

Eady edge waves, frontal wave-trains and type-B cyclogenesis

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ABSTRACT

Eady edge waves (EEWs) propagate on surface baroclinic zones and they resemble synoptic and sub-synoptic scale frontal wave-trains. The wave-trains are frequently depicted on surface charts, and occur *per se* on a front with a local baroclinicity extremum. The possible linkage of EEWs to frontal waves is explored within the framework of so-called ‘surface quasi-geostrophic dynamics’ by establishing the properties of free and forced EEWs propagating on an ambient flow comprising a laterally confined baroclinic zone. It is shown that the dispersion properties of the free EEWs modes are influenced significantly both by the zone’s baroclinicity extremum and by key features that are akin to those of cold and warm fronts. In particular the larger-scale waves are nondispersive and a local wave-packet can retain coherency, the leading modes on a pseudo-cold front possess a phase speed related to the baroclinicity extremum, whereas waves on a pseudo-warm front propagate in the reverse direction. Also theoretical considerations demonstrate that forced modes can undergo resonant secular growth. It is further shown that transient or sustained forcing of a front by an upper-tropospheric potential vorticity anomaly (sic. short-wave trough) can serve to instigate features akin to frontal wave-trains and type-B cyclogenesis. More generally the study hints at the significance for NWP of a front’s structure and its dispersion properties, and that an element of predictability is introduced by the latter features due to dynamical scale selection.

Keywords: Eady edge waves, frontal wave-trains, type-B cyclogenesis, surface quasi-geostrophic dynamics

1. Introduction

Two seminal features of extra-tropical atmospheric dynamics are the patterns of potential vorticity in the free atmosphere and the near-surface potential temperature (Hoskins et al., 2007). From this standpoint, Eady edge waves (EEWs) are integral to the dynamics because they constitute perturbations of an ambient surface baroclinic zone. In essence, the across-zone flow component of such a wave distorts the surface thermal pattern and thereby enables the wave to propagate along the zone.

On large synoptic scales, EEWs possess a mildly evanescent vertical structure and on this scale classical baroclinic instability of so-called type-A cyclogenesis (Petterssen et al., 1962; Petterssen and Smebye, 1971) can be viewed as the interaction between an EEW at the surface and an analogous wave propagating on the strong isentropic potential vorticity gradient at the tropopause break (Davies and Bishop, 1994).

On smaller synoptic and sub-synoptic scales, EEWs possess a stronger evanescent structure with significant wave amplitude only in the lower half of the troposphere. Such waves bear comparison with the depiction of frontal wave-trains on surface synoptic charts as envisaged inferentially by Fitzroy (1863), analysed empirically by Bjerknes and Solberg (1922) and encapsulated diagnostically by Shapiro and Keyser (1990). The wave-trains often evolve to form one or more cyclones, and it has frequently been asserted that their dynamics differs intrinsically from that of their larger synoptic-scale counterparts (see e.g. Charney, 1954; Hewson, 2009; Catto, 2016).

It is these shorter horizontal-scale Eady edge waves with strong vertical evanescence that forms the focus of the present study. The study is prompted by their resemblance to frontal waves because it hints at a possible role for EEWs in frontal wave cyclogenesis. It is motivated by two factors. *First*, the recognition that frontal waves occur *per se* within a richly structured ambient flow comprising a front with a local baroclinicity maximum. Hence, it is desirable to examine the influence of such a feature upon the character of EEWs. *Second*, the apparent similitude of the far-field

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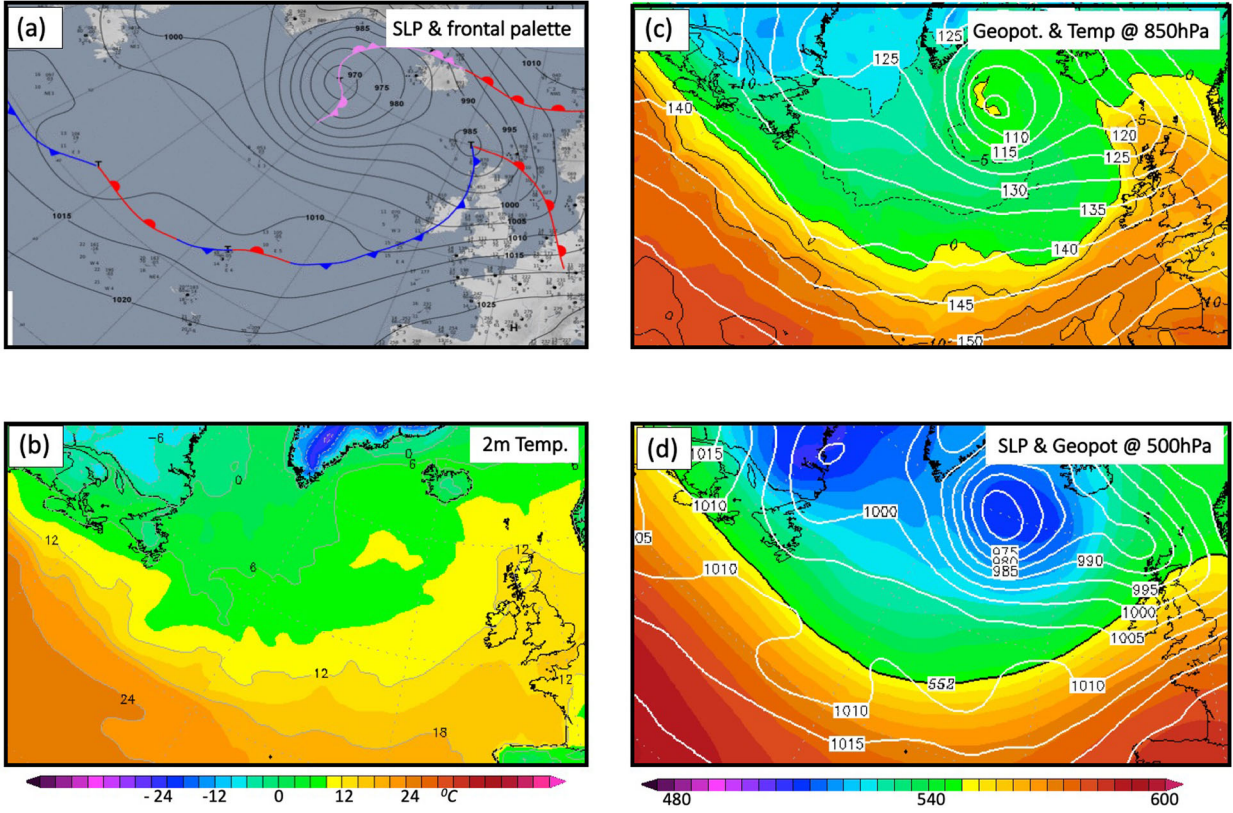


Fig. 1. Illustrative example of a frontal wave-train observed over the central North Atlantic and western Europe at 00Z on the 30 October 2020. Panel (a) displays the SLP pattern (contours interval 5 hPa) and the accompanying frontal palette. Remaining panels display the contemporaneous pattern of (b) the 2m temperature (coloured contours every 3 K), (c) the geopotential height (white isolines, gpm) and temperature (contours interval 5 K) on the 850 hPa surface and (d) the SLP field (white isolines every 5 hPa) and the geopotential field on the 500 hPa surface.

forcing of an EEW to the firmly established concept of type-B cyclogenesis whereby a tropopause-level short-wave trough (Petterssen and Smebye, 1971) or localised PV anomaly (Hoskins et al., 2007) approaching overhead of a surface front triggers frontal cyclogenesis.

In line with the above, we first document cursorily a realised atmospheric frontal wave-train that is akin to an EEW and set out the study’s theoretical framework (Section 2). Then, we proceed sequentially to:- derive and detail the properties of EEWs propagating on a uniformly sheared baroclinic zone (Section 3) and on an ambient state possessing a baroclinicity maximum (Section 4). Thereafter, we explore the character of EEWs forced by far-field features (Section 5), and discuss the possible link of EEWs to realised frontal waves and type-B cyclogenesis (Section 6).

2. Preliminaries

2.1. Features of a nascent frontal wave-train

Frontal wave-trains are a regular feature of extra-tropical surface synoptic charts for maritime and coastal regions.

There are numerous case-study analyses and climatological compilations that relate to aspects of such wave trains (Ford and Kent Moore, 1990; Appenzeller and Davies, 1992; Fehlmann and Davies, 1999; Zhang et al., 1999; Parker, 1998; Patoux et al., 2005; Boettcher and Wernli, 2011; Simmonds et al., 2012; Schemm and Sprenger, 2015). The objective in this subsection is merely to introduce and illustrate some characteristic features of frontal wave-trains as they pertain to the present study.

Figure 1 displays a wave-train on an elongated cold front that extended across the extra-tropical North Atlantic at 00Z on the 30 October 2020. The surface pressure field and accompanying frontal pattern (Fig. 1a) reveal a train with a sub-synoptic along-front wavelength ($\sim 1800\text{--}2400$ km) co-aligned with a strong surface baroclinic zone (Fig. 1b) of width approximately 800–1000 km. The contemporaneous 850 hPa and 500 hPa level charts (Fig. 1c,d) suggest that the wave-train was strongly evanescent in the vertical exhibiting only weak evidence of a pattern akin to a frontal wave-train aloft although satellite imagery (not shown) capture a cloud

pattern consistent with two incipient frontal waves. Other notable features at this time include a deep low-pressure system south-west of Iceland and an upper-level short-wave trough north of Newfoundland.

The aforementioned readily identifiable horizontal and vertical scales support the assertion that the wave-train structure can have features akin to that of an Eady edge wave. This example also underlines the need to establish the properties of EEWs propagating, not on a broad uniform baroclinic zone (Gill, 1982; Davies and Bishop, 1994; Wernli et al., 1999), but rather on a more compact frontal zone with a distinct baroclinicity maximum.

2.2. Theoretical framework

The fluid is assumed adiabatic and Boussinesq, and the flow is taken to be: confined to a channel flow on an f -plane bounded laterally by rigid walls at $y = [0, a]$; periodic in the x -direction; and semi-infinite above a flat bottom boundary located at $z = 0$. The dynamical framework is that of surface quasi-geostrophic (SQG) dynamics so that flow development is governed essentially by the evolving surface potential temperature distribution (Schär and Davies, 1990; Held et al., 1995). SQG dynamics further implies a uniform background Brunt–Väisälä stratification (N) with zero relative potential vorticity in the interior.

For this setting, small perturbations of a two-dimensional balanced basic steady state of a flow $U = U(y, z)$ and buoyancy, $B = B(y, z)$ is governed by two equations,

$$q' = 0 \quad \text{in the interior } (z > 0) \quad (1a)$$

and

$$D_G b' / Dt = 0 \quad \text{applied at } z = 0. \quad (1b)$$

Here, (q', b') are the perturbation potential vorticity and buoyancy, such that

$$q' = \left[\nabla_H^2 \psi' + (f/N)^2 \partial^2 \psi' / \partial z^2 \right], \quad b' = (g\theta^* / Q) = f \partial \psi' / \partial z \quad (2)$$

with ψ' denoting the perturbed flow's stream-function (i.e. $\mathbf{v}'_G = \mathbf{k} \wedge \nabla_H \psi'$), and θ^* the departure of the potential temperature from that of the ambient background state. The operator (D_G / Dt) is the linearised Lagrangian derivative following the geostrophic flow at the surface, so that

$$D_G b' / Dt = \left\{ \partial b' / \partial t + U(y, z) \partial b' / \partial x + v'(x, y, z) \partial B(y, z) / \partial y \right\}. \quad (3)$$

In effect for this linear setting SQG dynamics is governed by the relationship $q' = 0$ in the interior (Eq. (1a)) and

the thermodynamic equation (Eq. (1b)) applied at the surface. The latter takes the form,

$$\begin{array}{l} \partial b' / \partial t + U(y, 0) \partial b' / \partial x + v'(x, y, 0) \\ \partial B(y, 0) / \partial y = 0 \end{array} \quad (4)$$

I II III

Here, terms *II* and *III* relate, respectively, to the advection of the wave by the basic state surface flow, and the perturbation of the basic state's buoyancy by the across-front velocity.

3. Classic Eady edge waves

3.1. Derivation of dispersion properties

Consider the properties of an Eady edge wave propagating on a basic state of uniform baroclinicity with the buoyancy ($B = B_E$) of the form,

$$B_E(y, z) = (1/2) \Delta (1 - 2y/a), \quad (5a)$$

associated with a uniformly sheared basic state flow,

$$U_E(z) = (\Delta/a)(z/f), \quad (5b)$$

Here, Δ denotes the buoyancy difference of the mean state across the width of the channel, so that (Δ/a) is the measure of the baroclinicity.

Perturbations are governed by Eq. (4) but with no advective contribution from the mean flow (term *II*) because the surface velocity is zero. Modal solutions with an across-channel form of $\sin(\pi y/a)$, such that

$$b' = b_E \exp(-\mu z) \sin(kx - \omega t) \sin(\pi y/a), \quad (6a)$$

$$\text{with } \mu^2 = (N/f)^2 \left[(\pi/a)^2 + k^2 \right] \quad (6b)$$

satisfy the dispersion equation,

$$\omega = (\Delta/a)(1/f\mu)k. \quad (7a)$$

(Gill, 1982; Davies and Bishop, 1994). This relationship highlights the case for considering laterally confined EEWs. For an unconfined medium with $[k/(\pi/a)] \gg 1$, the wave frequency is almost constant, the group velocity zero, and hence, a local disturbance would remain stationary and pulsate (Müller et al., 1989). For a confined medium and in the 'long' wavelength limit, i.e. $[k/(\pi/a)] \ll 1$, the waves are almost nondispersive, and a local wave-packet would retain its coherency. This contrasting behaviour is central to our considerations.

In dimensionless form Eq. (7a) can be written as,

$$\omega^* = K / (1 + K^2)^{1/2} \quad (7b)$$

where, $\omega^* = \omega / [(\Delta/a)(1/N)]$ and $K = k/(\pi/a)$ are, respectively, nondimensional measures for the frequency and the along-front wavenumber.

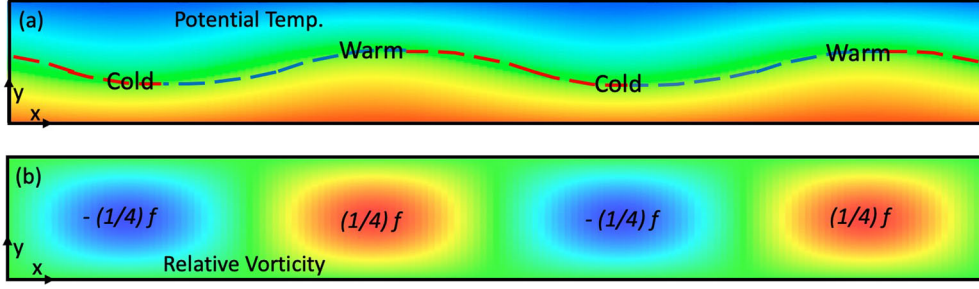


Fig. 2. Depiction of the surface patterns of a finite amplitude Eady edge wave on a zone of uniform baroclinicity confined by rigid lateral walls. Panels show the distribution of (a) the potential temperature and (b) the relative vorticity (See text for further details).

Note that for a broad baroclinic zone (say, $a \sim 900$ km) or a front-like zone (say, $a \sim 400$ km), a dimensionless wavenumber of $K = 1/2$ (i.e. located in the comparatively nondispersive range) would correspond, respectively, to wavelengths (L) of 3600 and 1600 km.

3.2. Further features

Several other features of these modal solutions are also pertinent to the present study. *First*, there is a critical height, $Z_c = (1/\mu)$, at which the wave's phase velocity matches that of the basic state flow. In dimensionless terms the critical height is governed by the relationship,

$$(N/f)(\pi Z_c/a)(K^2 + 1)^{1/2} = 1.$$

In effect, a large K implies a small critical height and/or a wide front. For example $(a, L) = (400, 1600)$ km corresponds to $Z_c \sim 1$ km for $N \sim 10^{-2} \text{ s}^{-1}$.

Second, the wave's vertical velocity field is given by

$$w'(x, y, z) = -(1/N^2)U(z)(\partial b'/\partial x).$$

so that this field leads the thermal field by a quarter of a wave-length, and its amplitude variation with height

$$|w'| = -(1/N^2)(\Delta/fa)z \exp(-\mu z),$$

attains an extremum at the critical height $Z_c = (1/\mu)$.

Third, an air parcel's vertical displacement, η' , is in phase with the thermal field, and its amplitude variation in the vertical is determined by

$$|\eta'| = -(1/N^2)(w - Uk)^{-1}/w'$$

so that the singular behaviour at Z_c is indicative of the criticality of that elevation.

Fourth, the wave's surface pressure and potential temperature are co-located in contrast to the relative westward shift of the surface pressure relative to the thermal field for conventional baroclinic instability.

Fifth, these particular waves are valid nonlinear SQG solutions. This follows from noting that, for an EEW on

a uniform baroclinic zone, the nonlinear terms take the form of the horizontal Jacobian, $J(\psi, \partial\psi/\partial z)$, which equates to zero because $\psi = -\mu\partial\psi/\partial z$. Note that a wave amplitude of $b_E \sim (1/8)\Delta$ has an accompanying vorticity (ζ') of $|\zeta'|/f \sim (1/8)\Delta/(Z_c N^2)$. It follows that for a basic state with an across-channel potential temperature difference of say $\sim 6^\circ\text{K}$ (i.e. $\Delta = 1/5$), corresponding to a jet of $\sim 30 \text{ ms}^{-1}$ at tropopause altitude and $(a, L) = (400, 1600)$ km would imply $|\zeta'|/f \sim (1/4)$ for typical tropospheric values of N . An illustration of such an EEW is provided in Fig. 2.

In effect a finite amplitude shallow EEW is associated with flow features and vorticity perturbations comparable to that of a nascent frontal wave cyclone. This lends credence to our rationale for exploring further the possible link of EEWs to frontal waves. Caveats to this association that relate to the temporal decay of EEWs due to turbulent boundary layer frictional effects and contrariwise to their possible growth due to the Diabatic Rossby Wave (DRW) effect, will be discussed later.

4. Eady edge waves on rudimentary baroclinic zones

The properties of the EEWs discussed in the previous section pertain to a basic state of uniform baroclinicity. Also the wave's lateral structure and that of its higher order eigenfunctions ($\sin(3\pi y/a)$, $\sin(5\pi y/a)$...) are linked directly the channel width. In contrast a realised frontal wave occurs on, and propagates along, an essentially laterally confined baroclinic zone possessing a baroclinicity maximum. Here, we examine the influence of the latter feature upon the structure and properties of an EEW in comparison with the classic EEW of the previous section.

4.1. Prototype cold and warm fronts

Two basic state configurations are considered that ostensibly bear some resemblance to, respectively, warm and

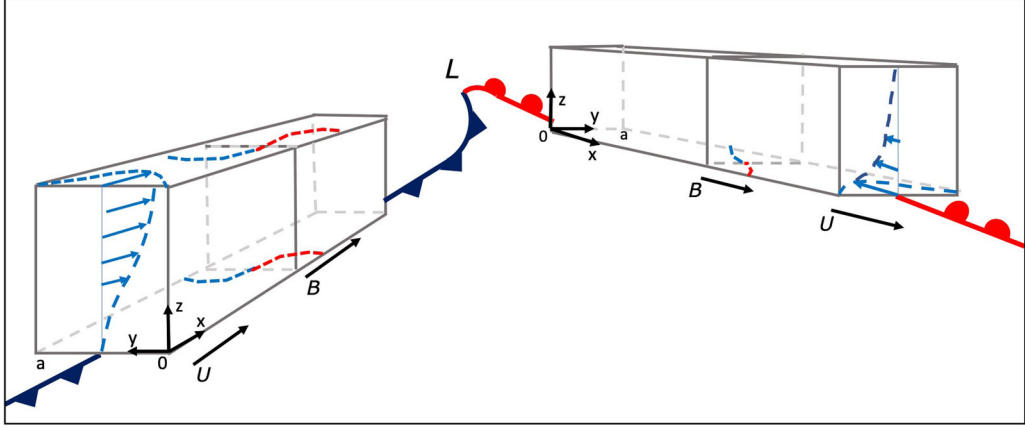


Fig. 3. A schematic of the basic states for the prototype cold and warm frontal configurations. The profiles refer, respectively, to the along-front velocity (U), and the across-front buoyancy pattern (B). The symbol L denotes the location of the parent cyclone.

cold frontal zones. Relative to a parent Low pressure system both have along-front flow components directed towards the parent system's centre. A schematic of their spatial structure is sketched in Fig. 3, and their mathematical specification is set out below.

Consider a shallow baroclinic zone confined to the lower troposphere with a buoyancy field ($b = B_S$) of the form

$$B_S(y, z) = (1/2)\Delta \cos(\pi y/a) \exp(-\mu_m z) \quad (8a)$$

where, $\mu_m = (N/f)(\pi/a)$

This pseudo-front has zero potential vorticity in the interior and, as in Section 3, a net buoyancy difference across the channel of Δ . In addition it has an evanescent vertical structure with an e-folding distance of $(1/\mu_m)$, a baroclinicity extremum of $(\partial B_S/\partial y) = -(\Delta/a)(\pi/2)$ located at $y = 1/2a$, and an along-front velocity (U_S) given by

$$U_S(y, z) = -(1/2)(\Delta/N) \sin(\pi y/a) \exp(-\mu_m z) \quad (8b)$$

We regard this basic state configuration as a *prototype warm front*. A net across-front potential temperature difference of 5°K with $N \sim 10^{-2}\text{s}^{-1}$ equates to a surface flow in mid-channel of $U_S(1/2a, 0) \sim -7.5\text{ms}^{-1}$ with the colder air on the flow's right flank. This is consistent with SQG dynamics and, apart from its vertical alignment, bears comparison with:- observed warm fronts; the transition region on an elongated front between its cold and warm segments; and idealised semi-geostrophic shallow fronts (c.f. Davies and Müller, 1988).

For a *prototype cold front* the configuration prescribed by Eq. (8) is modified by an additional contribution whose buoyancy (B_U) and flow (U_U) at the surface leaves unmodified the front's surface baroclinicity but offsets its along-front flow. This additional component mimics the surface contribution attributable to an elevated interior (sic. tropopause-level) potential vorticity pattern astride

the extra-tropical jet at the tropopause-break (Martius et al., 2010). In the present idealised setting this can be viewed as the contribution due to a similar lateral baroclinic structure, $\cos(\pi y/a)$, at a rigid upper lid and with q' zero in the interior, so that the full basic state frontal field takes the form,

$$B_U(y, z) = (1/2) \cos(\pi y/a) \cdot \cosh(\mu_m z) \quad (9a)$$

and

$$U_U(y, z) = (1/2)(\Delta/N) \sin(\pi y/a) \cdot \sinh(\mu_m z) \quad (9b)$$

For this setting, a localised jet exists at the rigid lid. Also the vertical shear is significantly weaker at the lower levels in comparison with that at the lid, and this in turn influences the location of the critical height (Z_c). We will further assume subsequently that the lid is sufficiently high so that the thermal field at that elevation is negligibly disturbed by highly evanescent surface wave perturbations.

4.2. EEWs on the prototype fronts

4.2.1. Derivation of dispersion equations. Small amplitude perturbations of the above basic states (Eqs. (8) and (9)) again satisfy the linearised Eq. (4), and to facilitate comparison with the results of the previous Section we proceed heuristically to seek EEW modal solutions of the form,

$$b' = \sum_{i=1,2} \{b_i \sin((2i-1)\pi y/a) \exp(-\mu_i z)\} \sin(kx - \omega t), \quad (10)$$

with $\mu_i^2 = (N/f)^2 [k^2 + (2i-1)^2(\pi/a)^2]$. In effect we assume that b' takes the form of a two-term limit of an infinite odd-powered sine series, and we later demonstrate this is a reasonable limit form.

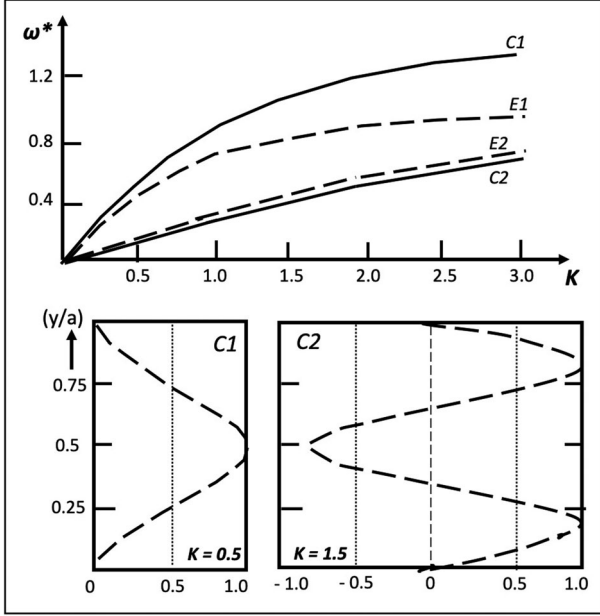


Fig. 4. Upper panel shows the dispersion diagram of the dimensionless frequency (ω^*) and dimensionless wavenumber (K) for Eady edge wave perturbations for two baroclinic settings. The continuous curves are for the $C1$ and $C2$ modes on an idealised cold front, whilst the dashed lines are for the counterpart $E1$ and $E2$ modes on a zone of uniform baroclinicity. Lower panels show the across-front buoyancy profiles of the $C1$ and $C2$ modes for, respectively, $K=0.5$ and 1.5 .

Substituting this expression for b' into the linear Eq. (4) produces contributions to terms *II* and *III* that are proportional, respectively, to $\sin^2(\pi y/a)$ and $\sin(\pi y/a) \sin(3\pi y/a)$. To retain the postulated wave form (i.e. Eq. (10)), we approximate

$$\begin{aligned} f_1(y) &= \sin^2(\pi y/a) \text{ by} \\ g_1(y) &= \alpha_1 \sin(\pi y/a) + \beta_1 \sin(3\pi y/a) \end{aligned} \quad (11a)$$

and

$$\begin{aligned} f_2(y) &= \sin(\pi y/a) \sin(3\pi y/a) \text{ by} \\ g_2(y) &= \alpha_2 \sin(\pi y/a) + \beta_2 \sin(3\pi y/a). \end{aligned} \quad (11b)$$

The (α, β) coefficients are determined with a ‘least squares function fit’ to the integral,

$$I = \int \{f(y) - g(y)\}^2 dy \text{ over the interval } [0, a].$$

This yields $[\alpha_1, \beta_1] = [8/(3\pi), -8/(15\pi)]$ and $[\alpha_2, \beta_2] = [-8/(15\pi), 72/(35\pi)]$, and the normalised ‘error’ measure, $I / \int \{f^2(y) dy\}$, for the two fits amount to modest values of, respectively, 0.024% and 2.7%.

Insertion of these decompositions into the linearised perturbation equation results in the following quadratic dispersion relationships for the EEWs:-

(a) the pseudo-cold front

$$v^2 + Sv + \gamma(\varphi_1 - 1)(\varphi_2 - 1) = 0, \quad (12)$$

(b) the pseudo-warm front

$$v^2 + (\alpha_1 \varphi_1 + \beta_2 \varphi_2)v + \gamma \varphi_1 \varphi_2 = 0. \quad (13)$$

Here, $v = (2/\pi)(\omega^*/K)$, is the wave’s nondimensional phase velocity with ω^* and K defined as before, and γ , φ_1 , φ_2 and S are given by

$$\begin{aligned} \gamma &= (\alpha_1 \beta_2 - \beta_1 \alpha_2), \varphi_1 = 1 - 1/(1 + K^2)^{1/2}, \\ \varphi_2 &= 1 - (1/3)/(1 + K^2/9)^{1/2}, \\ S &= \{\alpha_1(\varphi_1 - 1) + \beta_2(\varphi_2 - 1)\} \end{aligned}$$

4.2.2. Dispersion properties of EEWs on the pseudo-cold front. For the pseudo-cold front, the dispersion curves for its two modes ($C1$ and $C2$) are shown in the upper panel of Fig. 4. In the absence of an ambient surface flow the contribution of the baroclinicity (Term *III* of Eq. (4)) ensures that the phase velocity is positive, i.e. in the direction of the ambient flow overhead, and hence directed towards a parent cyclone. Both modes are nondispersive at longer wavelengths (the $C1$ modes for $K^2 < 1.5$, and the $C2$ modes for $K^2 < 9$), so that wave-packets of the modes in these restricted wave-bands would retain a measure of coherency as they propagate along the baroclinic zone.

Also shown in the upper panel are the dispersion curves for the classic EEW modes ($E1$ and $E2$) discussed in Section 3 that have a $\sin(\pi y/a)$ and $\sin(3\pi y/a)$ lateral structure. The $C1$ and $C2$ modes bear a strong qualitative resemblance to these counterpart uniform baroclinicity modes, but are less dispersive over a broader band of K and also the $C1$ mode possesses a distinctively higher frequency than the $E1$ mode so that the phase velocity is higher by a factor of $\sim (\pi/2)$. The lower panels show the across-front amplitude of the $C1$ mode for $K=0.5$ and the $C2$ mode for $K=1.5$. The profile for $C1$ is a mild but significant modification of a $\sin(\pi y/a)$ shape with $(b_1/b_2) \approx -5.5$. Its sharper peak in mid-channel and reduced amplitude on the flanks implies that it responds to the frontal zone’s baroclinicity extremum rather than the mean baroclinicity across the channel. This in turn accounts for its faster phase-speed compared with the counterpart ($E1$) mode. In contrast, the $C2$ mode with its rich across-front structure is closely akin to $\sin(3\pi y/a)$ with $(b_1/b_2) \approx +0.175$, but its weaker extremum in mid-channel mildly reduces its phase velocity relative to the $E2$ mode.

The qualitative similarity of the C and E modes suggests that the uniform baroclinicity limit is a reasonable first-order simplification for the study of frontal waves

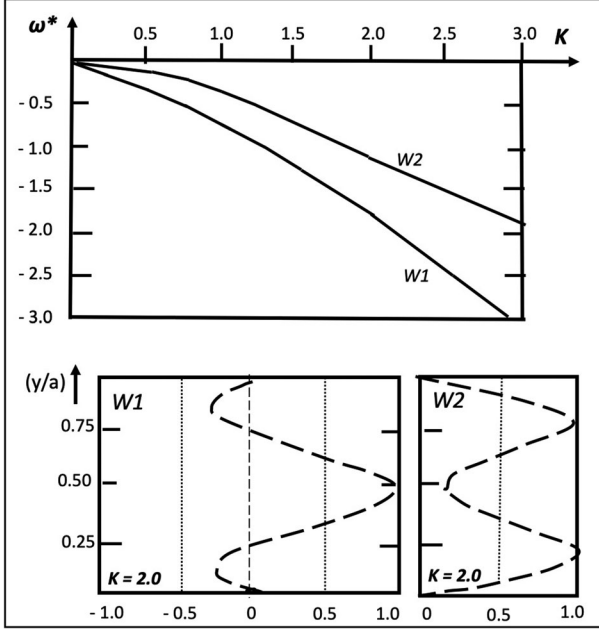


Fig. 5. As for Fig. 4, but now for the $W1$ and $W2$ modes on an idealised warm front, and the across-front buoyancy profiles are both for $K=2.0$.

provided account is taken of the front's finite lateral scale and baroclinicity extremum.

4.2.3. Dispersion properties of EEWs on the pseudo-warm front. For the pseudo-warm front, the dispersion diagram for the system's two modes ($W1$ and $W2$) is shown in the upper panel of Fig. 5. Both modes have a negative frequency and phase-velocity, so that the advection of the wave by the ambient flow (Term II of Eq. (4)) overcomes the contribution of the baroclinicity (Term III). In meteorological parlance, the modes would propagate towards the front's parent cyclone. Both W modes possess a comparatively low frequency for $K < 0.75$, whilst their group velocity tends to a constant for $K > 1.5$.

The lower panels of Fig. 5 show the across-front profile of the two modes for a wavenumber, $K=2$, in the comparatively nondispersive waveband, and possess comparable $\sin(\pi y/a)$ and $\sin(3\pi y/a)$ contributions. For the $W1$ mode, with $(b_1/b_2) \approx -1.066$, the inference is that because the wave structure on the flanks is out of phase with that in mid-channel the positive phase-velocity customarily associated with the mid-channel baroclinicity is weakened and the advective effect dominates. In contrast for the $W2$ mode, with $(b_1/b_2) \approx +1.1647$, the wave structure on the flanks is dominant but its influence upon the propagation is not countered by the flow in mid-channel, and hence, the resultant phase-velocity is less negative.

These warm-front dispersion properties differ radically from those of their classic counterparts even if allowance were made for a doppler-shift of the latter due to, say, a positive surface velocity. Trenchantly the negative surface velocity of the pseudo-warm front is an intrinsic feature of SQG dynamics whereby the velocity is determined exclusively by the surface potential temperature distribution. By default it also lends credence to our earlier inference that the along-front component of realised *cold* fronts is influenced significantly by the interior PV distribution.

4.2.4. Other features of EEWs on the prototype cold front. In addition to the dispersion properties noted above for EEWs on the prototype cold front, there are some other differences to the features of classic EEWs listed in Subsection 3.2. In particular the critical height will be at a slightly higher elevation due both to the decreased shear of the basic state at low levels (– see comments following Eq. (9)) and to the wave's larger phase speed.

Also the C modes, although excellent approximate solutions of the linearised Eq. (4), are not exact nonlinear SQG solutions because the two-term form of ψ (Eq. (10)) only half of the neglected nonlinear terms satisfy the relationship $J(\psi, \partial\psi/\partial z) = 0$. The other terms are small provided

$$[6b_2/\Delta; 2(\mu_1/\mu_2)b_2/\Delta]_{\max} \ll 1.$$

Hence, the $C1$ modes are for example acceptable finite amplitude SQG solutions provided their amplitude is small compared with the across-channel buoyancy difference (Δ).

5. Forced Eady edge waves

Here, we extend our previous results to examine the EEW response to far-field forcing for both uniform (c.f. Section 3) and nonuniform (c.f. Section 4) baroclinicity settings.

5.1. The uniform baroclinicity setting

Consider the triggering of an EEW on a uniform baroclinic zone forced by an interior PV anomaly that is localised in the vertical. The anomaly is assigned the form of a simple free-atmosphere perturbation potential vorticity, q' ,

$$q'(x, y, z) = P\delta(z - d) \cos k(x - U_d t) \sin(\pi y/a). \quad (14)$$

In effect q' has a wave-like pattern in the horizontal, a delta function structure in the vertical located at a height

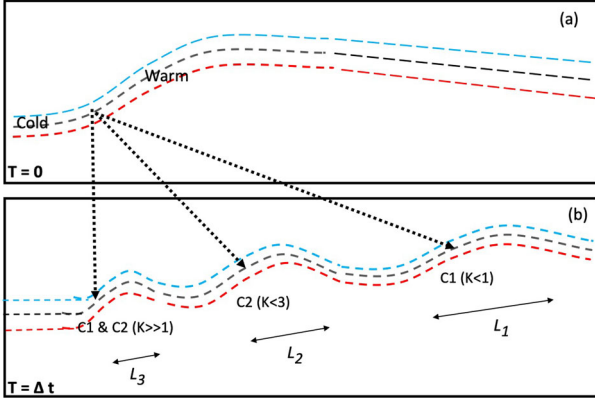


Fig. 6. Schematic of the potential temperature distribution at the surface following a transient forcing by an upper-level PV anomaly. Panels (a and b) depict, respectively, the initially induced cold-warm couplet, and the subsequent evolution of propagating coherent wave-packets of long (half width L_1) and intermediate (half width L_2), and the stationary pulsating short (half width L_3) wave-packet.

$z=d$ (c.f. Thorncroft and Hoskins, 1990; Davies and Bishop, 1994), and translates with the ambient mean flow at its level ($U = U_d$). For interpretative purposes the forcing, P , can be viewed as a uniform potential vorticity distribution extending over a small depth ε centred at $z=d$, such that $(P \varepsilon) = M$ with M remaining constant as $\varepsilon \rightarrow 0$.

In accord with the PV perspective we seek the stream function, ψ'_q , associated with Eq. (14) that has no thermal signal at the ground, and satisfies appropriate interface conditions at the height at $z=d$. The resulting stream function takes the form,

$$\begin{aligned} \psi'_q &= -[P * e^{-\mu d}] \cos \{k(x - U_d t)\} \sin(\pi y/a) \cosh(\mu z), \\ &\quad \text{for } 0 < z < d \\ \psi'_q &= -[P * e^{-\mu d} \cosh(\mu d)] \cos \{k(x - U_d t)\} \\ &\quad \sin(\pi y/a) e^{-\mu(z-d)} \text{ for } z > d. \end{aligned}$$

where, $P^* = (N/f)^2 M/\mu$.

A key feature of this configuration is that the across-front flow component of the interior anomaly at the surface influences the development of an EEW. In particular for a q' perturbation located at the critical height (i.e. $d = Z_c$), it follows from Eqs. (1a,b) and (4) that the forced EEW response is,

$$b' = (At) \cdot \sin \{k(x - U_d t)\} \sin(\pi y/a) \exp(-\mu z). \quad (15a)$$

with $A = \{(\Delta/a) k P^* e^{-\mu d}\}$

It follows that an EEW emerges with time that is in quadrature with, and leads, the free atmosphere perturbation potential vorticity (q'). It propagates with the phase

velocity of the corresponding EEW (Eq. (7)) but its amplitude undergoes secular growth.

Note that, for a more general distribution of q' forcing located at diverse heights and associated with a range of wavenumbers, it is the resonant wave that would eventually dominate the surface pattern. In effect, the flow would undergo a geometrical-cum-dynamical scale selection given by (c.f. Subsection 3.2),

$$[\pi(N/f)(d/a)] (K^2 + 1)^{1/2} = 1.$$

5.2. Setting of nonuniform baroclinicity

Consider a Type C1 wave mode on a cold frontal zone (i.e. Eq. (9)) forced by an across-front surface flow, $V(y,0)$, of the form,

$$V(y,0) = \phi \sin \{k(x - U_d t)\} \sin(\pi y/a).$$

This is taken to correspond to the flow associated with a localised q' of the form given by Eq. (14) located at a height $z=d$ and corresponding to the critical level of the C1 mode and advected by the flow at that level.

Resonance usually requires meeting particular criteria, and could in principle be hindered or even prohibited by the lateral structure of the C1 mode. For the present setting a resonant solution of the form,

$$\begin{aligned} b'(x, y, 0) &= (Bt) [b_1 \sin(\pi y/a) + b_2 \sin(3\pi y/a)] \\ &\quad \sin \{k(x - U_d t)\} \end{aligned} \quad (16a)$$

exists provided

$$\begin{aligned} (b_1/b_2) &= (\alpha_1/\beta_1) \cdot (\mu_2/\mu_1) \\ &= -5.0(K^2 + 9)^{1/2}/(K^2 + 1)^{1/2} \end{aligned} \quad (16b)$$

The free mode's structure is such that at most only a near-resonance condition can prevail with $K \sim 3$ being the most favourable.

6. Link to frontal waves and type-B cyclogenesis

Here, we explore the possible link of EEWs both to the occurrence of frontal wave-trains and to the manifestation of type-B cyclogenesis. To this end, we draw upon our previously derived results related to EEW dispersion and far-field forcing, and consider whether transient or sustained forcing by interior PV anomalies could result in the development of coherent EEW-related features at the surface.

6.1. Transient PV-forcing

Transient forcing of EEWs on a surface front would prevail if an interior PV-anomaly only influenced the surface flow significantly for a limited time. Such a scenario is frequently observed when an isolated fine-scaled tropopause-level PV anomaly embedded within a larger-scale baroclinic development moves rapidly towards the upper-level jet before being advected away by the near-jet flow (Kew et al., 2010). For the instigation of EEWs this equates to the rapid passage of the anomaly towards and away from an underlying elongated cold front. Its influence would create a cold-warm couplet (Fig. 6a) confined to the baroclinic zone, and the couplet itself can be viewed as a wave-packet comprising a broad band of EEWs.

In the linear limit, the subsequent evolution can be inferred from consideration of the dispersion properties of the $C1$ and $C2$ modes on a cold front (Fig. 4). The evolution entails the emergence of three coherent wave-packets (see Fig. 6):- one composed of $C1$ modes with $K < 1.5$ that would retain coherency and propagate comparatively quickly along the baroclinic zone; another composed of $C2$ modes with $K < 3$ would also retain coherency but propagate less rapidly along the zone; and a third composed of short wavelength $C1$ and $C2$ modes (with $K \gg 1$) that remain *in situ* at the initial location but pulsate with time.

The resulting pattern resembles a two (or three) carriage frontal wave-train with the spacing between the carriages increasing with time, whilst the individual systems would retain their disparate along-front half widths (L_1 , L_2 , L_3). The amplitude of the individual carriages would depend upon the strength of the initial couplet whereas the pattern's spatial scale would be determined by the front's lateral structure. In realised wave trains the foregoing space-time features would inevitably be influenced by other factors, but a measure of their prevalence could be readily assessed from observational analyses.

The foregoing constitutes a scale-dependent splitting of the initial surface couplet in the limit of linear flow dynamics. An intense perturbing upper-level PV-anomaly would instigate a more rapid nonlinear development of the surface couplet, and the pattern would be more akin to a type-B development (c.f. Schwierz et al., 2004).

6.2. Sustained PV-forcing

Sustained forcing would prevail if the interior PV-anomaly (or train of PV-anomalies) remained overhead of, and suitably phased with, EEWs on the surface front. However, isolated PV anomalies located close to the upper-level jet would be subject to rapid advection by the near-jet flow, and thus, probably be unable to remain

suitably phased with the EEWs. Likewise the anomaly would exert a strong dynamical perturbation upon the jet's attendant PV wave-guide (Schwierz et al., 2004), and this would mask or complicate its impact at the surface.

A possible scenario for sustained forcing can develop during the mature phase of a large-scale cyclogenesis when an elongated PV streamer extends downward and equatorward away from the jet into a region of weaker ambient flow and breaks up into a cut-off vortex or a train of individual vortices (Appenzeller and Davies, 1996). In this circumstance the vortex spacing is determined by the streamer's lateral width and is consistent with the observed scale of frontal waves.

For such a setting, the vortex/vortices would propagate with the more modest ambient velocity and retain a measure of spatial coherency. In tandem at the surface a coherent localised thermal disturbance composed of a wave-packet of $C1$ modes with nondispersive properties ($K^2 < 1.5$) could emerge if its associated translation velocity ($\sim 10 \text{ ms}^{-1}$) was comparable to that of the advection velocity of the vortex anomaly/anomalies. Again the resulting quasi-resonant response bears comparison to realised type-B cyclogenesis. As noted earlier the intensity of the upper-level PV anomaly would directly influence the initial strength of the surface signal but the latter's spatial scale would be governed by the contribution of the nondispersive and coherent EEWs.

7. Caveats and conclusions

Limitations to this study include the adoption of SQG dynamics, the assumption of channel flow, and the idealised form of the postulated frontal structures. The influence of these shortcomings is mitigated somewhat on noting that frontal waves are usually regarded as balanced systems confined to the vicinity of the front and that the idealised frontal structures capture features germane to realised fronts.

Also it was noted earlier that two caveats to the consideration of EEWs in their purest form relate to the temporal decay of EEWs due to turbulent boundary layer frictional effects and contrariwise their possible growth due to the so-called Diabatic Rossby Wave (DRW) effect. An EEW that is strongly evanescent in the vertical will be susceptible to boundary layer dissipation. To a leading order the latter effect can be included by incorporating into Eq. (1b) a conventional turbulent Ekman-layer vertical velocity, $w_{\text{top}} = \frac{1}{2} D_E \zeta_G \sin 2\alpha$, at the top of a 1 km deep boundary layer. (Here, D_E is a measure of the boundary layer depth, ζ_G is the *in situ* relative vorticity, and α the reorientation angle of the flow within the boundary layer.) This yields an e-folding decay time-scale for an EEW of

$$T \sim 4/(f\mu 10^3),$$

corresponding to approximately 1.5 and 2.5 days for evanescent scales of 3 and 5 km. However, the conventional formulation needs modification for the scenarios considered in the previous section of sub-synoptic scale, baroclinic, transient flow within a well-mixed maritime boundary layer, and a nonlinear formulation of w_{top} is likely to significantly increase the value of T .

In direct contrast an EEW's intrinsic vertical velocity field at low-level would be an integral feature of Diabatic Rossby Wave (DRW) dynamics by promoting cloud-diabatic heating and concomitantly the creation of interior PV anomalies with the latter in turn serving to force the EEW. Model and observational studies (see the list referenced in Moore et al., 2013) point to the realizability of this effect.

The relative importance of these counteracting effects has been specifically assessed in a model study (Moore and Montgomery, 2004) and indicated that the incipient growth phase of a DRW can prevail even in the presence of frictional effects. Note also that the vertical velocity of a pure Eady wave at its critical height (Subsection 3.2) is itself comparable to w_{top} .

The foregoing caveats and remarks suggest that the present study can at least provide a qualitative guide to the possible role of EEWs to:- frontal wave perturbations; the near-surface response to interior forcing; and precursor features of frontal cyclogenesis. In this context, the study highlights:- (a) the dependence of surface frontal structure and its perturbations upon the free-atmosphere potential vorticity distribution particularly at tropopause-levels; (b) the sensitivity of EEW dispersion properties to the lateral scale and the cold and warm front's low-level flow fields; (c) a front's zone of enhanced baroclinicity as the likely locale for a significant response to upper-level forcing; and (d) the possible role and the limitations of a localised interior PV anomaly in triggering features akin to frontal wave-trains and type-B like cyclogenesis. In addition, the study hints indirectly at the significance of frontal features (and hence, EEW dispersion properties) for NWP performance, and the element of predictability introduced by those same features due to a dynamical the scale-selection mechanism.

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panels are from the NCEP (CFS Reanalysis @ 0.5°) dataset.

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No potential conflict of interest was reported by the authors.

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