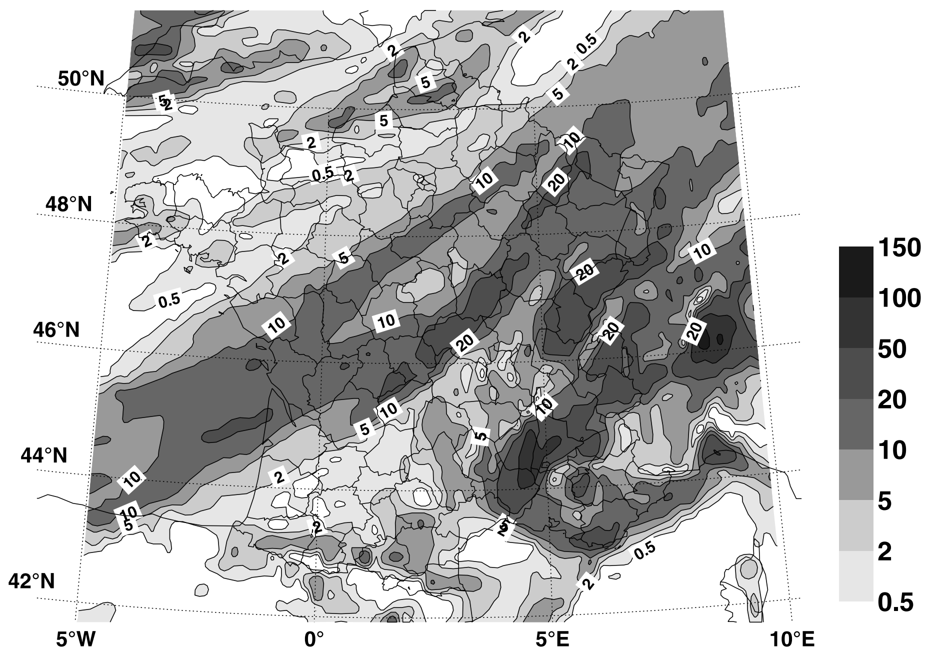

Fig. 12. Same as Fig. 9 but with ALADIN and the Lagrangian sedimentation algorithm.

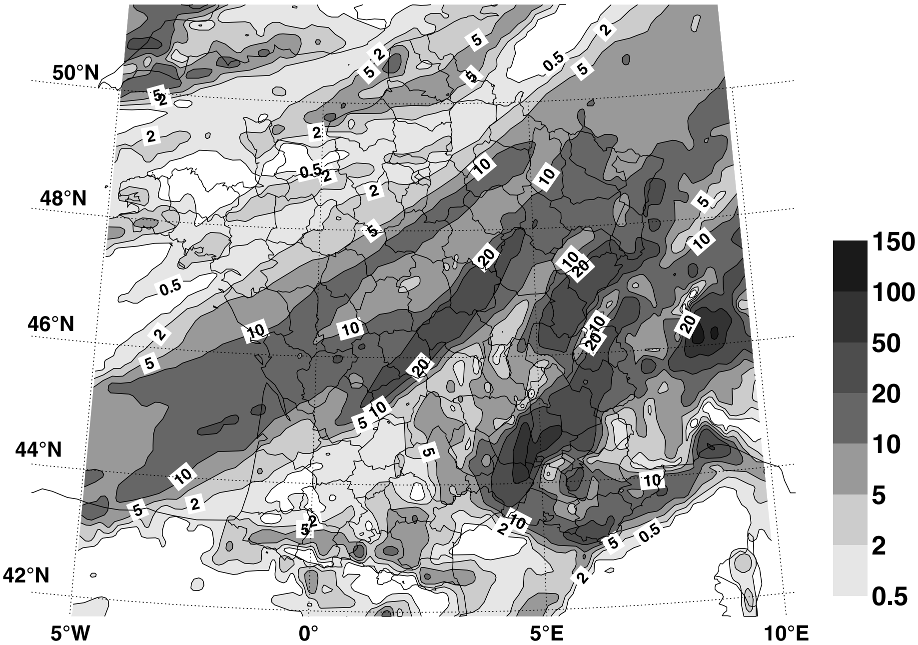

Fig. 13. Same as Fig. 12 but with the new operational scheme.

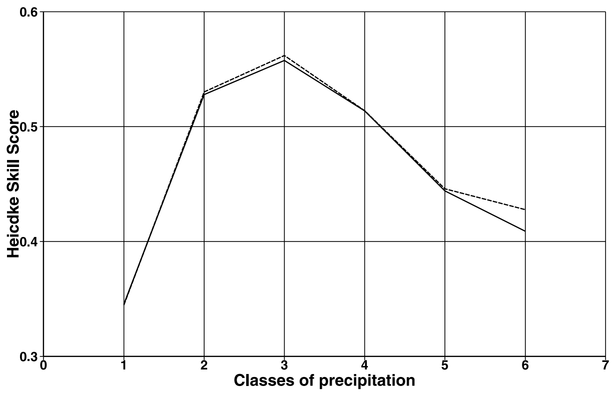

Fig. 14. Same as Fig. 11 but for two sets of ALADIN forecasts, during September 2008 over France. With the old Lagrangien algorithm (solid line) and with the new operational approach (dashed line).

5. Conclusions

We have demonstrated that the new operational sedimentation approach gives results very close to the original, Lagrangian or Eulerian, sedimentation schemes. That is a first positive point. There are two other very interesting points in the new scheme. The first one concerns the CPU cost of the scheme. In ARPEGE/ALADIN with 60 vertical levels, the total time saved per time step is around 6% of the total time spent to compute all model processes, including all dynamics and physical parameterizations. With this new formulation, the link between the cost and the vertical resolution is linear. In the Lagrangian approach, the link was quadratic. Therefore, the advantage of the new scheme will increase in coming years due to the improvement of the vertical resolution. In AROME, the time saved is also around 6% of the total time spent to compute all model processes. There is no double loop but the vertical CFL criterion that determines the number of time substeps is related to the vertical resolution. Again, this implies a quadratic dependence between CPU cost and vertical resolution and the gain will increase with more levels in the vertical direction.

The second interesting point is the simplicity of the scheme. The code is very simple, even in the ARPEGE/ALADIN case, more simple than the original Lagrangian scheme. This simplicity allows us to consider improvements to the physical aspects of the scheme like, for instance, variable sedimentation speed, which was difficult in the context of the old Lagrangian algorithm. This simplicity has made it possible to use this formulation in AROME. Because of the complex structure of the microphysical parameterization of AROME, and also in the intention of obtaining behaviour similar to that with the Eulerian algorithm, another approach is used. The formulation used in AROME was built to simulate the Eulerian scheme but also because it would not be easy to code a more complex approach. As has been shown in this paper for the two cases, the new algorithm gives results close to the original schemes.

But an open question remains: is it possible to improve AROME by using eq. (13) to diagnose the sedimentation flux? Or, is it possible to use the more simple eq. (12) in ALADIN?

On the one hand, there is no reason to simplify the formulation used in ARPEGE/ALADIN. The reduction in computation time would be very small. Given the time step used, 1800 s during the construction of the low-resolution trajectories inside the 4DVAR, it seems important to simulate the behaviour of the Lagrangian scheme as far as possible with a computation of the microphysical processes along the drops trajectory. Even if a deterioration was shown by an experiment using a simpler scheme, the only conclusion would be that, at the resolution and the time step used by ARPEGE and ALADIN, eq. (13) would be necessary. An extrapolation to other spatial and temporal resolutions would be very hazardous.

On the other hand, the microphysical scheme used in AROME has been validated and tested for many years in the research model Meso-NH. A sequential approach has never led to any practical problem. In AROME, due to the stability of the dynamical core, and to save computing time in operation, the time step is 60 s instead of 10 s for Meso-NH at the same resolution. However, experiments, not presented in this paper, have been performed with a time step of 10 s in AROME without any improvement in the quality of the precipitation fields. The two formulations must converge with the decrease of the time step. It is then reasonable to assume that the potential improvement of the scheme is very small in comparison with the large amount of work needed to use a more intricate formulation.

Finally, it is suggested that this algorithm can be used in any sedimentation scheme. Following Rotstayn (1997), the solution proposed in this paper can be obtained from an analytical resolution of the advection equation. The sedimentation algorithm described can also be considered as a special case of the PDF-based approach proposed by Geleyn et al. (2008). This last solution is also used in operation in some National Weather Services of the ALADIN consortium. The approach for sedimentation described in this paper seems to be very flexible and therefore able to be adapted to any type of NWP model.

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