



A Probabilistic Weighted Joint Aggregative Drought Index (PWJADI) criterion for drought monitoring systems

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ABSTRACT

Drought is a complex natural hazard. Its several adverse impacts are prevailing in almost all climatic zones around the world. In this regards, drought monitoring and forecasting play a vital role in making drought mitigation policies. Therefore, several drought monitoring tools based on probabilistic models had been developed for precise and accurate inferences of drought severity and its effects. However, risk of inaccurate determination of drought classes always exists in probabilistic models. To overcome this issue, we proposed a new system based Probabilistic Weighted Joint Aggregative Drought Index (PWJADI) criterion for three multi-scalar drought indices, namely Standardized Precipitation Index (SPI), Standardized Precipitation Temperature Index (SPTI), and Standardized Precipitation Evapotranspiration Index (SPEI) at one-month time scale. By the basic assumption of the Markov chain, the PWJADI is based on the temporal switched weights that are propagated from the transition probability matrix of each temporal classification of drought index. Application of the proposed method is made for three meteorological stations of Pakistan. We found that our proposed model has ability to restructure the drought classes by capturing and bending the information from the historical behaviour of each drought class. Consequently, to make accurate and precise drought mitigation policies, the proposed method may integrate into effective drought monitoring systems.

Keywords: drought index, Markov chain, drought classifications, drought monitoring systems

1. Introduction

Drought is a complex natural hazard that impacts on human activities by devastating natural system, water supply, socioeconomic and ecosystems (Heim, 2002; Wilhite and Buchanan-Smith, 2005). Worldwide perpetual increase in global warming and radical climate changes are the key factors that increase the risk of recurrent occurrences of drought hazard. About 8 billion dollar annually spent on the remedies of these adverse impacts of drought hazard (Ross and Lott, 2000). Further, the demand of water has been increased in developing

countries because of expanding agricultural and industrial sectors from last two decades. Therefore, prolonged and severe water shortfall may potentially be caused a major social and economic issues (Wilhite, 2000).

However, continuous and accurate drought monitoring is useful for making drought mitigation policies. From an operational point of view, drought characterization allows early warning drought risk analysis (Kogan, 2000; Hayes et al., 2004). In the last three decades, a comprehensive drought monitoring framework has been developed to overcome the risk associated with each type of drought. For this purpose, drought indices are the most commonly used tools for the quantification and

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assessment of drought. However, methodologies of each drought index are varied from zone to zone as it depends upon the availability of data and the nature of climatic zone. Svoboda and Fuchs (2016) provided a list of drought indices with their features about drought monitoring.

A drought index based on a single climatic variable (Vicente-Serrano et al., 2010) is insufficient for drought monitoring, as multiple factors (such as, precipitation, humidity, wind speed, temperature and heat waves, etc.) are involved in it (Wilhite, 1994; Sheffield et al., 2012; Hao and Singh, 2015; Vicente-Serrano et al., 2015). Initially, a Palmer Drought Severity Index (PDSI) based on precipitation, temperature, latitude and the available water capacity was introduced for the characterization and quantification of drought (Palmer, 1965). However, in later research, it was found that the PDSI cannot be used for defining droughts at various time scales (Wang et al., 2017).

Contrary to PDSI, McKee et al. (1993) proposed Standardized Precipitation Index (SPI) drought index based on monthly total precipitation. SPI has the ability to capture drought classification states at various time scales. Parallel to SPI, Tsakiris and Vangelis (2005) proposed Reconnaissance Drought Index (RDI) which incorporates evapotranspiration for the characterization of drought. Likewise SPI, RDI has the ability to monitor drought for various time scales. Moreover, Vicente-Serrano et al. (2010) proposed a new drought index – Standardized Precipitation Evapotranspiration Index (SPEI) that can accommodate more than one climatic variable. SPEI is an extension of SPI which uses water balance equation model. Several researchers compared the performance of SPEI with SPI in various region (Gurrapu et al., 2014; Homdee et al., 2016; Labudová et al., 2016). Ali et al. (2017a) proposed a novel drought index – Standardized Precipitation Temperature Index (SPTI) that overcome the deficiency of SPEI in low temperature regions. Additionally, estimation procedure and mathematical background of SPTI is same as used in (Vicente-Serrano et al., 2010) and (McKee et al., 1993).

The primary issue in making advance drought management and mitigation policies is the selection of appropriate drought index for accurate and precise drought monitoring and forecasting. However, the uncertainty in the estimation of drought classes always exists in probabilistic models of SPI, SPEI and SPTI. Further the selection of probability distribution for each indicator is purely subjective in nature. The choice of appropriate drought index depends on several factors such as availability of climatic data, type of drought and the periodic need of water in agricultural sectors (Wilhite and Buchanan-Smith, 2005). Further, six standards of drought indices (robust, tractability, transparency, sophistication,

extendibility and dimensionality) are available in the literature for drought assessment (Keyantash and Dracup, 2002; Quiring, 2009). Therefore, the influence of a particular drought index has been often a crucial consideration in the selection of drought indicators for drought monitoring and early warning risk analysis (Abebe et al., 1998; Hayes et al., 2012; De Stefano et al., 2015). The presence of several drought indices with their variant structures are challenging for comparative assessment and regional analysis of drought because of diverging trends (Bhuiyan, 2004; Stagge et al., 2016) and the use of general circulation model (Touma et al., 2015). The SPI drought index is commonly used due to its simplicity and worldwide acceptability (Guttman, 1999; Das et al., 2016; Juliani and Okawa, 2017; Mondol et al., 2017).

On the same rationale of SPI, SPEI is one of the most popular drought indices due to its multivariate capability for drought assessment. This multivariate quality of SPEI makes it more acceptable and superior than SPI, as SPI uses only one climatic variable (i.e. precipitation only). Still, the primary issue in SPEI is the existence of undefined values in a temporal series of low temperature regions/month (Quiring, 2009). In original proposal of SPEI, Vicente-Serrano et al. (2010) used Thornthwaite (TH) equation in PET (i.e. Penman Monteith equation (Jensen et al., 1990)) estimation. However, one drawback of TH equation is to produce undefined values of PET in low temperature regions/months (Wijk and Vries, 1954; Papadopoulou et al., 2003). In addition TH equation underestimate PET in arid and semi-arid regions, whereas, provides overestimates in semi-humid and humid regions (Van der Schrier et al., 2011). Although there are other approaches for the estimation of PET, these approaches require extra climatic data.

Besides the use of influential climatic factors in the estimation procedures of drought indices (SPI, SPEI and SPTI), statistical procedure is vital because of reasons for accurate and the precise determination of drought classification. First, Blain (2013) had shown that the performance of SPI for using only gamma distribution is affected when the gamma distribution is not well fitted on historical precipitation series. Further, the role of probability distributions is vital to explore the information about extreme drought classes. To overcome and handle this erroneous condition for accurate determination of drought classes, one way is to reduce the capacity of probability functions by including more flexible multi parameter type probability distributions. Ntale and Gan (2003) used Pearson type III (P3) distribution and probability plotting method to improve SPI values. Blain and Meschiatti (2015) provided an alternate procedure to quantify the risk of drought by introducing generalized normal distribution in Standardized Precipitation Index (SPI) model. Blain (2013) proposed a drought index based on generalized extreme value distribution instead of the

Table 1. Drought classification criteria of SPI, SPEI and SPTI.

SPI/SPEI/SPTI values	Classes	Cumulative probabilities
≥ 2	Extremely Wet (EW)	0.977–1.00
1.5 to 1.99	Severe Wet (SW)	0.933–0.977
1 to 1.49	Moderate Wet (MW)	0.841–0.933
.99 to -0.99	Normal Drought (ND)	0.159–0.841
-1 to -1.49	Moderate Drought (MD)	0.067–0.159
-1.5 to -1.99	Severe Drought (SD)	0.023–0.067
≤ -2	Extreme Drought (ED)	0.000–0.023

gamma distribution (McKee et al., 1993). Recently, Stagge et al. (2015) provided a new rationale of using multiple and more flexible probability distributions for various drought indicators. However, in each probabilistic models, the uncertainty always exists in accurate and precise estimation (Parker, 2014). Moreover, the selection of probability distributions for each indicator at their various time scales is purely subjective in nature. Therefore, several recent studies proposed different distributions for the assessment and categorization of drought (Blain, 2011; Karavitis et al., 2011; Angelidis et al., 2012; Raible et al., 2017).

In this paper, we aimed to develop a new criterion – the PWJADI to overcome the uncertainty in the accurate determination of drought classes. The PWJADI has capability to give joint decision on the classification of the region under study based on various drought indices. The developed model is on the drought classification of SPI, SPEI and SPTI at one month time scale. Description of the proposed method and its application are given in Section 2 and 3. While, Section 4 consists on the concluding remarks about this research.

2. Methodology

In this section, we discussed the methodological structure of the proposed drought classification criterion termed as PWJADI.

2.1. The multi-scalar drought indices

McKee et al. (1993) developed an SPI drought index, which is based on long-term precipitation record to quantify the precipitation scarcity for different time scales of the single monitoring station. One of the major advantages of using SPI index is that it can be used to monitor drought for various time scales (e.g. 1-, 3-, 6-, 9-, 12-, 24-months). Estimation of quantitative values of SPI can be made by normalizing appropriate probability distributions of the observed monthly cumulative precipitation time series using standard inverse Gaussian functions. Negative and positive SPI values designate less than or greater than median

precipitation, respectively (Bordi and Sutera, 2007). To characterize meteorological drought, in the meeting on “Lincoln Declaration on Drought Indices” experts also suggest to use SPI drought index for all National Meteorological and Hydrological Services (NMHSs) around the world (Svoboda et al., 2012). Moreover, in past research, due to simple and probabilistic mathematical structure, several applications of drought monitoring were based on SPI drought index.

Vicente-Serrano et al. (2010) developed a multi-scalar drought index: SPEI, which is based on both temperature and precipitation data. In SPEI, water balance model (the deficient D) based on the difference between precipitation and potential evapotranspiration (PET) was used in the same methodological framework of SPI for the characterization of drought. Though, in the original paper of (Vicente-Serrano et al., 2010) used Thornthwaite equation (Thornthwaite, 1948) to estimate PET values which have a drawback in over-estimation and under-estimation of PET in low temperature regions. Yet, several other procedures like Penman equation (Allen et al., 1998) and Blaney-Criddle (Allen and Pruitt, 1986) can be used for accurate estimation of PET values (Begueria and Vicente-Serrano, 2013). In several research articles, instead of its comparison with SPI (Gurrapu et al., 2014; Maca and Pech, 2016), individual application of SPEI for drought monitoring are available (Su and Li, 2012; Banimahd and Khalili, 2013; Ali et al., 2017b).

However, empirical analysis of this research is based on Hargreaves equation (Hargreaves and Samani, 1985) where the estimation of PET values is computed using *hargreaves* function of SPE (Begueria and Vicente-Serrano, 2013) R package.

Ali et al. (2017a) developed a drought monitoring index called SPTI. Like precipitation and water deficient index D, they have suggested Demartone aridity index (Martonne, 1920) in multivariate context. In SPTI model, Demartone aridity index were standardized to obtain quantitative values of SPTI. Application of SPTI and its comparison with SPI and SPEI were made in seventeen meteorological stations of Northern Areas (northernmost administrative territory in Pakistan) and Khyber Pakhtunkhwa (KPK) (the province Pakistan). Similar to SPEI, SPTI has ability to characterize the drought region by admitting the role of temperature without any mathematical conflict. However, unlike SPEI, effectiveness of SPTI and its strong association with SPI in the low temperature regions suggest its candidacy for drought monitoring in multivariate context.

2.2. Transition probabilities matrix: drought index as a Markov process

A stochastic process, or random process $\{Z = Z_t, t \in T\}$ is a set of random variables indexed by time. That

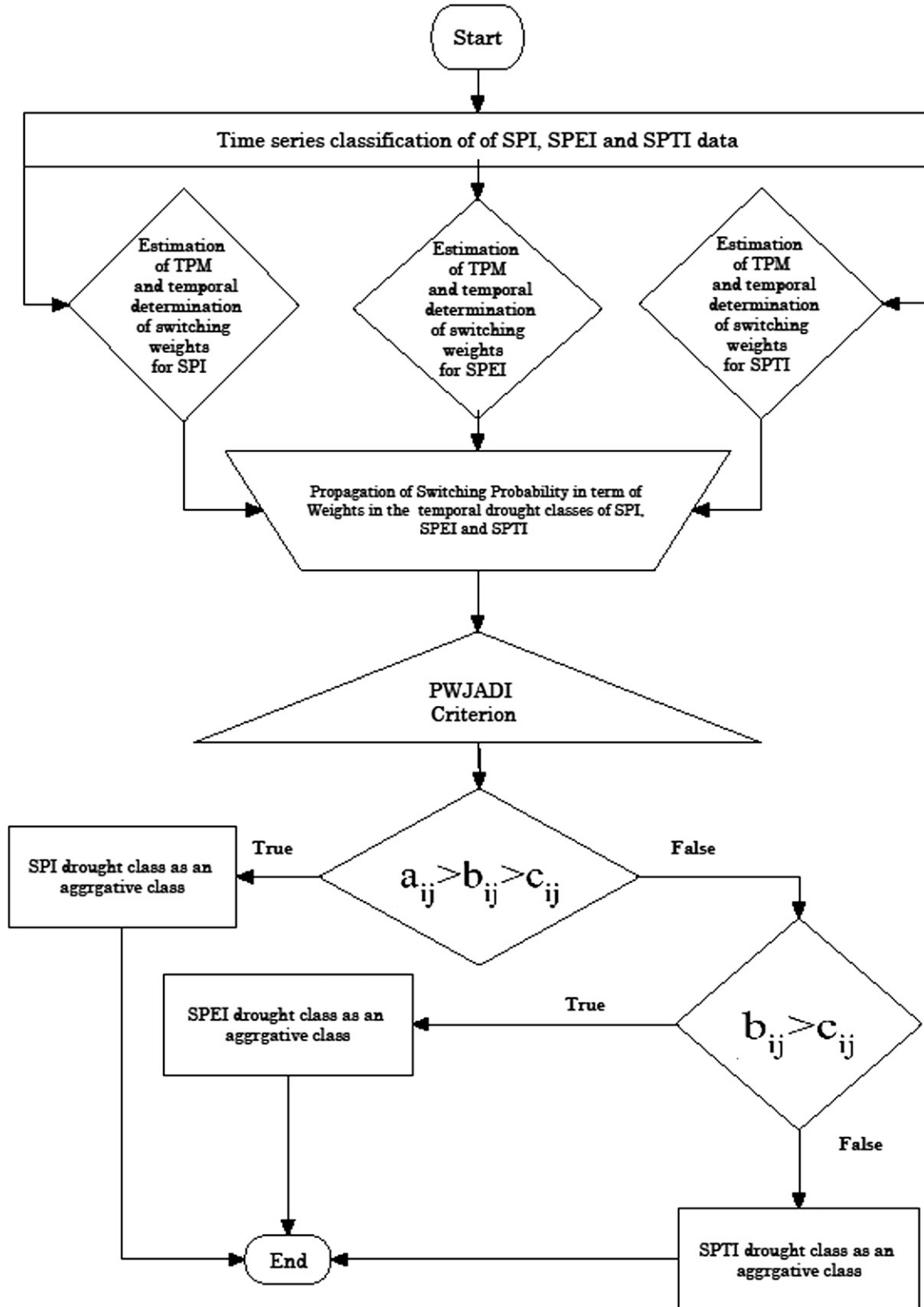


Fig. 1. Flow chart of the proposed framework.

Table 2. Algorithm evaluation of temporal drought class using PWJADI criterion.

Time	SPI (d_{1t})	SPEI (d_{2t})	SPTI (d_{3t})	PWJADI (d_{jt})
2	$p_{1c/c-1}, d_{12}$	$q_{1c/c-1}, d_{22}$	$r_{1c/c-1}, d_{32}$	$\max(p_{1c/c-1}, q_{1c/c-1}, r_{1c/c-1}) \in d_{j2}$
3	$p_{1c/c-1}, d_{13}$	$q_{1c/c-1}, d_{23}$	$r_{1c/c-1}, d_{33}$	$\max(p_{1c/c-1}, q_{1c/c-1}, r_{1c/c-1}) \in d_{j3}$
4	$p_{1c/c-1}, d_{14}$	$q_{1c/c-1}, d_{23}$	$r_{1c/c-1}, d_{34}$	$\max(p_{1c/c-1}, q_{1c/c-1}, r_{1c/c-1}) \in d_{j4}$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
n	$p_{1c/c-1}, d_{1n}$	$q_{1c/c-1}, d_{2n}$	$r_{1c/c-1}, d_{3n}$	$\max(p_{1c/c-1}, q_{1c/c-1}, r_{1c/c-1}) \in d_{jn}$

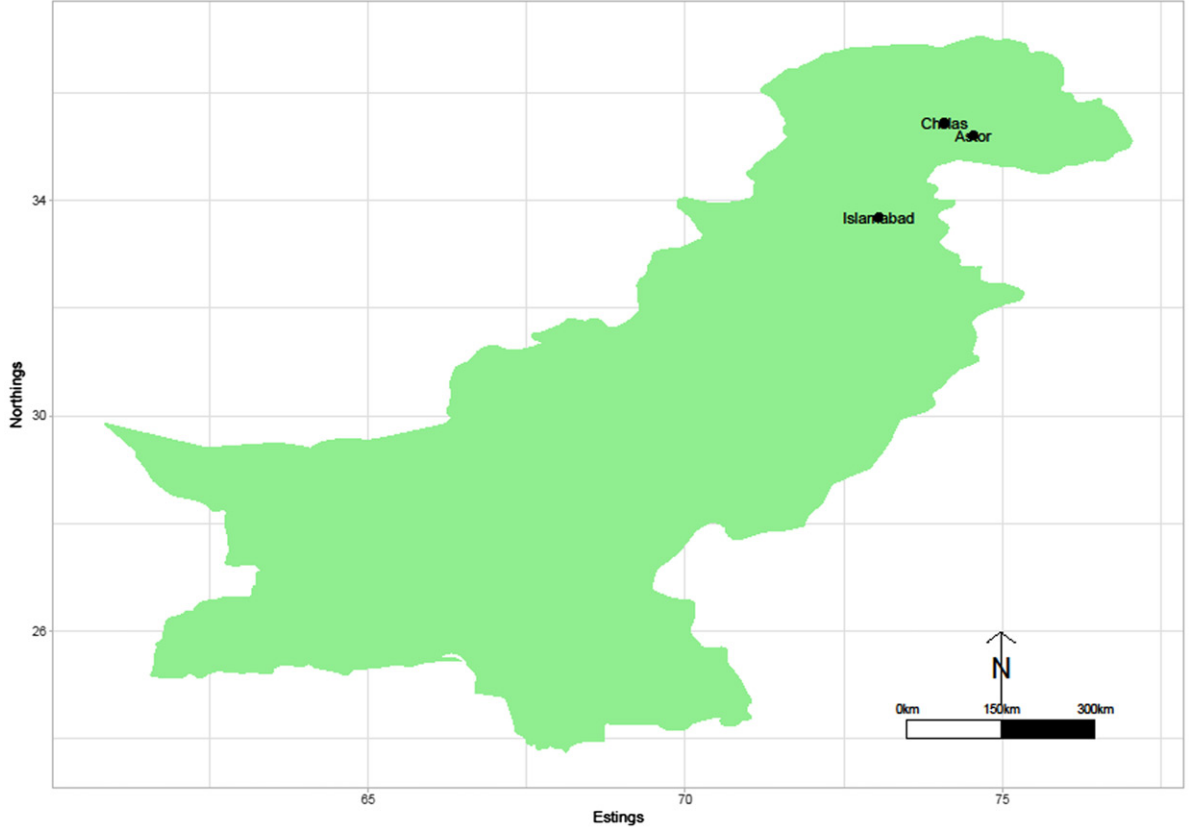


Fig. 2. Geographical locations of the selected stations.

is, for all t in the index set T , $\{Z_t\}$ is a random variable. If the index set T is a countable set, we call $\{Z_t\}$ a discrete-time stochastic process. If T is a continuous, we call it as continuous-time stochastic process. All possible values that $\{Z_t\}$ can assume are called its state space (Chiang, 1968). In any discrete stochastic phenomena, transition probability matrix gives the probability of moving one state to another state. Transition probabilities play very important role for modelling and prediction of stationary as well as non-stationary stochastic process. The transition probability matrix is an outcome of Markov chain. A Markov chain is a

stochastic process $\{Z_t\}$, having the property that the future states of the process depended on the previous states only through the present state of the process $\{Z_t\}$ (Gilks et al., 1995; Haan, 2002; Paulo et al., 2005).

Thus, in the current scenario, time series data on drought classes determined by SPI, SPEI and SPTI for a single station can be considered as a sequence of drought classes and formulated it in a discrete Markov model. Each Markov chain is characterized by a transition probability matrix that represents the probability of transition from one drought state to another drought state. By the

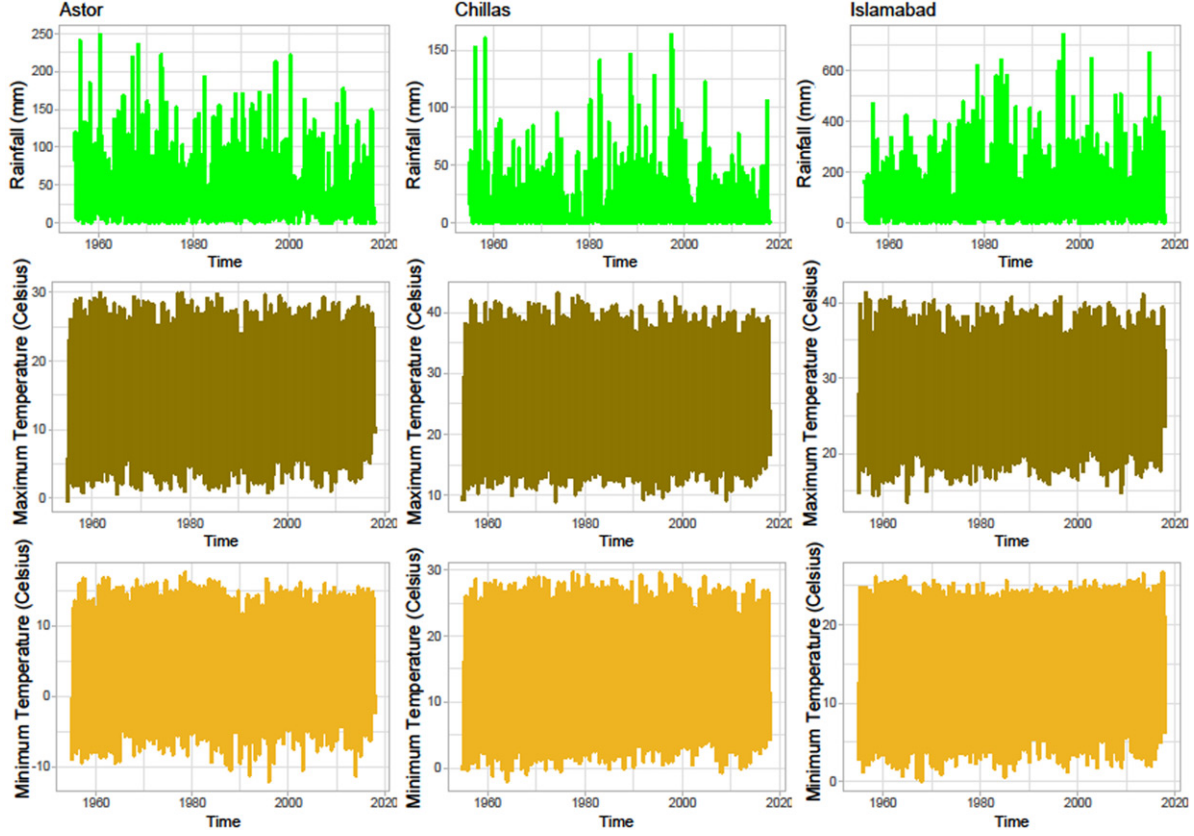


Fig. 3. Temporal behaviour of rainfall, minimum temperature, maximum temperature.

assumption of first order Markov chain, this research assume that, given the present month drought classes, the future drought classes/class at particular month/station are conditionally independent (Sanusi et al., 2015). This means that the probabilities of transitions are homogeneous over time. It is just a statistical compliance that allows us to consider each drought class as a first order Markov chain. However, the second or higher order Markov models can be considered if the time series of drought classes depends on its previous two or n drought classes. Further, in its mathematical structure, each transition probability matrix must have to satisfy the following two conditions;

$$\sum p_{ij} = 1 \quad (1)$$

and

$$p_{ij} \leq 1 \quad (2)$$

for all i and j . In matrix form transient probabilities of moving one state to another state is described in the following ways.

Let $Y_{ij}^{(t)}$ be the total number of transitions of drought class S_i to drought class S_j in t time steps. Moreover, Y_i

is to total number of particular drought stats. Hence, the probabilities of moving one drought class to another class can be computed using Equation (3),

$$P_{ij}^{(t)} = \frac{Y_{ij}^{(t)}}{Y_i} \quad i, j = 1, 2, 3, 4, \dots, m \quad (3)$$

Further, the transient behaviours of all drought classes are presented by the following transition probability matrix.

$$P^{(t)} = \begin{bmatrix} P_{11}^{(t)} & P_{12}^{(t)} & \cdot & \cdot & \cdot & \cdot & P_{1m}^{(t)} \\ P_{21}^{(t)} & P_{22}^{(t)} & \cdot & \cdot & \cdot & \cdot & P_{2m}^{(t)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ P_{m1}^{(t)} & P_{m2}^{(t)} & \cdot & \cdot & \cdot & \cdot & P_{mm}^{(t)} \end{bmatrix}$$

In this article, extremely wet (EW), very wet (VW), moderate wet (MW), normal drought (ND), moderate drought (MD), severe drought (SD), and extreme drought (SD) classes are considered as the states of the discrete stochastic process (i.e. $n=7$). Table 1 provides the range of drought

Table 3. Climatology of the selected stations (Monthly data length: 1955–2017).

Stations	Variables	Jab	Jan	Feb	March	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Astor	Rainfall	Avg.	39.3	47.3	78.1	86.5	69.8	25.1	24.8	24.9	23.6	23.2	17.0	28.1
		Std.	35.4	32.9	53.3	50.4	58.2	20.3	21.1	20.2	28.2	30.2	23.3	33.7
		Sk.	1.2	0.7	1.2	0.5	1.2	1.5	2.1	2.0	3.0	2.5	2.1	2.3
	Minimum temperature	Avg.	−7.3	−5.2	−0.9	3.8	7.4	11.2	14.5	14.6	10.3	4.6	−0.4	−4.6
		Std.	1.9	1.9	1.3	1.3	1.4	1.4	1.2	1.2	1.3	1.0	1.1	1.4
		Sk.	−0.3	0.3	0.0	−0.1	0.1	0.8	−0.1	−0.3	0.5	−0.4	−0.1	0.1
	Maximum Temperature	Avg.	2.9	4.5	9.0	15.0	19.8	24.6	27.2	26.9	23.4	17.7	11.4	5.5
		Std.	1.5	1.7	2.0	1.9	2.3	1.6	1.4	1.5	1.5	1.7	1.7	1.7
		Sk.	0.7	0.7	0.6	0.1	0.3	0.5	−0.1	−0.4	0.3	−0.4	−0.3	0.2
	Potential Evapotranspiration	Avg.	25.9	33.1	59.7	93.3	129.6	153.2	162.8	145.4	111.0	74.9	42.4	27.6
		Std.	2.9	4.1	7.5	8.4	12.9	8.5	8.6	9.4	7.1	6.2	3.9	3.2
		Sk.	−0.4	0.2	−0.4	0.3	0.2	−0.4	−0.9	0.0	−0.7	0.6	−0.1	0.0
Chilas	Rainfall	Avg.	5.4	9.4	14.8	23.7	27.0	9.6	17.1	21.1	12.6	7.7	3.2	4.7
		Std.	7.3	12.8	16.1	23.2	26.0	9.9	17.8	21.4	16.7	16.8	7.7	8.4
		Sk.	2.5	2.8	2.1	1.5	1.4	1.2	2.2	1.8	2.3	5.5	3.9	3.7
	Minimum temperature	Avg.	1.2	3.7	8.5	13.6	18.5	23.8	27.1	26.3	22.4	14.7	7.3	2.5
		Std.	1.3	1.2	1.3	1.5	2.2	1.9	1.4	1.5	1.5	1.3	1.1	1.4
		Sk.	0.1	0.3	0.1	−0.7	−0.2	−0.1	−0.2	−0.8	0.0	0.2	−0.1	−0.1
	Maximum Temperature	Avg.	12.1	14.6	20.0	25.9	31.1	37.0	39.2	38.5	34.9	28.4	20.7	13.9
		Std.	1.4	1.8	2.0	1.9	2.7	1.9	1.7	1.7	1.6	1.7	1.4	1.3
		Sk.	−0.2	0.4	0.3	0.2	0.3	0.5	−0.1	−0.3	0.0	−0.5	−0.2	−0.1
	Potential Evapotranspiration	Avg.	41.7	54.0	94.0	133.6	177.3	205.1	210.4	188.8	145.2	103.8	62.0	41.7
		Std.	3.9	5.3	8.8	10.2	15.0	11.2	12.3	10.4	8.2	6.7	4.4	3.4
		Sk.	−0.6	−0.1	−0.7	0.3	0.5	4.2	1.8	0.7	1.3	−0.1	0.3	−0.1
Islamabad	Rainfall	Avg.	56.2	80.1	89.6	55.9	36.3	66.3	276.8	314.4	119.3	33.5	16.3	32.9
		Std.	46.5	61.2	57.6	53.6	28.3	58.0	152.3	134.5	83.8	35.0	20.2	41.4
		Sk.	0.7	1.4	1.4	1.9	1.1	1.3	0.9	0.6	1.2	2.0	1.8	1.7
	Minimum temperature	Avg.	2.9	5.6	10.5	15.5	20.3	24.0	24.6	23.7	21.1	14.6	8.1	3.9
		Std.	1.2	1.5	1.6	1.5	1.8	1.2	0.8	0.7	1.2	1.4	1.5	1.4
		Sk.	0.2	0.1	−0.5	0.2	0.4	−0.1	0.1	0.0	−0.2	0.3	0.5	−0.5
	Maximum Temperature	Avg.	17.6	19.6	24.3	30.2	35.7	38.6	35.2	33.6	33.4	30.8	25.4	20.0
		Std.	1.5	2.0	2.3	2.2	2.3	1.3	1.7	1.0	1.2	1.2	1.3	1.5
		Sk.	−0.2	0.4	0.7	−0.2	0.1	−0.3	0.6	−0.3	−0.2	−0.7	−0.8	−0.1
	Potential Evapotranspiration	Avg.	58.6	71.7	115.4	160.1	209.8	219.3	182.8	158.5	141.9	119.7	80.2	60.6
		Std.	4.7	7.1	11.0	12.2	14.6	12.6	15.9	8.6	10.2	6.1	4.8	4.8
		Sk.	−0.1	−0.5	0.2	−0.2	−0.4	0.2	0.2	−0.3	2.6	1.7	0.2	−0.7

Note: Avg. (Average), Std. (Standard deviation), and Sk. (Skewness); Rainfall and temperature are measured in millimetre and centigrade, respectively.

classes with corresponding to their cumulative probabilities. The final stage of the proposed criterion consists on the individual data of temporal classifications and the transient behaviour in each index.

2.3. Outlines of the proposed criterion of PWJADI

Here, we proposed a new criterion termed as; the PWJADI to utilise the varying methodologies of commonly used three drought indices (SPI, SPEI and SPTI). As these drought indices have uniform mathematical structure and classification states, therefore the use of these indices to report joint characterization is

reasonable. Following three steps are involved in the proposed framework (see Fig. 1):

1. The first step is straightforward. In this step historical quantitative data on SPI, SPEI and SPTI for single station is classified according to the basic drought classification criteria (McKee et al., 1993; Vicente-Serrano et al., 2010; Ali et al., 2017a). Table 1 provides the range of SPI, SPEI and SPTI values associated with classified drought classes.
2. In the second step, separate transition probability matrices are calculated from each temporal classified series. Here, following the work of Paulo et al. (2005)

Table 4. BIC of various distributions.

Stations Distributions	Astor			Chillas			Islamabad		
	SPI	SPEI	SPTI	SPI	SPEI	SPTI	SPI	SPEI	SPTI
2P Beta	-853.70	-586.86	-280.10	-400.09	-501.31	-379.15	-649.02	-798.62	-175.52
3P Weibull	-1248.54	-742.71	-480.83	-802.14	-499.76	-377.99	-957.96	-794.19	-530.14
4P Beta	-1243.31	-745.71	-469.79	-804.11	-596.01	-338.36	-950.07	-999.42	-524.50
Arcsine	-976.81	-677.68	-272.85	-573.72	-535.82	-334.56	-733.50	-869.80	-276.20
Burr	-897.92	-685.22	-347.29	-552.51	-580.56	-333.51	-685.97	-921.57	-349.46
Cauchy	-1062.66	-705.69	-367.96	-609.06	-570.14	-266.33	-815.15	-1039.25	-339.64
Chi	-891.67	-592.94	-247.93	-529.75	-505.56	-263.65	-685.96	-804.31	-186.58
Chi – Square	-898.99	-593.03	-441.42	-561.03	-505.52	-233.31	-686.20	-803.99	-400.85
Cosine	-955.91	-749.20	-192.31	-420.16	-588.46	-218.93	-691.23	-912.17	-221.50
Curvilinear Trapezoidal	-919.08	-692.83	-375.44	-465.58	-531.02	-200.00	-662.30	-802.74	-347.39
Exponential	-1184.40	-592.94	-401.81	-541.19	-505.52	-148.24	-685.99	-803.98	-416.79
F–	-853.70	-588.68	-457.56	-666.41	-501.38	-118.30	-734.53	-799.56	-397.90
Gamma	-1231.37	-597.08	-477.56	-676.12	-501.38	-97.89	-958.03	-807.46	-529.91
Generalized Extreme Value	-1136.72	-744.03	-456.48	-710.95	-600.36	-97.11	-910.00	-1002.13	-429.33
Generalized normal	-1157.38	-741.93	-476.01	-757.31	-596.88	-94.60	-940.23	-1005.17	-463.18
Gumbel	-1111.02	-730.27	-389.62	-575.20	-579.37	-62.38	-837.26	-1001.09	-355.68
Inverse Chi – Square	-977.24	-597.63	-273.05	-691.27	-505.58	-58.44	-745.12	-808.03	-277.75
Inverse Gamma	-1040.61	-598.37	-396.36	-693.05	-501.46	-43.66	-666.58	-809.94	-363.11
Inverse Gaussian	-992.97	-586.86	-356.23	-634.16	-501.31	-39.21	-855.36	-798.62	-301.65
Johnson SB	-1154.09	-748.91	-478.62	-753.49	-618.70	-31.88	-937.22	-842.69	-494.07
Johnson SU	-1152.55	-737.66	-471.37	-752.90	-592.94	-23.93	-935.91	-1040.57	-458.60
Laplace	-1071.73	-717.40	-382.24	-586.94	-571.37	-23.01	-830.09	-1064.06	-352.81
Logistic	-1080.45	-736.36	-365.07	-568.66	-582.07	-16.07	-809.03	-1023.31	-329.32
Log – normal	-1144.79	-597.90	-478.07	-707.29	-501.46	-16.00	-943.26	-808.57	-464.28
Normal	-1076.88	-743.47	-357.27	-559.95	-585.22	7.74	-801.45	-1008.64	-320.81
Rayleigh	-1099.33	-741.32	-373.83	-563.77	-584.08	13.94	-820.24	-987.28	-338.22
Scaled/shifted t	-1075.95	-739.20	-369.08	-620.14	-581.08	18.20	-813.32	-1039.68	-336.58
Skewed – normal	-1126.18	-743.67	-384.42	-561.25	-603.16	33.01	-838.66	-1006.44	-354.71
Trapezoidal	-1111.66	-761.06	-361.71	-543.75	-606.25	41.18	-820.53	-1004.38	-336.16
Triangular	-1116.49	-749.46	-366.35	-539.98	-608.48	105.68	-824.84	-1008.51	-339.99
Uniform	-860.67	-657.41	-274.09	-487.34	-598.51	194.16	-739.90	-817.55	-158.35
von Mises	-874.06	-590.16	-364.93	-552.56	-501.56	207.11	-649.02	-800.34	-332.92

and Khalili et al. (2011), we assume that each temporal series of drought classes follow first order Markov Chain.

3. In the third step, a new criterion of decision aggregation is suggested in such a way that in each time step (i.e. each month), each drought class of SPI, SPEI and SPTI receives transient probabilities as a weight. Among the three drought indices, succeeding drought category which receives maximum weight from the list of switching probabilities is declared as a joint determined drought class. The basic idea of using transient probabilities is the assumption that next drought class is somehow having a great chance of dependence on the previous drought class.

Following section described the development procedure of the PWJADI.

2.4. Probabilistic weighted joint aggregative drought index

Let $A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_n$ and $C_1, C_2, \dots, C_n$ be the monthly time series of drought classes estimated from SPI, SPEI and SPTI drought index, respectively. Moreover, let each drought classification series being as a discrete stochastic process that follow the first order Markov chain. In the past, several authors used Markov chain models to determine the behaviour of drought in different climatic regions (Paulo et al., 2005; Khalili et al., 2011; Rahmat et al., 2016; Rezaeianzadeh et al., 2016). Here, transient behaviour of each drought class is determined from particular temporal classification of drought index.

Hence, to account the effect of the transient behaviour of each drought class in each index, drought class receiving maximum value of switching probability is declared

(a): Selected probability distribution for SPI, SPEI and SPTI in Astor

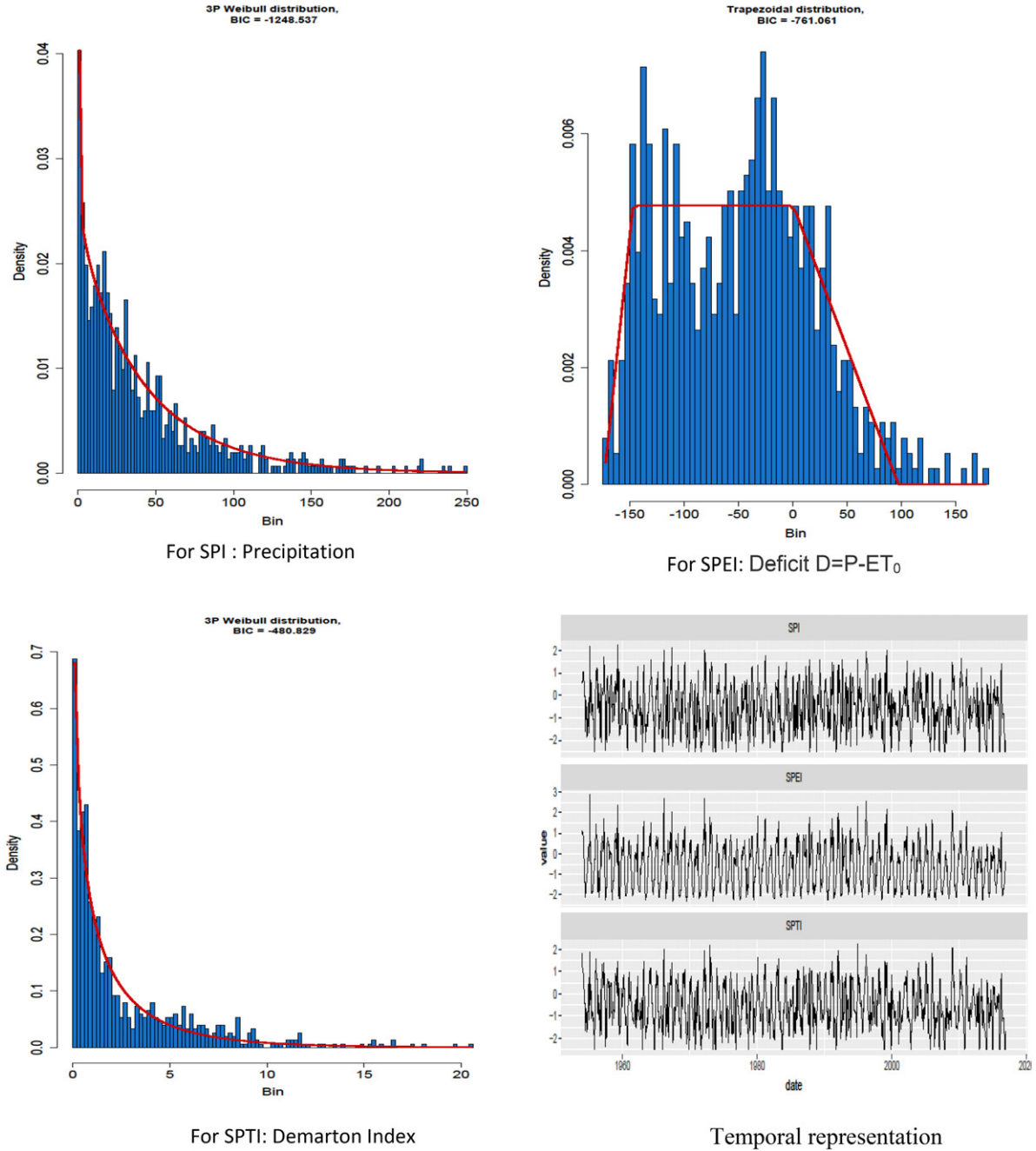


Fig. 4. (a) Selected probability distribution for SPI, SPEI and SPTI in Astor. (b) selected probability distribution for SPI, SPEI and SPTI in Chillas. (c) selected probability distribution for SPI, SPEI and SPTI in Islamabad.

as a joint aggregative drought class. Here, switching probabilities are considered as a weight in the candidacy of respective drought class. Therefore, for our proposed framework, separate transition probability matrices are required to obtain the probability of

moving from one drought class to another under the Markov chain framework. Following matrices show the mathematical structure for the selection of switching probabilities from one drought class to another drought class for each index.

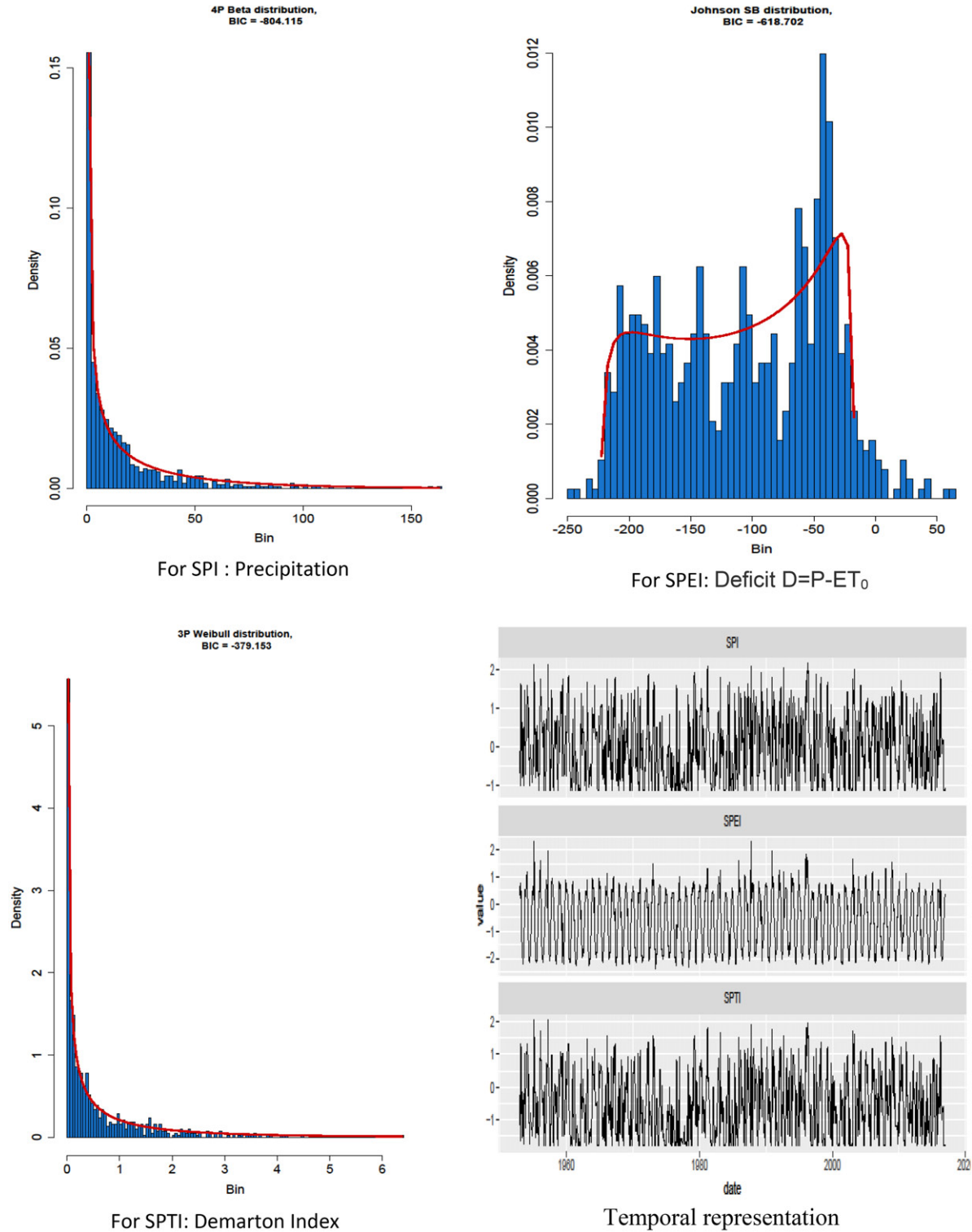
(b): Selected probability distribution for SPI, SPEI and SPTI in Chillas

Fig. 4. Continued.

(c): Selected probability distribution for SPI, SPEI and SPTI in Islamabad

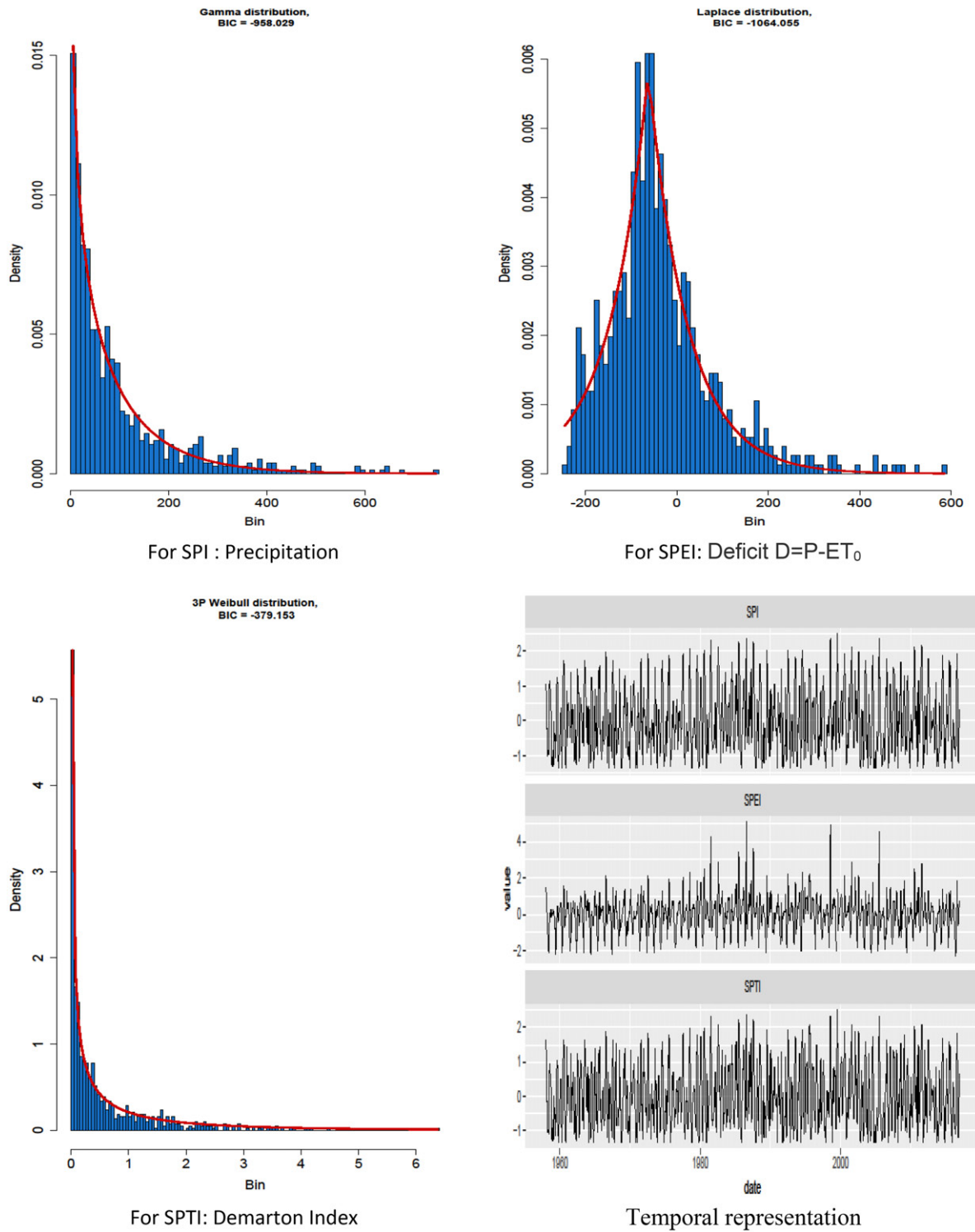


Fig. 4. Continued.

<i>SPI Index</i>	<i>EW</i>	<i>VW</i>	<i>MW</i>	<i>NN</i>	<i>MD</i>	<i>SD</i>	<i>ED</i>
<i>EW</i>	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}
<i>VW</i>	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}
<i>MW</i>	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}
<i>NN</i>	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}
<i>MD</i>	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}
<i>SD</i>	a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}
<i>ED</i>	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}
<i>SPEI Index</i>	<i>EW</i>	<i>VW</i>	<i>MW</i>	<i>NN</i>	<i>MD</i>	<i>SD</i>	<i>ED</i>
<i>EW</i>	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}	b_{17}
<i>VW</i>	b_{21}	b_{22}	b_{23}	b_{24}	b_{25}	b_{26}	b_{27}
<i>MW</i>	b_{31}	b_{32}	b_{33}	b_{34}	b_{35}	b_{36}	b_{37}
<i>NN</i>	b_{41}	b_{42}	b_{43}	b_{44}	b_{45}	b_{46}	b_{47}
<i>MD</i>	b_{51}	b_{52}	b_{53}	b_{54}	b_{55}	b_{56}	b_{57}
<i>SD</i>	b_{61}	b_{62}	b_{63}	b_{64}	b_{65}	b_{66}	b_{67}
<i>ED</i>	b_{71}	b_{72}	b_{73}	b_{74}	b_{75}	b_{76}	b_{77}
<i>SPTI Index</i>	<i>EW</i>	<i>VW</i>	<i>MW</i>	<i>NN</i>	<i>MD</i>	<i>SD</i>	<i>ED</i>
<i>EW</i>	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}
<i>VW</i>	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}
<i>MW</i>	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}
<i>NN</i>	c_{41}	c_{42}	c_{43}	c_{44}	c_{45}	c_{46}	c_{47}
<i>MD</i>	c_{51}	c_{52}	c_{53}	c_{54}	c_{55}	c_{56}	c_{57}
<i>SD</i>	c_{61}	c_{62}	c_{63}	c_{64}	c_{65}	c_{66}	c_{67}
<i>ED</i>	c_{71}	c_{72}	c_{73}	c_{74}	c_{75}	c_{76}	c_{77}

where a_{ij} , b_{ij} and c_{ij} are one-step switching probabilities that are indexed by their corresponding transient classes, and i, j represents drought classes in Markov chain settings.

In the next step, weights in terms of switching probability that belong to particular classes of particular index, are arranged in chronological order. These weights will help us to determine next month drought class by accounting the effect of historical transient behaviour of drought classes in each index. In Equation (4), aggregation criteria of maximum value of the weights used to select the respective drought class from particular index is given as;

$$PWJADI = \max(a_{ij} \in SPI, b_{ij} \in SPEI, c_{ij} \in SPTI) \quad (4)$$

Those drought categories which have maximum switching probabilities from corresponding transition probability matrices of each drought index is considered as a joint aggregative drought class. However, this process will be repeated to all the three temporal vectors of drought classes and weights for the generalization purpose, and to generate historical time series of newly generated index. Table 2 shows the temporal structure and the execution of the proposed algorithm.

In Table 2, $p_{c|c-1}$, $q_{c|c-1}$, $r_{c|c-1}$ denotes the switching probabilities for previous and present month drought classes based on the existence of previous month drought classes for SPI, SPEI and SPTI, respectively. And d_{ij} denotes the general symbol defined for the drought index corresponding particular drought class in respective series

Table 5. Pairwise correlation between quantitative values.

Station	Pairs	Correlation	Test-statistics	p-value
Astor	SPI vs SPEI	0.6233	21.891	2.2e-16
	SPI vs SPTI	0.9361	73.136	2.2e-16
	SPEI vs SPTI	0.7775	33.949	2.2e-16
Chilas	SPI vs SPEI	0.2323	6.6386	5.994e-11
	SPI vs SPTI	0.9793	133.98	2.2e-16
	SPEI vs SPTI	0.3892	11.697	2.2e-16
Islamabad	SPI vs SPEI	0.6433	23.258	2.2e-16
	SPI vs SPTI	0.9181	115.23	2.2e-16
	SPEI vs SPTI	0.7395	29.363	2.2e-16

(i.e. i denotes drought index and j denotes drought classes). Here, the repetition of this step for monthly drought classes determined by each index yield new vectors of drought years. These newly drought classes have a maximum probability of its occurrences.

3. Application and discussion

To evaluate the framework of PWJADI for defining drought classes, we first estimated a time series data on SPI, SPEI and SPTI at one month time scale for three meteorological stations Astor (Latitude: 35.367, Longitude: 74.850, Elevation: 2600m), Chilas (Latitude: 35.43, Longitude: 74.083, Elevation: 950m) and Islamabad (Latitude: 33.738, Longitude: 73.084, Elevation: 540m) located in different climatic regions of Pakistan. Figure 2 presents the geographical locations of the selected study regions. In the proposed method, we required long term time series data on monthly precipitations, minimum and maximum temperature. Therefore, the secondary data on these variables ranging from January 1955 to December 2017 is collected from Karachi Data Processing Center (KDPC) through Pakistan Meteorological Department (PMD). This data set fulfil the WMO requirements, where, errors, scrutiny, tabulation and quality control is done by KDPC (http://www.pmd.gov.pk/rmc/RMCK/Services_Climatology.html). Figure 3 and Table 3 provide the summary statistics of the monthly time series of the total recorded amount of precipitation, minimum and maximum recorded temperature. The discrepancies between Astor and Chilas statistics are due the differences between the elevations of these meteorological stations.

In the estimation procedure, several probability distributions are fitted to check their appropriateness on the respective time series of each index. In current research, Kolmogorov-Smirnov, chi-squared and Anderson-Darling tests were used to check the goodness of fit at the most commonly used level of significance 0.05 by using Easyfit

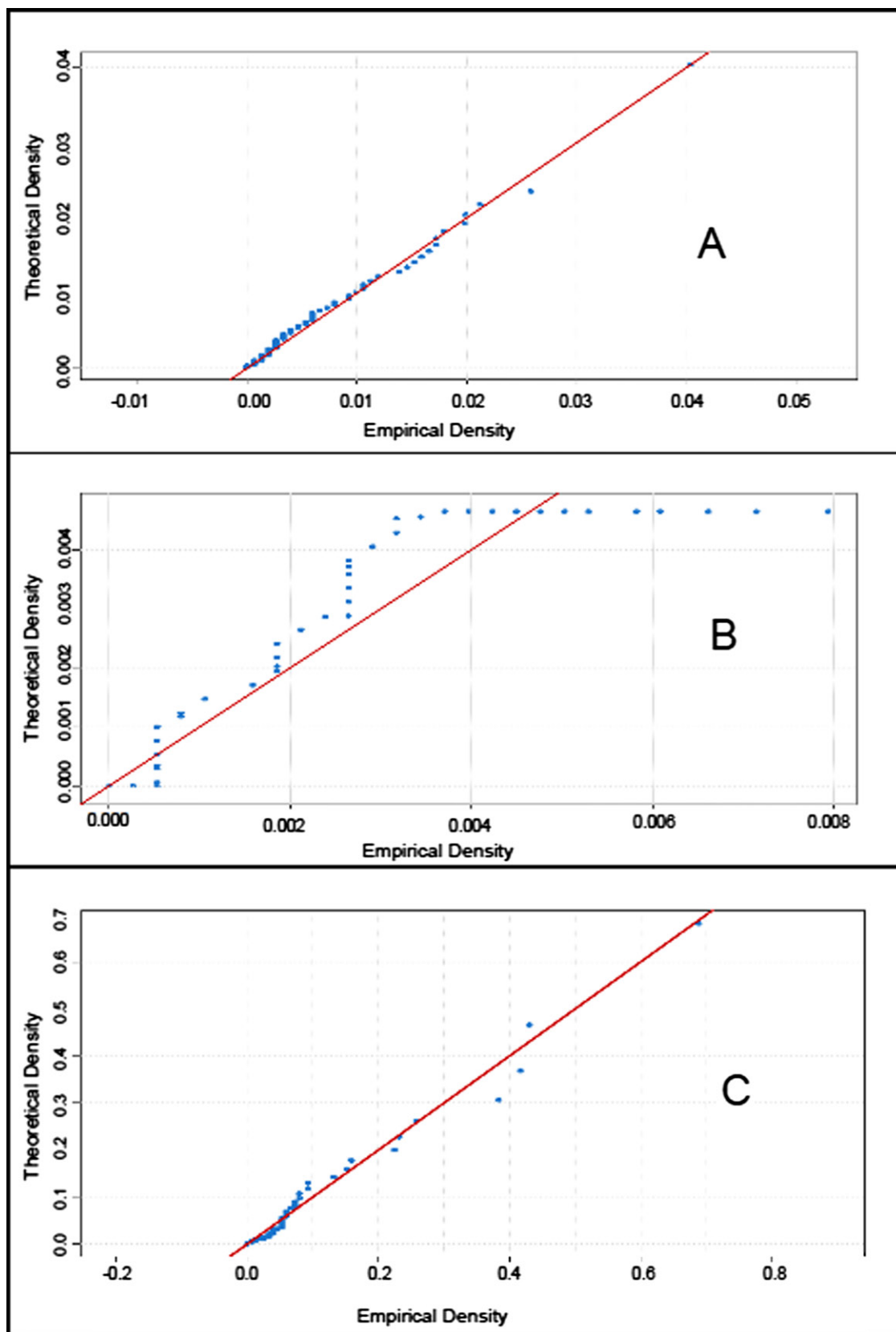


Fig. 5. Q-Q plots of Astor station. A- for precipitation, B- for deficient index (D), and C-for DAI index.

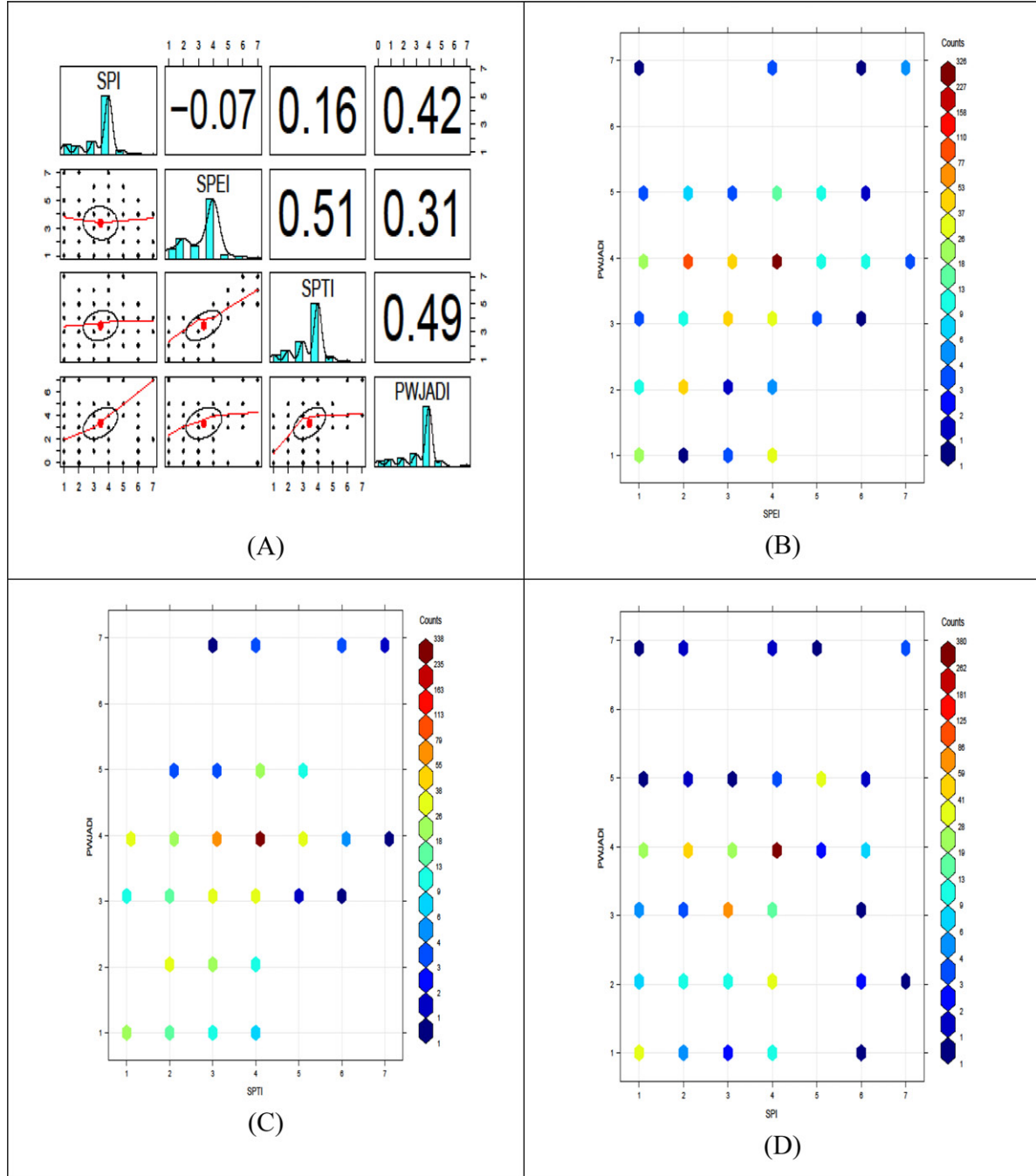


Fig. 6. Joint aggregation analysis of PWJADI criterion with SPI, SPEI and SPTI for Astor stations. The correlation among each categorical observed classes is present in top left panel. The scatter plot of historical accumulated drought classes are displayed in top right panel and bottom panel for SPI, SPEI and SPTI, respectively.

software (Schittkowski, 2002) and Propagate (Spiess, 2014) package of R statistical language.

Table 4 provides the Bayesian Information Criterion (BIC) values of all the tested probability distributions. Probability distributions that have minimal value of BIC is then standardized to obtain temporal values of each

index accordingly. Figure 4 presents the best distribution for SPI, SPEI and SPTI for all the selected regions.

In the Fig. 4a–c, the best fitted probability distribution based on BIC, and corresponding standardized values are presented for SPI, SPEI and SPTI. We found a strong correlation between SPI and SPTI at each station,

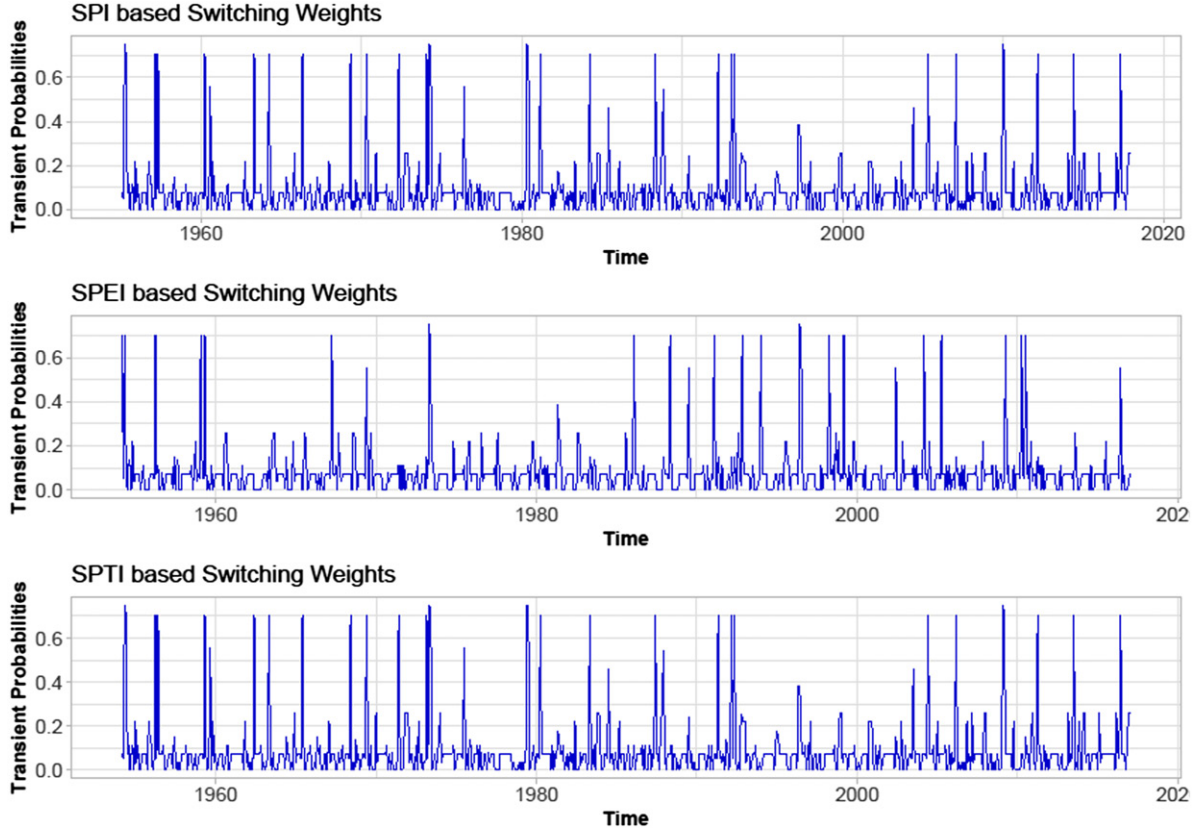


Fig. 7. Temporal representation of switching weights for historical observed Drought classes of SPI, SPEI and SPTI at Astor station.

Table 6. Transition probability matrix for Astor station.

		<i>ED</i>	<i>EW</i>	<i>MD</i>	<i>MW</i>	<i>ND</i>	<i>SD</i>	<i>SW</i>
<i>TPM_{of SPI}</i>	<i>ED</i>	0.269	0.000	0.154	0.000	0.436	0.128	0.013
	<i>EW</i>	0.000	0.000	0.200	0.000	0.800	0.000	0.000
	<i>MD</i>	0.103	0.000	0.187	0.047	0.495	0.150	0.019
	<i>MW</i>	0.029	0.000	0.057	0.114	0.714	0.057	0.029
	<i>ND</i>	0.081	0.009	0.131	0.056	0.623	0.081	0.018
	<i>SD</i>	0.123	0.014	0.164	0.014	0.575	0.110	0.000
	<i>SW</i>	0.071	0.000	0.143	0.000	0.571	0.071	0.143
		<i>ED</i>	<i>EW</i>	<i>MD</i>	<i>MW</i>	<i>ND</i>	<i>SD</i>	<i>SW</i>
<i>TPM_{of SPEI}</i>	<i>ED</i>	0.258	0.000	0.136	0.000	0.061	0.545	0.000
	<i>EW</i>	0.111	0.000	0.000	0.111	0.556	0.111	0.111
	<i>MD</i>	0.080	0.000	0.218	0.000	0.460	0.241	0.000
	<i>MW</i>	0.037	0.000	0.037	0.074	0.704	0.148	0.000
	<i>ND</i>	0.039	0.022	0.053	0.051	0.751	0.056	0.029
	<i>SD</i>	0.174	0.000	0.254	0.007	0.174	0.384	0.007
	<i>SW</i>	0.000	0.000	0.067	0.067	0.800	0.000	0.067
		<i>ED</i>	<i>EW</i>	<i>MD</i>	<i>MW</i>	<i>ND</i>	<i>SD</i>	<i>SW</i>
<i>TPM_{of SPTI}</i>	<i>ED</i>	0.211	0.000	0.175	0.018	0.404	0.000	0.000
	<i>EW</i>	0.000	0.000	0.000	0.333	0.667	0.000	0.000
	<i>MD</i>	0.088	0.000	0.293	0.027	0.435	0.000	0.014
	<i>MW</i>	0.026	0.000	0.103	0.128	0.718	0.000	0.000
	<i>ND</i>	0.051	0.002	0.169	0.058	0.597	0.000	0.022
	<i>SD</i>	0.120	0.012	0.241	0.024	0.494	0.000	0.000
	<i>SW</i>	0.000	0.083	0.000	0.167	0.750	0.000	0.000

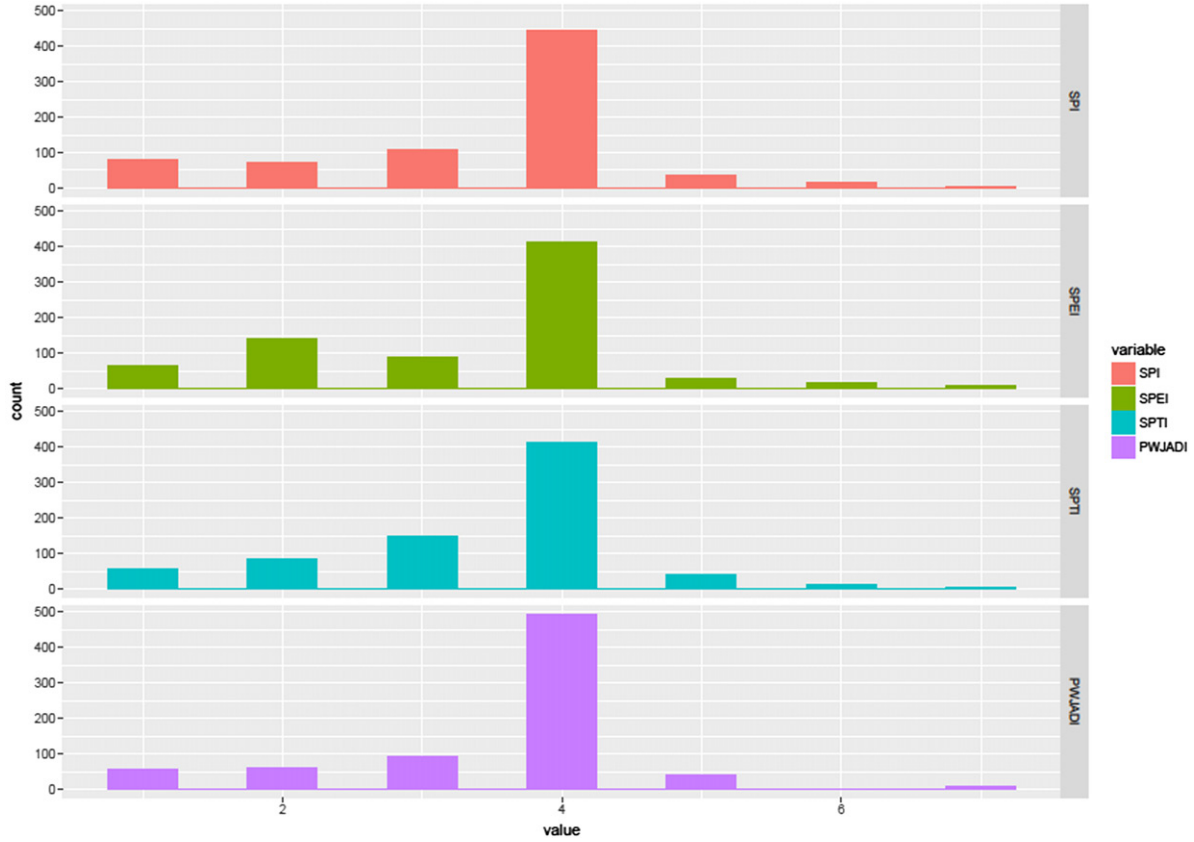


Fig. 8. Bar plot of Drought Categories at Astor station: where 1, 2, 3, 4, 5, 6, and 7 represent ED, EW, MD, MW, ND, SD, and SW drought categories, respectively.

although SPEI has small correlation with SPI and SPTI (see Table 5). However, this correlation is not necessarily hold among each drought index in terms of classified categories of drought classes. It is due to the inappropriateness of probability function. Figure 5 clearly indicates that trapezoidal distribution is not well fitted on the deficient index D. However, among all the available probability distributions, trapezoidal distribution has minimum BIC value. This proves that subjective choice or absence of appropriate probability distribution may leads error in accurate drought characterization.

In our experimental results, we found a negative correlation (0–.07) between classified values of SPI and SPEI (see Fig. 6), which might be because of the cold temperature of Astor station where the effect of evapotranspiration were negligible in the estimation of SPEI index.

Further, in terms of accumulated drought classes in their corresponding historical series, each drought index has significantly different behaviour. Although there is significant correlation between SPTI and SPEI, however, a significant drop in the agreement between SPTI and SPI that was found when quantitative values were transformed into qualitative drought classes. To observe the

Table 7. Shapiro–Wilk normality test.

Index	Test Statistics (W)	P-Values
SPI	0.80525	2.2e-16
SPEI	0.83744	2.2e-16
SPTI	0.83333	2.2e-16
PWJADI	0.76353	2.2e-16

disagreement among the temporal behaviour of drought classes, correlation among each indices is presented in Fig. 6a. Here, the correlation between SPI and SPTI is dropped from 0.936 to 0.160, which raises several questions on their individual usage in drought monitoring paradigm. There are several reasons of the discrimination in the historical drought classes determined by the three methods. However, the main reason is the methodology of each drought index which is totally based on the probabilistic models, and errors in each model always exist. Further, there may exist a good probability among beyond the candidate list in the selected model. Another main hypothetical reason which can be further extended to all the monitoring network is the errors in historical data.

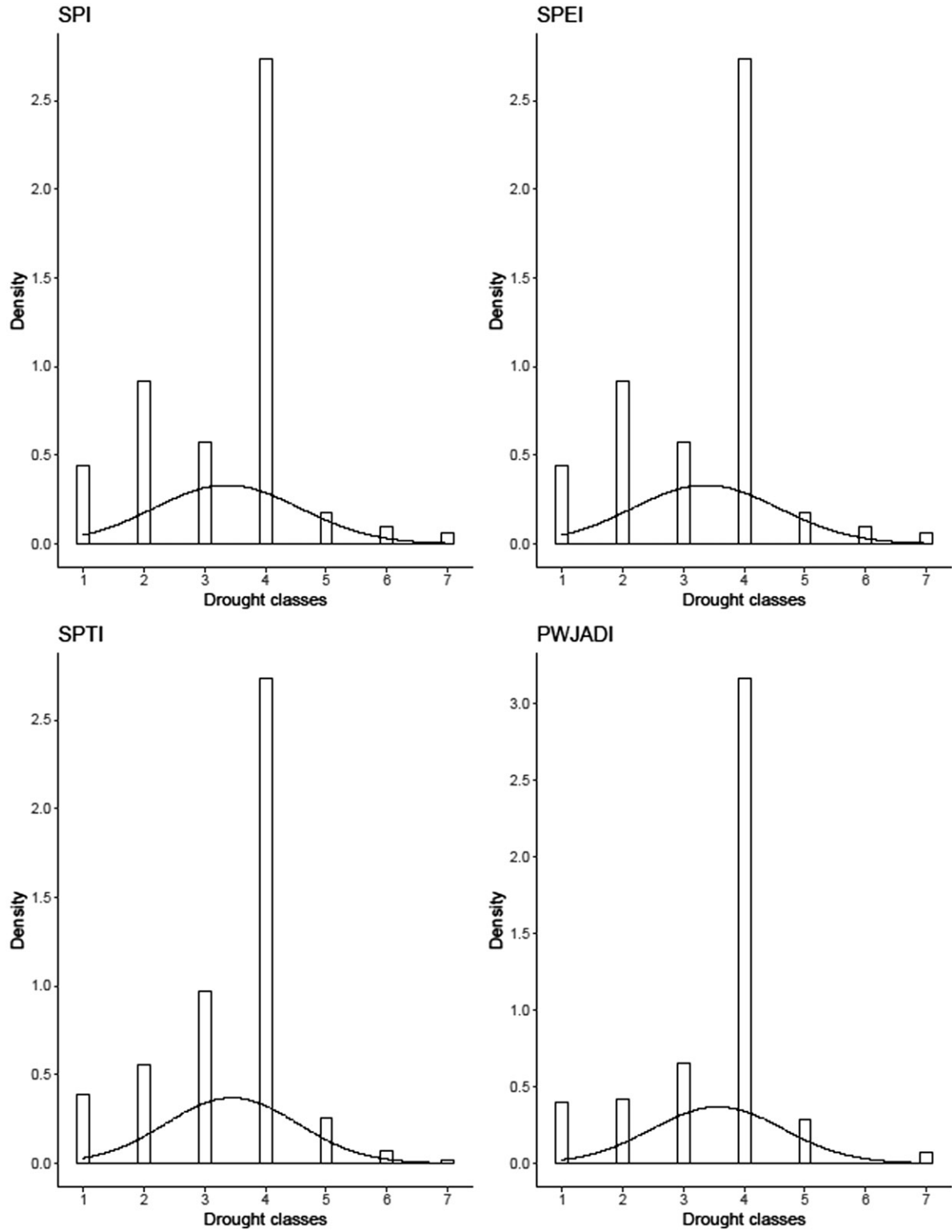


Fig. 9. Histograms of SPI, SPEI, SPTI and PWJADI for Astor.

To accumulate the joint effect of all the drought indices by considering their memory effect, outcomes associated with the proposed criterion of new drought indices

show positive correlation among SPI, SPEI and SPTI. Selection of drought classes, which have maximum transient probability among the available vector of three classes

reduced the incorrect determination drought classes. Fig. 6a shows that proposed criterion of PWJADI shared significant part of an agreement with SPI, SPEI and SPTI.

Further, to observe the in depth behaviour of PWJADI with other selected indices, scatter plots of qualitative drought classes are presented in Fig. 6b–d. One can observe that each drought class of SPEI, SPI and SPTI behaves consistent with PWJADI.

To assign weights, we prepared separate transition probability matrices for each index. In this research, the assumption of first order Markov chain are validated on small segments of the total of time data. We used chi-square test by “*verify Markov Property*” function of *Markov chain* package of R language on small segments of data. Figure 7 presents the chronological behaviour of these weights for each drought class of each index. Therefore, incorrect determination of drought classes can be adjusted by maximum weighting criterion using Equation (4).

Table 6 provides the transition probability matrices of SPI, SPEI and SPEI drought classification series for Astor observatory. In the case study, these transient probabilities are switched with the corresponding drought class to each temporal series of drought index. Figure 8 represents the how PWJADI produce identical drought categorical sequences, where the proportion of each drought category is more or less consistent with each index. Strong positive association of PWJADI with SPI, SPEI and SPTI shows its effectiveness in determining drought class without any conflict with other indices. In bar plot analysis, the behaviour in the historical drought classes is same as in SPI, SPTI and SPEI. This shows the consistency of the proposed methodology with SPI, SPEI and SPTI. However, due to high frequency of normal drought classes, the behaviour of each drought classes is not normal. This result is due to Shapiro-Wilk test of normality (see Table 7 and Fig. 9).

4. Concluding remarks

In this study, we introduced a new joint aggregative criterion for assessing accurate drought classes by using SPI, SPEI and SPTI drought indices. We found that aggregate decisions based on three drought indices (SPI, SPEI and SPTI) can be useful for accurate and precise drought monitoring. We concluded from the analysis of three meteorological stations as follows:

1. The choice of appropriate probability distribution for each drought indicator increase its efficiency for exact drought category.
2. Although there are positive correlations among each quantitative value of SPI, SPEI and SPTI, but it does

not guarantee that in a particular month each drought indicator produces same drought class.

3. The transient memories as a weight help to reduce the error rate of inaccurate drought class.
4. Utilization of more than one drought index for drought monitoring, the proposed model can be considered for making reliable drought mitigation policies.

Further, the inferences and numerical computations can be generalized for other time scales and other drought indices such as RDI.

However, the limitation of the proposed methods is not to consider the nonstationary behaviour of Markov chain on whole data length. Moreover, in the computations, the study assumed each Markov chain as first-order Markov process.

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Disclosure statement

No potential conflict of interest was reported by the authors. The manuscript is prepared by using secondary data and authors have not received any financial support. The authors of manuscript certify that they have no affiliations with or involvement in any organization or entity with any financial interest, or non-financial interest in the subject matter or materials discussed in this manuscript.

Ethical statement

The manuscript is prepared in accordance with the ethical standards of the responsible committee on human experimentation and with the latest (2008) version of Helsinki Declaration of 1975.

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