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Method of evaluation and quality control by means of Y performance curves and RPC curves of Six Sigma metrics of a concurrent system

Tomás Fontalvo[1], Ana Banquez[1], Andrea Fontalvo[2]

[1]University of Cartagena, Avenida del Consulado #Calle 30 No. 48 152 Cartagena, Colombia; tfontalvoh@unicartagena.edu.co (TF); abanquezm@unicartagena.edu.co (AB)
[2]Polytechnic University of Madrid, Calle Ramiro De Maeztu, 7, 28040, Madrid, España; a.fontalvo@alumnos.upm.es
*Correspondence: abanquezm@unicartagena.edu.co

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Abstract
The objective of this research was to evaluate and monitor a concurrent production system by means of Y performance curves and RPC curves. As a theoretical conception it articulated the concepts of concurrent production system, Six Sigma metrics, Y performance curves and RPC curves of Six Sigma metrics. The research was of basic type supported a rational, quantitative, propositive analysis structured in mathematics, statistics, multivariable statistical control, curves and operation of Y performance and of average performance of RPC run of Six Sigma metrics. As a result, Y-performance curves and RPC Average Run Performance curves are proposed to evaluate and control the concurrent system. Additionally, it was determined that the best performing process is 1, with a sigma level ranging between 4.15 <Z<5.28, which allows an increase in Y performance of 99.61% and 99.99%. As future research it is invited to replicate the proposed method in other business contexts integrating other multivariable statistical control tools and Six Sigma.

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1. Introduction

Six Sigma metrics are one of the most widely used statistical quality control tools currently used to improve the performance of companies, using real process information to eliminate process problems (Caballero et al., 2023). It should be noted that the application of Six Sigma metrics has allowed to improve the evaluation of different sectors associated to efficiency, quality, performance, among other criteria that arise as a prerequisite at the moment of analyzing any organizational context (Ndrecaj et al., 2023).

In that order of ideas, as the use of Six Sigma metrics improves and develops the quality and performance of products or services, companies manage to satisfy customers, achieve goals and maintain a competitive position in the market (Pierce et al., 2023). It is pertinent to mention that these metrics have been complemented by other statistical control tools such as multivariable capability indicators, used in the articles by Khadse and Shinde (2021), Joshi and Patil (2022) and Goharshenasan et al. (2022) to evaluate different production systems, achieving a significant increase in the sigma level, but still with aspects to improve. On the other hand, studies such as Fontalvo and Banquez (2023) implemented the operation curves of Six Sigma metrics to carry out a comparative evaluation of yields in serial and parallel production systems. This research quantified that, although the process results are similar, serial production systems exhibit a slightly better performance than parallel systems, evidenced in their multivariable capability indicators (1.078960271 vs. 1.074608998).

It is important to highlight that other researchers have analyzed the use of Six Sigma metrics and their operating curves in various production systems, including serial, parallel, multiline, and concurrent parallel systems. These studies have demonstrated the relevance of the operating curves of Six Sigma metrics as tools to establish criteria for quality evaluation and control. Such criteria facilitate the identification of variable operating conditions and the analysis of a system's capability to achieve the quality parameters necessary to meet customer expectations. This contributes to informed decision-making based on process monitoring and control (Fontalvo et al., 2024a; Fontalvo et al., In press).

Skalli et al. (2024), in their article, present a framework that integrates Lean Six Sigma and Industry 4.0, with the purpose of optimizing operational excellence. The results obtained include a reduction in defects, a 20% increase in process efficiency, an improvement in equipment effectiveness to 85%, and a 30% decrease in unplanned downtime, thus achieving significant improvements in productivity and reduction of operating costs in the sectors evaluated.

On the other hand, Cruz et al. (2024) evaluated the impact of the Six Sigma methodology in a production system, achieving outstanding results. These include a 24.7% increase in productivity, an increase in Sigma level from 2.9 to 3.8, an improvement in process capability from 0.60 to 1.16, and a significant reduction in defects per million opportunities, reflecting the effectiveness of the methodology in production environments.

Also, Faishal et al. (2024) focused on analyzing and reducing defects in production using the Lean Six Sigma (DMAIC) methodology. Initial results reflected a Sigma level of 3.19 and defects equivalent to 10.2% of total production. After the implementation of improvements, such as the 5S method and new SOPs, defects were reduced by 50%, achieving a Sigma level higher than 4, which represents a significant improvement in the quality and efficiency of the production process. Similarly, other authors have shown that, through the use of Six Sigma, it is possible to evaluate a production system with the aim of reducing variability and consequently improving system compliance under specific conditions (Ezewu et al., 2024). Therefore, it is necessary for companies to monitor their performance variability using appropriate measures and implement continuous improvement strategies to support relevant decision-making (Langle et al., 2024).

Likewise, other studies have explored Six Sigma metrics and the operating curves generated from them to evaluate the performance of production systems, as illustrated in Figure 1. In this line of analysis, Garg and Agrawal (2023) emphasize that the operating curves of Six Sigma metrics reflect the relationship between process quality, its variability, and the minimization of defects in the delivery of products or services to customers. This approach facilitates informed decision-making regarding potential adjustments and improvements.

Additionally, various authors have pointed out how the integration of diverse approaches has significantly advanced the practical applications of Six Sigma for statistical quality control (Escobar et al., 2024). For instance, Fontalvo et al. (2024c) conducted an analysis of multiple multivariate capability indicators based on Six Sigma metrics, addressing quality dimensions both specifically and holistically.

The literature review reveals that although studies exist on the use of Six Sigma metrics, multivariate capability indicators, and operating curves applied in industrial and service companies, no research has been identified that analyzes the average run-length operating curves of Six Sigma metrics in concurrent production systems. This gap limits the identification of a process's sensitivity to changes in the sigma level (Z), an aspect addressed in this study. Similarly, within the scientific domain, there are no models associated with statistical quality control systems that allow for an integral and holistic evaluation of process performance and control through operating curves (Y) and average run-length performance (RPC) of Six Sigma metrics. Moreover, it is essential that these proposals be structured from an integral conception of the analyzed system, enabling a detailed balance of defects to calculate Six Sigma metrics and propose the corresponding operating curves. This underscores the originality of this research by presenting an innovative and structured method that can be replicated in various business, scientific, and academic contexts.

The purpose and intention are to establish an integral statistical quality control system for a concurrent production system that allows for the evaluation (Z vs. Y curves) and control (Z vs. RPC(n) curve) of quality. Based on the issues discussed, the following research questions arise for this study: How can an integral statistical control system be established to evaluate and control quality? How can a system performance evaluation for concurrent systems be established using Six Sigma metric operating curves for performance (Y)? How can a statistical control system be proposed using RPC curves of Six Sigma metrics? To address these research questions, the following objectives are defined: i) to establish an integral statistical quality control system for quality evaluation and control, ii) to establish a concurrent system performance evaluation system using Six Sigma metric operating curves for performance (Y), and iii) to propose a statistical control system using RPC curves of Six Sigma metrics.

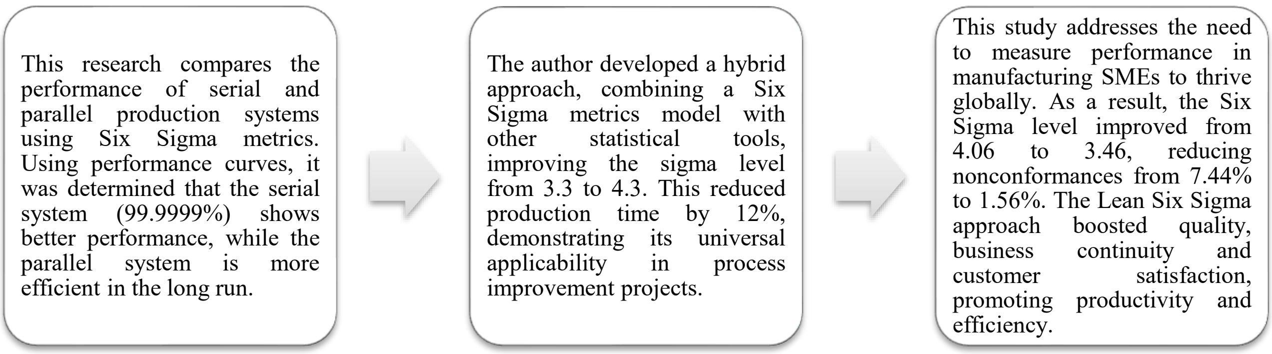


Fig 1. Research related to the methodology proposed in this study (Benavidez et al,2023;Krishnan et al, 2022;Dhingra et al,2022).

2. Theoretical framework

2.1. Concurrent production system

The concept of concurrency originates with concurrent engineering, designed to mitigate and control the complexities associated with the development and implementation of new products (Gan et al., 2022). This engineering modality proposes a continuous link between phases, orchestrated by a multidisciplinary team working on the evolution of the product in an interactive manner, from the beginning to the culmination of the process (Goli & Alinaghian, 2020). However, successful implementation of this methodology can be challenging, and even unfeasible, without the presence of a product development manager who possesses the influence and backing of top management, enabling him or her to make decisions that effectively impact the organization's functions (Zhang et al., 2023).

In this sense, the concurrent production approach, by requiring greater articulation in data processing, model generation and multidisciplinary coordination from the initial design phases to the final implementation of the product, proves to be an essential driver in the optimization of processes and resources in the industry (Gai et al., 2022). The unification of languages and tools across disciplines strengthens coherence and efficiency in product development, ensuring a more effective and competitive outcome in the marketplace. Likewise, this strategy addresses the incompatibilities that emerge in the different phases of the system by thoroughly studying each stage of analysis, definition and/or design specification, promoting a collaborative and preventive

approach to avoid setbacks in the detailed design phases (Tang et al., 2023). It is imperative to accompany This process with systematic and collaborative tools that enhance creativity and analysis, ensuring the optimal execution of each phase until the culmination of the final process.

2.2. Six Sigma Metrics

The paradigm of concurrent production, in its eagerness to optimize efficiency and quality in organizational processes, finds an ally in statistical control tools, among which the Six Sigma metrics stand out. These metrics have proven to be an invaluable resource to evaluate the performance of concurrent productive systems, covering vital aspects such as efficiency, quality and performance, crucial elements for the exhaustive analysis of the organizational situation (Sallam & Snygg, 2023). The Six Sigma philosophy, oriented to the achievement of optimal performance levels, has been consolidated as a widely adopted approach in several organizations, thanks to its systematic and evidence-based approach, which is based on exhaustive analysis of inputs and procedures (Shokry et al., 2023). This continuous improvement methodology has transcended the manufacturing sphere, finding applicability and effectiveness in service environments with remarkable results (Badawi et al., 2023).

Six Sigma, as a process improvement method, channels its focus toward reducing the variability inherent in processes (Buestan & Perez, 2022). This technique adopts a defect measurement approach, where defect percentages are quantified and evaluated, converting them to defect rates per million, which are then translated into specific sigma metrics using a referential table system (Najm et al., 2022). The importance of these metrics lies in their ability to accurately quantify the quality and efficiency of processes, allowing the identification of deviations, areas for improvement and optimization opportunities in concurrent production (Patel & Desai, 2023). It should be noted that the effective adoption of Six Sigma demands an organizational commitment to a culture of quality and excellence, where training, committed leadership and the integration of methodologies that promote efficiency and quality are encouraged (Sodhi, 2023).

2.3. Y-performance curves

Y-performance curves are fundamental tools in the comprehensive evaluation and continuous improvement of production systems. These curves visually represent the relationship between multiple operating variables and system performance (Fontalvo & Banquez, 2023). They are often used to show the relationship between capacity, quality, efficiency, time and other key factors that influence the production process (Benavides et al., 2023). These curves allow analysts and decision makers to gain a deeper understanding of how the system behaves under different operating conditions. The graphical representation facilitates the identification of patterns, trends and anomalies in system performance, which is essential for detecting areas for improvement and optimization (Lin et al., 2022).

In that order of ideas, the construction of Y performance curves involves the analysis of data collected from the different stages of the production process. These data are visualized in graphs that show, for example, the relationship between product quality and production speed or the relationship between efficiency and resources used (Fontalvo & Banquez, 2023). This visual representation provides a clearer and more detailed understanding of how changes in certain parameters affect overall system performance. By allowing a comprehensive evaluation of multiple variables simultaneously, Y-performance curves become a crucial tool for identifying critical points in the production chain, allowing informed decisions to be made and specific improvement strategies to be developed (Verheyen & Stoks, 2023).

Now, the usefulness of Y-performance curves extends to different industries and types of production systems. From manufacturing to services, these curves provide a complete and detailed view of system performance (Menacé et al., 2023). For example, in manufacturing, these curves can help identify the relationship between production speed and product quality, allowing efficiency to be optimized without compromising quality (Lin et al., 2022). In services, these curves can be used to assess the relationship between response time and customer satisfaction, enabling continuous improvement in user experience. Ultimately, Y-performance curves provide a powerful visual representation that facilitates data-driven, continuous improvement-oriented decision making in diverse and complex production systems.

Specifically, it can be specified and pointed out that the performance operation curve of Six Sigma metrics Z vs Y, provides as value the level of quality performance Z in function of the yield, understood as the proportion of good product that the global system and each one of its processes yields, which allows to identify the real capacities of each process and system to reach a quality level according to the operation conditions of the productive system analyzed in changing conditions of Z according to the particularities of each system and its operation variables (Figure 6).

2.4. Operation curves and operation curve performance of Six Sigma metrics RPC run

The operation curves of the Six Sigma metrics allow analyzing the performance of a statistical control system valued by the defects in parts per million DPMO and the Y yield when analyzing the variability of the sigma level in variable conditions. Six Sigma metrics operating curves are useful for monitoring quality control processes, determining the number of samples needed to detect a significant change in the process. It allows analyzing how the Six Sigma performance metrics DPMO and Y change as a function of the variability of the sigma level of Z (Fontalvo et al., 2024a). The authors in their study Evaluation with operation curves of Six Sigma metrics the performance of production systems analyze the behavior of a productive system through a sensitivity analysis modifying the performance level Z (Sigma level of the Six Sigma metrics) and analyzing its effect on the defects in parts per million and the performance Y of the Six Sigma metrics, and they study the performance in different operation curves in the system under study. These curves reflect the capacity of the system to produce good goods or services once the variability measured through the Z metric or sigma level of the Six Sigma metrics is increased (Fontalvo et al., 2024b).

This tool becomes a crucial parameter to compare and evaluate the capacity of a system to produce goods or services, considering the installed operating conditions of the control system according to the variability of the Z level of the Six Sigma metrics (Fontalvo et al., 2023). In other researches, operation curves are used to evaluate productive systems and services with Six Sigma metrics. In the research of the authors Fontalvo et al. (2024a), they show that when increasing the Sigma level in the system, the defects per million opportunities increase. In other researches, Fontalvo et al. (2024b) and Fontalvo et al. (2023), analyze the performance of operations system where they show the variable performance of DPMO and the Y performance as a function of the variability of the signa Z level of the Six Sigma metrics according to the operating conditions of the studied systems, with which it is possible to analyze the quality performances in variable conditions, which allows the decision making for the improvement of a system.

Consistent with the concept of operation curve in this research, we work the Average Run Performance with $RPC = \frac{1}{1-Yi}$, for the mathematical model of the Six Sigma metrics, this is a curve that allows to establish control criteria for once Z changes, we can analyze which is its effect on the number of products or services n to be produced to detect that the level of Z (Sigma level of the Six Sigma metrics) changed. The Average Run Performance curves allow to analyze how many samples must be produced or services rendered to detect that Z changed from Z1 to Z2. The Mathematical model of the RPC is given by:

$$\mathrm{Y_i} = \text{Yield of the production process (\% good product)}$$

$$1 - \mathrm{Y_i} = \text{\% Defective products}$$

$$\mathrm{RPC} = \frac{1}{1 - \mathrm{Yi}} = \text{Number of units to be produced to achieve Yi and Zi yields}$$

$$\mathrm{RPCmayor} = \frac{1}{1 - \mathrm{Ymayor}} = \text{Number of units produced to show that Z reached Z higher and Y higher.}$$

$$\mathrm{RPCmenor} = \frac{1}{1 - \mathrm{Ymenor}} = \text{Number of units produced to evidence that Z reached Zminor and Yminor}$$

$$\mathrm{RPCmayor} - \mathrm{RPCmenor} = \text{Units that } \frac{1}{1-\mathrm{Ymayor}} \text{ and } \frac{1}{1 - \mathrm{Ymenor}} \text{ produced to detect that Z change.}$$

Once the RPC is calculated, under variable conditions of Z (with a sensitivity analysis of Z), its effect on Y, and therefore on RPC= n, can be seen. These variables calculated in variable conditions lead us to the construction of the Six Sigma metrics operation curve of Z Vs (RPC= n), and represents the concept of RPC operation curve, as shown in Figure 2. Therefore, the Six Sigma metrics operation curve Z Vs RPC= n shows the number of products or services to produce, to detect that the level of z changes as stated in the mathematical model presented, as shown in Figure 2.

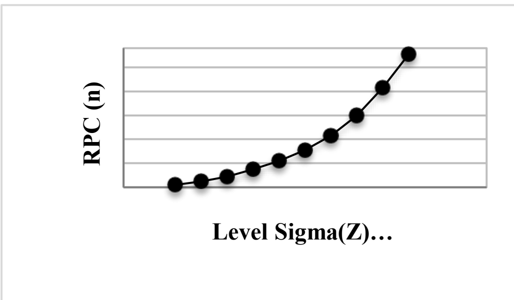


Fig 2. RPC curves for Six Sigma metrics

The above allows analyzing the performance of a standardized production or service delivery system and specifying its operating conditions according to the units produced, the processes considered, the units required to be produced in order to detect that the sigma level Z changes, considering an integral statistical control system. This allows informed decision making on the variables of the system to achieve quality improvement.

3. Methodology

This research was developed from a basic type of study, supported by mathematics, statistics, multivariate statistical control, Y performance operation curves and average performance curves of RPC run of Six Sigma metrics. This implied establishing a mathematical model from a rational propositive approach, which integrated the proposed tools and the variables of the system, in order to develop a simulation and sensitivity analysis, starting from the performance level Z of the operating conditions of the concurrent production system, which under the operational conditions and the variables of the system under study showed a global quality performance that ranged between 3 and 4.7. With the proposed mathematical model varying Z, the critical units and defects were generated and the performances of the processes under investigation were calculated, with the response variables of the sensitivity analysis and the simulation. The n (Defects), Y yield, and DPMO per process were generated, which allowed i) determining the overall DPMO, ii) calculating the overall and sub-process defects for each level Z of the concurrent system, iii) an overall balance of defective units of the processes and the system, iv) propose the Six Sigma metrics Y-performance operation curves, v) establish the Six Sigma metrics RPC run average performance operation curves to control the system, and v) analyze the findings and results of the proposed method. This is summarized in Figure 3.

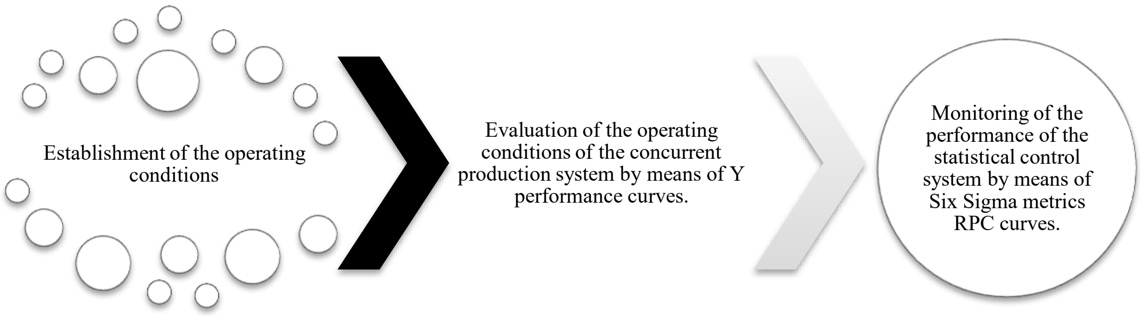


Fig 3. Method to evaluate the quality of the concurrent production system

To carry out the analysis of the performance of the concurrent production system, the Six Sigma metrics presented in Table 1 were used, which provide a consistent way to measure and compare different processes.

Table 1. Six Sigma metrics used

Metrics	Definition
DPMO	Number of defects per million opportunities
U	Number of critical units reviewed
O	Opportunity of error per unit
Z	Quality performance level of systems and processes
N	Number of nonconformities or failures present in the process
Y	Process performance

On the other hand, Figure 3 presents the method for evaluating the quality of the system studied in this research.

The analyzed system is built by nine processes, which are calculated the quality performance level of the systems and processes (Z) and generate n defective units for all processes, respectively. In addition, different initial quantities of units (U) are processed in each of the processes of the system. In this concurrent system, the initial quantity entering process 1 is divided and enters processes 2 (20%), 3 (40%) and 6 (40%), and the outputs of processes 2, 5 and 8 enter process 9 as shown in Figure 5.

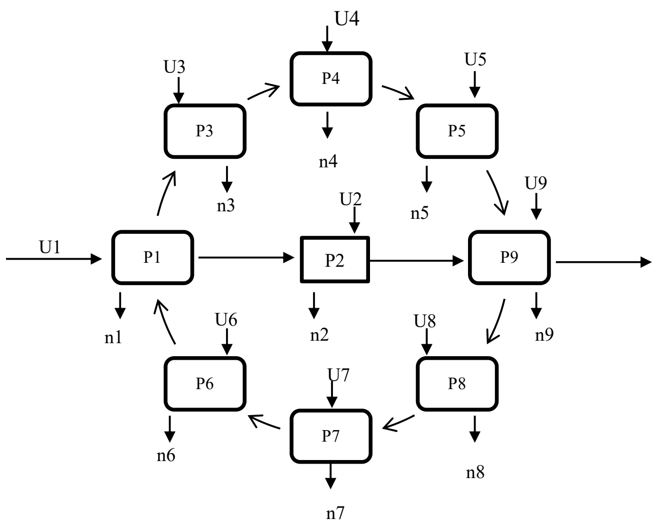


Fig 4. Diagram of the concurrent production system

To evaluate the system and calculate the global and sub-process defects, we start from the sigma level Z = 3, considering that this is the acceptable level at which a production system should operate according to Table 2.

Due to the characteristics of the model used for the research, it is necessary to calculate the number of defects generated in the system in each process using the value of the sigma level in Formula 1:

$$DMPOg = e^{\frac{29.3-(Zg-0,8406)^2}{2,221}} \quad (1)$$

After that, we proceed to find the value of the global defects of the system ng by clearing Formula 2:

$$DMPOg = \frac{n}{u * o} * 1.000.000 \quad (2)$$

This results in Formula 3:

$$n = \frac{u * o(DMPO)}{1.000.000} \quad (3)$$

After calculating DPMO and applying the result in the formula of ng, the total amount of defective products generated in the system globally was obtained. From this value and taking into account that ng is also the sum of all defects in the system for each process. We proceeded to calculate a global balance of defects that considers the defects of each subprocess of the system and Formula 4 was cleared as a function of n9:

$$ng = \frac{1}{4}n9 + \frac{1}{3}n9 + \frac{1}{2}n9 + \frac{1}{3}n9 + 2n9 + \frac{1}{2}n9 + \frac{1}{2}n9 + 2n9 + n9$$

$$ng = \frac{1}{4}n9 + \frac{3}{3}n9 + \frac{2}{2}n9 + 5n9$$

$$ng = \frac{1}{4}n9 + n9 + n9 + 5n9$$

$$ng = \frac{1}{4}n9 + 7n9$$

$$ng = 7.25n9$$

$$n9 = \frac{ng}{7.25} \quad (4)$$

Subsequently, n9 is replaced in each defect formula n for each of the processes as shown in Formula 5:

$$n1 = \frac{1}{4}n9 \quad (5)$$

After determining the number of defective units generated in each process, we proceed to obtain the number of units entering each process with the following formulas:

For the case of processes 2, 3 and 6 Formula 6 is used:

$$E = (u - n) * \% \quad (6)$$

Consequently, Formula 7 is used for process 9:

$$E9 = (u2 - n2) + (u5 - n5) + (u8 + n8) \quad (7)$$

Formula 8 is used for the rest of the processes, as shown in the example of process 4:

$$E4 = u3 - n3 \quad (8)$$

These inputs help us to determine the overall number of critical units for each process with Formula 9, as an example we will use the formula for process 4:

$$Ug4 = E4 + u4 \quad (9)$$

Once we have all the values of n and U, we proceed to find the sigma level of each process with Formula 10, as well as its yield with Formula 11:

$$Z = \sqrt{29.3 - 2.221 * ln(DPMO)} + 0.8406 \quad (10)$$

$$y = 1 - \frac{n}{u - o} \quad (11)$$

This mathematical model was used in the sensitivity analysis for the Z performance range for this global system, which oscillates between 3 and 4.7 according to the system variables studied and the system operating conditions. This allowed the construction of the operation curves shown in Figure 6 and 7. Then we proceed to determine the number of samples that must be produced so that the sigma level Z presents a change from Z1 to Z2 with Formula 12:

$$ARL = \frac{1}{1 - Yi} \quad (12)$$

Where,
Y= Percentage of good product
1 -Yi = Percentage of defective product

The whole process mentioned above was repeated with different sigma levels to compare the performance of each of the processes and the overall system. To evaluate the performance of the system and the processes, a sensitivity analysis was performed on the Six Sigma scale, in the established range of 3.0 to 4.7 according to the structured model. In this order of ideas, to determine the performance of each sigma level according to the result obtained, Table 2 was taken into account.

Table 2. Performance criteria for the concurrent system

Level sigma (Z)	Performance
$\boldsymbol{Z} < 3.0$	Deficient
$3.0 \leq \boldsymbol{Z} \leq 3.5$	Acceptable
$3.5 < \boldsymbol{Z} \leq 4.6$	Good
$\boldsymbol{Z} > 4.6$	Excellent

4. Results

This research delved into the application of a quantitative and analytical approach to address a comprehensive analysis of the concurrent production system, using a method that is based on the balance of inputs, defects and outputs and structured several key stages. These stages include: i) the establishment of operating conditions, ii) the thorough evaluation of operating conditions through Y-performance curves, and iii) the rigorous monitoring of system performance through statistical control, using RPC curves that align with Six Sigma metrics. These phases become the structural basis on which the detailed analysis of this study is developed, starting from Figure 5.

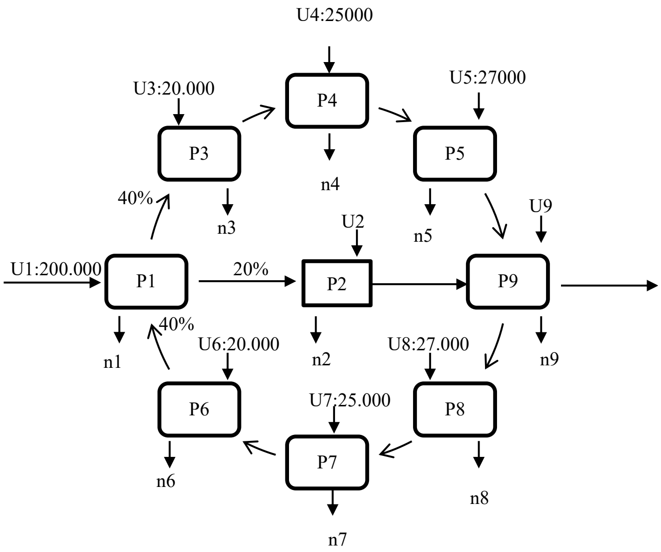
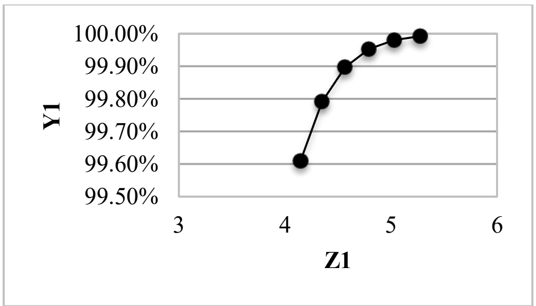
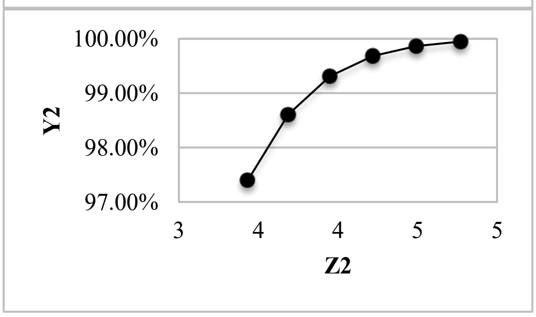


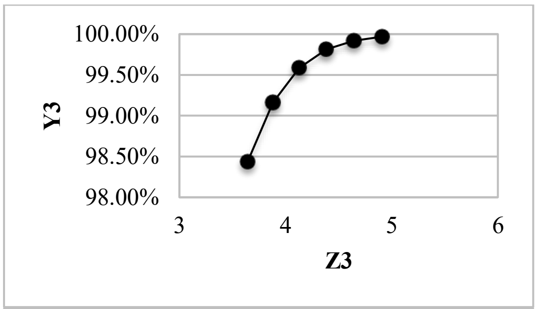
Fig 5. Diagram of the concurrent production system with values.

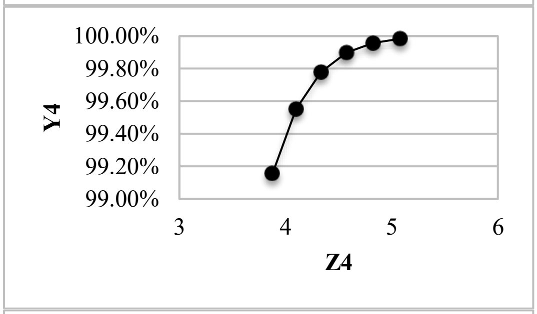
4.1. Performance evaluation curves and Six Sigma metrics

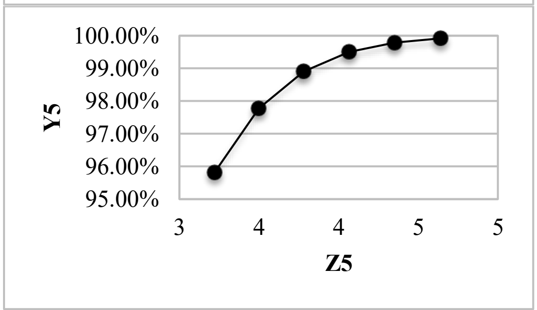
Figure 6 shows the Z vs Y operating curves, which allow evaluating and analyzing the individual performance of each process under the operating conditions of the concurrent system. These curves facilitate the identification of the processes with the best performance levels (Y) and those that present opportunities for improvement. This information is necessary to guide decision making focused on quality improvement in an efficient manner and aligned with the needs of the system.

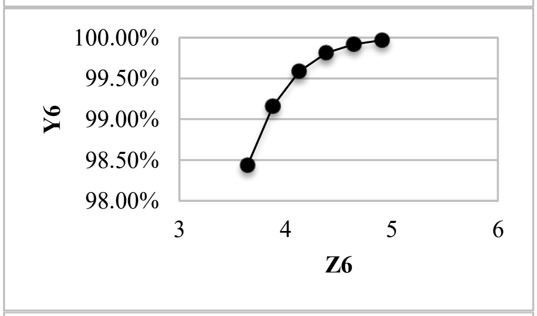


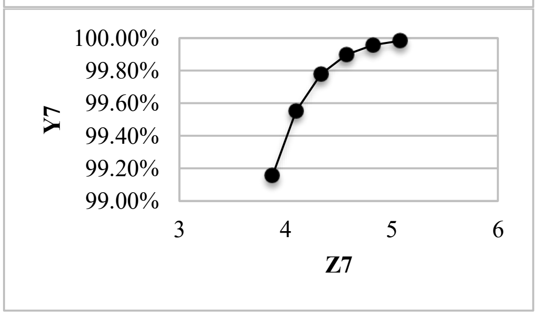












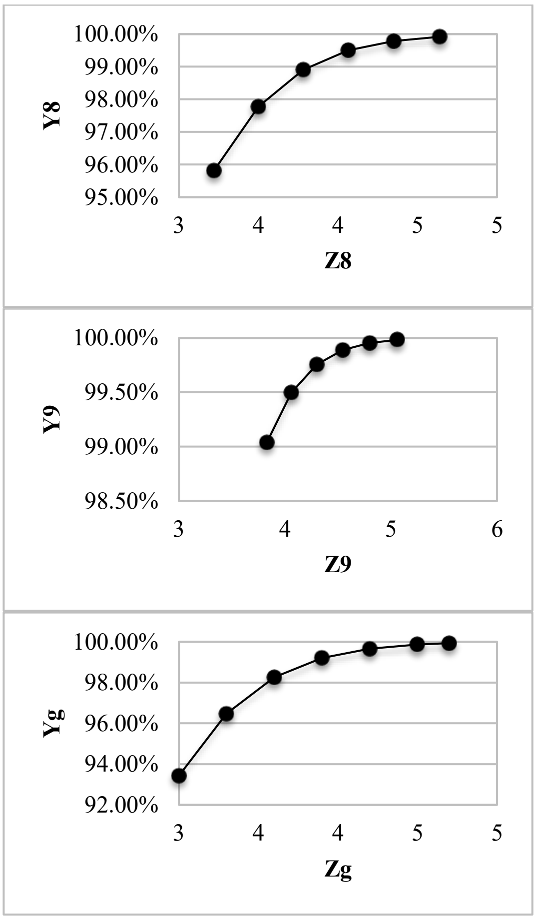


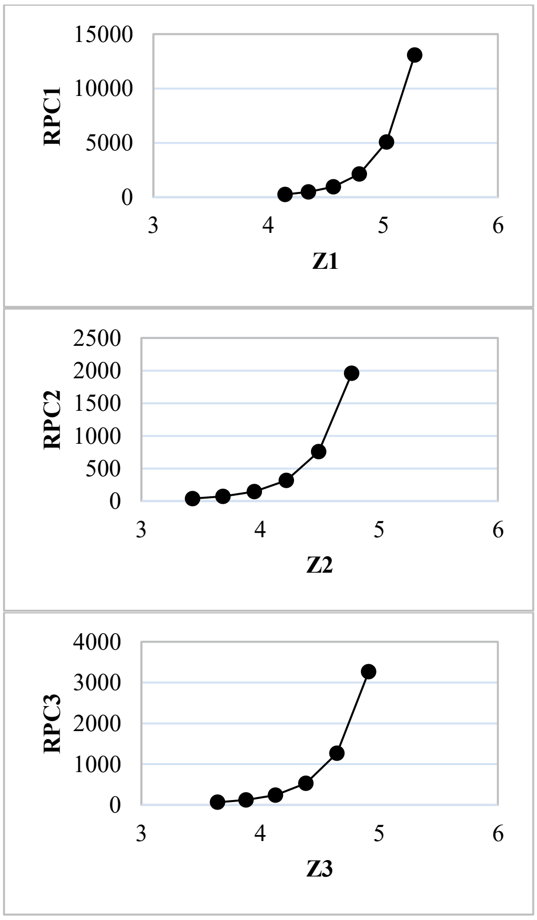
Fig 6. Z Vs Y operating curves of Six Sigma metrics

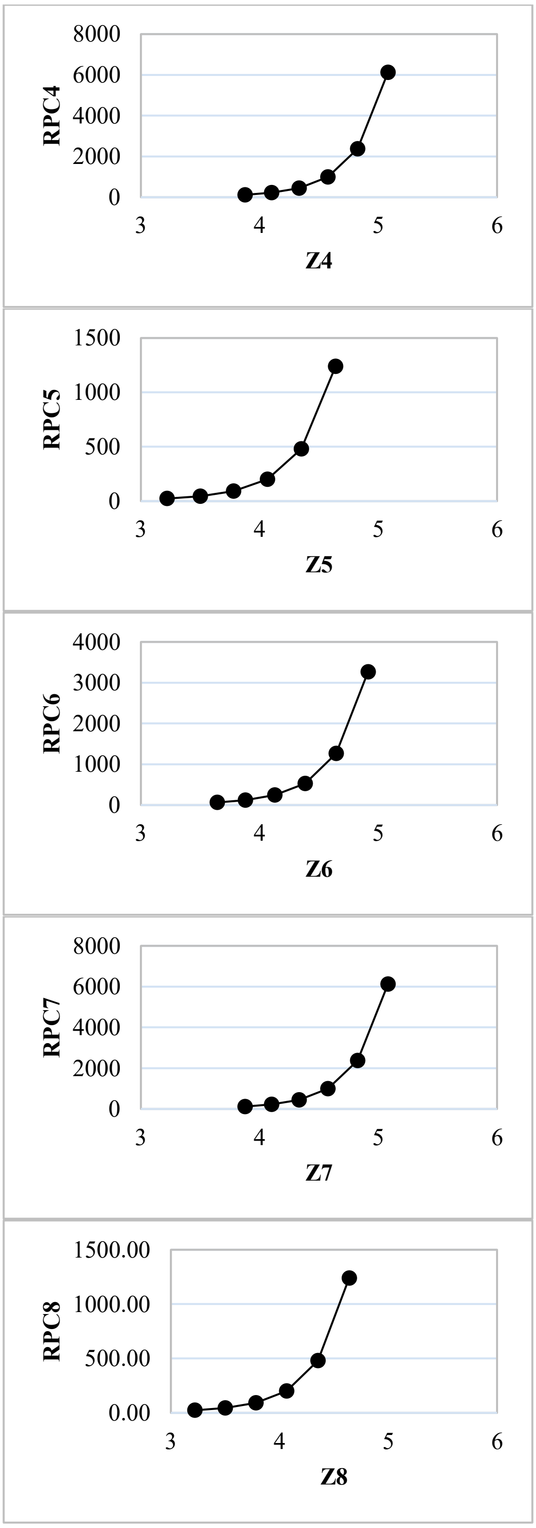
As can be seen in the figures above, there is a direct correlation between the increase in the sigma Z level in the processes and the consequent increase in yield. In process 1, an increase in the sigma Z level from 4.15 to 5.28 results in a notable increase in yield, ranging from 99.61% to 99.99%. Similarly, the increase in the sigma Z level in process 2, from 3.43 to 4.77, results in a noticeable increase in yield ranging from 97.39% to 99.95%. In process 3, the change in Z, rising from 3.64 to 4.91, allows an increase in yield from 98.44% to 99.97%. In process 4, increasing the Z sigma level from 3.88 to 5.08 generates an appreciable increase in yield, ranging from 99.16% to 99.98%. Similarly, increasing the sigma Z level in process 5 from 3.22 to 4.64 results in an outstanding increase in yield, ranging from 95.82% to 99.92%. In process 6, increasing the sigma Z level from 3.64 to 4.91 generates a significant increase in yield, ranging from 98.44% to 99.97%. Similarly, the increase in sigma level Z in process 7, from 3.88 to 5.08, results in a noticeable increase in yield, ranging from 99.16% to 99.98%. The change in Z in process 8, from 3.22 to 4.64, allows an increase in yield from 95.82% to 99.92%. Similarly, in process 9, increasing the sigma Z level from 3.83 to 5.06 generates a noticeable increase in yield, ranging from 99.04% to 99.98%. In addition, the increase in the sigma Z level in the overall system, from 3 to 4.7, translates into an appreciable increase in performance, ranging from 93.43% to 99.93%.

Of the concurrent system processes analyzed, it is evident that process 1 presents the best performance range to generate quality products according to the operating conditions of the system, showing a sigma Z level that ranges from 4.15 to 5.28. On the other hand, process 8 is the process that presents more opportunities for improvement in order to offer quality to the company, considering that the performance range of the sigma Z level oscillates between 3.22 to 4.64, that is to say, it shows the lowest performance with respect to the other processes under study. In practical terms, this assessment by means of the Y performance operating curves makes it possible to identify priorities that allow those responsible to prioritize improvement actions in the context where applicable.

4.2. RPC monitoring curves for the statistical control system

In Figure 7, the operation curves of average run performance Z Vs RPC (Units to be produced to reach the Z level) are provided and integrated, which facilitate the analysis for each process of the relationship between the quality performance level Z and the units that must be produced to detect that the Z level changes to Zi. With these curves the company has control criteria to identify what should be the production level to detect changes in each process of the analyzed system for a change in the specified Z level.





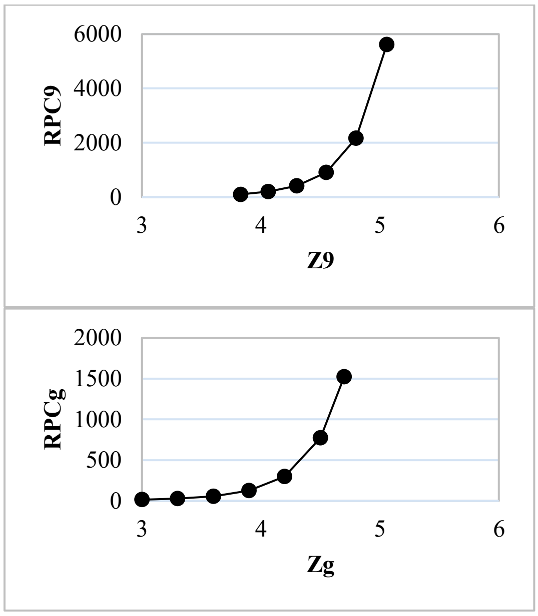


Fig 7. Run Z average performance operating curves Vs RPC of Six Sigma metrics

Based on Figure 7, it can be inferred that in process 1, 13064 units must be produced to generate a significant change in the sigma Z level from 4.15 to 5.28. For process 2, 1959 units must be produced to detect a noticeable change in Z from 3.43 to 4.77. For process 3, 3266 units are needed to identify a relevant change in the Z sigma level increasing from 3.64 to 4.91. For process 4, 6122 units are required to experience a noticeable change in Z from 3.88 to 5.08. For process 5, 1241 units are required to observe a considerable change in the sigma Z level from 3.22 to 4.64. In process 6, 3266 units are required to cause a noticeable change in Z from 3.64 to 4.91. In process 7, 1240 units are required for the process to show a noticeable change in the Z sigma level from 3.22 to 4.64. Regarding process 9, 5611 units are required to detect a noticeable change in Z from 3.83 to 5.06. In the overall system, 1525 units must be produced to observe a change in Z from 3 to 4.7.

From the results of the Z Vs RPC curves, it can be analyzed which processes have a higher level of sensitivity to changes in the Z level, which allows in business terms to determine which process of the proposed statistical control system allows to control and detect faster a change in terms of variability in the statistical control system, once the Z variability of this changes. Also, from the results it can be observed that the most sensitive process to detect a change in the Z measurement is process 5 with 1241 units to be produced to detect the change in its operating range. In the case of process 4, 6122 units are required to reach its operating range of Z that ranges between 3.88 and 5.08, which identifies it as the process with the lowest level of detection to survive that Z change in its operating range.

Table 3. Lower and higher yields with their variations.

Criteria	Z1	Y1	Z2	Y2	Z3	Y3	Z4	Y4	Z5	Y5
Minimum	4.15	99.61%	3.43	97.39%	3.64	98.44%	3.88	99.16%	3.22	95.82%
Maximum	5.28	99.99%	4.77	99.95%	4.91	99.97%	5.08	99.98%	4.64	99.92%
Variation	*1.13*	*0.38%*	*1.34*	*2.56%*	*1.27*	*1.53%*	*1.2*	*0.82%*	*1.42*	*4.10%*
Criteria	**Z6**	**Y6**	**Z7**	**Y7**	**Z8**	**Y8**	**Z9**	**Y9**	**Zg**	**Yg**
Mínimos	3.64	98.44%	3.88	99.16%	3.22	95.82%	3.83	99.04%	3	93.43%
Máximo	4.91	99.97%	5.08	99.98%	4.64	99.92%	5.06	99.98%	4.7	99.93%
Variation	*1.27*	*1.53%*	*1.2*	*0.82%*	*1.42*	*4.10%*	*1.23*	*0.94%*	*1.7*	*6.50%*

From Table 3 we can point out as the main finding that the optimal processes, identified as 1, 4, 7 and 9, with sigma levels of 5.28, 5.08, 5.08 and 5.06, respectively. These processes showed significant improvements in their yields: from 99.61% to 99.99% (process 1), from 99.165% to 99.98% (process 4), from 99.16% to 99.98% (process 7) and from 99.04% to 99.98% (process 9). In contrast, the processes with worst returns were 5, 8 and 2, with sigma levels of 4.64, 4.64 and 4.77, respectively, resulting in yields of 99.92% (processes 5 and 8) and 99.95% (process 2). Globally, a low sigma level of 3 results in a low performance of 93.43%, while a noticeable increase in the sigma level to 4.7 achieves an increase in performance to 99.93%. It can also be seen that an overall increase of 1.7 in Z results in a significant increase in total yield, from Y1=93.43% to Y2=99.93%, representing an increase of 6.5%. Similarly, an increase of 1.42 in Z in processes 5 and 8 generates a 4.10% increase in the yields of these processes. On the other hand, an increase of only 1.13 in Z in the first process results in a yield increase of only 0.38%. The Average Run Rate (MPR) curves allow you to determine how many samples should be produced or services provided to detect a change in Z from Z1 to Z2. In process 1, 13,064 samples must be produced for Z to vary from 4.15 to 5.28. In process 2, 1,959 samples are needed for Z to change from 3.43 to 4.77.

In process 3, 3,266 samples are required to modify Z from 3.64 to 4.91. In process 4, it is necessary to produce 6,122 samples for Z to vary from 3.88 to 5.08. In process 5, 1,241 samples are needed to adjust Z from 3.22 to 4.64. In process 6, 3,266 samples must be generated to change Z from 3.64 to 4.91. In process 7, 6,122 samples are required to vary Z from 3.88 to 5.08. In process 8, it is necessary to produce 1,240 samples for Z to change from 3.22 to 4.64. In process 9, 5,611 samples are needed to set Z from 3.83 to 5.06. Finally, in the overall process, 1,525 samples must be produced to make Z vary from 3.00 to 4.70.

The most sensitive process is 1, as it requires the largest number of samples (13,064) for Z to undergo a significant change from 4.15 to 5.28. In contrast, the least sensitive process is 8, since the smallest number of samples (1,240) are needed for Z to undergo a noticeable variation from 3.22 to 4.64. This analysis based on RPC curves allows you to understand the impact of Z variations on production required to maintain quality control according to Sigma metrics.

5. Discussion

Other authors have pointed out the relevance of Six Sigma metrics to establish processes for assessing quality performance through Six Sigma metrics (Fontalvo et al, 2023). However, the results found in this research could be analyzed the relevance of the performance operation curves AND Six Sigma metrics, This introduces a new and different approach by providing the performance of the process under changing conditions of the sigma Z level, which provides the proportion of good product in terms of the Sigma Z level achieved by the process. This allowed us to identify the process with the best performance and the process with more opportunities for improvement.

In relation to the operation curves of Six Sigma metrics, the contribution of this research is relevant considering that although there are other studies that have proposed Six Sigma metric curves Z Vs n, Z Vs DPMO, Z Vs Y to evaluate performance. As a differentiating element this research provides the Z curve vs RPC(n) which allows to establish control criteria for a statistical quality control system for a concurrent production system. And in turn, it allowed the identification of the most and least sensitive processes to detect a change at Z level, which is practical, novel and useful for making improvement decisions in the systems where it is applied.

Previous research on Six Sigma metrics has mainly used operation curves to assess the performance of production systems, analyzing the variability of the Z-level sigma and its impact on defects per million opportunities (DPMO) and performance Y (Fontalvo et al., 2024a). In contrast, this research introduces a novel methodology by using Average Run Rate (RPC) curves, which allows determining how many samples should be produced to detect changes in Z from Z1 to Z2.

The RPC approach is based on the formula RPCmayor – RPCmenor = units that $\frac{1}{1-\text{Ymayor}}$ y $\frac{1}{1-\text{Ymenor}}$ produce to detect that Z change. This method provides a more dynamic and accurate assessment of the impact of changes in the sigma level on the number of products needed to demonstrate these changes. For example, in process 1 13,064 samples must be produced to have Z vary from 4.15 to 5.28, while in process 8 only 1,240 samples are required to change Z from 3.22 to 4.64.

In comparison, previous studies such as those of Fontalvo et al. (2024b), Fontalvo et al. (2024c) and Fontalvo et al. (2023) focused on operation curves, showing that increasing the sigma level reduces defects and improves performance, but without explicitly addressing the number of samples required. Fontalvo and Banquez's research (2023) compared serial and parallel systems, highlighting that serial systems offer better performance in certain contexts, but did not explore RPC curves.

This research is novel and useful for the business sector in providing new criteria to measure the performance of a statistical control system by providing criteria i) for evaluating quality performance using metrics and curves of operation Y Six Sigma metrics and ii) propose a tool for quality control as the Z vs RPC Average Yield curve. Therefore, this methodology provides an additional and complementary tool to traditional operation curves, offering a more detailed and practical view of quality control in variable conditions with Six Sigma metrics. This is a statistical quality control tool for production process systems, enables informed and relevant decision making by being able to detect the sensitivity level of a process to a change in Z and the number of units to be produced to detect the change as a function of Y.

The proposed method is also novel in that it provides a statistical quality control system for a concurrent production system which allows quality assessment and control to be analysed under changing conditions. It closes a gap and brings a new vision of using statistical control tools differently from traditional ones where tools are applied in a punctual and individual way to the feature, characteristics or processes evaluated and not in an integral and systematic way as proposed in this research.

6. Conclusion

As a unique and differentiating contribution, this research provides an effective and reliable methodology that allows: i) to establish the operating conditions; ii) to evaluate the operating conditions of the concurrent production system by means of the Y performance curves; ii) monitor the performance of the statistical control system using RPC Six Sigma metrics curves. This approach facilitates the evaluation and monitoring of production systems, identifying maximum and minimum process capacities as well as determining the number of samples needed to detect changes in Z performance, crucial for quality analysis. This allows for early detection of defects, avoiding significant costs associated with poor quality.

As a theoretical and methodological contribution, this study discusses concepts related to concurrent production system, Six Sigma metrics, Y-curves and RPC curves of Six Sigma metrics, integrated with an effective method based on the Six Sigma metrics, the Y-Performance curves and the Six Sigma metric RPC curves, which allow you to independently and globally evaluate and monitor your production processes. It also makes it easier to determine the influence of quality level (Z) on defects (n), defects per million opportunities (DPMO) and performance (Y). Also, of the RPC curves at the sigma level Z.

As a practical contribution, this research proposes the Y-yield curves and the Six Sigma Average Run Yield curves to evaluate and control the global system of concurrent production and its independent processes. It also sets the best performance of concurrent system production processes and the sigma Z level to be achieved in order to meet the target. And additionally the proposed method allows to establish through the RPC curve of Six Sigma metrics and the performance range Z for on this basis analyze the number of units to be produced to detect the change in the production system, what becomes a new and different statistical control evaluation system from how they are being implemented with traditional approaches.

As future research, the academic, business and scientific community is invited to replicate the proposed method in various business, academic, production process and standardized service contexts, The aim is to integrate different control tools into a structured statistical quality control system with Six Sigma metrics and metric operation curves that allow the control to be measured and intervened to reduce defects by parts million and any operating system, Establishing methods that integrate other univariate and multivariate statistical control techniques in addition to the tools used.

The limitation of this research and criterion to consider for the success of the method proposed in this research is the fact that it can establish a good system of collection standardized over time, of the primary information to be able to calculate the performance metrics and Y operation curves and the average performance curves of run RPC in any organizational context. Knowledge and understanding of the tools and techniques of the human talent that will implement the method. The availability of technological resources that allow the systematization of the integral system of measurement and control for a good analysis and decision-making of performance indicators to guarantee quality in the industrial context and/or service to be evaluated and managed.

It can also be pointed out as a limitation that the performance of quality levels Z, yields Y and average yield allow only to evaluate and control univariabe processes and system, It is therefore important to apply and complement this type of system with a multivariate measurement to assess overall performance. As a limitation, the proposed method requires that information collected in a business context is collected for all processes in a timely and timely manner, to ensure that the defect balance raised in the mathematical model is dimensionally consistent.

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Annexes

Z1	DPMO1	n1	Y1	U1	RPC1	Z2	DPMO2	n2	Y2	U2	RPC2
4.15	3896	779	99.61%	200000	257	3.43	26077	1039	97.39%	39844	38
4.35	2088	418	99.79%	200000	479	3.69	13948	557	98.61%	39916	72
4.57	1032	206	99.90%	200000	969	3.95	6885	275	99.31%	39959	145
4.79	470	94	99.95%	200000	2127	4.22	3136	125	99.69%	39981	319
5.03	198	40	99.98%	200000	5062	4.49	1317	53	99.87%	39992	759
5.28	77	15	99.99%	200000	13064	4.77	510	20	99.95%	39997	1959
Z3	**DPMO3**	**n3**	**Y3**	**U3**	**RPC3**	**Z4**	**DPMO4**	**n4**	**Y4**	**U4**	**RPC4**
3.64	15634	1559	98.44%	99688	64	3.88	8438	1039	99.16%	123130	119
3.88	8365	835	99.16%	99833	120	4.10	4490	557	99.55%	123998	223
4.13	4130	413	99.59%	99917	242	4.33	2210	275	99.78%	124505	453
4.38	1881	188	99.81%	99962	532	4.58	1005	125	99.90%	124774	995
4.65	790	79	99.92%	99984	1265	4.82	422	53	99.96%	124905	2371
4.91	306	31	99.97%	99994	3266	5.08	163	20	99.98%	124963	6122
Z5	**DPMO5**	**n5**	**Y5**	**U5**	**RPC5**	**Z6**	**DPMO6**	**n6**	**Y6**	**U6**	**RPC6**
3.22	41814	6234	95.82%	149091	24	3.64	15634	1559	98.44%	99688	64
3.50	22205	3341	97.78%	150441	45	3.88	8365	835	99.16%	99833	120
3.78	10915	1651	98.91%	151230	92	4.13	4130	413	99.59%	99917	242
4.07	4960	752	99.50%	151649	202	4.38	1881	188	99.81%	99962	532
4.35	2081	316	99.79%	151852	480	4.65	790	79	99.92%	99984	1265
4.64	806	122	99.92%	151943	1241	4.91	306	31	99.97%	99994	3266
Z7	**DPMO7**	**n7**	**Y7**	**U7**	**RPC7**	**Z8**	**DPMO8**	**n8**	**Y8**	**U8**	**RPC8**
3.88	8438	1039	99.16%	123130	119	3.22	41814	6234	95.82%	149091	23.92
4.10	4490	557	99.55%	123998	223	3.50	22205	3341	97.78%	150441	45.03
4.33	2210	275	99.78%	124505	453	3.78	10915	1651	98.91%	151230	91.61
4.58	1005	125	99.90%	124774	995	4.07	4960	752	99.50%	151649	201.61
4.82	422	53	99.96%	124905	2371	4.35	2081	316	99.79%	151852	480.43
5.08	163	20	99.98%	124963	6122	4.64	806	122	99.92%	151943	1240.58
Z9	**DPMO9**	**n9**	**Y9**	**U9**	**RPC9**	**Zg**	**DPMOg**	**ng**	**Yg**	**Ug**	**RPCg**
3.83	9605	3117	99.04%	324519	104	3.00	65693	22598	93.43%	344000	15
4.06	5007	1670	99.50%	333561	200	3.30	35202	12110	96.48%	344000	28
4.30	2436	825	99.76%	338841	411	3.60	17395	5984	98.26%	344000	57
4.55	1101	376	99.89%	341649	908	3.90	7927	2727	99.21%	344000	126
4.80	461	158	99.95%	343012	2170	4.20	3331	1146	99.67%	344000	300
5.06	178	61	99.98%	343617	5611	4.50	1291	444	99.87%	344000	775
						4.70	656	226	99.93%	344000	1525