

River Nile discharge, the Pacific Ocean and world climate – a seasonal synchronization perspective

By WOOSOK MOON^{1,2*}, and ABDEL HANNACHI³, ¹*Nordic Institute for Theoretical Physics, Stockholm, Sweden;* ²*Department of Mathematics, Stockholms Universitet, Stockholm, Sweden;* ³*Department of Meteorology, MISU, and Bolin Centre for Climate Research, Stockholm Universitet, Stockholm, Sweden*

(Manuscript Received 11 February 2021; in final form 15 June 2021)

ABSTRACT

The Nile is the longest river in Africa stretching over around 6650 km through 11 countries. From the times of the ancient Egyptian Pharaonic civilization, the Nile is known to be a blessing, which provides major resources including water and fertile soil for agriculture, and facilitates transportations and international trades in nearby countries. Due to its invaluable importance to local economy and agriculture, it is undoubtedly of paramount importance to know how the variability of the Nile is controlled by local and global climate and its morphological characteristics. Here, we utilize a newly developed time-series analysis method applied to monthly Nile river inflow data to reveal various factors changing the river inflow from seasonal to inter-annual, decadal and beyond. On seasonal time-scales a positive feedback, associated mostly with river's morphological change driven by summer precipitation, is identified as a main mechanism for maximal variability in September leading to major flooding or drought. In particular, the positive feedback is quite similar in its mechanism to major climate feedbacks observed, e.g. with ice albedo and Bjerknes feedbacks. The slow time-evolution of the positive feedback explains human endeavour history to control nature, such as the control of the Nile annual flooding through dam construction. The analysis of climate association reveals clear link with large-scale and low-frequency forcing. Decadal and multi-decadal timescales of local precipitation and associated teleconnection with atmospheric and oceanic circulation can be traced back to the Pacific Ocean, and involve mostly the El-Nino Southern Oscillation and the Pacific Decadal Oscillation.

Keywords: planetary geostrophic motion, quasi-geostrophic motion, multi-scale analysis

1. Introduction

Most ancient world civilizations, e.g. ancient China, Maya and central Asia, emerged and flourished around big rivers. These rivers grant various resources for agriculture and transportation. The Nile is an excellent example, which was the backbone of the great ancient Egyptian civilization. In particular, its agriculture was entirely dependent upon the seasonal evolution of the Nile river runoff (Kees, 1961). Lands in the neighbourhood of the river are flooded every summer, bringing up silt to the Nile delta from upstream and saturating the soil with moisture. Soon after the flood water is lowered down and gets infiltrated into the ground. The soil then becomes saturated with adequate moisture and enough nutrient, opening the way to sowing and harvesting

before the dry season. In this tight schedule of seasonal cycle of irrigation, the most important factor for a higher crop yield is to predict the degree of flooding in advance, which requires an identification of the parameters controlling the seasonal flooding.

The Nile has two main sources, namely the Great Lake in Central Africa, making the White Nile, and the Ethiopian high lands, making the Blue Nile (Fig. 1a) (Said, 2013). The water streams from these two sources and converge near the capital city of Sudan, Khartoum, making the Nile, which continues flowing downstream through Aswan area until reaching the Mediterranean Sea. In the Ethiopian high lands, in particular, tropical precipitation controls the amount and variability of water provided to the Nile (Camberlin, 1997), which is reflected in the monthly river discharge data measured at Aswan (Fig. 1b). The contribution of the White Nile is

*Corresponding author. e-mail: woosok.moon@gmail.com

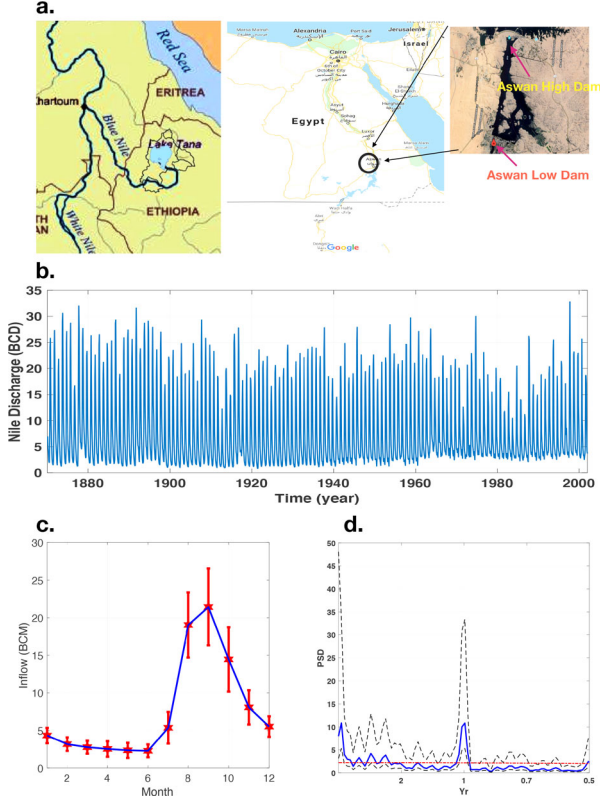


Fig. 1. The Nile has two streams flowing from two sources, Lake Victoria and Ethiopian highlands, which converge in Khartoum and continue its journey to the Mediterranean Sea (Gebre and Ludwig, 2015). The Nile river discharge was measured and recorded monthly from 1870 to 2002 in Aswan area, where two major dams, Aswan Low dam and Aswan High dam, were constructed during this time-period (a). The entire time-series is shown in (b), which suggests various time-scales including the major seasonal cycle. The climatological seasonal cycle of the river discharge, constructed by averaging 133 seasonal cycles of the time-series, is shown in (c) along with standard TWO deviation bars representing confidence interval. The seasonal cycle shows a dramatic change from June (3 BCD) to September (23 BCD). This is reflected in the power spectrum (d) showing one dominant peak associated with the annual cycle, and a low frequency (decadal) variability. The 95% confidence limits (dashed lines) are included with the spectrum of the red noise background (red) for comparison.

controlled mostly by Lake Victoria and is therefore more or less steady. The monthly river discharge data measured at Aswan span from 1870 to 2002 and contain various timescales ranging from seasonal to decadal and multi-decadal. The variability of the Nile discharge, including seasonality, ranges from just below 5 billion cubic meter (BCM) to about 30 BCM, yielding around 20–25 BCM change in seasonal time scales (Fig. 1b). The power spectrum analysis (Fig. 1d) shows a prominent peak around 1 year, suggesting that the seasonal cycle of the Nile river

discharge is the most dominant timescale. The climatological mean seasonal cycle of the river discharge (Fig. 1c), which is calculated by averaging 133 seasonal cycles in the entire data record, describes large contrast of river discharge in the year, with a maximum in September and a minimum in June. At the same time, the degree of seasonal variability is also maximized in September and minimized in June (Fig. 1c). A major concern here is to understand the Nile discharge variability and to be able to predict the degree of the summer flooding in September.

The flooding of the Nile originates mainly from the precipitation in Ethiopian highlands, where tropical monsoon mostly controls precipitation (Conway, 2000). The variability of the Nile discharge in the downstream could be understood as the result of a combination of the tropical precipitation, which varies on diurnal, seasonal and decadal time scale and the river dynamics controlled by the spatial structure of the Nile river including elevations and local catchments (Kummu and Varis, 2007). In particular, remote teleconnections (e.g. Hannachi, 2021) could be crucial to shape the decadal and multi-decadal variability of the Nile river discharge because a tropical monsoon climate contains considerable decadal and multi-decadal variability (Krishnan and Sugi, 2003; Kamae et al., 2017). Furthermore, the Nile river provides vast amount of fresh water to the Mediterranean and acts as a main controller of the salinity of the Mediterranean (Borghini, 2014), which suggests the active role of the Nile river on large-scale climate variability of the areas surrounding the Mediterranean. Therefore, man-made dams in the Nile river can influence the salinity of the Mediterranean Sea leading to the change of local climate (Nof, 1979). Numerous processes distinguished by different time-scales control the Nile river discharge, thus it is necessary to adopt a proper method to distinguish and categorize dominant processes depending on the time-scales. A proper way to address this concern is to construct a reliable stochastic model that is able to simulate the seasonal variability of the Nile.

In climate science, statistical models have been widely applied for the analysis and prediction of various data sets, e.g. Imkeller and Von Storch (2012). One of the basic and most successful implementation is the Ornstein-Uhlenbeck process represented by a simple Langevin equation, which has been introduced to explain the ubiquitous emergence of red noise in climate data including sea surface temperature (Hasselmann, 1976; Frankignoul and Hasselmann, 1977). The Langevin equation, $\dot{x} = -\lambda x + \sigma \zeta$, consists of two parts: the deterministic part, $-\lambda x$, interpreted as a climate providing the overall stability measured by the sign and magnitude of the parameter λ and the noise forcing, $\sigma \zeta$, implying the contribution of weather-related processes. This equation yields a

stationary Gaussian process when ξ is a white noise. The Langevin equation has been generalised recently to encompass seasonality, which is externally linked to astronomical conditions. Instead of the constant parameters λ and σ , two periodic functions $a(t)$ and $N(t)$ are introduced, hence a periodic non-autonomous stochastic ordinary differential equation $\dot{x} = a(t)x + N(t)\xi$ (Moon and Wettlaufer, 2013, 2017a, 2017b). In particular, the periodic function $a(t)$ reflects the seasonal evolution of the stability of a given system, and enables us to understand how the seasonal change of significant positive feedbacks, such as sea ice albedo or Bjerknes feedbacks, shapes the seasonal variability of climate systems. Moreover, the remainder from the time-series defined by $\dot{x} - a(t)x$ contains long-term variabilities together with the noise contribution $N(t)\xi(t)$. The construction of a periodic non-autonomous stochastic model from the Nile river discharge data enables to categorize dominant processes according to time-scales.

In this research, the periodic non-autonomous Langevin equation $\dot{x} = a(t)x + N(t)\xi$ will be used, for the first time, to study the seasonality of the Nile inflow with the inclusion of an extra long-term forcing $d(\tau)$, where τ is a slow yearly varying time-scale and $d(\tau)$ will be related to large-scale indices such as Pacific Decadal Oscillation (PDO) (Mantua and Hare, 2002) and El-Nino Southern Oscillation (ENSO) (Philander, 1983). Using a monthly-averaged inflow time-series measured at Aswan high dam spanning the period 1871–2002, the stochastic model is constructed to regenerate the seasonal statistics of the data. The model therefore consists of three terms representing the seasonal stability $a(t)x$, the contribution of weather-related processes $N(t)\xi$ and the long-term forcing $d(\tau)$, which makes it possible to decompose fluctuations by the three different time-scales and then deduce the major physics for the fluctuations. The data and methodology are described in section 2. The model results are discussed in detail and with various perspectives in section 3. Conclusion and discussion are presented in the last section.

2. Data and methods

2.1. Data

The Nile runoff was measured at Aswan, Egypt, and archived monthly from 1870 to 2002. The SLP data come from the National Center of Atmospheric Research/ National Center for Environmental Prediction (NCAR/ NCEP) reanalysis. They are distributed on a $2.5^\circ \times 2.5^\circ$ lat-lon resolution and span the period 1948 to date. The Sea Surface Temperature (SST) data come from the Hadley Centre Sea Ice and Sea Surface Temperature (HadISST), which are combination of globally complete

monthly field of SST and sea ice on a $1^\circ \times 1^\circ$ lat-lon grid from 1870 to date (Rayner et al., 2003). The Nino-3.4 index is an anomalous SST over a specific area (5°N–5°S, 170°W–120°W) in the tropical Pacific Ocean. The index was downloaded from the University Corporation for Atmospheric Research (UCAR) website, <https://climate-dataguide.ucar.edu/climate-data/nino-sst-indices-nino-12-3-34-4-oni-and-tni>.

The PDO index is defined by the leading principle component of North Pacific monthly SST variability (poleward of 20°N) and the index data was downloaded from the same UCAR site: <https://climatedataguide.ucar.edu/climate-data/pacific-decadal-oscillation-pdo-definition-and-indices>. Ancient Nile height records (yearly maximum and minimum heights), measured during medieval Egypt from 622 to 1469 A.D., were recovered and corrected. These records are taken from the book by William Popper, The Cairo Nilometer (131 pp. 138 pp.) (Popper, 1951). The PDO index and the Nile river height are compared during the overlapping time-period (1009 A.D. to 1469 A.D.).

2.2. Method

The method is based on the construction of a periodic non-autonomous Langevine equation to regenerate the seasonal statistics of a given climate data such as surface air temperature spanning several decades. The detailed description of the method can be found in Moon and Wettlaufer (2017b, 2019).

2.2.1. Theoretical background. Most of climate data have the seasonal cycle as the largest signal. Because of the dominant seasonal cycle, other variabilities such as weather-related fluctuations and decadal and multi-decadal variability are represented as slight updates of the seasonal cycle. The Nile discharge is not an exception. In Fig. 1b and c, we can interpret the overall variability of the Nile river discharge as a dominant seasonal cycle with small (2nd order) fluctuations from various other sources of variability. This is quite typical for climate variables including surface air temperature, precipitation, and sea ice thickness and extent. Thus, climate scientists subtract a climatological mean seasonal cycle, often constructed by a composite of several decades average of seasonal cycles, from original data to enable obtain other variabilities in the data, e.g. the impact of climate change. This strong characteristic can be described by a mathematical model,

$$\frac{dx}{dt} = F(x, t) + N(t)\xi(t) + D(\tau), \quad (1)$$

where x is a climate variable, $F(x, t)$ is a main periodic forcing satisfying $F(x, t) = F(x, t + T)$ where $T = 1$ year, $N(t)\xi(t)$ is a noise forcing mimicking the effect of

weather-related processes, and $D(\tau)$ is a long-term forcing representing the impact of slowly-evolving ocean dynamics or global warming. Here, $\xi(t)$ is a Gaussian white noise implying that there exists a clear separation between weather and season. The intensity of weather varies seasonally, which is considered by the seasonally-varying magnitude of the periodic noise forcing $N(t)$ satisfying $N(t) = N(t + T)$. The additional forcing $D(\tau)$ is much slower than seasonal dynamics, so we introduce a slow time τ distinguished from the main time t . Due to the dominance of seasonal cycle in the overall variability of the main variable x , it is assumed that the magnitude of $F(x, t)$ is much larger than the other terms.

As seen in Fig. 1c, there exists a dominant seasonal cycle of the Nile river discharge, which could be understood as a periodic steady-state (or cyclostationary) solution $[x]$ satisfying $d[x]/dt = F([x], t) + \bar{D}$ with a mean value $\bar{D}$ of $D(\tau)$. Hence, $x = [x] + \eta$ where $|[x]| \gg |\eta|$ implying the dominance of the climatological seasonal cycle $[x]$ compared with other variabilities. Equation (1) becomes

$$\begin{aligned} \frac{d}{dt}([x] + \eta) &= F([x] + \eta, t) + N(t)\xi(t) + \bar{D} + d(\tau) \\ &\simeq F([x], t) + \left. \frac{\partial F}{\partial x} \right|_{x=[x]} \eta + N(t)\xi(t) + \bar{D} + d(\tau), \end{aligned} \quad (2)$$

which leads to the equation for η ,

$$\frac{d\eta}{dt} = a(t)\eta + N(t)\xi(t) + d(\tau), \quad (3)$$

where $a(t) \equiv \partial F / \partial x|_{x=[x]}$ and satisfies $a(t) = a(t + T)$.

Typically, in climate science, a climatological seasonal mean cycle is subtracted from an original monthly-averaged data to analyze other variabilities and their associated timescales. And, in principle, Eq. (3) should provide an approximation of the variability resulting after the removal of the climatological seasonal cycle should be approximated by Eq. (3). The goal is to construct two periodic functions $a(t)$ and $N(t)$ and the slowly-evolving function $d(\tau)$ (which can be derived from the above equation by $d(\tau) = \int_t^{t+T} (\frac{dx}{dt} dt' - a(t')x) dt'$) from the monthly-averaged time-series of the Nile river discharge after removing the climatological seasonal cycle.

2.2.2. Time-series with dominant inter-annual variability and no clear long-term trends. This case is applied to tropical climate indices such as ENSO and Indian Ocean Dipole (IOD). Even though these contain decadal and multi-decadal variability, the overall contribution of these latter time scales is negligible compared with the dominant inter-annual variability, mainly caused by air-sea interactions in tropical areas (Jin, 1997; Jin et al., 2007). The simplest model representing the air-sea interaction is

a two-dimensional non-autonomous linear stochastic dynamical system,

$$\begin{aligned} \frac{d\eta}{dt} &= a(t)\eta + N(t)\xi(t) + \omega_E h \\ \frac{dh}{dt} &= -\omega_E \eta. \end{aligned} \quad (4)$$

For the case of ENSO, for example, η and h can be understood, respectively, as the Nino-3 index and equatorial heat content (i.e. surface temperature and thermocline depth). In the above model, the inter-annual variability of η is originated from $\omega_E h$. Multiplying both sides by η on both sides and taking long-term average leads approximately to

$$\begin{aligned} \frac{1}{\Delta t} (\langle \eta(t + \Delta t)\eta(t) \rangle - \langle \eta^2(t) \rangle) \\ = a(t) \langle \eta^2(t) \rangle - \frac{1}{2} \left\langle \frac{d}{dt} \eta^2 \right\rangle, \end{aligned} \quad (5)$$

where $\Delta t = 1/12$ year and we assume that $\eta(t)$ and $\xi(t)$ are uncorrelated, i.e. $\langle \eta(t)\xi(t) \rangle = 0$, and $\langle \cdot \rangle$ stands for long-term mean of yearly data. In theory, $\langle \cdot \rangle$ should be interpreted as an ensemble average, but in application we have to use a long-term average instead of the ensemble average. The calculation of the monthly statistics is based on the yearly sampling of $\eta(t)$ at a specific month. Similarly, $\langle dh^2/dt \rangle$ refers to an average of various positive and negative values of dh^2/dt caused by the dominant inter-annual variability in a several-decadal long time-series. It is normally quite small compared with other terms. Therefore,

$$a(t) \simeq \frac{\langle \eta(t + \Delta t)\eta(t) \rangle - \langle \eta^2(t) \rangle}{\langle \eta^2(t) \rangle \Delta t}. \quad (6)$$

Now, we can construct a residual variable $y(t) = \eta(t + \Delta t) - \eta(t) - a(t)\eta(t)\Delta t$ from the given data $\eta(t)$, to obtain approximately

$$N(t) \simeq \sqrt{\frac{1}{\Delta t} \langle y^2(t) \rangle}. \quad (7)$$

Even though $y(t)$ contains inter-annual and decadal variability together with the noise $N(t)\xi(t)$, the magnitude of the noise is much larger than that of long-term variability (deduced from the auto-correlation of $y(t)$, which decreases sharply with time.) Hence, in Eq. (7), the contribution from long-term variability is ignored.

Due to the dominance of the seasonal variability in the data of η and the inter-annual variability in h , we can construct $a(t)$ and $N(t)$ from Eqs. (6) and (7). The validity of Eq. (6) comes from the time average $\langle \cdot \rangle$, where the contribution of the inter-annual variability has been neglected. For several decades, the ups and downs of $h^2(t)$ generated by the inter-annual variability are nearly equally distributed, leading to the negligible net in the

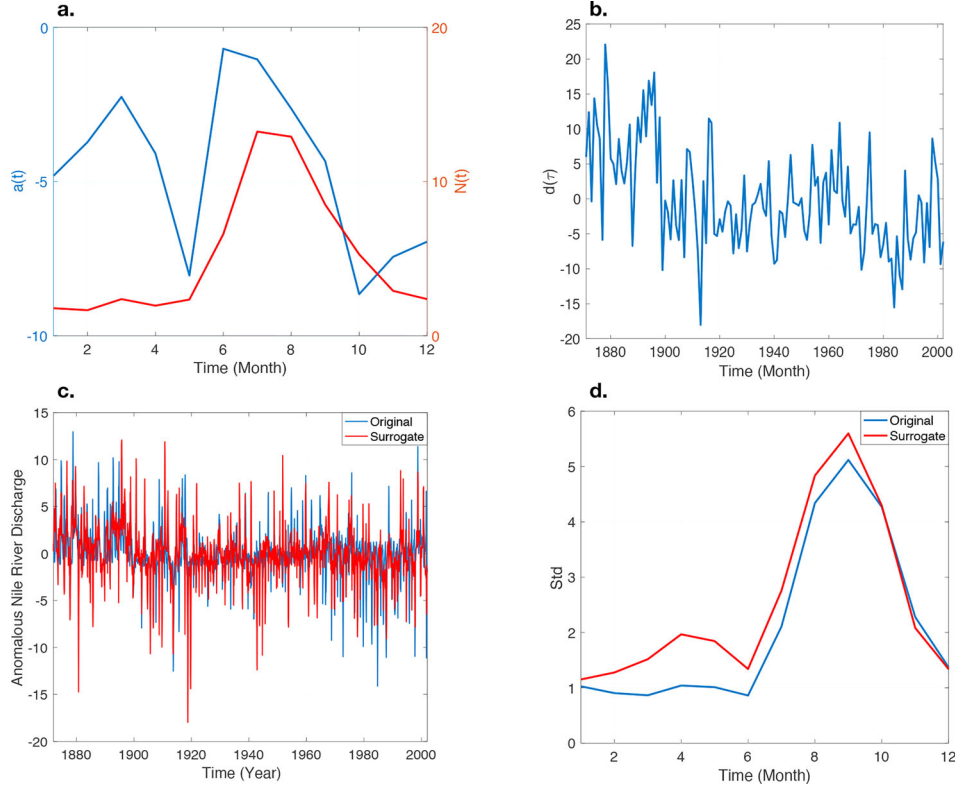


Fig. 2. Stochastic model, $\dot{\eta} = a(t)\eta + N(t)\xi(t) + d(\tau)$, construction based on the Nile river discharge for the period 1870–2002. Equations (11), (13), (14) are used to construct the three functions $a(t)$, $N(t)$, and $d(\tau)$. (a) The two periodic functions: the stability function $a(t)$ (blue), and the noise amplitude $N(t)$ (red). (b) The long-term forcing $d(\tau)$ spanning from 1870 to 2002 calculated from Eq. (14). (c) Time series of the Nile river discharge data (blue), and a model simulation (red). (d) Monthly standard deviation of the observation data (blue) and the model (red).

time-average. This idea was validated using a simple 2-dimensional stochastic dynamical system representing the ENSO dynamics, which could be found in the [supplementary material](#) of Moon and Wettlaufer (2017b). The residual variable $y(t)$ contains long-term variability. We can also use $y(t)$ to investigate the characteristics of the inter-annual and decadal variability contained in the time-series $\eta(t)$. In particular, $y(t)$ is a filtered data without the influence of major seasonal variability represented by $a(t)\eta$.

On seasonal time scales the Nile river discharge is dominantly influenced by precipitation in Ethiopian high lands located in tropical areas. Even though the Aswan area is located far away from the tropics it is still reasonable to treat the Nile river discharge as being part of tropical climate on seasonal time-scales. The climate variability of tropical phenomena is largely shaped by air-sea interactions on seasonal to inter-annual time-scales. The seasonal variation of the stability function $a(t)$ and the magnitude of weather time-scale processes $N(t)$ are combined with inter-annual variability encompassing the interaction with thermocline depth. This

basic tropical dynamics in tropics is embedded in the Nile river discharge, hence the method suggested in this section is used for the Nile river discharge data and works pretty accurately.

2.2.3. Stochastic model construction. Let's assume that $x_{i,k}$ is the monthly-averaged Nile river discharge data, where i , $1 \leq i \leq N$, is an integer representing the year and k is an integer between 1 and 12 representing a specific month in a year. First, we have to calculate a climatological seasonal mean $[x]_k$,

$$[x]_k = \frac{1}{N} \sum_{i=1}^N x_{i,k}. \quad (8)$$

The deviation $\eta_{i,k}$ is calculated by subtracting $[x]_k$ from $x_{i,k}$, i.e.

$$\eta_{i,k} = x_{i,k} - [x]_k. \quad (9)$$

The periodic functions $a(t)$ and $N(t)$ characterizing the stochastic model are then derived based on $\eta_{i,k}$.

The next step consists in calculating the covariance/variance terms,

$$\begin{aligned} V_k &= \frac{1}{N-1} \sum_{i=1}^N \eta_{i,k}^2 \\ A_k &= \frac{1}{N-1} \sum_{i=1}^N \eta_{i,k} \eta_{i,k+1} \end{aligned} \quad (10)$$

Note that in Eq. (10) when $k=12$, instead of $\eta_{i,k+1}$, $\eta_{i+1,1}$ is used. Eq. (10) consists of 12 real values of V and A , which can be considered as periodic functions 1-year period. Combining Eqs. (6) and (10) yields:

$$a_k = \frac{A_k - V_k}{V_k \Delta t}, \quad (11)$$

where $\Delta t = 1/12$. Now, we construct the residuals $y_{i,k}$ by

$$y_{i,k} = \eta_{i,k+1} - \eta_{i,k} - a_k \eta_{i,k} \Delta t, \quad (12)$$

(for $k=12$, $k+1$ is set to 1 and i to $i+1$). The noise magnitude N_k is then approximated by:

$$N_k = \sqrt{\frac{1}{\Delta t} \frac{1}{N-1} \sum_{i=1}^N y_{i,k}^2} \quad (13)$$

Finally, we can formulate the long-term forcing $d(\tau)$, which is embedded in $y_{i,k}$ with the noise $N(t)\xi(t)$. Hence, as an approximation, $d(\tau)$ can be deduced by a moving-average with 1-year time window,

$$d_i = \sum_{k=1}^{12} y_{i,k}. \quad (14)$$

Note that the term $d(\tau)$ also depends on the starting month (e.g. April to March of next year), which could lead to yearly difference, but decadal and multi-decadal variabilities are consistent regardless of the choice of the starting month. Eqs. (11)–(14) are then used to construct a_k , N_k , $k = 1, \dots, 12$, and d_i , $i = 1, \dots, N$, from the time-series $\eta_{i,k}$. Using these three functions, we set out to draw physical characteristics of the Nile river discharge on seasonal, inter-annual and decadal time-scales.

To simulate the river discharge, we construct a stochastic model $\dot{\eta} = a(t)\eta + N(t)\xi(t) + d(\tau)$ using the monthly Nile river discharge data spanning from 1870 to 2002. The results are shown in Fig. 2. The two periodic functions $a(t)$ and $N(t)$ constructed from Eqs. (11, 13) are shown in Fig. 2a, and the long-term forcing $d(\tau)$ from Eq. (14) in Fig. 2b. A stochastic realization from the constructed model $\dot{\eta} = a(t)\eta + N(t)\xi(t) + d(\tau)$ is compared with the original river discharge time-series (Fig. 2c). The reliability of the model could be deduced by the match between the monthly standard deviation of the stochastic model and that of the original data (Fig. 2d).

The Nile river is located far away from tropical areas. However, the seasonal variability of the Nile river discharge is similar to that of tropical phenomena influenced by air-sea interaction in tropical ocean. The Nile river discharge is largely controlled by precipitation in the Ethiopian High

lands, the main source of the Blue Nile. The weather of the region is largely affected by tropical monsoon, thus the influence of inter-annual variability of air-sea interaction in tropics is significant. Therefore, the Nile river discharge belongs to a climate phenomenon with a dominant inter-annual variability. The stochastic model construction introduced in this section fits quite well with the monthly Nile river discharge time-series, which is shown in Fig. 2. The $a(t)$ provides the information of the seasonal sensitivity of the Nile river and the noise magnitude $N(t)$ shows how much short-time weather impacts upon the Nile river discharge. The $d(\tau)$ can represent how the Nile river is influenced by inter-annual and decadal forcing.

Positive values of (negative) $a(t)$ imply exponential increase (decrease) of fluctuations. Unlike autonomous dynamical systems where a single sensitivity a determines the overall stability, the stability of a periodic non-autonomous dynamical system is determined by the integral of the sensitivity $a(t)$. Thus, a stable system satisfies the positivity condition, i.e. $\int_0^T a(t)dt < 0$, where T is the period of the system. Unless we violate the integral condition for stability, it is possible to have positive $a(t)$ during a certain time in a period. In climate, there are two well-known relevant examples; Arctic sea ice and ENSO. The two phenomena have positive feedbacks called sea ice albedo feedback and Bjerknes feedback, respectively, which act upon the associated systems during a specific season. The sea ice albedo feedback operates during summer with strong sunlight and accelerates melting of sea ice in Arctic ocean thereafter. The Bjerknes feedback is turned on in the eastern Pacific from July to November, which is strongly linked with ENSO phase locking with the annual cycle and spring predictability barrier. The noise magnitude $N(t)$ represents the magnitude of the contribution from short-time scale processes related to weather. The white noise forcing $N(t)\xi(t)$ is an approximation of chaotic behaviour of weather-related processes, which is normally quite challenging to simulate using numerical models. Hence, the noise term $N(t)$ is closely related to seasonal predictability together with the function $a(t)$. In particular, large $N(t)$ during early spring for ENSO is strongly related to the low spring predictability of ENSO. The construction of two periodic functions $a(t)$ and $N(t)$ from the Nile river discharge could give us physical insights of how the seasonal variability of the Nile river discharge is shaped by the interaction between the seasonal stability and the short-time scale forcing.

3. Results

3.1. Phase locking of the Nile discharge seasonality and feedback mechanism

The Aswan dam construction, that started in 1899, went through different phases. One of the primary ingredients

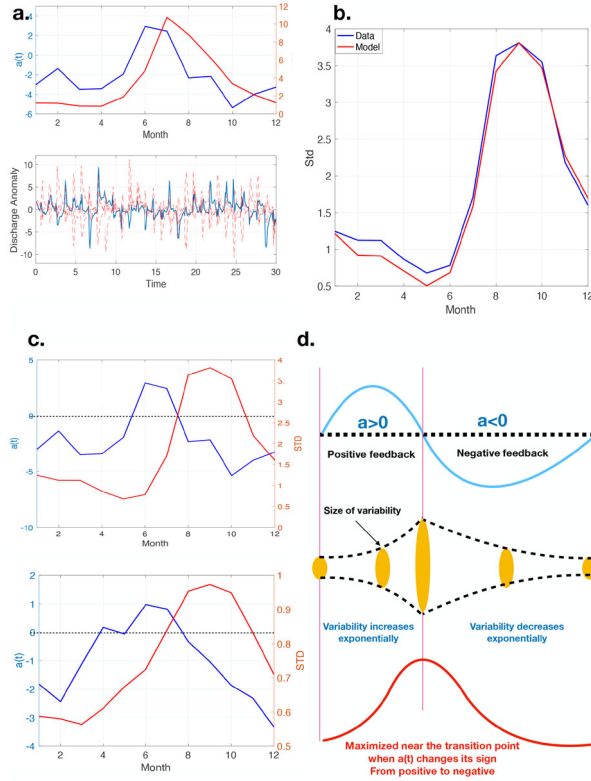


Fig. 3. Stochastic model, $\dot{\eta} = a(t)\eta + N(t)\xi(t)$, constructed based on the Nile river discharge for the period 1870–1899. (a) The two periodic functions: the stability function $a(t)$, and the noise amplitude $N(t)$ (top) along with a stochastic realization (red) to be compared with the original time-series (blue) (bottom). (b) Monthly standard deviations derived from the ensemble members (red) and the original data (blue). (c) Seasonal stability $a(t)$ (blue) along with the standard deviation (red) for ENSO. Positive feedback is associated with positive $a(t)$, yielding exponential increase of variability and corresponds to June–August for the Nile and July–Nov for ENSO. Maximum variability occurs approximately near the transition when $a(t)$ changes sign, i.e. September (Nile) and December (ENSO). The occurrence of extreme phases of the Nile discharge (flooding and drought) and ENSO (El-Nin and La-Nin) takes place at specific time of the year due to the feedback mechanism strongly associated with phase locking, as summarized in (d).

of this analysis is to learn a basic mechanism of seasonal flooding and drought prior to any artificial interruption, i.e. dam. We start first by determining the periodic functions $a(t)$ and $N(t)$ using the Nile discharge time-series spanning from 1870 to 1899 (Fig. 3a), during which there was no dam constructed in Aswan. Hence, the variability of river discharge was naturally controlled without any artificial controlling medium. The seasonal stability function $a(t)$ in Fig. 3a (blue line), representing the monthly stability of the river discharge, is positive during summer from June to August. The positive sign of $a(t)$ results in

an exponential growth of $\eta(t)$ implying the existence of a positive feedback in leading-order physics. The two-season characteristic, in terms of the sign of the seasonal stability $a(t)$, appears in several major climate phenomena including El-Nino and Southern Oscillation (ENSO) (Stein et al., 2014; Moon and Wettlaufer, 2017b) and sea ice albedo feedback (Moon and Wettlaufer, 2011, 2017a). In particular, the existence of a positive feedback represented by the positive function $a(t)$ is strongly related to the phase locking of the above phenomena (Tziperman et al., 1998), maximizing the variability when the stability function $a(t)$ changes its sign from positive to negative (Moon and Wettlaufer, 2017b). The stability function $a(t)$ of the Nile river discharge changes its sign from positive to negative in August and then the variability gets maximized in September (Fig. 3b and c (top panel)). Moreover, the magnitude of the noise forcing $N(t)$ (Fig. 3a, red line), which should be strongly related to local precipitation, is also maximized during summer. This implies that the positive (negative) noise forcing is magnified exponentially during summer, which causes extreme flooding (drought) near the end of summer (Fig. 1c). This is equivalent to the phase locking of the ENSO and Arctic sea ice.

Consider, for example, the familiar case of ENSO. The seasonal stability $a(t)$ is positive from July to November, which is equivalent to the positive feedback in the Nile river (Fig. 3c). This implies a seasonal change of the Bjerknes feedback (Bjerknes, 1966), which is summarized by the positive interaction between upwelling in eastern tropical Pacific and surface tropical easterly wind. Due to the system instability, the response to various short-time forcing magnifies exponentially during the time from July to November (Fig. 2c), which is the main physical reason why the degree of the response to a noise forcing is maximized near the end of the year, hence phase locking. The peaked phases of ENSO, i.e. El-Nino and La-Nina, are therefore expected near Christmas time more than any other time of the year due precisely to the effect of the positive feedback during summer and fall, and also the change of sign of the stability function near the end of November. With the same mechanism, we can explain the timing of the maximum variability of the Nile river discharge near the end of summer. As shown in the top panel of Fig. 3c, the positive feedback is operating from May to late July, and the standard deviation of the Nile river discharge increases from June and reaches a maximum in September. Similarly, in Arctic Ocean, sea ice albedo feedback is active during summer until the end of August (because of the clear sky and solar short wave reflection). Hence, the seasonal variability of Arctic sea ice extent, or thickness, is maximized near the end of summer (Moon and Wettlaufer, 2013). In terms of the

seasonal stability characterized by the stability function $a(t)$, the Nile also possesses a similar mechanism to that of ENSO and Arctic sea ice for the maximum of seasonal variability. Therefore, the maximum variability of the Nile river discharge can be understood as a phase locking and the annual flooding and drought of the Nile is analogous to El-Nino and La-Nina phases of ENSO (see the schematic in Fig. 3d).

3.2. The history of dam constructions in the Aswan reflected in the Nile river discharge

Because the stability function $a(t)$ is by construction time-dependent, the degree of the positive feedback can vary slowly on decadal and multi-decadal time-scales. In addition, external forcing can also affect the characteristics of multi-scale dynamical systems. The interaction between global warming and ENSO or Arctic sea ice natural decadal and multi-decadal variability, can necessarily lead to slow changes of their variability and seasonal stability. The Nile is also expected to behave in a similar fashion. But unlike ENSO and Arctic sea ice, human-made infrastructures, especially dams, have been introduced to alter the morphological characteristics of the river (Petts, 1979), which is directly related to the positive feedback causing flooding and drought in summer. To trace the decadal and multi-decadal time-evolution of the stability function $a(t)$, we apply a 30-yr moving time-window from 1870 to 1973, where $a(t)$ is constructed for each time window, to enable following its time-evolution (Fig. 4). The positive feedback, mirrored by the positive sign of $a(t)$ during summer can be clearly seen from 1884 to 1930, after which the stability started to decrease until it becomes almost neutral (close to 0) around 1960. In particular, starting from around 1960, a dramatic shift is observed (Fig. 4), implying that flooding or drought, normally happening near the end of summer, sharply weakens. In other words, the seasonal flooding or drought was likely to be under control.

Flooding and drought have been prevented or controlled by constructing dams upstream. In Aswan two major dams, the Aswan low dam and the Aswan high dam, have been constructed and amended from 1901 (Cook, 2013). Aswan low dam was completed and in full operation in 1902, but was not satisfactory in its storage for preventing flooding, which brought up two further amendment works by raising up the dam during 1907–1912 and 1929–1933, respectively. Again, the dam was heightened in 1946 due to unsatisfactory performance even after the previous two amendments. Finally, Aswan high dam was built between 1960 and 1970 and was fully operational in 1976. During this time-period of dam constructions, the seasonal stability $a(t)$ changed

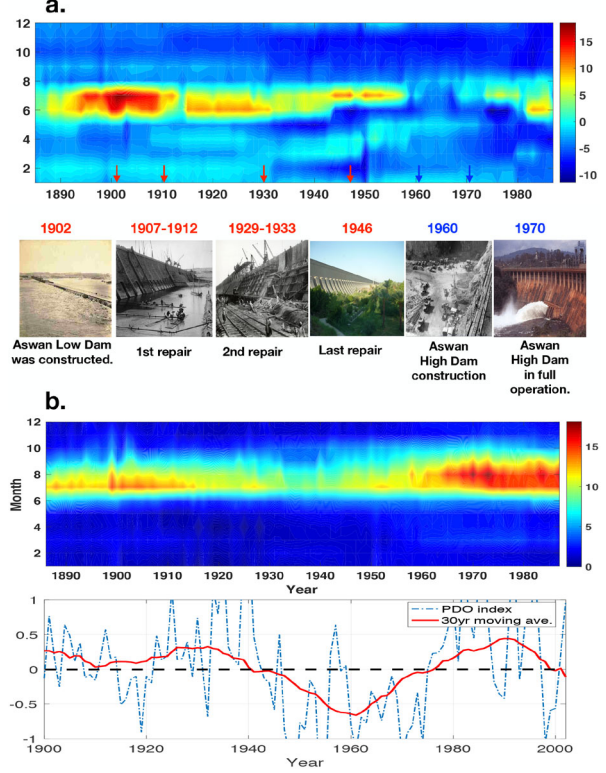


Fig. 4. 30-yr window characteristics of monthly stability and noise of the Nile discharge. (a) Time evolution of the seasonal stability $a(t)$ (top) and various stages of Aswan dam construction/amendment (bottom). Positive feedback is shown by the hot colour (positive $a(t)$) and occurs between June and August, characterized by its slow time evolution, which is related to the history of dam construction. A decrease of $a(t)$, during summer, started from 1930. From 1946, corresponding to the last stage of raising Aswan low dam, the dominant positive summer feedback starts to weaken as a result. Aswan high dam was constructed during 1960 to 1970. The difference between $a(t)$ before 1902 and after the construction of the two dams suggests an impact of the dams on the Nile river variability. (b) Time evolution of the noise amplitude $N(t)$ (top) and the annual as well as a 30-yr moving average of the PDO index (bottom). The noise amplitude $N(t)$ shows multi-decadal variability, and is correlated with the 30-year PDO average, where positive (negative) PDO is associated with large (small) noise amplitude on multi-decadal time scales.

dramatically. The degree of the positive feedback was weakened as the construction or amendment were progressing. Around the year 1960 the feedback became almost neutral (Fig. 4). Bearing in mind that no remarkable climate change signal was recorded around this time (Fig. 1b), this suggests that the change to neutrality can only be the result of the last Aswan low dam amendment. In the meantime, the complete amendment of Aswan high dam continued during the time period 1960–1979. Moreover, the Floquet exponent $\lambda = \int_0^T a(t) dt$ constantly

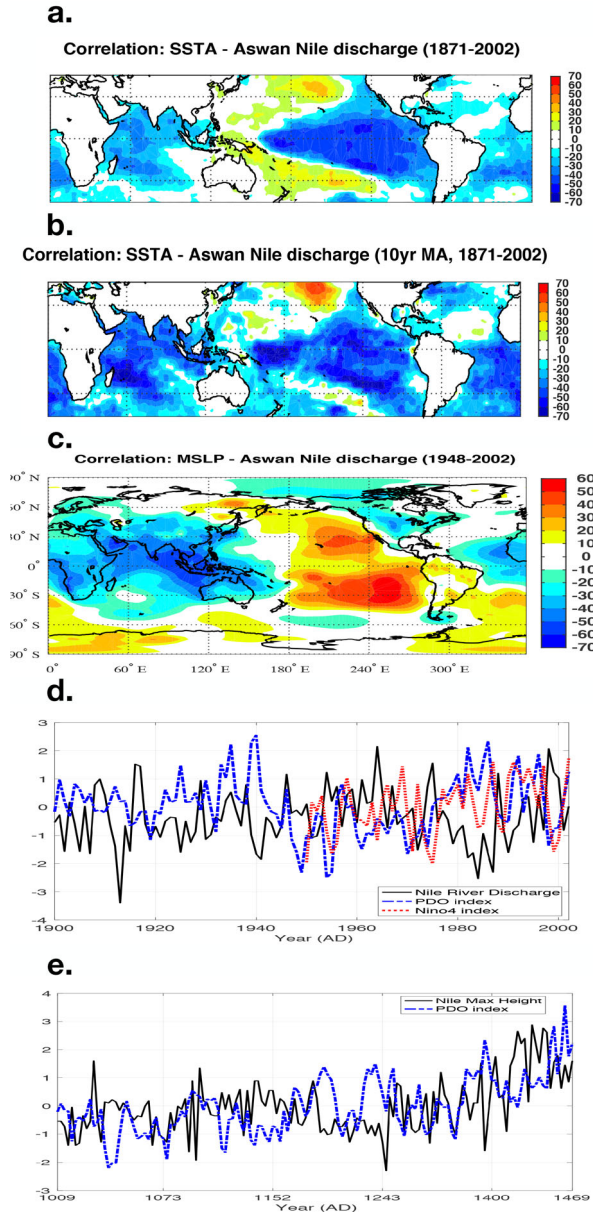


Fig. 5. One point correlation map between the annual average of the Nile river discharge and that of SST (a), showing similar spatial pattern with ENSO, and the same map between 10-year moving averaged Nile river discharge and SST (b) showing a PDO pattern. The correlation of the annual mean of the Nile river discharge with SLP is also shown in (c). The normalized annual mean Nile river discharge (thick black) is compared with the normalized PDO index (dashed blue) and the normalized Nino4 index (red dots) (d). The normalized Nile maximum height recorded by the Nilometer for the last millennium is compared with the normalized PDO index deduced from tree rings (e). The Nile river discharge (or height) as given by the Nilometer is negatively correlated with the PDO on inter-annual time-scales, but positively correlated with the PDO on multi-decadal time-scales.

decreases as the dam constructions evolved toward the completeness of the Aswan high dam. The morphological changes of the river resulting from the summer precipitation is suggested to be strongly related to the seasonal stability of the river discharge, and the dam constructions alter the river morphology, which is reflected in the alteration of the seasonal stability $a(t)$ (Fig. 4).

3.3. The influence of the tropical Pacific on inter-annual and decadal time-scales

Besides the man-made intervention affecting the seasonal stability of the Nile river discharge, various other climate processes, with distinct timescales, also control the river discharge. Large scale teleconnections (Hannachi et al. 2017), such as ENSO, act as a connecting linkage between physically remote areas and play the role of atmospheric bridges. We have investigated the relationship between the world ocean surface temperature, i.e. sea surface temperature (SST) as well as sea level pressure (SLP), with the Nile discharge. On inter-annual time scales, Figure 5 shows the correlation of the annual average of the Nile discharge with SST anomalies (Fig. 5a) and SLP anomalies (Fig. 5c). A clear ENSO signal stands out, describing the connection with the Nile river discharge (Wang and Eltahir, 1999). During La-Nina phase the centre of convection over the equatorial central Pacific is shifted westward over the west equatorial Pacific and the Indonesian archipelago. All the region extending from the tropical and subtropical Pacific west of the dateline westward and reaching Africa is dominated by a large-scale low pressure system, i.e. the positive phase of the Southern Oscillation (SO) (Fig. 5c). During El-Nino, the opposite phase of the SO takes place and yields smaller Nile discharge. The ENSO signal is known to have a timescale of the order 3–7 years. The power spectrum of the river discharge (Fig. 1d), however, also shows that the data sustain lower frequency than that of the ENSO. We computed the same correlation with SST but using a 10-yr moving average. Figure 5b shows the spatial structure of the correlation between the decadal average of the Nile river discharge and that of SST, which is quite similar to, and reflects the Pacific Decadal Oscillation (Mantua and Hare, 2002). We have also compared the annual mean of the Nile river discharge with the Nino4 (Trenberth and National Center for Atmospheric Research Staff, 2019) and PDO indices (Fig. 5d). It is seen, in particular, that the annual mean of the Nile river discharge is negatively correlated with the both indices in inter-annual time-scales. The correlation between PDO index and the annual Nile river discharge is -0.3720 , and the correlation between Nino4 index and the annual Nile river discharge is -0.5109 . In

the recent period when the PDO and the Nino4 indices were mostly positive, the annual mean discharge was strongly correlated with the PDO.

About 5000 years ago, the ancient Egyptians invented and used Nilometer to measure the water level of the Nile river during its annual flood, which provided the almost yearly records of the height of the Nile river during the Pharaoh reign. Even at a much later time when Egypt was invaded by, e.g. Greeks and Romans, the Nilometer was still in use due to its importance. Regular measurements were recorded and converted to modern units (Popper, 1951). To complement the analysis the PDO index, spanning from 993 A.D. to 1996 A.D., is also retrieved from tree rings of *pinus flexilis* in California (MacDonald and Case, 2005). The comparison between the two time-series shown in Fig. 5e also tells us that the Nile river height is negatively correlated with the PDO on inter-annual time-scales in the last millennium, on the other hand, positively correlated on multi-decadal time-scales. The correlation between the PDO index from the tree rings and the height of the Nile river is 0.2412, which mainly comes from the contribution on multi-decadal time-scales and beyond to compensate the negative correlation on inter-annual time-scales. Moreover, the magnitude of the noise forcing $N(t)$ (Fig. 4b), reflecting the degree of weather-related forcing, was small before 1960, but increased sharply between 1960 and 1970, which seems to be approximately coincident with the sign change of the PDO from negative to positive in 1970s, often referred to as a climate regime shift (d'Arrigo et al, 2001). The positive PDO, which was dominant in the later period, is suggested to be related to the increased variability of the seasonal precipitation on multi-decadal time-scales. A recent model-based research predicts that the variability of the Nile discharge will increase in the near future under the influence of ongoing global warming (Siam and Eltahir, 2017). The question of how does the combination of the natural variability of the PDO and the global warming influence the future Nile river discharge is an important one whose answer helps predict the long-term evolution of the Nile river discharge.

4. Summary and conclusion

Historically the Ornstein-Uhlenbeck (OU) process $\dot{x} = -\lambda x + \sigma \xi$ was introduced as a way to discriminate between climate and weather and then interpret climate variability by a linear combination between the two time-scales. The deterministic term $-\lambda x$, representing the overall stability of a system, is implied as the climate and the noise forcing $\sigma \xi$ as weather effects. The red-noise from the OU process is ubiquitous in climate including sea surface temperature. However, the original OU process

cannot accommodate the seasonality which comes from external astronomical conditions. To include the seasonality of shortwave radiative flux, the OU process should be generalised to become periodic non-autonomous.

Considering the strong seasonality of the Nile inflow, we applied a periodic non-autonomous OU process $\dot{x} = a(t)x + N(t)\xi + d(\tau)$ with a long-term forcing $d(\tau)$. The three unknown functions are constructed to match the time-evolution of the second-order statistics of the data. It is shown in particular that the model reproduces reasonably well the seasonal evolution of the statistics in the data. The model also enables us to interpret the essence of the major characteristics of the Nile inflow in terms of weather, seasonal and decadal time-scales separately.

The Nile river discharge data used in this research have been measured at Aswan, a mere characteristic geological location belonging to the river. Even with the spatial-domain restriction, the data spanning around 130 years contains various climate timescales. Local precipitation is a dominant process controlling short-time variability approximated as a noise forcing in a stochastic model. But, of course, the river flow (spatially) integrates the precipitation over the river catchment, preserving more or less the temporal structure of the forcing. The seasonal evolution of the river morphology provides a positive feedback during summer, which explains the observed high variability of the Nile near the end of summer characterized by flooding or drought. To reduce the possible negative impact, due to the high variability of the Nile river during summer, two major dams, Aswan low and high dams were constructed and/or amended. Their effect is detected through the observed change of the seasonal stability of the constructed stochastic model (see Fig. 4 and related discussions).

Inter-annual and long-term variability of the river discharge is found to be associated with ENSO and the decadal and multi-decadal variability of the SST in the Pacific Ocean. The Nile river discharge is not merely a local phenomenon confined to one restricted area. It also contains a footprint of human-striving history to control and utilize the natural variability of seasonal flooding and drought. At the same time, the inter-annual, decadal and multi-decadal fluctuations of the Pacific Ocean are also imprinted in the Nile river discharge record. This leads us to suggest that the Nile can be seen as a barometer of past and current climate. Historic data of the Nile river discharge must be a good proxy to reveal past human intervention history and climate, particularly the Pacific Ocean, e.g. PDO signal, and the future prediction of the Nile river discharge will require forecasts of global climate, in particular, in the Pacific Ocean.

Acknowledgements

We thank J. S. Wettlaufer for discussion of this study and helpful feedback. WM acknowledges the support of Swedish Research Council grant no. 638-2013-9243. Two anonymous reviewers provided constructive comments that helped improve the manuscript. We thank Prof. H. A. Basset for sharing the Nile discharge time series.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

Vetenskapsrådet.

References

- Bjerknes, J. 1966. A possible response of the atmospheric Hadley circulation to equatorial anomalies of ocean temperature. *Tellus* 18, 820–829.
- Borghini, M., Bryden, H., Schroeder, K., Sparnocchia, S. and Vetrano, A. 2014. The Mediterranean is becoming saltier. *Ocean Sci.* 10, 693–700. doi:10.5194/os-10-693-2014
- Camberlin, P. 1997. Rainfall anomalies in the source region of the Nile and their connection with the Indian summer monsoon. *J. Clim.* 18, 219–224.
- Conway, D. 2000. The climate and hydrology of the upper blue Nile river. *Geographical J.* 166, 49–62. doi:10.1111/j.1475-4959.2000.tb00006.x
- Cook, T. 2013. Engineering Modernity: The Aswan Low Dam and Modernizing the Nile. Western Oregon University.
- d'Arrigo, R., Villalba, R. and Wiles, G. 2001. Tree-ring estimates of Pacific decadal climate variability. *Clim. Dyn.* 18, 219–224. doi:10.1007/s003820100177
- Frankignoul, C. and Hasselmann, K. 1977. Stochastic climate models, part II application to sea-surface temperature anomalies and thermocline variability. *Tellus* 29, 289–305.
- Gebre, S. L. and Ludwig, F. 2015. Hydrological response to climate change of the upper Blue Nile river basin: Based on IPCC fifth assessment report (AR5). *J. Climatol. Weather Forecast.* 3, 1–15.
- Hannachi, A. 2021. *Pattern Identification and Data Mining in Weather and Climate*, Berlin: Springer-Verlag, 416 pp.
- Hannachi, A., Straus, D. M., Franzke, C. L. E., Corti, S. and Woollings, T. 2017. Low-frequency nonlinearity and regime behaviour in the Northern Hemisphere extratropical atmosphere. *Rev. Geophys.* 55, 199–234. doi:10.1002/2015RG000509
- Hasselmann, K. 1976. Stochastic climate models part I. Theory. *Tellus* 28, 473–485.
- Imkeller, P. and Von Storch, J. S. 2012. *Stochastic Climate Models*. Progress in Probability, 1st edn. Basel: Birkhäuser, Vol. 49.
- Jin, F. F. 1997. An equatorial ocean recharge paradigm for ENSO. Part I: Conceptual model. *J. Atmos. Sci.* 54, 811–829. doi:10.1175/1520-0469(1997)054<0811:AEORPF>2.0.CO;2
- Jin, F. F., Lin, L., Timmermann, A. and Zhao, J. 2007. Ensemble-mean dynamics of the ENSO recharge oscillator under state-dependent stochastic forcing. *Geophys. Res. Lett.* 34.
- Kamae, Y., Li, X., Xie, S. P. and Ueda, H. 2017. Atlantic effects on recent decadal trends in global monsoon. *Clim. Dyn.* 49, 3443–3455. doi:10.1007/s00382-017-3522-3
- Kees, H. 1961. *Ancient Egypt: A Cultural Topography*. Chicago: University of Chicago Press.
- Krishnan, R. and Sugi, M. 2003. Pacific decadal oscillation and variability of the Indian summer monsoon rainfall. *Clim. Dyn.* 21, 233–242. doi:10.1007/s00382-003-0330-8
- Kummu, M. and Varis, O. 2007. Sediment-related impacts due to upstream reservoir trapping, the lower Mekong river. *Geomorphology* 85, 275–293. doi:10.1016/j.geomorph.2006.03.024
- MacDonald, G. M. and Case, R. A. 2005. Variations in the Pacific decadal oscillation over the past millennium. *Geophys. Res. Lett.* 32(8).
- Mantua, N. J. and Hare, S. R. 2002. The Pacific decadal oscillation. *J. Oceanogr.* 58, 35–44. doi:10.1023/A:1015820616384
- Moon, W. and Wettlaufer, J. S. 2011. A low-order theory of Arctic sea ice stability. *EPL* 96, 39001. doi:10.1209/0295-5075/96/39001
- Moon, W. and Wettlaufer, J. S. 2013. A stochastic perturbation theory for non-autonomous systems. *J. Math. Phys.* 54, 123303. doi:10.1063/1.4848776
- Moon, W. and Wettlaufer, J. S. 2017a. A stochastic dynamical model of arctic sea ice. *J. Clim.* 30, 5119–5140. doi:10.1175/JCLI-D-16-0223.1
- Moon, W. and Wettlaufer, J. S. 2017b. A unified nonlinear stochastic time series analysis for climate science. *Sci. Rep.* 7, 44228. doi:10.1038/srep44228
- Moon, W. and Wettlaufer, J. S. 2019. Coupling functions in climate. *Philos. Trans. A Math. Phys. Eng. Sci.* 377, 20190006.
- Nof, D. 1979. On man-induced variations in the circulation of the Mediterranean Sea. *Tellus* 31, 558–564.
- Petts, G. E. 1979. Complex response of river channel morphology subsequent to reservoir construction. *Prog. Phys. Geogr.* 3, 329–362. doi:10.1177/030913337900300302
- Philander, S. G. H. 1983. El Niño southern oscillation phenomena. *Nature* 302, 295–301. doi:10.1038/302295a0
- Popper, W. 1951. *The Cairo Nilometer*. University of California Press, Berkeley, 269 pp.
- Rayner, N. A. A., Parker, D. E., Horton, E. B., Folland, C. K., Alexander, L. V. and co-authors. 2003. Global analyses of sea surface temperature, sea ice, and night marine air temperature since the late nineteenth century. *J. Geophys. Res.* 108,
- Said, R. 2013. *The River Nile: Geology, Hydrology and Utilization*. Oxford: Elsevier.
- Siam, M. S. and Eltahir, E. A. 2017. Climate change enhances inter-annual variability of the Nile river flow. *Nature Clim. Change* 7, 350–354. doi:10.1038/nclimate3273
- Stein, K., Timmermann, A., Schneider, N., Jin, F. F. and Stuecker, M. F. 2014. ENSO seasonal synchronization

- theory. *Journal of Climate* 27, 5285–5310. doi:[10.1175/JCLI-D-13-00525.1](https://doi.org/10.1175/JCLI-D-13-00525.1)
- Trenberth, K, and National Center for Atmospheric Research Staff (Eds.) 2019. “The Climate Data Guide: Nino SST Indices (Nino 1+2, 3, 3.4, 4; ONI and TNI).” 11 January. Online at: <https://climatedataguide.ucar.edu/climate-data/nino-sst-indices-nino-12-3-34-4-oni-and-tni>.
- Tziperman, E., Cane, M. A., Zebiak, S. E., Xue, Y. and Blumenthal, B. 1998. Locking of El Nino’s peak time to the end of the calendar year in the delayed oscillator picture of ENSO. *J. Clim.* 11, 2191–2199. doi:[10.1175/1520-0442\(1998\)011<2191:LOENOS>2.0.CO;2](https://doi.org/10.1175/1520-0442(1998)011<2191:LOENOS>2.0.CO;2)
- Wang, G. and Eltahir, E. A. 1999. Use of ENSO information in medium-and long-range forecasting of the Nile floods. *J. Clim.* 12, 1726–1737. doi:[10.1175/1520-0442\(1999\)012<1726:UOEIIM>2.0.CO;2](https://doi.org/10.1175/1520-0442(1999)012<1726:UOEIIM>2.0.CO;2)