

Sensitivity experiments with the spectral HIRLAM and its adjoint

By NILS GUSTAFSSON*, *Swedish Meteorological and Hydrological Institute, S-60176 Norrköping, Sweden*, and XIANG-YU HUANG, *Danish Meteorological Institute, Lyngbyvej 100, DK-2100 Copenhagen Ø, Denmark*

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ABSTRACT

The tangent-linear and the adjoint of the spectral High Resolution Limited Area Model (HIRLAM) have been derived as a first step in the development of a 4-dimensional variational data assimilation system for HIRLAM. The adjoint of the spectral HIRLAM was applied successfully to test the sensitivity of short-range forecast errors to initial conditions. These sensitivity experiments were carried out for a particular case study in addition to a full 5-day period. The results of the sensitivity experiments indicate an ability of the adjoint model to improve the assimilation of baroclinically developing systems and this may open possibilities for application of the adjoint model in a “Poor mans 4-dimensional variational data assimilation” in advance of the implementation of the full 4-dimensional variational data assimilation.

1. Introduction

A key requirement for the successful application of numerical weather prediction at all time ranges is the availability of high quality initial data. The lack of relevant high quality observations over large portions of the globe, e.g., over the Atlantic and the Pacific in the Northern Hemisphere, makes it necessary to utilize the forecast model in a data assimilation process in order to integrate the few available observations into a time sequence of consistent atmospheric forecast model states.

The HIRLAM (High Resolution Limited Area Modelling) project is a common research effort among the weather services in the Nordic countries, Ireland and The Netherlands for development of forecast models and data assimilation techniques to be used for short range operational numerical weather prediction. A cooperation between the HIRLAM project and the weather services in France and Spain has also been estab-

lished. The problem of designing an optimal data assimilation scheme for HIRLAM was recently studied by Gustafsson et al. (1995), and they suggested that a new data assimilation system for HIRLAM should be based on 4-dimensional variational data assimilation (Le Dimet and Talagrand, 1986; Lewis and Derber, 1985). A necessary component of any 4-dimensional data assimilation based on variational techniques is the adjoint of the tangent linear forecast model. When the adjoint of the forecast model is available, it is possible to relate forecast errors, as determined by observations at any future time within the forecast range, back to possible errors in the initial conditions for the forecast. This makes it possible to combine observed data from different observing times in a way that is consistent with the forecast model.

The atmosphere is governed by non-linear physical laws and is chaotic. With regard to the numerical weather prediction problem, this has the effect that relatively small changes in the initial state for the model integration may lead to a

* Corresponding author.

drastically altered forecast even on short time scales (Källén and Huang, 1988). The availability of the adjoint of the numerical forecast model makes it possible to carry out efficient experiments to study the sensitivity of forecast errors to the initial conditions. Rabier et al. (1996) utilized the adjoint of the ECMWF forecast model to study the sensitivity of 48-h forecast errors to initial data. They found, for example, that 48-h forecast errors over Europe were extremely sensitive to small scale, small amplitude and baroclinically tilting changes to the initial conditions over North America. Errico and Vukicevic (1992) used the adjoint of a limited area model to study the sensitivity of artificially imposed forecast errors to the initial conditions.

The scientific and computer coding work for the development of the HIRLAM 4-dimensional data assimilation started during the spring 1995. Now, after half a year of intensive development work, the adjoint of the adiabatic part of the HIRLAM spectral forecast model has been finished, allowing us to carry out sensitivity experiments with regard to the initial conditions. The results of such experiments to study the sensitivity of 6, 12 as well as 48 hour forecast errors to initial data are reported on here. These results have encouraged us to start thinking of a much simpler approach to 4-dimensional variational data assimilation. Such as "Poor mans 4-dimensional variational data assimilation" may possibly be utilized for operational purposes well in advance of the time when computational resources and scientific progress will allow for introduction of the full scale HIRLAM 4-dimensional variational data assimilation.

2. The spectral HIRLAM

Main components of the HIRLAM forecasting system are the forecast model, the Optimum Interpolation analysis scheme and the non-linear normal mode initialization, see Machenhauer (1988), Gustafsson (1993) and Källberg (1989). The operational version of the HIRLAM forecast model is based on a gridpoint representation for the model variables and approximation of spatial derivatives by second order finite differences. A semi-implicit scheme, that treats the fastest gravity waves implicitly, is used for the time-stepping in

addition to the explicit leap-frog scheme, used for the remaining dynamical processes and a forward time-stepping scheme used for the parameterized processes.

Haugen and Machenhauer (1993) developed a spectral limited area model for the shallow water equations based on the use of Fast Fourier Transforms (FFT). The principal idea of this model was to extend the model variables in both horizontal dimensions in order to obtain bi-periodic variations. Gustafsson (1991) extended this spectral technique to the full HIRLAM primitive equations. Utilizing an idea of Radnoti (1995) for the coupling of the spectral HIRLAM to the global model providing the lateral boundary conditions, the spectral HIRLAM now provides higher order accuracy to the same computational cost as the gridpoint HIRLAM.

For the development of the first version of the adjoint HIRLAM, we decided to use the spectral formulation of the model. There are two reasons for starting with the spectral version: (1) The spectral version of HIRLAM is a more "modern" code based on Fortran 90 and utilization of automatic arrays, (2) In general, it is easier to develop adjoints of spectral models since Fourier transforms are self-adjoint and since no efforts are needed to develop adjoints of complicated finite difference operators. In addition, since parallel computers very soon will be available at some of the weather services, we have chosen to start from the MPP (Massively Parallel Processors) version of the spectral HIRLAM (Gustafsson and Salmond, 1994). This model code can also be run on workstations and traditional vector computers, provided the Fortran compiler accepts automatic arrays.

3. The tangent linear and adjoint of the spectral HIRLAM

We will introduce the concepts of tangent linear and adjoint models in a symbolic form only to be able to discuss our technique for derivation of the adjoint of the spectral HIRLAM, more comprehensive introductions to adjoint models may be found elsewhere (Thepaut and Courtier, 1991). We will use the standard notations suggested by Ide et al. (1995). Consider a non-linear model M for the forecast of a model state vector $x(t)$ from

time $t = t_0$ until time $t = t_1$:

$$x(t_1) = \mathbf{M}(x(t_0)). \quad (1)$$

Provided we know a non-linear solution $x(t)$, we may introduce a tangent-linear model $\mathbf{M}(t_0, t_1)$ for small perturbations $\delta x(t)$ added to this non-linear solution:

$$\delta x(t_1) = \mathbf{M}(t_0, t_1) \delta x(t_0). \quad (2)$$

Assume that we have observations available at time $t = t_1$. As a measure of the forecast error at time t_1 we introduce a cost function J . The cost function is defined as a quadratic norm of the difference between the analysis and the forecast with dimension energy, see Rabier et al. (1996). For the 4-dimensional variational data assimilation problem, we need to calculate gradients of this cost function with respect to the initial conditions. Introducing a scalar product $\langle x, y \rangle$, first order variations δJ of this cost function with respect to a perturbation $\delta x(t)$ may be expressed by the gradient of the cost function with respect to initial conditions $x(t_0)$ or with respect to the forecast $x(t_1)$ at the observation time t_1 :

$$\delta J = \langle \nabla_{x(t_0)} J, \delta x(t_0) \rangle, \quad (3)$$

$$\delta J = \langle \nabla_{x(t_1)} J, \delta x(t_1) \rangle = \langle \nabla_{x(t_1)} J, \mathbf{M}(t_0, t_1) \delta x(t_0) \rangle. \quad (4)$$

Introducing the adjoint $\mathbf{M}^*(t_0, t_1)$ of the tangent-linear model $\mathbf{M}(t_0, t_1)$, the second expression may be modified as follows:

$$\delta J = \langle \mathbf{M}^*(t_0, t_1) \nabla_{x(t_1)} J, \delta x(t_0) \rangle. \quad (5)$$

In case the tangent-linear model is a complex-valued matrix, the adjoint model is simply the complex-conjugate and transpose of the tangent-linear model matrix. Identification between the two expressions for the first order variation δJ gives us the equation for calculation of the gradient of the cost function with respect to the initial conditions:

$$\nabla_{x(t_0)} J = \mathbf{M}^*(t_0, t_1) \nabla_{x(t_1)} J. \quad (6)$$

The time integration of the tangent-linear model normally is carried out by a number of timesteps forward in time. This can be considered as a sequence of matrix multiplications, one for each time-step. Due to the matrix transpose, this means that the adjoint model runs backward in time from time t_1 to time t_0 .

We used a manual coding technique to develop

the first version of the adjoint of the adiabatic part of the spectral HIRLAM including horizontal diffusion. The tangent-linear and the adjoint of the non-linear normal mode initialization were developed as well and utilized for the sensitivity experiments described below. For each subroutine containing any non-linear expressions, we first coded the corresponding tangent-linear subroutine. Then the adjoints of each tangent-linear (and originally linear) subroutine were coded in a statement-by-statement fashion. By considering each statement of the tangent-linear and linear subroutines as a complex matrix operator, the corresponding adjoint statement(s) were derived by taking the complex conjugate and transpose of this matrix operator. An important and very time-consuming phase in the development of the adjoint code by this manual technique was of course the testing and verification of the correctness of each subroutine. Thus, all tangent-linear and adjoint subroutines were tested for the scalar product identity

$$\langle T \delta \psi, T \delta \psi \rangle = \langle T^* T \delta \psi, \delta \psi \rangle, \quad (7)$$

where T is the tangent-linear operator of a particular subroutine, T^* the corresponding adjoint operator and $\delta \psi$ a perturbation that is applied in different directions. The same scalar product identity test was also applied successfully for complete tangent-linear and adjoint model timesteps as well as to a sequence of such timesteps over a 6-h period. The tangent-linear model was simply verified by comparing the evolution of a small perturbation in the non-linear and the tangent-linear models.

4. Design of sensitivity experiments

In order to validate the adjoint of the spectral HIRLAM, we have chosen to run experiments to test the sensitivity of short-range forecast errors to the initial conditions. The basic idea of such sensitivity experiments is to carry out an adjoint model integration backward in time, starting from forecast errors as given by differences between the verifying analysis and the forecast for a given forecast range. This adjoint backward integration provides us with an estimate of the gradient of a quadratic cost function of the forecast errors with respect to the initial conditions. By subtraction of a fraction of this gradient from the original initial

data fields, it is possible to derive an alternative initial state that, hopefully, should result in an improved forecast.

The following energy-related quadratic cost-function J was used in our experiments to measure the differences between the verifying analysis and the forecast:

$$J = \frac{1}{2} \sum_x \sum_y \sum_p \left[(\Delta u)^2 + (\Delta v)^2 + R_d T_r (\Delta \ln p_s)^2 + \frac{C_p}{T_r} (\Delta T)^2 \right]. \quad (8)$$

The summation in this cost function is simply taken over all the horizontal and vertical grid-points after spectral truncation. Δu , Δv , $\Delta \ln p_s$ and ΔT are the differences between analyzed and forecasted values of the wind components, logarithms of surface pressure and temperature in the individual model transform gridpoints. R_d is the gas constant for dry air, C_p is the specific heat at constant pressure for dry air and T_r is a reference temperature. The spatial variation of the grid volumes was ignored in the definition of the cost-function given above, but this should not be too serious since the model is applied on limited areas in a rotated latitude-longitude geometry. The equator of the transformed geometry runs through the center of the integration area.

A sensitivity experiment over a L_{win} hour assimilation window is carried through the following steps.

(1) Non-linear HIRLAM forecast, including horizontal diffusion and all physical parameterization schemes, ("reference" forecast run) from initial data valid at year YY, month MM, day DD and hour HH.

(2) The difference between a verifying analysis, valid at YYMMDDHH + L_{win} hour, and the L_{win} hour HIRLAM forecast is used to calculate the gradient of the quadratic forecast error cost function J , see above, with respect to the forecast model variables at + L_{win} hour.

(3) Backward projection of the gradient field with the adiabatic adjoint HIRLAM including horizontal diffusion from + L_{win} hour to +00 hour in order to obtain the gradient of the quadratic forecast error cost function with respect to the initial conditions.

(4) The gradient of the forecast error cost function with respect to the initial conditions,

together with the definition of this cost function, is used to estimate the error of the initial analysis fields. Then, "improved" initial conditions are obtained by subtracting a fraction α of these estimated analysis errors from the original initial condition fields.

(5) Non-linear HIRLAM forecast, including horizontal diffusion and all physical parameterization schemes, from the modified initial conditions ("sensitivity" forecast run).

In the experiments described below, the scaling factor α was chosen to be 0.01 for the 48-h assimilation window. This factor corresponds to a mode which grows by a factor of 10 in 48 h. For the experiment with a 12-h assimilation window, a larger scaling factor of 0.03 was utilized. Experiments with different scaling factors have also been conducted. Within the present range of selected values, the "sensitivity" forecast modifications are approximately proportional to the scaling factor.

For all the experiments reported on below, a horizontal resolution corresponding to 80 km in the spectral model transform grid was utilized (shortest resolved wave-length 240 km). The number of vertical model levels was 16.

5. Results

5.1. A case study. The "Avalanche" forecast failure

An intensive mesoscale low pressure system hit Northern Iceland in the morning of 16 January 1995. Very strong winds caused serious avalanche accidents with loss of several human lives in the villages along the narrow fiords with surrounding steep mountains. Synoptic surface observations from Iceland for 0000 UTC 16 January 1995 are given in Fig. 1. The numerical forecasts for this mesoscale low pressure development were rather poor. The operational SMHI (Swedish Meteorological and Hydrological Institute) HIRLAM forecast was approximately 15 hPa in error already at the 12-h forecast range with initial data from 1200 UTC 15 January 1995, see Fig. 2. It is interesting to note that the 36-h forecast from 1200 UTC 14 January 1995 (not shown) is slightly better than the 12-h forecast shown in Fig. 2. This indicates that the forecast error is likely to be

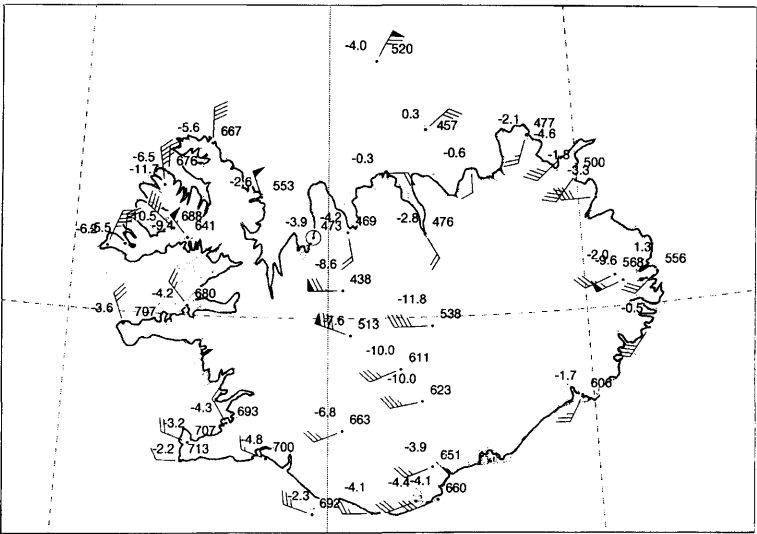


Fig. 1. Surface observations for 0000 UTC 16 January 1995.

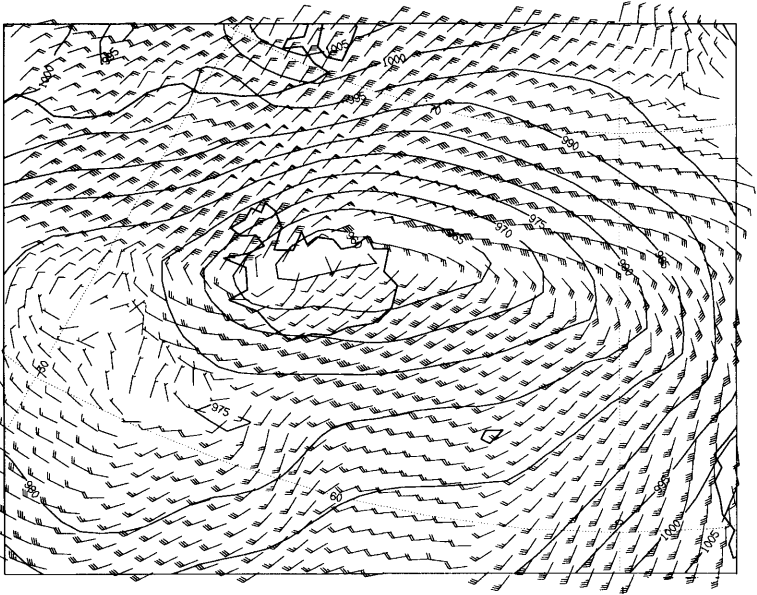


Fig. 2. Operational SMHI HIRLAM forecast for mean sea-level pressure and 10 m winds, 1200 UTC 15 January 1995 + 12 h.

related to poorly described baroclinic structures in the initial conditions.

5.1.1. Sensitivity experiment for a 12-h period.
A sensitivity experiment with an assimilation window of 12 h and starting from 1200 UTC

15 January 1995 was carried out through the steps described above. The differences between the mean sea-level pressure analysis for 0000 UTC 16 January 1995 and the 12 h reference forecast valid at the same time are shown in Fig. 3. Notice the significant mean sea level pressure forecast

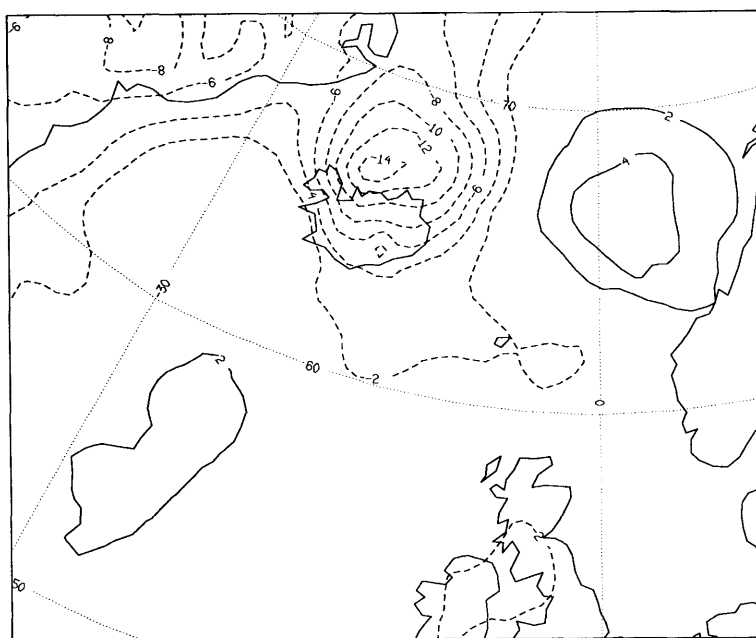


Fig. 3. Differences between the operational SMHI verification analysis and the 12-h forecast of mean sea-level pressure, 0000 UTC 16 January 1995.

errors north of Iceland. The difference between the 12-h sensitivity forecast and the 12-h reference forecast of surface pressure is shown in Fig. 4.

It is obvious that the sensitivity forecast manages to compensate for most of the errors in the reference forecast run, due to the inherent information in the gradient of the quadratic forecast error cost function obtained by the backward adjoint model integration. It is also of interest to note that the modifications introduced by the sensitivity forecast have significantly smaller horizontal scales than the forecast errors of the reference forecast. An explanation may be that the backward adjoint model integration + the forward model integration corresponds to larger weights given to the most unstable forecast error eigenmode than to the less unstable forecast error eigen-modes (see Rabier et al. (1996) for a more elaborate discussion). In this way, the backward + forward model integrations may be looked upon as a "dynamical filter" applied to the statistically determined analysis increments. In the particular case of a mesoscale low pressure development examined here, the smaller horizontal scale analysis increments provided by this "dynamical" filter seem to com-

pen-
sate in a positive sense the tendency of the HIRLAM Optimum Interpolation to filter the analysis increments too much in cases of mesoscale structures.

Another aspect of this sensitivity experiment is the small amplitude of the modifications added to the initial data of the sensitivity forecast. These modifications are so small, that it would be difficult to utilize such small amplitude observational increments by any 3-dimensional analysis scheme. This points to the potential benefits of 4-dimensional data assimilation. By looking at a particular meteorological disturbance at a later stage, when the instability conditions have forced it to grow significantly, we have a chance to recover it from observed data. We may thus consider the time dimension of 4-dimensional data assimilation to be a "magnifying glass".

A potential problem in relation to this sensitivity experiment is the use of an adjoint model without any of the physical parameterization schemes included. It was demonstrated by Navon et al. (1992) that it is important to include horizontal diffusion as well as parameterization of surface friction in order to obtain reasonable results for,

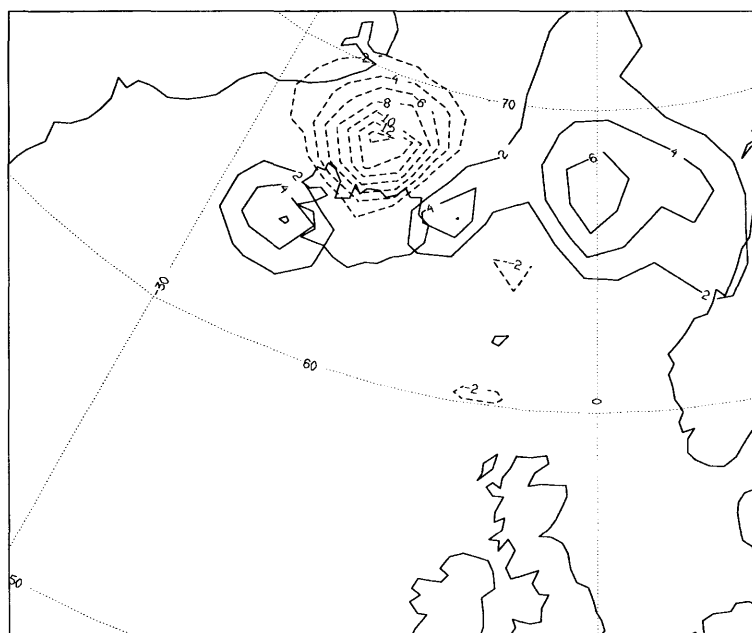


Fig. 4. Differences between "sensitivity" and "reference" forecasts of surface pressure, 1200 UTC 15 January 1995 + 12 h.

e.g., low level winds in sensitivity experiments, similar to those described in this paper. We have included horizontal diffusion in our adjoint model. Another difference to the experiments described by Navon et al. (1992) is the application of all the physical parameterization schemes in our non-linear forward model. The effect of, e.g., surface friction is thus included in the time-dependent basic state, around which the tangent-linear and the adjoint models are formed by linearization. Another sensitivity experiment with utilization of the adiabatic non-linear HIRLAM model for the forward integration (results are not shown) confirmed the unreasonable low level winds, obtained by Navon et al. (1992).

The initial perturbation created by the adjoint model integration in our 12-h sensitivity experiment was inspected carefully. A shallow baroclinic perturbation structure with maximum amplitude in the lower troposphere was found in the temperature field and there was a well-balanced perturbation in the wind field. The wind field perturbation at the lowest model levels had a negligible amplitude and thus there were no obvious negative effects of ignoring surface friction in the adjoint

model. The unperturbed and the perturbed initial temperature fields of model level 11, approximately at 700 hPa, are given in Fig. 5. The amplitude of the initial perturbation is approximately 1°K .

5.1.2. Sensitivity experiments for a 48-h period.

The assimilation window used above is a 12-h period. The aforementioned ECMWF sensitivity experiments used a 48-h assimilation window. Our sensitivity experiment was repeated for a 48-h period starting at 0000 UTC 14 January 1995 and ending at 0000 UTC 16 of January (as above). The differences between the verifying mean sea level pressure verification analysis and the 48-h forecast are shown in Fig. 6. The 48-h sensitivity forecast was less successful in compensating for details in the errors of the very poor 48-h reference forecast. The reasons may be several. For a 48-h forecast period, the linearity assumption of a single adjoint model integration is certainly less accurate and it may also be necessary to introduce some diabatic effects into the adjoint model. In addition, we have not yet understood the effects of using (dynamically inconsistent) analysis boundary conditions in our sensitivity experiments. It is, how-

ever, of interest to examine the growth during 48 h of the modifications introduced into the sensitivity forecast run, see Fig. 7.

5.2. Statistical evaluation. Sensitivity experiments over a 5-day period.

5.2.1. Synoptic situation. In order to get a statistical evaluation of the sensitivity forecasts, a 5-day period was chosen in which forecasts similar to those described in earlier sub-sections were run $4 \times$ a day with a forecast length of 36 h. This 5-day period, between 0000 UTC 13 September 1994 and 0000 UTC 18 September 1994, was characterized by an intensive cyclone development. The operational DMI (Danish Meteorological

Institute) HIRLAM data assimilation was re-run to produce analyses every 6th hour. The analyzed mean sea level pressure fields at 0000 UTC are given in Fig. 8. In the beginning of the period, a small low pressure system was found over the Atlantic ocean, south-west of Ireland. Three days later, this system developed into a major cyclone which hit Denmark. At the end of the period, the cyclone was considerably weakened. The forecast verification scores were closely related to the evolution of the cyclone with relatively poor forecasts during the most intensive development phase.

5.2.2. Initial state and boundaries. The analyses produced in the data assimilation were used

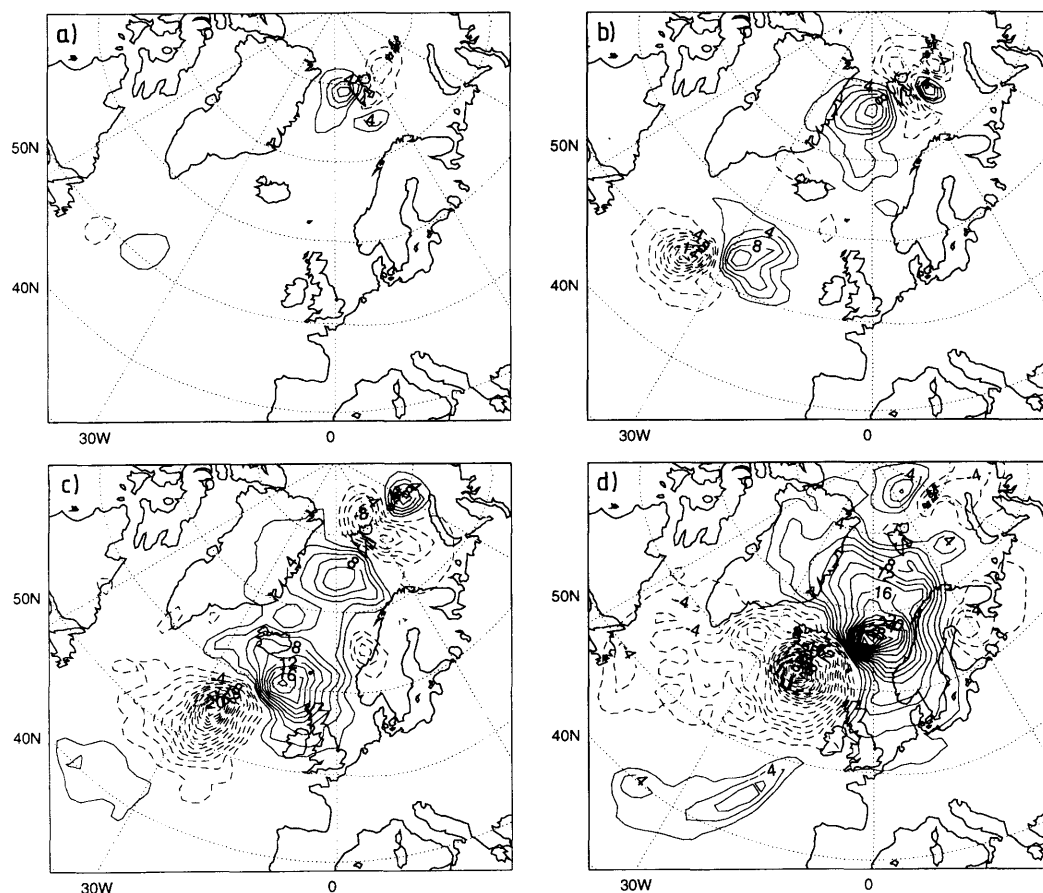
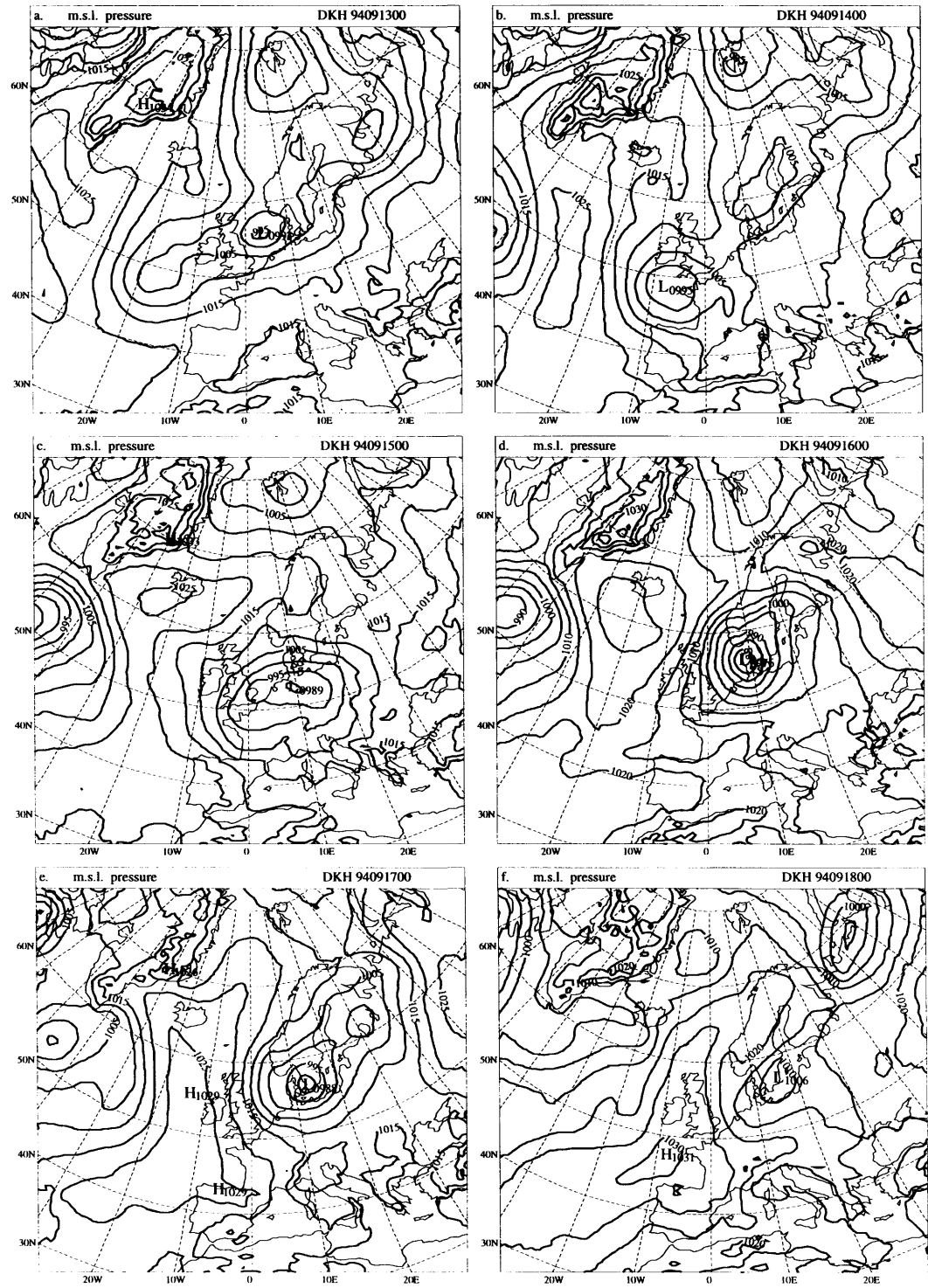


Fig. 7. Differences between "sensitivity" and "reference" forecasts of surface pressure, (a) 0000 UTC 14 January 1995 + 12 h. (b) 0000 UTC 14 January 1995 + 24 h. (c) 0000 UTC 14 January 1995 + 36 h. (d) 0000 UTC 14 January 1995 + 48 h.



both as initial model state and as boundaries for the sensitivity forecasts. It should be pointed out here that this experiment setup is a good test bed for the spectral HIRLAM model, as both the initial and lateral boundary conditions for the model are from the grid point model. In all experiments conducted so far, no problems have been found which relate to the different formulations in the grid point model and in the spectral model. The use of analyses as boundaries provides the sensitivity forecasts with the best boundary conditions, which may remove problems sometimes occurring in operational limited area model forecasts with too old boundary conditions (Gustafsson, 1990).

5.2.3. Overview of the experiments. Five experiments were conducted. A summary is given in Table 1. As a reference, denoted as FCST, the spectral HIRLAM was run from the analyses without any modifications. In the control experiment, SENA, the assimilation window was set to 6 h, the scaling coefficient α was set to 0.1, only one iteration (i.e., only one backward adjoint integration and one forward sensitivity integration) was used. The results are compared with those of the reference runs both on a case-to-case basis and by the average verification scores. The remaining three experiments were run to evaluate the impact of a larger scaling coefficient ($\alpha=0.5$, SENB), a longer assimilation window ($L_{win}=12$ h, SENE) and more iterations ($N_{cyc}=10$, SEND). Note that the application of several iterations with our sensitivity experiments is equivalent to a minimization with the steepest descent algorithm.

Table 1. Summary of 5-day forecast experiments

Experiment	α	N_{cyc}	L_{win}
FCST	—	—	—
SENA	0.1	1	6
SENB	0.5	1	6
SEND	0.1	10	6
SENE	0.1	1	12

Fig. 8. Operational DMI HIRLAM analyses at (a) 0000 UTC 13 September 1994, (b) 0000 UTC 14 September 1994, (c) 0000 UTC 15 September 1994, (d) 0000 UTC 16 September 1994, (e) 0000 UTC 17 September 1994, (f) 0000 UTC 18 September 1994.

5.2.4. Verification using analyses. The analyses were taken as the “truth” in this study. Therefore, the most natural validation was to verify the forecasts against the corresponding analyses. The simplest quantity to use was the cost function evaluated every 6th hour in the forecast.

A one-to-one comparison between FCST and SENA is made in Fig. 9. With regard to the synoptic development shown in Fig. 8, it is evident that the forecast error is large during the fast development stage. The improvement by SENA seems also larger for the poor forecasts during this period. Several other interesting features can be noticed in Fig. 9. First of all, sensitivity forecasts based on initial data from 0600 UTC and 1800 UTC generally have a stronger impact than sensitivity forecasts based on 0000 UTC and 1200 UTC initial data. This confirms that the forecast errors at 0000 UTC and 1200 UTC, used as input to the adjoint model integration, contain more reliable observed information than at 0600 UTC and 1800 UTC. It is also interesting to note that there is

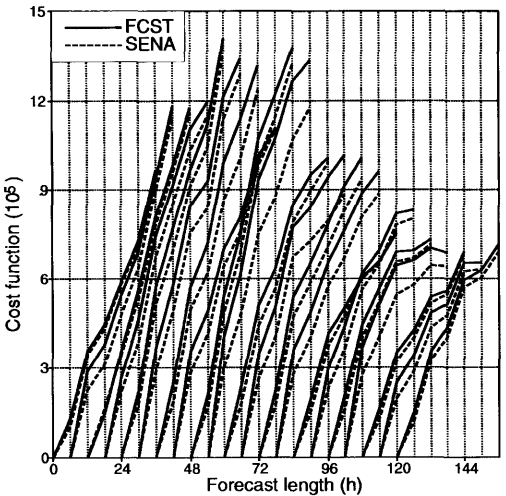


Fig. 9. Cost function as a function of forecast length for FCST (full lines) — original forecast and SENA (dashed lines) — sensitivity forecast with $\alpha=0.1$ and with one 6-h assimilation cycle. The left two curves are for FCST- and SENA-forecasts started at 13 September 1994 0000 UTC. The rest curves are shifted with a 6-h interval.

one reference forecast (8th) out-performing the reference forecast made 6 h later (9th), indicating that the analysis scheme even can do harm when adding new observations to the first-guess field. There are a few sensitivity forecasts which out-perform the forecasts made 6 h later. In these cases, the sensitivity fields calculated by the adjoint model amplify significantly and the sensitivity forecasts can be used directly in the real time operational practise to issue better forecasts. In the overall performance statistics, however, 6 h later forecasts are better than the sensitivity forecasts. The average scores for all 21 forecasts are given in Fig. 10. It is obvious that one iteration of the minimization scheme (SENA) gives a consistent and significant improvement over the original forecast (FCST).

A number of experiments have been run with different scaling coefficients α . Generally speaking, the improvement in the cost function is proportional to the coefficient. However, the minimization process may diverge if the coefficient is too large. The largest coefficient with which the 21 forecasts can run through without any problem is

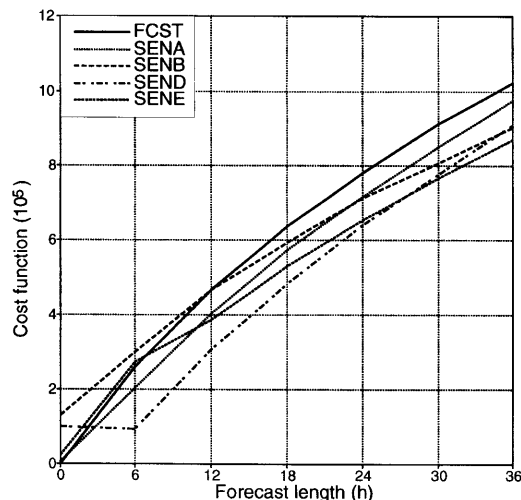


Fig. 10. Cost function as a function of forecast length for FCST (full line) — original forecast, SENA (dotted line) — sensitivity forecast with $\alpha=0.1$ and one 6-h assimilation cycle, SENB (dashed line) — with $\alpha=0.5$ and one 6-h assimilation cycle, SEND (dot-dash line) — with $\alpha=0.1$ and with ten 6-h assimilation cycles and SENE (short dash line) — with $\alpha=0.1$ and with one 12-h assimilation cycle over the 5-d period (13–18 September 1994).

0.5 (SENB). In Fig. 10, a clear improvement in SENB over SENA can be noticed after +12 h. However, longer assimilation windows and more iteration cycles cannot be used together with $\alpha=0.5$. A close inspection of the figure also reveals that on the average the one-step minimization actually diverged: the cost function for SENB at +6 h is larger than that of FCST.

By iterating the minimization process a few times more, a more consistent improvement can be achieved (experiment SEND). The first few iterations lead to changes in the cost function almost in a linear fashion. However, as the cost function approaches the one from the 6-h later forecasts, the modification from each iteration becomes smaller and smaller. It seems very difficult for the sensitivity forecasts to out-perform the forecasts made 6 h later by just increasing the number of iteration cycles. This is easy to understand, since the basis for our minimization is to approach the analysis 6 h later with the +6 h sensitivity forecast.

By using a longer assimilation window, $L_{\text{win}}=12$ h in experiment SENE, a different slope in the evolution of the cost function is obtained. This leads to a larger improvement after 12 h, out-performing 6-h later forecasts. However, if it is to be used in an operational context, the comparison should be made with 12-h later forecasts, since analyses valid 12 h later are used in the sensitivity experiments. We should not expect, however, the sensitivity forecasts to win the forecasts in real time. As discussed earlier with regard to experiment SEND, running many iterations only makes the cost function closer to that of a 12-h later forecast but not better than it.

To summarize the verification scores based on the difference between the forecasts and the analyses, it can safely be stated that the sensitivity fields produced by running the adjoint model are capable to capture the main features of the error growth and the sensitivity forecasts are better than the original forecasts.

5.2.5. Verification using observations. Another way to evaluate forecasts is to verify them against observations from European radiosonde and synoptic stations (Hall, 1987). The initial perturbation for an adjoint model integration is derived from the difference between a short-range forecast and the verifying analysis. The influence of observa-

tions comes indirectly into the sensitivity forecast experiments via this analysis, which is a combination of a first-guess field and observations derived by the OI scheme.

The observation verification scores are summarized in Fig. 11. Both mean error (the lower group of curves in each panel) and rms error (the upper group of curves in each panel) are shown as functions of forecast length. Average verification scores are calculated over all 21 forecasts during the 5-day period. 12 parameters have been chosen: geopotential height Z , temperature T , and wind V at 250 hPa, 500 hPa and 850 hPa, mean sea level pressure $MSLP$, 2-m temperature $T02M$, and 10-m wind $V10M$.

It is encouraging to see from the figure that the rms errors of the first-guess fields have been reduced by all sensitivity experiments. The largest improvements were found in mass fields. Using FCST as a reference and SENA as the control experiment, the following can be concluded: (1) the sensitivity forecasts (SENA) reduce the rms error; (2) the reduction in rms error is larger when a larger scaling coefficient is used (SENB), more iteration cycles are run (SEND) or a longer assimilation window is chosen (SENE).

The forecast improvements obtained through the sensitivity experiments seem to indicate stronger impact at upper levels than at lower levels, see Fig. 11. This may tell us two things: The change in initial conditions mainly improve the upper level flow which is coupled to large scale baroclinic structures. At lower levels, where diabatic effects become more important, the improvements are smaller or nonexistent, see, e.g., verification scores for 2-m temperature and 10-m wind. Furthermore, we see that degrading of the bias only occurs in the height field and in the MSL pressure while the wind bias is almost unchanged. This also points toward the neglect of diabatic processes as the main limiting factor of these experiments.

The improvement in the rms scores seems easy to understand: the observations influence the sensitivity field indirectly through the analyses. The minimization is based upon a cost function which can be considered as a rms measure of the error. However, the bias of the forecast normally has a smaller amplitude than forecast errors with spatial and temporal structures. The forecast bias will therefore contribute less to the quadratic cost

function. Thus, there is less control over the bias in our sensitivity experiments. This is another possible reason why there is no systematic improvement in the bias.

The verification scores at 6 h are of particular importance: they give the quality of the first-guess field to be used by the OI scheme in the next cycle. One way to improve the analysis is to use a better first-guess. A sensitivity forecast may be considered a step towards that. From the results presented here, sensitivity forecasts could give a better first-guess field only in rms sense. In order to proceed with the idea, we need first to understand better and try to avoid the degrading of the bias scores.

6. Discussion and concluding remarks

The HIRLAM project is aiming for development of a 4-dimensional variational data assimilation system to be applied for operational numerical weather prediction. This paper deals with the first step of this development effort, the derivation of the tangent-linear and the adjoint of the adiabatic part of the spectral HIRLAM model and with the application of the adjoint HIRLAM to test the sensitivity of short-range forecast errors to initial conditions. These sensitivity experiments, reported on here for one particular case study and for a complete 5-day period, have shown that the adjoint of the spectral HIRLAM may be used to improve the initial conditions by knowledge on forecast errors at +6–+48-h forecast range. For selected cases we have in particular shown that we are able to utilize the future analyses to improve the initial conditions on small amplitude baroclinically growing model modes, that would have been impossible to recover from present observations only.

There are certainly limitations to the present approach. Using a cost function based on deviations between analysis and forecast fields imposes in principle the spatial structures created by the particular analysis scheme as a weak constraint. Running the minimizations through a sufficient number of iteration cycles would eventually result in a reconstruction of the analysis at the end of the assimilation window, provided this analysis is a forecast model solution. Thus, the final sensitivity forecast will be close to the forecast started

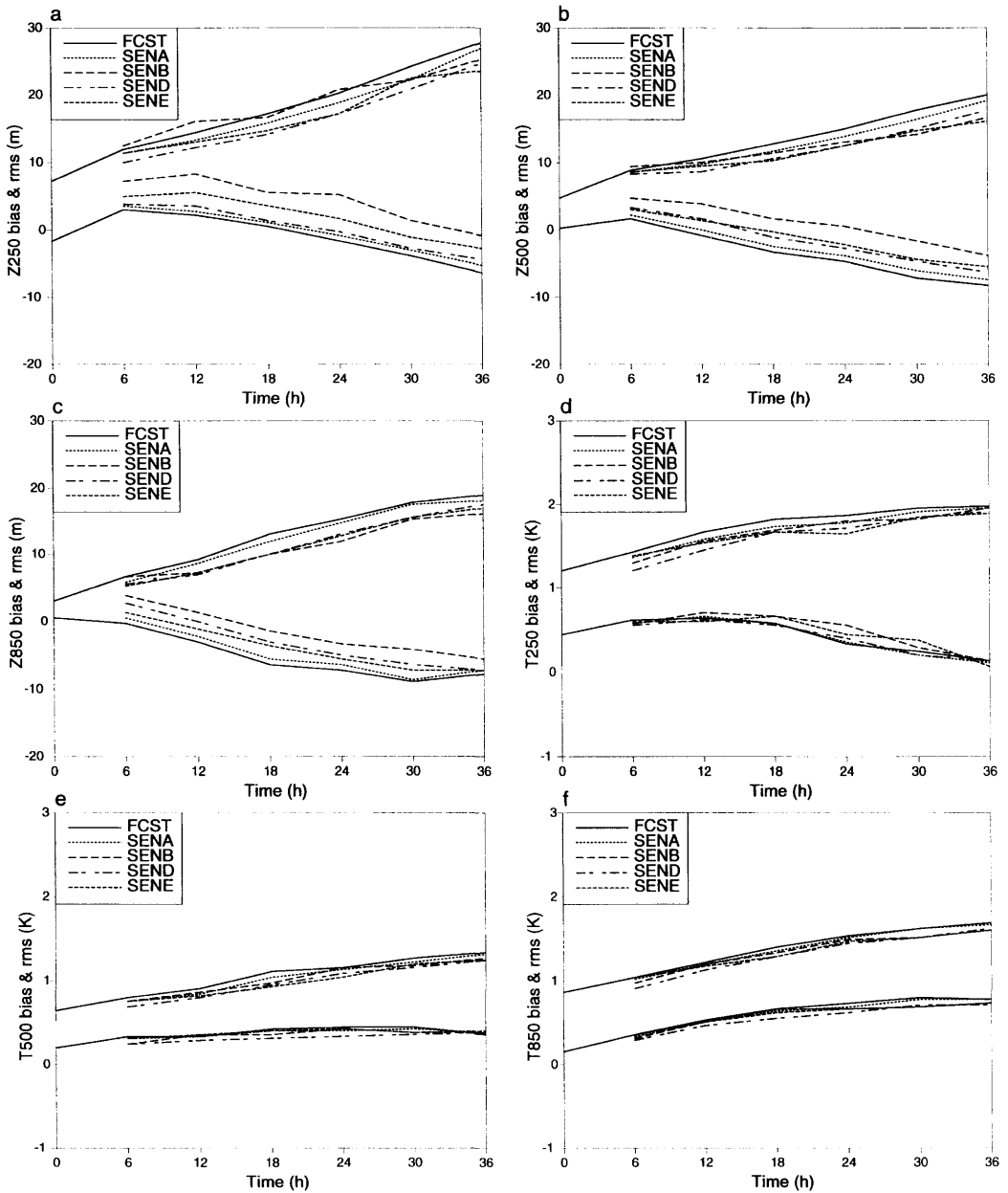


Fig. 11. (a–f) Observation verifications as functions of forecast length for FCST (full line), SENA (dotted line), SENB (dashed line), SEND (dot-dash line) and SENE (short dash line) over the 5-day period (13–18 September 1994). (a) 250 hPa height (b) 500 hPa height (c) 850 hPa height (d) 250 hPa temperature (e) 500 hPa temperature (f) 850 hPa temperature (g) 250 hPa wind (h) 500 hPa wind (i) 850 hPa wind (j) Mean sea level pressure (k) 2-m temperature (l) 10-m winds.

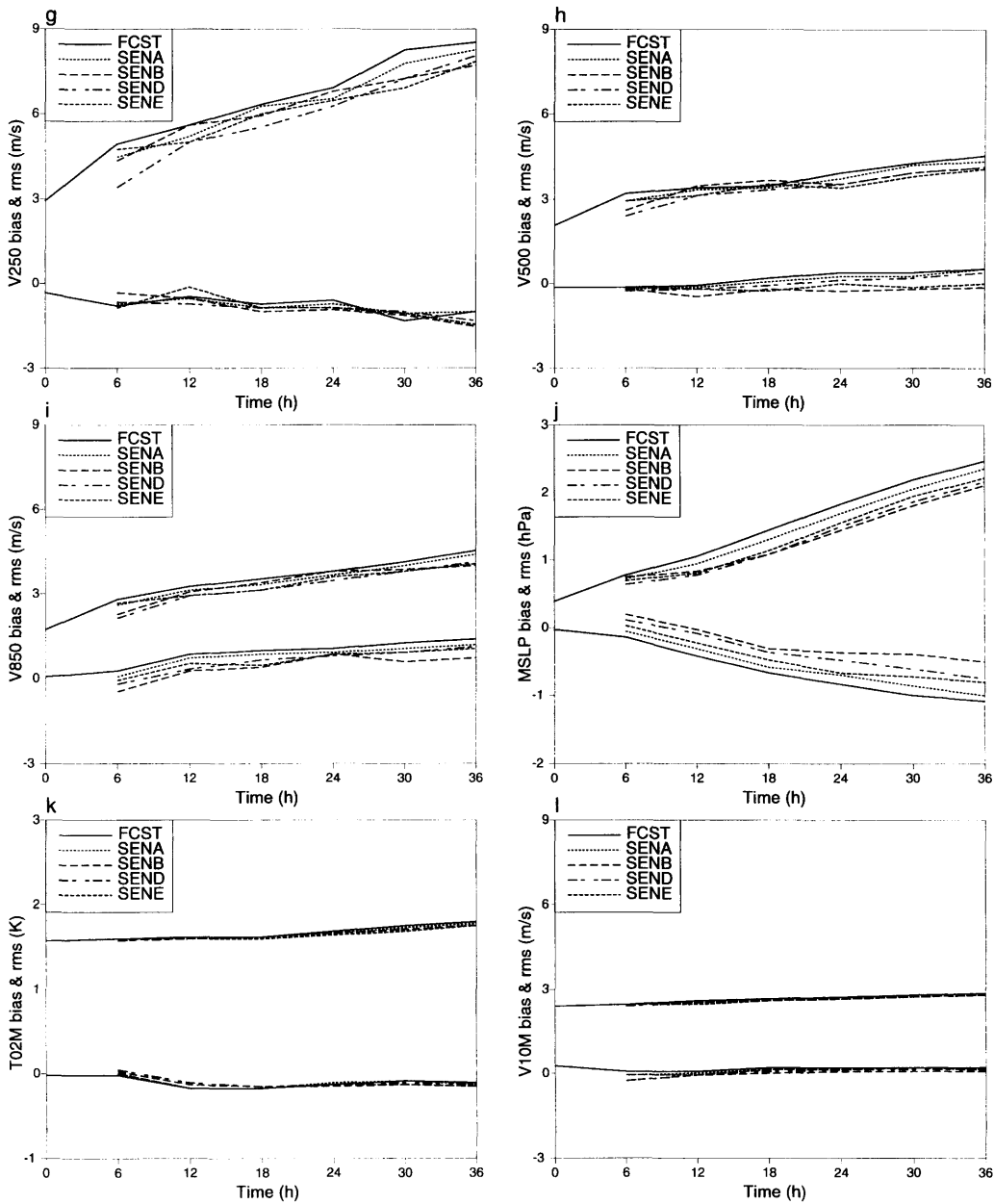


Fig. 11. (cont'd)

from the later analysis. However, there are still improvements to be expected by running this simple minimization scheme. Since many of the parameterized physical processes are irreversible, the forward time integration of a non-linear

forecast model is not fully invertible. Our applied minimization procedure can therefore only be considered to be a pseudo-inverse. For instance, when the final sensitivity forecast passes the analysis time, it has already been run for a few hours

(e.g., 6 h) and therefore it should have less spinup problem than the original forecast. In this sense, the minimization scheme can be used as a diabatic initialization scheme. Another application of the minimization scheme could be the initialization of some non-analyzed fields like cloud water content (Huang, 1996). Through the iterations of adiabatic backward and diabatic forward integrations, a dynamic balance between the non-analyzed variables and analyzed variables could be obtained. In the cases when the initial balance and spinup are important, running the sensitivity forecast iteratively should result in better forecasts.

A main limitation of the present study is the neglect of diabatic processes in the calculation of the tangent linear and adjoint operators. We see possible effects of this both when extending the time window over which the model is linearized and in degrading the height field bias in the perturbed forecast. Diabatic processes primarily affect the temperature field and a systematic neglect of diabatic heating/cooling over the entire model domain should give rise to a systematic bias in the height and sea level pressure fields. Furthermore, diabatic effects contribute more to baroclinic developments over longer time scales while rapid growth over shorter time scales to a large extent is governed by adiabatic processes at upper levels.

The ultimate goal of our development efforts is of course to introduce the remaining components of the 4-dimensional variational data assimilation, an observation cost function to measure the distances to the spatially and temporally scattered observations and a background error cost function to assure a proper spatial spectrum of the assimilation increments.

The concept of a "poor mans" 4-dimensional data assimilation was first introduced by Eugenia Kalnay (personal communication). She suggested the use of "breeding" modes in forward intermittent data assimilation to improve the baroclinic structures of analysis first guess fields. Our success-

ful first sensitivity experiments with the adjoint HIRLAM have, however, stimulated us to start thinking on the possibilities of introducing an alternative "poor mans" 4-dimensional data assimilation. One could, e.g., consider the following assimilation scheme: (1) 6-h forward forecast model run, (2) preliminary OI analysis, (3) 6-h adjoint model backward run, (4) 6-h forward forecast model run and (5) final OI analysis. The motivation for this backward and forward assimilation cycle is to create an improved analysis first guess for the final OI analysis. As we have shown by our sensitivity experiments, such a backward and forward cycling improves the forecast first guess and it may also act as a dynamical filter on the OI analysis increments. The analysis increments in the second OI analysis application at the end of the assimilation window would certainly result in smaller amplitude analysis increments and, consequently, the ensuing forecast would be associated with a less serious spinup of, e.g., baroclinic structures as well as diabatic processes.

Limited area model forecasts are sensitive to the quality of the initial conditions and to the quality of the lateral boundary conditions. The sensitivity to the lateral boundary conditions may be studied with the same adjoint model technique that was applied for the sensitivity to the initial conditions in this paper. This will be the subject for further experimentation with the adjoint of the spectral HIRLAM.

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