



Corrigendum

Pannekoucke, O., Ricci, S., Bathelemy, S. Ménard, R., and Thual, O. (2016). Parametric Kalman filter for chemical transport models. *Tellus A*, 68, 31547.
<https://doi.org/10.3402/tellusa.v68.31547>

1. Introduction

In our previous contribution Pannekoucke et al. (2016) (P16), an error has been made in the derivation of the error diffusion tensor dynamics Eq. (20). This involves an error in the Lagrangian dynamics of the uncertainty given in the Algorithm 2, while leaving the Eulerian dynamics unchanged.

The corrigendum is organized as follows. The modification of the Lagrangian dynamics is presented in Section 2, where a new version of the Algorithm 2 of P16 is presented. The computation leading to the Eulerian dynamics is described in Section 3, which gives the dynamics presented in Eq. (26) of P16.

2. Lagrangian dynamics of the diffusion tensor

The advection over a small time step δt can be viewed as equivalent of the deformation of the error field ε^b by the transformation $D(x) = x + u(x, t)\delta t$ (Pannekoucke et al., 2014). Hence, it follows that the metric tensor field $g_x(t)$ evolves in time as $\hat{g}_x(t + \delta t) = \mathbf{D}_x^{-T} g_{D^{-1}(x)}(t) \mathbf{D}_x^{-1}$, where $\mathbf{D}_x^{-1}$ is the gradient of the inverse deformation D^{-1} at x (Pannekoucke et al., 2014, see Eq. (35)). This can be formulated considering the diffusion tensor v , defined by $v_x = \frac{1}{2} g_x^{-1}$, as:

$$\hat{v}_x^b(t + \delta t) = (\mathbf{D}_x^{-1})^{-1} v_{D^{-1}(x)}^b(t) (\mathbf{D}_x^{-T})^{-1}. \quad (1)$$

Eq. (1) can be simplified as follows. Since the derivative of the identity $D[D^{-1}(x)] = x$ is $\mathbf{D}_{D^{-1}(x)} \mathbf{D}_x^{-1} = \mathbf{I}$, it results that $(\mathbf{D}_x^{-1})^{-1} = \mathbf{D}_{D^{-1}(x)}$. Hence, the dynamics of the diffusion tensor Eq. (1) writes

$$\hat{v}_x^b(t + \delta t) = \mathbf{D}_{D^{-1}(x)} v_{D^{-1}(x)}^b(t) (\mathbf{D}_{D^{-1}(x)})^T. \quad (2)$$

Considering Eq. (2), the Algorithm 2 of P16 now writes Algorithm 1.

Algorithm 1. Iteration process to forecast the background covariance matrix at time $t = \tau$ from the analysis covariance matrix given at time $t = 0$, under local homogeneity assumption.

Require: Fields of v^a and V^a . $\delta t = \tau/N$, $t = 0$

for $k = 1 : N$ **do**

1- *Pure advection*

$$D(x) = x + u(x, t)\delta t$$

$$\hat{v}_x^b(t + \delta t) = \mathbf{D}_{D^{-1}(x)} v_{D^{-1}(x)}^b(t) (\mathbf{D}_{D^{-1}(x)})^T$$

$$\hat{v}_x^b(t + \delta t) = V^b[D^{-1}(x), t]$$

2- *Pure diffusion*

$$v_x^b(t + \delta t) = \hat{v}_x^b(t + \delta t) + 2\kappa(x)\delta t$$

$$V_x^b(t + \delta t) = \hat{v}_x^b(t + \delta t) \frac{|\hat{v}_x^b(t + \delta t)|^{1/2}}{|v_x^b(t + \delta t)|^{1/2}}$$

3- *Update of the background statistics*

$$V_x^b(t) \leftarrow V_x^b(t + \delta t)$$

$$v_x^b(t) \leftarrow v_x^b(t + \delta t)$$

$$t \leftarrow t + \delta t$$

end for

Return fields $v_x^b(\tau)$ and $V_x^b(\tau)$

3. Eulerian dynamics of the diffusion tensor

This expression modifies the derivation of the Eulerian dynamics in Appendix D of P16, where this time

$$\mathbf{D}_{D^{-1}(x)} = \mathbf{D}_{x - \delta t u(x, t) + o(\delta t)} = \mathbf{I} + \delta t \nabla u(x - \delta t u(x, t) + o(\delta t)),$$

leading to

$$\mathbf{D}_{D^{-1}(x)} = \mathbf{I} + \delta t \nabla u(x, t) + o(\delta t). \quad (3)$$

With Eq. (3), the computation in P16 leading to Eq. (D1) applies, and Eq. (D1) is found again: for the advection process, the dynamics of the error diffusion tensor writes:

$$\partial_t v^b + \mathbf{u} \nabla v^b = v^b (\nabla \mathbf{u})^T + (\nabla \mathbf{u}) v^b. \quad (4)$$

4. Conclusion

In this corrigendum, the dynamics of the metric tensor and of the diffusion tensor have been corrected. Algorithm 2 in Pannekoucke et al. (2016) is now replaced by the present Algorithm 1.

This modification does not alter the numerical results presented in P16.

References

- Pannekoucke, O., Emili, E. and Thual, O. 2014. Modeling of local length-scale dynamics and isotropizing deformations. *Q. J. R. Meteorol. Soc.*, 140, 1387–1398.
- Pannekoucke, O., Ricci, S., Barthelemy, S., Menard, R. and Thual, O. 2016. Parametric kalman filter for chemical transport model. *Tellus*, 68, 31547.