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DESIGN OF OFFSET–FREE PREDICTIVE CONTROL WITH STATE–SPACE MODELS: EQUIVALENCES AND NEW RESULTS

PIOTR TATJEWSKI [a]

[a] Institute of Control and Computation Engineering
Warsaw University of Technology
ul. Nowowiejska 15/19, 00-665 Warsaw, Poland
e-mail: piotr.tatjewski@pw.edu.pl

Design of offset-free model predictive control (MPC) with a linear state-space process model is discussed in the paper, for deterministic constant or asymptotically constant external and internal (modeling errors) disturbances. The three existing, established methods assuring offset-free control are briefly compared and relations between them are recalled. The main result of the paper is a new, more flexible but still simple formula defining unmeasured disturbance estimates in the model-based prediction error method (abbreviated as MPE). The new formulation eliminates the only deficiency of this approach, i.e., being possibly highly sensitive to noise. Equivalence of the proposed technique with the augmented process-and-disturbance model method (AM for short) is proved, indicating how to define arbitrary choice of disturbance matrices in the equivalent formulation of the AM approach. With the new formulation, the MPE method seems to be a competitive, best design choice. Theoretical results are validated and illustrated by extensive simulation results of a MIMO control problem, with unmeasured disturbances from the mentioned class and also with noises added. The capability of the proposed method to tune sensitivity to noises of the MPC control system is shown.

Keywords: model predictive control, offset-free control, state-space model, disturbance estimate, noise sensitivity.

1. Introduction

Model predictive control (MPC) is now a well established advanced control technology, resulting in a variety of very successful control techniques applied in practice (see, e.g., Camacho and Bordons, 1999; Maciejowski, 2002; Qin and Badgwell, 2003; Rossiter, 2003; Blevins *et al.*, 2003; Tatjewski, 2007; Wang, 2009; Rao and Rawlings, 2009). Model predictive control had also a great impact on directions of research in the area of multivariable feedback control, including its potential to interact with economic aspects of control (see, e.g., Tatjewski, 2008; 2010; Ben Aicha *et al.*, 2013; Blevins *et al.*, 2013; Grosso *et al.*, 2016; Karimi Pour *et al.*, 2018). MPC implementations can use various models of the controlled processes. Nonparametric step response models and discrete transfer function ones lead to well established MPC structures, DMC and GPC, respectively. However, the third very popular approach to linear modeling, the use of state-space models, results in a variety of possible design versions, mainly due to different handling of deterministic disturbances to get offset-free control. The main factor here is the way in which estimates of the disturbances are defined and placed in the process model. The problem has attracted significant attention in the literature (Muske and Badgwell, 2002; Pannocchia and Rawlings, 2003; Tatjewski, 2007; 2011; 2014; Pannocchia and Bemporad, 2007; Gonzalez *et al.*, 2008; Wang, 2009; Maeder and Morari, 2010; Morari and Maeder, 2012; Lawryńczuk, 2015; Pannocchia, 2015), also in the last decade (Pannocchia *et al.*, 2015; Tatjewski, 2017; Jimoh *et al.*, 2020; Tatjewski and Ławryńczuk, 2020; Tuan *et al.*, 2022; Khoury *et al.*, 2022; Yaghini *et al.*, 2024; Kuntz and Rawlings, 2024).

In this paper, asymptotically constant external disturbances and internal disturbances (modeling errors) will be considered, which can lead to non-zero steady-state errors (offsets) if controllers do not use offset-free techniques. Three approaches to the problem of offset-free attenuation of disturbances from the defined class, within the MPC with state-space models, were established. The first one uses an augmented process-and-disturbance state model and observer. It was introduced and analysed by Muske and Badgwell (2002), and will be shortly described as the AM method further in the paper. The second one uses the velocity form model (for detailed presentation see, e.g., the work of Wang (2009)) and it will be described as the VM method. The third one was introduced by Tatjewski (2007) and finally fully analysed by the same author (2014), and it relies on model-based prediction error to estimate influence of disturbances, using the observer of process state only (not augmented). It will be described as the MPE method further in the paper.

The deficiency of the AM method is the necessity to choose arbitrarily matrices defining the way the disturbance estimates enter the process model, under the restriction that dimensionality of these estimates does not exceed the number of measured outputs. The VM method also uses an augmented state model, but in a different way. Due to the velocity form state, estimates of disturbances are not needed but the resulting augmented model is more complicated and has inferior properties which can lead to problems, especially for longer prediction horizons. Pannocchia (2015) showed the equivalence of these all three methods, provided the model extension matrices and the observer gain matrix in the AM method and the observer matrix in the VM method have structures appropriately related to the process observer gain matrix $\mathbf{K}$ used in the MPE method (see Section 3.4). In particular, the mentioned equivalence shows that the MPE method uses, implicitly, a dead-beat observer for disturbance estimation. As is well known in control theory, the dead-beat observer assures the quickest estimation (closed-loop poles equal to zero), but can lead to increased sensitivity to noises. This may be treated as a deficiency in noise affected situations—the only one of the possibly best MPE method. The aim of this paper is to propose, analyse and verify an extension of the MPE method leading to elimination of this deficiency. It allows shifting the closed-loop poles away from zero, but still keeps the simplicity of the approach.

The structure of the paper is as follows. First, in Section 2 the process itself and its model are defined and the possible state observers are introduced. MPC with a linear state-space model is briefly recalled, to introduce formulations needed for further deliberations. In Section 3 the three methods assuring offset-free control are recalled and equivalence conditions are given. In Section 4 the main result of the paper is presented. New, extended formulae for the state and output disturbance estimates are given, containing previous formulation (Tatjewski, 2014) as a special case. Then equivalence conditions of the new approach with the AM method are given, but now without the restriction for the disturbance observer to be dead-beat. Finally, in Section 5 theoretical results of the paper are validated and justified by an extensive simulation study using a multivariable (2×2) sample process model. In particular, the new ability of the proposed MPE method to tune noise sensitivity of the MPC control system is illustrated.

2. Preliminaries

2.1. Controlled system, nominal model, state observer. We shall consider the following linear state-space description of the process to be controlled:

$$x(k+1) = \mathbf{A}_p x(k) + \mathbf{B}_p u(k) + z(k), \tag{1}$$

$$y(k) = \mathbf{C}x(k) + w(k), \tag{2}$$

where x is the state vector of dimension n_x, y is the vector of measured outputs of dimension n_y, while z and w are state and output disturbance vectors, representing unknown asymptotically constant disturbances, possibly also with noises added.

We shall assume that the process is stabilizable and controllable and there is a process input u such that the process output y can reach any set-point y^{sp}. A necessary condition for that is (see, e.g., Pannocchia, 2015)

$$\operatorname{rank} \begin{bmatrix} \mathbf{A}_p - \mathbf{I} & \mathbf{B}_p \\ \mathbf{C} & 0 \end{bmatrix} = n_x + n_y, \tag{3}$$

which implies $n_u \geq n_y$. We shall further assume $n_u = n_y$.

We shall also presuppose that the following nominal state-space model of the process (1)–(2) is known:

$$\hat{x}(k+1) = \mathbf{A}\hat{x}(k) + \mathbf{B}u(k), \tag{4}$$

$$\hat{y}(k) = \mathbf{C}\hat{x}(k), \tag{5}$$

where $\hat{x}(k)$ denotes an estimate of the generally unmeasured process state $x(k)$ and $\hat{y}(k)$ is an estimate of the measured output $y(k)$. Differences between the estimates $\hat{x}(k)$, $\hat{y}(k)$ and the process values $x(k)$, $y(k)$ are due to unknown state and output disturbances and possible parametric differences between matrices $\mathbf{A}_p$, $\mathbf{B}_p$ and $\mathbf{A}$, $\mathbf{B}$.

Certainly, there are cases when the process state is measured, but we shall consider a more general situation when the process outputs are measured only and the process state must be estimated using an observer (filter). We shall assume the following form of the observer:

$$\hat{x}(k) = \mathbf{A}\hat{x}(k-1) + \mathbf{B}u(k-1) + \mathbf{K}(y(k) - \hat{y}^*(k)), \tag{6}$$

where

$$\hat{y}^*(k) = \mathbf{C}(\mathbf{A}\hat{x}(k-1) + \mathbf{B}u(k-1)). \tag{7}$$

Equations (6)–(7) can describe both the well known Luenberger current observer and the stationary Kalman filter (see, e.g., Astrom and Wittenmark, 1997; Anderson and Moore, 2005). In the latter case the matrix $\mathbf{K}$ is a solution of the corresponding Ricatti equation.

2.2. Model predictive control. Model predictive control is presented in many papers and books, in numerous formulations including those with discrete-time state-space process models we are interested in (see, e.g., Maciejowski, 2002; Rossiter, 2003; Tatjewski, 2007; Wang, 2009). Therefore, we shall only briefly recall what will be needed for further presentation.

The principle of MPC is to evaluate the current process control input by minimizing, at each sampling instant k, a cost (performance) function over the future prediction horizon of N samples. The following form of this function is widely used:

$$\begin{aligned} J(k) = &\sum_{p=1}^{N} \|y^{sp}(k+p|k) - y(k+p|k)\|_{\mathbf{\Psi}}^2 \\ &+ \sum_{p=0}^{N_u-1} \|\triangle u(k+p|k)\|_{\mathbf{\Lambda}}^2 , \end{aligned} \tag{8}$$

where $\|x\|_{\mathbf{R}}^2 = x^T\mathbf{R}x$, $\mathbf{\Psi} \geq \mathbf{0}$ and $\mathbf{\Lambda} > \mathbf{0}$ are square diagonal scaling matrices of dimensions corresponding to n_y and n_u, respectively; N_u denotes the length of the control horizon, $N_u \leq N$; vectors $y^{sp}(k+p|k)$ and $y(k+p|k)$ are set-points (reference values) and outputs predicted for a future sample $k+p$, but calculated at the current sample k. Increments of the control inputs, $\triangle u(k+p|k) = u(k+p|k) - u(k+p-1|k)$, $p = 0, \ldots, N_u - 1$, are decision variables of the optimization of $J(k)$. A more general form of (8) is possible and often used in theoretically oriented papers, namely, with $N_u = N$ and an additional quadratic term defined for final state, to get the problem equivalent to an infinite horizon one. We shall avoid this to simplify the presentation.

The minimization of $J(k)$ is performed subject to the constraints

$$\begin{aligned} -\triangle u_{\max} \leq \triangle u(k+p|k) \leq \triangle u_{\max}, \\ p = 0, \ldots, N_u-1, \end{aligned} \tag{9}$$

$$u_{\min} \leq u(k+p|k) \leq u_{\max},\ p = 0, \ldots, N_u-1, \tag{10}$$

$$y_{\min} \leq y(k+p|k) \leq y_{\max},\ p = 1, \ldots, N. \tag{11}$$

A more general form of constraints, including any linear functions of all variables used, is possible but avoided here for presentational simplicity.

To get offset-free control, an appropriate representation of unknown disturbances is necessary in the prediction equations. We assume that every process state and output can be influenced by disturbances (including model inaccuracies), therefore vectors of state and output disturbance estimates, $d_x(k)$ and $d_y(k)$, will be added in the prediction equations.

For a given vector of inputs on the prediction horizon, $u(k+p|k), p = 0, \ldots, N-1$, state and output predictions can be calculated in a recursive way using the model (4)–(5), with added disturbance estimates:

$$\begin{aligned} x(k+1|k) &= \mathbf{A}\hat{x}(k) + \mathbf{B}u(k|k) + d_x(k), \\ x(k+p|k) &= \mathbf{A}x(k+p-1|k) + \mathbf{B}u(k+p-1|k) \\ &\quad + d_x(k), \qquad p = 2, \ldots, N, \\ y(k+p|k) &= \mathbf{C}x(k+p|k) + d_y(k), \quad p = 1, \ldots, N. \end{aligned} \tag{12}$$

How the values $d_x(k)$ and $d_y(k)$ are defined and computed depends on the method applied; this will be clarified in the next section.

Elimination of the recursion in (12) leads to the well known closed (explicit) prediction formulae

$$\begin{aligned} y(k+1|k) &= \mathbf{C}\left[\mathbf{A}\hat{x}(k) + \mathbf{B}(u(k-1) + d_x(k))\right] \\ &\quad + \mathbf{CB}\triangle u(k|k) + d_y(k), \\ y(k+p|k) &= \mathbf{C}\left[\mathbf{A}^p\hat{x}(k) + \mathbf{C}\mathbf{V}_p(\mathbf{B}u(k-1) + d_x(k))\right] \\ &\quad + \mathbf{M}_p\triangle U(k) + d_y(k),\ p = 2, \ldots, N, \end{aligned} \tag{13}$$

where

$$\mathbf{V}_p = \mathbf{A}^{p-1} + \mathbf{A}^{p-2} + \cdots + \mathbf{A} + \mathbf{I}, \tag{14}$$

$\mathbf{M}_p$ is the p-th block row of the dynamic matrix $\mathbf{M}$ and $\triangle U(k)$ is the vector of decision variables of the MPC optimization problem, i.e., the problem of minimization of (8) subject to the prediction equality constraints (13) as well as the inequality constraints (9), (10) and (11),

$$\begin{aligned} \triangle U(k)^T \ = \ &[\triangle u(k|k)^T \ \triangle u(k+1|k)^T \cdots \\ &\cdots \triangle u(k+N_u-1|k)^T] \end{aligned} \tag{15}$$

(see, e.g., Tatjewski, 2007).

The described MPC optimization problem is a strictly convex quadratic programming (QP) one, thus with a well defined, unique solution, provided the set defined by the constraints assures feasibility (is non-empty).

The only and vital problem which remains to be presented to complete the controller design is to show how to formulate disturbance estimates $d_x(k)$ and $d_y(k)$ assuring offset-free control despite unmeasured disturbances acting on the process, of the class considered in the paper. In further sections, existing formulations are briefly described and compared, and then an improved design is proposed.

3. Methods assuring offset-free control

Three different methods assuring offset-free MPC design with a linear state-space process model are briefly reviewed in this section, following the way of presentation and including the results given by Pannocchia (2015).

3.1. AM method: The augmented model and its observer. In this approach, the state of the system is augmented by additional integrating variables d of dimension n_d, leading to the state-space model (4)–(5) augmented to

$$\begin{bmatrix} \hat{x}(k+1) \\ \hat{d}(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_d \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \hat{x}(k) \\ \hat{d}(k) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u(k) = \mathbf{A}_a \hat{x}_a(k) + \mathbf{B}_a u(k), \tag{16}$$

$$\hat{y}(k) = \begin{bmatrix} \mathbf{C} & \mathbf{C}_d \end{bmatrix} \begin{bmatrix} \hat{x}(k) \\ \hat{d}(k) \end{bmatrix} = \mathbf{C}_a \hat{x}_a(k), \tag{17}$$

where $x_a^T = [x^T \ d^T]$ is the augmented state (Muske and Badgwell, 2002; Pannocchia and Rawlings, 2003; Pannocchia and Bemporad, 2007; Maeder *et al.*, 2009; Pannocchia, 2015).

It has been proven that the augmented system is detectable if and only if the non-augmented system is detectable and the following condition holds:

$$\operatorname{rank} \begin{bmatrix} \mathbf{A} - \mathbf{I} & \mathbf{B}_d \\ \mathbf{C} & \mathbf{C}_d \end{bmatrix} = n_x + n_d, \tag{18}$$

which implies the inequality $n_d \leq n_y$. In practice we always assume the equality, as too small a number of disturbances can be a drawback, not vice versa, therefore $n_d = n_y$. It has also been shown (Muske and Badgwell, 2002; Pannocchia and Rawlings, 2003) that a pair of matrices $(\mathbf{B}_d \ \mathbf{C}_d)$ can always be found such that (18) is true and that the stated condition assures offset-free control.

The dynamics of the disturbance vector stems from that of the augmented state observer,

$$\hat{x}_a(k) = \mathbf{A}_a \hat{x}_a(k-1) + \mathbf{B}_a u(k-1) + \mathbf{K}_a(y(k) - \hat{y}^*(k)), \tag{19}$$

where now

$$\hat{y}^*(k) = \mathbf{C}_a(\mathbf{A}_a \hat{x}_a(k-1) + \mathbf{B}_a u(k-1)), \tag{20}$$

$$\mathbf{K}_a = \begin{bmatrix} \mathbf{K}_x \\ \mathbf{K}_d \end{bmatrix}. \tag{21}$$

The current value of $\hat{d}(k)$ is needed to calculate the current values of state and output disturbances $d_x(k)$ and $d_y(k)$ to be used in the prediction equations (12) or (13),

$$d_x(k) = \mathbf{B}_d \hat{d}(k), \ \ d_y(k) = \mathbf{C}_d \hat{d}(k). \tag{22}$$

3.2. MPE method: The model-based prediction error and process observer. The method was proposed by Tatjewski (2007) for cases with measured state and then extended to the general case of state estimation (Tatjewski, 2014). The extended model is not used and the process state observer is only applied, as given by Eqns. (6)–(7).

The state and output disturbance estimates used in the prediction equations (12) or (13) are defined and computed as follows:

$$d_x(k) = \hat{x}(k) - [\mathbf{A}\hat{x}(k-1) + \mathbf{B}u(k-1)], \tag{23}$$

$$d_y(k) = y(k) - \mathbf{C}\hat{x}(k), \tag{24}$$

where $y(k)$ is the measured process output.

It should be pointed out that this method does not require arbitrary definitions of matrices, like $\mathbf{B}_d$ and $\mathbf{C}_d$ in the previous approach, which define how the disturbances d enter the state and output equations; see (22).

It should be also noticed that, when the entire state is measured, then the observer is not needed at all in this method, i.e., the disturbance observer is not needed, as in the AM method.

3.3. VM method: The velocity form model and its observer. The method is based on the velocity form model, where state increments are used in place of the state (see the work of Wang (2009) for the most elaborate presentation of MPC with such a model). Defining the augmented state vector x_δ,

$$x_\delta(k) = \begin{bmatrix} \delta x(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} x(k) - x(k-1) \\ y(k) \end{bmatrix}, \tag{25}$$

the velocity form nominal model can be easily derived from the model (4)–(5):

$$x_\delta(k+1) = \mathbf{A}_\delta x_\delta(k) + \mathbf{B}_\delta \delta u(k), \tag{26}$$

$$y(k) = \mathbf{C}_\delta x_\delta(k), \tag{27}$$

where $\delta u(k) = u(k) - u(k-1)$,

$$\mathbf{A}_\delta = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{CA} & \mathbf{I} \end{bmatrix}, \ \mathbf{B}_\delta = \begin{bmatrix} \mathbf{B} \\ \mathbf{CB} \end{bmatrix}, \ \mathbf{C}_\delta = \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix}. \tag{28}$$

Assuming $\mathbf{A}_p = \mathbf{A}$, $\mathbf{B}_p = \mathbf{B}$, the process description (1)–(2) in velocity form is as follows:

$$x_\delta(k+1) = \mathbf{A}_\delta x_\delta(k) + \mathbf{B}_\delta \delta u(k) + \begin{bmatrix} \delta z(k) \\ \mathbf{C}\delta z(k) + \delta w(k+1) \end{bmatrix}, \tag{29}$$

$$y(k) = \mathbf{C}_\delta x_\delta(k). \tag{30}$$

This indicates that the velocity form model does not need any disturbance estimates in prediction equations to assure zero offset, because the increments $\delta z(k)$, $\delta w(k)$ of constant disturbances are zero.

However, the MPC optimization problem must be reformulated for the reformulated model. In particular, prediction equations must be written for the velocity model (26)–(27), which can lead to numerical problems, especially for long prediction horizons, as the velocity state matrix $\mathbf{A}_\delta$ has all additional n_y eigenvalues equal to 1 (i.e., on the stability boundary); see the structure of the matrix $\mathbf{A}_\delta$ in (28). Moreover, an observer for the extended velocity state (25) must be used,

$$\hat{x}_\delta(k) = \mathbf{A}_\delta \hat{x}_\delta(k-1) + \mathbf{B}_\delta u(k-1) + \mathbf{K}_\delta(y(k) - \hat{y}^*(k)), \tag{31}$$

where

$$\hat{y}^*(k) = \mathbf{C}_\delta \left(\mathbf{A}_\delta \hat{x}_\delta(k-1) + \mathbf{B}_\delta u(k-1)\right), \tag{32}$$

$$\mathbf{K}_\delta = \begin{bmatrix} \mathbf{K}_{\delta x} \\ \mathbf{K}_y \end{bmatrix}. \tag{33}$$

It can be shown that the pair $(\mathbf{C}_\delta, \mathbf{A}_\delta)$ is detectable if and only if the pair $(\mathbf{C}, \mathbf{A})$ is detectable.

3.4. Equivalences between the methods. It was shown by Pannocchia (2015) that the following relations between the three presented methods are true:

1. The AM method is equivalent to the MPE approach with

   $$\mathbf{B}_d = \mathbf{K},\ \mathbf{C}_d = \mathbf{I} - \mathbf{CK},\ \mathbf{K}_x = \mathbf{K},\ \mathbf{K}_d = \mathbf{I}, \tag{34}$$

   and then it holds that

   $$\hat{d}(k) = \mathbf{C}d_x(k) + d_y(k). \tag{35}$$

   Moreover, the detectability condition (18) is satisfied provided the pair $(\mathbf{A}, \mathbf{C})$ is detectable.

2. The VM method with $\mathbf{K}_y = \mathbf{I}$ is equivalent to the AM approach with

   $$\mathbf{B}_d = \mathbf{K}_{\delta x},\ \mathbf{C}_d = \mathbf{I} - \mathbf{CK}_{\delta x},\ \mathbf{K}_x = \mathbf{K}_{\delta x}, \mathbf{K}_d = \mathbf{I}. \tag{36}$$

3. The VM method is equivalent to the MPE approach provided $\mathbf{K}_{\delta x} = \mathbf{K}$, $\mathbf{K}_y = \mathbf{I}$.

4. Main result: The extended MPE method

It follows from the previous sections that the MPE method is superior to the AM approach in the sense that it does not require arbitrary choice of matrices $\mathbf{B}_d$ and $\mathbf{C}_d$, and needs the observer for the non-augmented model only, thus of lower order. The only difference is that it constrains the choice of the $\mathbf{K}_d$ part of the augmented observer matrix (21) to the unity matrix $\mathbf{I}$. Under this condition, the state matrix of the closed-loop dynamics of the augmented observer (19)–(20) is

$$\begin{aligned}
&\mathbf{A}_a - \mathbf{K}_a\mathbf{C}_a\mathbf{A}_a \\
&= \begin{bmatrix} \mathbf{A} & \mathbf{K} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} - \begin{bmatrix} \mathbf{K} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{C} & \mathbf{I} - \mathbf{CK} \end{bmatrix} \begin{bmatrix} \mathbf{A} & \mathbf{K} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{A} & \mathbf{K} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} - \begin{bmatrix} \mathbf{KCA} & \mathbf{K} \\ \mathbf{CA} & \mathbf{I} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{A} - \mathbf{KCA} & \mathbf{0} \\ -\mathbf{CA} & \mathbf{0} \end{bmatrix}.
\end{aligned} \tag{37}$$

That is, the first n_x poles of this matrix are poles of the closed-loop dynamics of the process observer (6)–(7) and the remaining $n_d = n_y$ poles describing the disturbance observer dynamics are zero, i.e., correspond to a dead-beat observer. This assures quick attenuation of the disturbance estimation error but, as indicated by Pannocchia (2015), may lead to aggressive changes of the control signals and thus may be sensitive to noise.

We shall eliminate this possible deficiency of the MPE method by its redesign in such a way that the equivalent disturbance part of the observer dynamics can have non-zero stable poles, not only zero poles as in the original formulation. Let us assume that these poles should be equal to $1-\alpha$, $0 < \alpha \leq 1$. To achieve this goal, we redefine the state and output disturbance estimates (23)–(24) used in the prediction equations (12) or (13), as follows:

$$\begin{aligned} d_x(k) = {} & (1-\alpha)d_x(k-1) \\ & + \alpha(\hat{x}(k) - [\mathbf{A}\hat{x}(k-1) + \mathbf{B}u(k-1)]), \end{aligned} \tag{38}$$

$$d_y(k) = (1-\alpha)d_y(k-1) + \alpha\left(y(k) - \mathbf{C}\hat{x}(k)\right), \tag{39}$$

where $y(k)$ is the measured process output. We have the following result.

Proposition 1. *If the disturbance estimates in the MPE method are calculated according to (38)–(39) and the gain matrix of the process observer (6) is* $\mathbf{K}$*, and if the AM method is designed with*

$$\mathbf{B}_d = \mathbf{K},\ \mathbf{C}_d = \mathbf{I} - \mathbf{CK},\ \mathbf{K}_x = \mathbf{K},\ ,\mathbf{K}_d = \alpha\mathbf{I}, \tag{40}$$

then the methods are equivalent,

$$\hat{d}(k) = \mathbf{C}d_x(k) + d_y(k), \tag{41}$$

and closed-loop poles corresponding to the disturbance part of the observer in the AM method are all $1-\alpha$.

To prove this result we shall first calculate poles of the closed loop dynamics of the extended model observer used in the AM metod. We have

$$\begin{aligned}
&\mathbf{A}_a - \mathbf{K}_a\mathbf{C}_a\mathbf{A}_a \\
&= \begin{bmatrix} \mathbf{A} & \mathbf{K} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} - \begin{bmatrix} \mathbf{K} \\ \alpha\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{C} & \mathbf{I} - \mathbf{CK} \end{bmatrix} \begin{bmatrix} \mathbf{A} & \mathbf{K} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{A} - \mathbf{KCA} & \mathbf{0} \\ -\alpha\mathbf{CA} & (1-\alpha)\mathbf{I} \end{bmatrix},
\end{aligned} \tag{42}$$

which shows that the poles of the disturbance part of the closed-loop observer dynamics are all $1-\alpha$. To prove the equivalence of the methods we analyse dynamics equations of the observer (19)–(20) used in the AM method with the matrices as in (40):

$$\begin{aligned}
&\begin{bmatrix} \hat{x}(k) \\ \hat{d}(k) \end{bmatrix} \\
&\quad = \begin{bmatrix} \mathbf{A} & \mathbf{K} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \hat{x}(k-1) \\ \hat{d}(k-1) \end{bmatrix} \\
&\qquad + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u(k-1) + \begin{bmatrix} \mathbf{K} \\ \alpha\mathbf{I} \end{bmatrix} \\
&\qquad \times \Bigg(y(k) - [\mathbf{C}\ \ \mathbf{I}-\mathbf{C}\mathbf{K}] \Bigg(\begin{bmatrix} \mathbf{A} & \mathbf{K} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \hat{x}(k-1) \\ \hat{d}(k-1) \end{bmatrix} \\
&\qquad\quad + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u(k-1) \Bigg) \Bigg). \qquad (43)
\end{aligned}$$

It can be easily seen, by direct calculation, that the equation for the process state estimate $\hat{x}(k)$ in (43) is precisely Eqn. (6) of the process state observer used in the MPE method. Calculation of the equation which describes the dynamics of the disturbance estimate $\hat{d}(k)$ leads to the following result:

$$\begin{aligned}
\hat{d}(k) &= \hat{d}(k-1) \\
&\qquad + \alpha\Big[y(k) - \mathbf{C}\big(\mathbf{A}\hat{x}(k-1) \\
&\qquad + \mathbf{B}u(k-1)\big) - \hat{d}(k-1) \Big] \\
&= (1-\alpha)\hat{d}(k-1) + \alpha\,[y(k) - \mathbf{C}\hat{x}(k)] \\
&\qquad + \alpha\Big[\mathbf{C}\hat{x}(k) - \mathbf{C}\big(\mathbf{A}\hat{x}(k-1) \\
&\qquad + \mathbf{B}u(k-1)\big) \Big]. \qquad (44)
\end{aligned}$$

Anticipating $\hat{d}(k-1)$ consistent with (41), we finally get

$$\begin{aligned}
\hat{d}(k) &= (1-\alpha)d_y(k-1) + \alpha(y(k) - \mathbf{C}\hat{x}(k)) \\
&\qquad + \mathbf{C}\,[(1-\alpha)d_x(k-1) \\
&\qquad + \alpha\,(\hat{x}(k) - \mathbf{A}\hat{x}(k-1) - \mathbf{B}u(k-1))] \\
&= d_y(k) + \mathbf{C}d_x(k), \qquad (45)
\end{aligned}$$

where $d_x(k)$ and $d_y(k)$ are precisely as in (38) and (39) in the MPE method. This completes the proof.

We shall further denote the presented augmented formulation of the MPE method by MPEα, when distinction between the new augmented formulation and the previous one with $\alpha = 1$ is needed.

The MPE method in the improved MPEα formulation seems to have clear advantages over the AM approach, as it does not require arbitrary definitions of matrices $\mathbf{B}_d$ and $\mathbf{C}_d$ defining how the disturbances d enter the state and output equations, and it allows appropriate and separate choice of poles of both the closed loop process observer system and (equivalent) disturbance observer dynamics. Certainly, an equivalent formulation of the AM method according to (40) is possible, but then an increased order process-and-disturbance observer is used.

Analysing the influence of parameter α on the performance of the whole MPC feedback system, it should be pointed out that the value of α does not influence basic properties of the MPC closed loop dynamics (stability, robustness) when attenuating external state, input or output disturbances. The reason is that it influences only the behaviour of disturbance estimates $d_x(k)$ and $d_y(k)$ (or $\hat{d}(k) = d_y(k) + \mathbf{C}d_x(k)$ in the equivalent formulation (41)). This can be clearly seen if we analyse the analytic (explicit unconstrained) formulation of the MPC algorithm (see Tatjewski, 2007). The explicit MPC closed loop state feedback gains matrix can be then formulated. It depends on the process model (matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$), weighting parameters of the cost function (8) (matrices $\mathbf{\Psi}$ and $\mathbf{\Lambda}$)) and on the length of prediction and control horizons, N and N_u, respectively, but it does not depend on $d_x(k)$ and $d_y(k)$. All the enumerated parameters directly influence basic dynamic properties of the MPC feedback loop. In most cases full state is not measured: the feedback is from the state estimate. But in the MPE method the process state observer/filter (not augmented) is used, which acts independently from the disturbance estimation. The value of α influences directly only disturbance estimation dynamics, the speed of construction of the additive disturbance estimates and thus also the speed of disturbance attenuation. Therefore, it should also influence sensitivity to noises, being part of external disturbances. We shall further address this point analysing results of simulations in the next section.

The situation is similar when influence of inaccuracy of model matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ is analysed, in particular, the state matrix $\mathbf{A}$. We can then write the relation between the process matrix $\mathbf{A}_p$ and its model $\mathbf{A}$ as $\mathbf{A}_p = \mathbf{A} + \triangle\mathbf{A}$. Hence $\triangle\mathbf{A}x(k)$ can be treated as an additional state disturbance vector attenuated by the MPE (or other) method, similarly to the differences between other matrices of the process and its model. We shall address also this problem by appropriate design of simulations in the next section. But, certainly, inaccuracies in the process model influence dynamics of the feedback control loop and need appropriate robust control design. This well known general problem is out of the scope of this paper.

We did not manage to achieve a clear relation between the VM method and the MPEα one. However, it seems to be not important from the practical point of view, because the VM method is inferior to both remaining approaches for the reasons discussed in Section 3.3.

5. Example

We simulated MPC control systems with different MIMO processes and their state-space models, including the 2×2

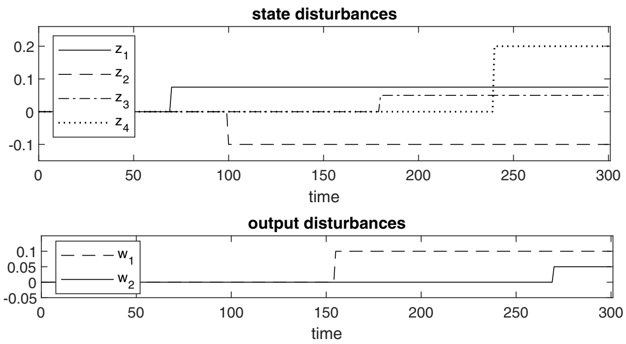

Fig. 1. Scenario of step changes of state and output disturbances over the simulation horizon.

model by Tatjewski (2007) and Pannocchia (2015). All the results were qualitatively similar. In this paper, the following discrete-time model will be considered, which yielded representative simulation results:

$$x(k+1) = \begin{bmatrix} 0.5 & 0.2 & 0 & 0 \\ 0 & 0.7 & 0.1 & 0 \\ 0 & 0 & 0.75 & 0.3 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u(k), \quad (46)$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k). \quad (47)$$

We simulated control of the presented system by the MPC controller with both the AM and the MPEα methods of disturbance attenuation to get offset-free control. The obtained results were identical for equivalent formulations of these methods (according to (40)), validating computationally Proposition 1. The chosen parameters of MPC were as follows: prediction horizon $N = 10$, control horizon $N_u = 5$ and weighting parameters in the cost function (8), i.e., $\mathbf{\Psi} = \mathbf{I}$, $\mathbf{\Lambda} = \lambda\mathbf{I}$, with $\lambda = 0.1$ or $\lambda = 1$. The poles of the process state observer (6)–(7) were chosen at 0.2, 0.21, 0.22, 0.23 and its initial point was [0.1 0.1 0.1 0.1]. All simulations started with the system at the zero equilibrium point. The simulation scenario contained step changes of both set-points and unmeasured state and output disturbances $z(k)$ and $w(k)$, according to the disturbance scenario shown in Fig. 1.

The results of closed-loop control system simulations obtained for MPC with $\lambda = 0.1$ and with the MPEα method of disturbance attenuation, for different values of α, are shown in Table 1 in an integrated way. That is, in terms of values of four control performance indices: ISE—the integral of squared control errors, IAE—the integral of absolute values of control errors, ISCI—the integral of squared increments (moves) of control signals (controller outputs) and IACI—the integral of absolute values of increments of control signals. Analogous results, but for a larger value of $\lambda = 1$, are shown

Table 1. Values of control performance indices for MPC with the MPEα method for different values of α, under step changes of set-points and disturbances, $\lambda = 0.1$.

α	ISE	IAE	ISCI	IACI
1	0.4771	3.600	1.0221	4.8810
0.9	0.4822	3.658	0.8886	4.6326
0.8	0.4882	3.718	0.7747	4.3797
0.7	0.4955	3.779	0.6764	4.3797
0.6	0.5049	3.860	0.5913	3.8932
0.5	0.5175	3.996	0.5176	3.6291

Table 2. Values of control performance indices for MPC with the MPEα method for different values of α, under step changes of set-points and disturbances, $\lambda = 1$.

α	ISE	IAE	ISCI	IACI
1	0.6447	4.9668	0.2135	2.4659
0.9	0.6541	5.0434	0.1916	2.3915
0.8	0.6654	5.1292	0.1722	2.3005
0.7	0.6793	5.2306	0.1545	2.2166
0.6	0.6969	5.3578	0.1380	2.1536
0.5	0.7200	5.5193	0.1226	2.0536

in Table 2. Additionally, process output and input trajectories for $\lambda = 0.1$ and two values of α, $\alpha = 1$ and $\alpha = 0.6$, are shown in Fig. 2.

The presented results are intentionally without noises affecting the states or outputs, to show first the clear impact of the α values on control system performance. The results are fully consistent with the theoretical background. The best control performance, in terms of the ISE or IAE values, is for $\alpha = 1$. This corresponds to zero poles of the disturbance part of the equivalent extended model observer, i.e., to a dead-beat disturbance observer (the poles are $1 - \alpha$; see (42)). But, at the same time, the control inputs are most aggressive, yielding the largest values of the ISCI and IACI indices. Reducing α leads to slightly larger values of the ISE and IAE, but at the same time significantly smaller values of the ISCI and IACI, i.e., more quiet control signals, with smaller deviations between sampling instants.

The choice of zero or close to zero disturbance observer poles can be harmless when states and outputs are noise-free and aggressive moves of control signals are acceptable. But, in practice, the opposite is usually true. For noisy systems, the choice of the Kalman filter and not the Luenberger observer is a common practice. The disturbance part of the (equivalent) augmented system observer should have closed-loop poles not too close to zero, i.e., values of α smaller than one should be chosen in the MPEα disturbance attenuation method. Such simulations were also performed for our example system, to verify the above considerations.

We added to the process states and outputs additional

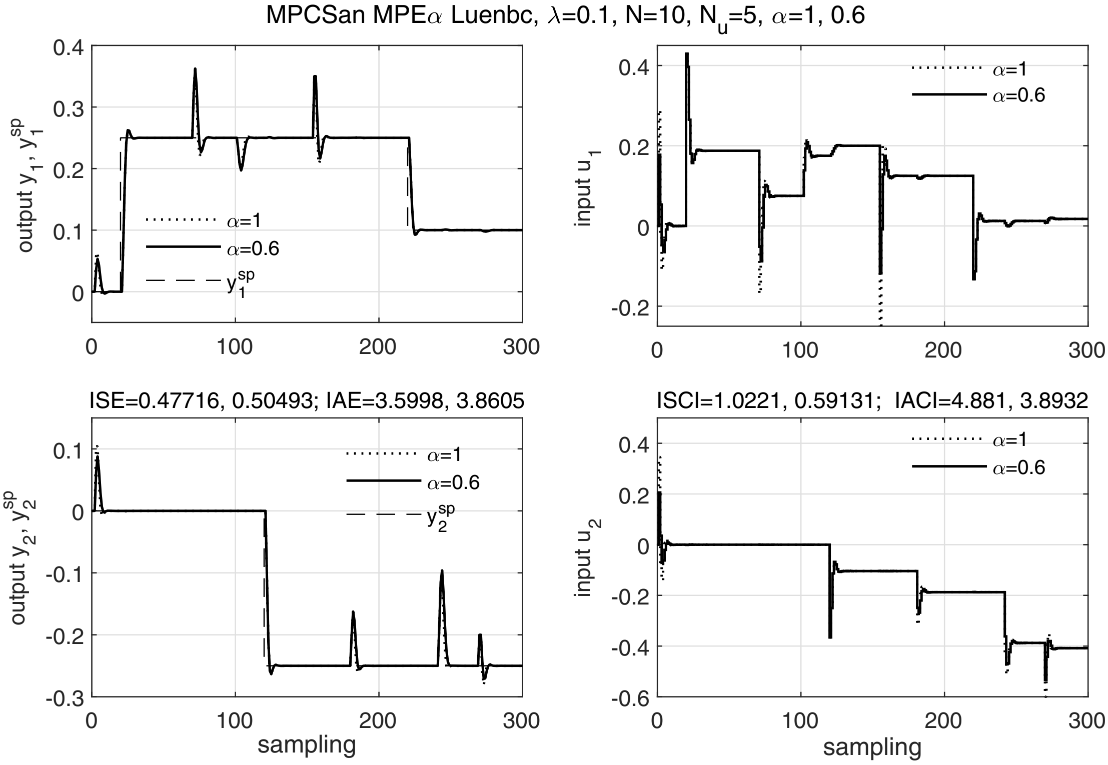

Fig. 2. Process outputs and inputs for MPC-MPEα under step changes of set-points and disturbances, for $\alpha = 1$ (dotted line) and 0.6 (solid line), with a Luenberger current observer (noise-free case).

noises with normal distributions, zero means and standard deviations of 0.01, in addition to the unmeasured piecewise constant disturbances described by the scenario as in Fig. 1. Then the stationary Kalman filter for the process only was designed (using the "idare" Matlab function), with matrices $\mathbf{Q} = 10^{-3}\mathrm{diag}\{0.5\ 0.5\ 0.5\ 0.5\}$ and $\mathbf{R} = 10^{-4}\mathrm{diag}\{0.2\ 0.2\}$, which correspond to appropriately tuned covariance matrices of the noises. The control system with this filter was tested with the MPEα method for different values of α, with $\lambda = 0.1$. We intentionally assumed noises with relatively large amplitudes, to obtain a clear view of the influence of different values of α on the control system performance. The results are presented in Table 3, in an analogous way as in the previous tables. Additionally, the output and input trajectories for two chosen values of α, $\alpha = 1$ and $\alpha = 0.6$, are shown in Figs. 3 and 4, respectively, where also noises added to the corresponding outputs are shown.

The presented results clearly indicate decrease of sensivity to noises with decrease of the α value. It is interesting that the ISE criterion appeared to be not very sensitive, and it even first decreases with the decrease of α; the impact of smaller α is only seen for significantly smaller values and just slightly. On the other hand, the IAE criterion values decrease monotonously, contrary to

Table 3. Values of control performance indices for MPC with the MPEα method and a Kalman filter, for different values of α, with step changes of set-points and disturbances and noises added to states and outputs, $\lambda = 0.1$.

α	ISE	IAE	ISCI	IACI
1	0.6800	12.003	3.8500	35.583
0.9	0.6792	11.917	3.0442	31.411
0.8	0.6795	11.823	2.3966	27.645
0.7	0.6813	11.717	1.8740	24.096
0.6	0.6857	11.597	1.4529	20.807
0.5	0.6944	11.457	1.1163	17.740

the case without noises. Comparing the IAE columns in Tables 1 and 3, it can be clearly seen that the negative effect of slower disturbance dynamics is even reversed when noises are added, due to smaller variation of the parts of disturbance estimates corresponding to noises. Certainly, the smaller the amplitudes of noises, the less visible this effect should be. The control signals performance, given by values of the ISCI and IACI indices, very strongly depends on α, improving significantly with its decrease. This can be clearly seen in Table 3 and is also illustrated in detail when comparing

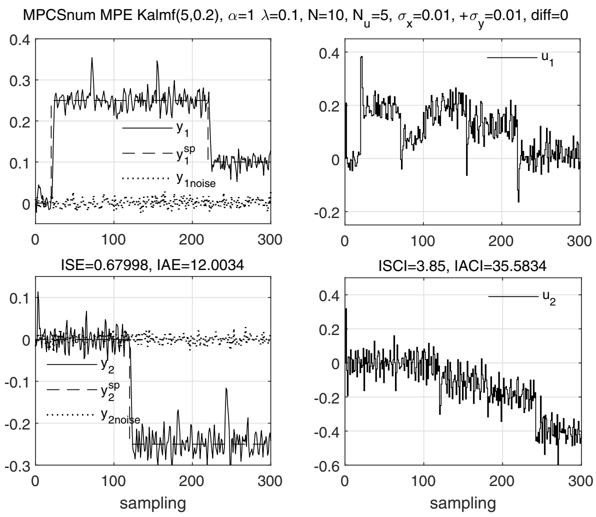

Fig. 3. Process outputs and inputs for MPC-MPEα with step changes of set-points and disturbances and with noises added to states and outputs, $\alpha = 1$.

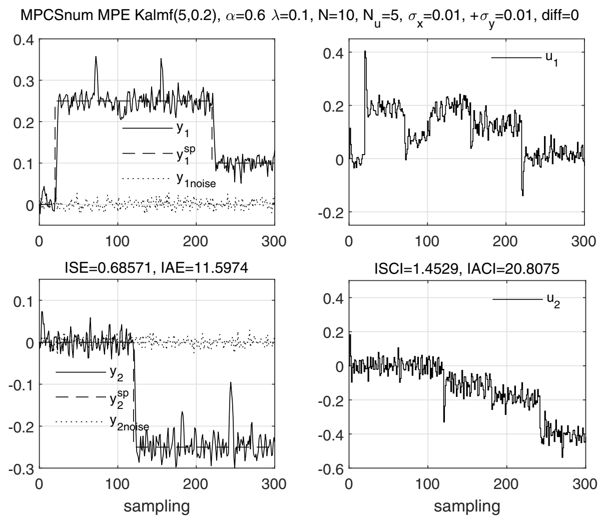

Fig. 4. Process outputs and inputs for MPC-MPEα with step changes of set-points and disturbances and with noises added to states and outputs, $\alpha = 0.6$.

Figs. 3 and 4. Furthermore, sensitivity of the ISCI is higher, which indicates that decrease of α leads to stronger suppressing of larger changes of control signals.

Until now, simulation results were presented without model inaccuracies, i.e., for the same matrices in the process description (1)–(2) and its model (4)–(5), to show clearly the impact of α on control performance when attenuating the disturbances, including noises. But we also performed simulations adding various differences between the process description and its model. Selected, representative results are given in Table 4 for the following state matrix of the process description (1)–(2):

$$\mathbf{A}_p = \begin{bmatrix} 0.55 & 0.18 & 0 & 0 \\ 0 & 0.65 & 0.2 & 0 \\ 0 & 0 & 0.7 & 0.33 \\ 0 & 0 & 0 & 0.55 \end{bmatrix} \tag{48}$$

whereas the matrix $\mathbf{B}_p$ remains unchanged and equal to $\mathbf{B}$ in the model, while keeping the model as given in (46)–(47). Comparing the results given in Tables 3 and 4, we can conclude that they are qualitatively very similar, and even the ISE index behaves better in the latter case. Certainly, this difference can be the opposite for other model inaccuracies, as we encountered performing various simulations. But the qualitative results were always similar.

The presented example simulation results confirm theoretical analysis provided in the paper. Based on these, and others not presented here due to limited space, it can be concluded that the method proposed in the paper (the MPEα extension of the MPE method) can be recommended as a simple, sound and competitive technique.

Table 4. Values of control performance indices for MPC with the MPEα method and a Kalman filter, for different values of α, with step changes of set-points and disturbances and noises added to states and outputs, with model inaccuracies, $\lambda = 0.1$.

α	ISE	IAE	ISCI	IACI
1	0.7172	12.359	4.0179	36.020
0.9	0.7131	12.224	3.1889	31.816
0.8	0.7098	12.086	2.5132	27.937
0.7	0.7081	11.952	1.9625	24.336
0.6	0.7096	11.815	1.5164	20.996
0.5	0.7166	11.696	1.1595	17.915

6. Conclusions

The paper addressed the problem of proper handling of deterministic, asymptotically constant disturbances in the design of offset-free structures of MPC controllers with linear state-space process models. The presentation and comparison of three well established methods was briefly handled, necessary as a background for derivation of the new result. This result is an extension of the MPE method—with model-based prediction errors as state and outputs disturbance estimates and with the process observer only, which is simplest in design and implementation out of the three existing methods. The extension contains new, augmented formulae for state and output disturbance estimates, yet still simple and containing those from the previous version as a special case. Conditions assuring equivalence of the new method with the augmented process-and-disturbance state approach are given in the paper and the equivalence is proved. The proposed new, extended formulation of the MPE method removes the only possible deficiency of the previous formulation: the dynamics of the disturbance observer being of dead-beat type only, which could lead to increased sensitivity to noises. The new method seems to be a competitive, possibly best design solution, the only one not using augmented state models as in the two remaining methods. Theoretical results of the paper were validated and illustrated with results of an extensive simulation study with a multivariable (2×2) sample process model. The simulations show the impact of the tuning of the α parameter on control system performance, including positive impact on noise sensitivity, also in the case with model-reality differences.

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Piotr Tatjewski, MSc 1972, PhD 1976, DSc 1988 in control engineering from the Warsaw University of Technology (WUT). Since 1972 he has been with the Institute of Control and Computation Engineering (ICCE) at the WUT Faculty of Electronics and Information Technology. Since 1993 a professor (since 2006 tenured), 1996–2008: director of the ICCE, 2012–2020: faculty vice-dean for research. Member of the Committee on Automatics and Robotics of the Polish Academy of Sciences (since 2003), 2007–2011: member of the EUCA (European Union Control Association) Administrative Council, 2017–2020: member of the Polish Central Comission for Scientific Degrees and Titles. Author of eight books (three in English), 22 book chapters, 48 journal papers and over 75 published conference papers. Leader of several research projects, including direct cooperation with industry. Main scientific interests: advanced control, linear and nonlinear predictive control, multilayer control and on-line set-point optimization, soft computing techniques in control, hierarchical methods in optimization and control.

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