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# A NOVEL METHOD FOR MODELING AND PREDICTING TRANSPORTATION DATA VIA MULTIDEEP ASSESSMENT METHODOLOGY AND FRACTIONAL CALCULUS

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Aviation is one of the most global industries, and if we can model and predict a country's air transportation flow and indicators ahead of time, we may be able to use it as a key decision-making tool for the management and operation process. This study proposes a new modeling, and prediction method that employs both fractional calculus and Multi Deep Assessment Methodology (MDAM) techniques. For the application, air passengers carried, air freight, available seat kilometers, number of flights, destination points, international travelers, international destination points, and international flight data between 2011 and 2019 for eight countries with the busiest airports were chosen. As a result, the highest modeling error was discovered to be Germany's air transport freight factor expressed as a percentage of 1,59E-02. The percentage of predictions with errors less than 10% was 90.278. We also compared the performance of two different MDAM methodologies. The novel MDAM wd methodology proposed in this paper has a higher accuracy in aviation factors prediction and modeling.

**Keywords:** Air transportation, deep assessment methodology, fractional calculus, time series prediction, modeling, applied mathematics

## 1. Introduction

Every day, about 12 million passengers and around USD 18 billion worth of commodities are transported by over 100.000 planes. With the help of 170 air navigation service providers, 1.303 scheduled airlines now fly 31.717 aircraft, serving 3.759 airports. Air traffic numbers are expected to more than double in the next 20 years, according to current projections. It is important to recognize the socioeconomic contributions made by the aviation sector on a national and international scale. Because demand for air travel is expanding faster than system capacity can keep up with, the operation and health of the future global air traffic system are dependent on efficient and effective management of system capabilities. Despite substantial advancements in the last three decades in the relevant fields of air traffic flow management and airport capacity modeling, research on issues related to air traffic management has lagged in methodological quality (Kira *et al.*, 2019).

Evaluating the prediction of complex systems, such as aviation revenue management, is a challenging task. When regarding aviation as a time-continuous system measured in value discrete time series via performance indicators and certain metrics, it is important to use sufficiently targeted mathematical models within the analysis. Consistent identification of economic dynamics at the evaluation level, without dealing with the actual physical events of the system, transforms the analysis of time series into a system identification process, which ensures control of a partially known system.

The majority of transportation statistics are generally released with significant time delay. "Nowcasting" refers to estimating their current values before they are published (Giannone *et al.*, 2008). Previous studies have forecasted transportation and economic data using real-time internet data (Carrière *et al.*, 2011; Preis *et al.*, 2013; Da *et al.*, 2009; Vosen and Schmidt, 2011; Bollen and Mao, 2011; Botta *et al.*, 2015; Moat *et al.*, 2013; Mestyán *et al.*, 2013).

Better nowcasts are extremely valuable, as major economic policy tools like interest rates can have a 20-month lag before fully impacting the economy (Bernanke and Gertler, 1995). When confronted with shocks such as the 2008 financial crisis, and the 2019 pandemic policymakers must respond as quickly as possible, which necessitates accurate knowledge of the aviation economy's current state. Failure to do so has exacerbated previous recessions (Claveria *et al.*, 2020).

Recent years have witnessed a growing interest in understanding and analyzing transportation data. To uncover hidden insights from complicated and dynamic datasets, data analysis has become essential. Deep evaluation methods have seen significant improvements, enabling their applicability in a variety of fields and enabling corporations and other organizations to forecast the future and make wiser decisions. The best possible use of resources is needed in aviation to improve customer satisfaction and safety records (Abu-shady and Kaabar, 2022).

Many studies have been published in the literature that involves the modeling of transportation economics and also transportation data (Baltagi, 2021; Banister and Berechman, 2001; Berechman *et al.*, 2006; Karaçuha *et al.*, 2021; Karaçuha *et al.*, 2020; Hong *et al.*, 2011). In recent years, the importance of studies such as artificial intelligence, machine learning, artificial neural networks, fractional calculus, deep learning, and deep assessment has grown. Because of its memorization property, fractional calculus, which has recently been used in many different domains, has also begun to be used in predictive studies. Studies conducted with this method involve transportation data and financial prediction (Lau and Sin, 1997; Lavee *et al.*, 2011; Yamaguchi, 2007; Cowie *et al.*, 2009; Önal Tuğrul *et al.*, 2022; Önal Tuğrul *et al.*, 2021).

In the previous studies; the studies were also conducted in different industries (transportation economics, economy, healthcare, telecommunication sectors) to highlight the wide range of applications for fractional calculus and economic projections. Korbel and Luchko (2016) propose a new model for financial processes using a space-time fractional diffusion equation with varying order. The model is applied to analyze financial data. Kristjanpoller and Minutolo (2018) introduce a hybrid model combining Artificial Neural Networks (ANN) and Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) to forecast the price volatility of Bitcoin, the largest cryptocurrency. Laskin (2000) presents an extension of the fractality concept in financial mathematics. The paper introduces a new fractional Langevin-type stochastic differential equation that incorporates fractional derivatives and a generalized "shot noise" stochastic force. Liu (2013) develops a system dynamics model to study the relationship between human capital and economic development. The model simulates the future development trend of various index variables related to human capital and the economy.

Tenreiro and Maria Eugénia Mata (2015) utilize Pseudo Phase Plane (PPP) and Fractional Calculus (FC) mathematical tools to model world economies. The paper explores the synergies between these tools, aiming to better understand the dynamics of world economies and estimate future states. Cortell and Peterson (1999) outline a framework for studying institutional transformation, accounting for both radical and episodic changes. The paper emphasizes the role of crises in precipitating structural change. Meerschaert and Scalas (2006) discuss the application of Continuous Time Random Walks (CTRWs) in finance to model anomalous diffusion. CTRWs represent log returns as particle jumps and waiting times as delays between transactions. Nosratabadi *et al.* (2020) provide a comprehensive investigation of recent advances in data science applied to emerging economic applications. The analysis focuses on deep learning models, hybrid deep learning models, hybrid machine learning, and ensemble models. These papers collectively contribute to the understanding and application of mathematical and statistical methods in various domains, including finance, economics, transportation and data science.

The models which have developed thus far involved a multiple-input system (Önal Tuğrul and Karaçuha, 2022), but in the current study a mathematical model with multiple-input, and deep assessment with first-order derivative structure is suggested. Novel modeling and prediction methodologies were developed by plugging multiple factor effects on a single factor and historical data into the system, and air transportation data will be the first time used for this novel method in our paper.

In the current study, Air transport passengers carried, Air transport freight, Number of flights, Number of destination points (airport), International travelers, Number of International destination points (airport), International number of flights, ASK (Available seat kilometer) International, and ASK data of eight the busiest airport owner countries; China, France, Germany, India, Turkey, Singapore, UK and USA from 2011 to 2019 years are used (OECD Data Statistic, 2021; World Bank Open Data, 2021).

In our research, we first determine the finest fractional order value of the derivative for each factor, which will allow us to develop effective modeling with and without derivatives. Second, we developed a formulation for forecasting future steps. To implement the mathematical models, we also run an example application on data from eight countries' air transportation and economic factors. This mathematical model is coded on Matlab.

In the next section, a problem formulation will be given for the modeling and prediction. The third section will give applications. The fourth section will include the results and we demonstrate the superiority of our model using figures of modeling, and prediction implementations. In the fifth and last section, a general evaluation will take place, and finally, we conclude and give research directions for the future.

## 2. The Proposed Approaches

### 2.1. The Formulation of the factors modeling

In this section we will explain modeling via multi factors in which expressing $m$th factor will be like the following (2.1a).

$$f^{(m)}(x) = \sum_{k=1}^{l}\sum_{j=1}^{r}\left[\alpha_k^{(j)} f^{(j)}(x-k) + \beta_k^{(j)} f^{(j)'}(x-k)\right], \tag{2.1a}$$

where $f^{(m)}(x)$ is the factor. Here, $m$ represents whichever factor we are currently modeling and j represents the factor that we think contributes, and x stands for the time. For example, if we formulize the 1st factor concerning other factors, we take $m = 1$. $j$ stands for the factor that is modeled. Our r value is 9 as air transport passengers carried, air transport freight, number of flights, number of destination points (airport), international travelers, number of international destination points (airport), international number of flights, ASK (Available seat kilometers) international, and ASK.

$$f^{(j)}(x-k) = \sum_{n=0}^{M} a_n^{(j)} (x-k)^{n\alpha_j}, \tag{2.1b}$$

$0 < \alpha_j < 1$ , and $j = 1,2,\dots,r$.

Here, we can express the (2.1a) equation as (2.1c) below.

$$f^{(m)}(x) = \sum_{k=1}^{l}\sum_{j=1}^{r}\sum_{n=0}^{M}\left[\alpha_k^{(j)} a_n^{(j)} (x-k)^{n\alpha_j} + \beta_k^{(j)} a_n^{(j)} n\alpha_j (x-k)^{n\alpha_j-1}\right], \tag{2.1c}$$

$k = 1,2,\dots,l$ and $n = 1,2,\dots,M$.

In our formula by using 2.1a, 2.1b, and 2.1c and expressing fractional derivative order as $\nu$, if we take $F^{(m)}(s)$ as the Laplace transformation of $f^{(m)}(x)$ we can write,

$$\frac{\partial^\nu f^{(m)}(x)}{\partial x^\nu} \triangleq \sum_{k=1}^{l}\sum_{j=1}^{r}\sum_{n=1}^{M}\left[\alpha_k^{(j)} a_n^{(j)} (n\alpha_j)(x-k)^{n\alpha_j-1} + \beta_k^{(j)} a_n^{(j)} (n\alpha_j)(n\alpha_j-1)(x-k)^{n\alpha_j-2}\right]. \tag{2.1d}$$

$$F^{(m)}(s) = \frac{f^{(m)}(0)}{s} + \sum_{k=1}^{l}\sum_{j=1}^{r}\sum_{n=1}^{M}\left[a_{kn}^{(j)}\Gamma(n\alpha_j+1)\frac{e^{-ks}}{s^{n\alpha_j+\nu}} + b_{kn}^{(j)}\Gamma(n\alpha_j+1)\frac{e^{-ks}}{s^{n\alpha_j+\nu-1}}\right]. \tag{2.1e}$$

For the numerical calculation, the infinite summation is converted into a finite summation. Here, $a_{kn}^{(j)} = \alpha_k^{(j)} a_n^{(j)}$ *and* $b_{kn}^{(j)} = \beta_k^{(j)} a_n^{(j)}$ . If we take the inverse Laplace transformation of equation (2.1e),

$$f^{(m)}(x) = f^{(m)}(0) + \sum_{j=1}^{r}\sum_{k=1}^{l}\sum_{n=1}^{M}\left[a_{kn}^{(j)} C_{kn}^{(j)}(x) + b_{kn}^{(j)} D_{kn}^{(j)}(x)\right], \tag{2.2}$$

where $0 < \nu < 1$ and $x > k$.

Here, we obtain:

$$C_{kn}^{(j)}(x) = \frac{\Gamma(n\alpha_j+1)}{\Gamma(n\alpha_j+\nu)}(x-k)^{n\alpha_j+\nu-1} \tag{2.3}$$

and

$$D_{kn}^{(j)}(x) = \frac{\Gamma(n\alpha_j+1)}{\Gamma(n\alpha_j+\nu-1)}(x-k)^{n\alpha_j+\nu-2}. \tag{2.4}$$

Modeling of any factor will be obtained by using equation (2.2) and calculating the $a_{kn}^{(j)}$ and $b_{kn}^{(j)}$ coefficients with the least squared error method. The most important step here is changing the fractional order $\nu$ to a value ranging between (0,1) and finding the $\nu$ value that will give the least modeling error.

$$\epsilon_T^2 = \sum_{i=z}^{t} \epsilon_i^2 = \sum_{i=z}^{t} \left[ \left( P_i^{(m)} - f^{(m)}(0) - \sum_{j=1}^{r} \left[ \sum_{k=1}^{l} \sum_{n=1}^{M} a_{kn}^{(j)} C_{kn}^{(j)}(x) + b_{kn}^{(j)} D_{kn}^{(j)}(x) \right] \right) \right]^2, \tag{2.5}$$

$t$ (years): [2011, 2012, ..., 2019],
$i$ (points): [1, 2, ..., 9],
$P_i^{(m)}$ (value of $i$): [$P1$, $P2$, …, $P9$],
$P_i^{(m)}$: It shows the mth factor of each country in each $ith$ year. For example, $P_1^{(1)}$ is the Air transport, passengers carried of the chosen country in 2011,
$i$: It is the corresponding number for each year. For example, $i$ = 1 for 2011, $i$ = 2 for 2012, and $i$ = 9 for 2019.

The main purpose of the modeling region is to minimize $\epsilon_T^2$ . To minimize the square of the total error, the following are required. The first one of the data to be modeled is $x = z$ and the last one is $x = t$.

$$\frac{\partial \epsilon_T^2}{\partial f^{(m)}(0)} = 0, \ \frac{\partial \epsilon_T^2}{\partial a_{pw}^{(s)}} = 0 \ \text{and} \ \frac{\partial \epsilon_T^2}{\partial b_{pw}^{(s)}} = 0 \quad s = 1,2,..r, \quad p = 1,2,\dots l, \quad w = 1,2,..M. \tag{2.6}$$

Here $w$ represents years (2011, 2012, …, 2019), s represents factors and p represents the modeling time range as years. To obtain the optimum modeling, $l$ value is investigated. The value of $l$ is found as 2. In other words, for the modeling of the factors of each country, the required previous data of past years used in the algorithm differs. When the test region is considered, the optimal $M$ values are found to be 2.

The modeling process is completed by finding the $\nu$ with the least error, $f^{(m)}(0), a_{kn}^{(j)}$, and $b_{kn}^{(j)}$ values.

## 2.2. A Proposal for Prediction

Let us define the factor ($m = 1,2,\dots,r$) we want to model with the target year and historical values below,

$$\begin{aligned} f^{(m)}(x) &= \sum_{k=1}^{l} \sum_{j=1}^{r} \left[ \alpha_k^{(j)} f^{(j)}(x-k) + \beta_k^{(j)} {f^{(j)}}'(x-k) \right], \\ f^{(m)}(x-1) &= \sum_{j=1}^{r} \sum_{k=1}^{l} \left[ \alpha_k^{(j)} f^{(j)}(x-(k+1) + \beta_k^{(j)} {f^{(j)}}'(x-(k+1) \right], \\ &\quad \vdots \qquad\qquad\qquad \vdots \\ f^{(m)}(x-l) &= \sum_{j=1}^{r} \sum_{k=1}^{l} \left[ \alpha_k^{(j)} f^{(j)}(x-(k+l) + \beta_k^{(j)} {f^{(j)}}'(x-(k+l) \right]. \end{aligned} \tag{2.7}$$

Here, $x > 2l$.

$$\epsilon_T^2 = \sum_{i=z}^{t} \epsilon_i^2 = \sum_{i=z}^{t} \left[ f^{(m)}(x-i) - \sum_{j=1}^{r} \sum_{k=1}^{l} \left[ \alpha_k^{(j)} f^{(j)}(x-(k+i) + \beta_k^{(j)} {f^{(j)}}'(x-(k+i) \right] \right]^2. \tag{2.8}$$

$$\frac{\partial \epsilon_T^2}{\partial \alpha_y^{(s)}} = 0, \quad \frac{\partial \epsilon_T^2}{\partial \beta_y^{(s)}} = 0, \quad s = 1,2,..r, \quad y = 1,2,\dots l. \tag{2.9}$$

$(l * r)$ number of equations are obtained from (2.9) equation. After finding the $\beta_k^{(j)}$ values from these equations, future predictions of any factor can be made by using the first set of (2.7) equations. In other words, values in the next step are obtained by $x = x + 1$. Factors that are used for eight countries are given below:

**Table 1.** Factors

| No | $f^{(m)}$ | Series Name |
|---|---|---|
| 1 | $f_1$ | Air transport, passengers carried (million) |
| 2 | $f_2$ | Air transport, freight (million ton-km) |
| 3 | $f_3$ | Number of flights (thousand) |
| 4 | $f_4$ | Number of destination points (airport) |
| 5 | $f_5$ | International passengers carried (million) |
| 6 | $f_6$ | Number of International destination points (airport) |
| 7 | $f_7$ | International number of flights (thousand) |
| 8 | $f_8$ | ASK (Available seat kilometers) International (billion) |
| 9 | $f_9$ | ASK (Available seat kilometers) (billion) |

## 3. Results

### 3.1. Modeling Results

Table 2 displays the results obtained by applying fractional calculus and MDAM with and without derivative methodologies to all countries, taking into consideration the impact of each factor between $f^{(1)}(x) - f^{(9)}(x)$. Table 2 shows the modeling results for the values $M = 2,\ l = 2$ for two different modeling types MDAM wd (multi input deep assessment methodology with first order derivative) and MDAM (multi input deep assessment methodology). Using value $l = 2$ means that the years between 2014 and 2019 were modeled using data from the previous two years. For instance, for model year 2017; 2015 and 2016 years data are used, and to model year 2015; 2014 and 2013 years data are used.

**Table 2.** Modeling 2014-2019 years by using economic factors and aviation data for eight of the busiest airport owner countries

| FACTORS | | CHINA | | FRANCE | | GERMANY | |
|---|---|---|---|---|---|---|---|
| | | MAPE (%) | | MAPE (%) | | MAPE (%) | |
| | | MDAM wd | MDAM | MDAM wd | MDAM | MDAM wd | MDAM |
| $f^{(1)}$ | Air passengers carried | 2,96E-06 | 1,30E-02 | 3,37E-05 | 3,96E-01 | 3,57E-03 | 2,68E-01 |
| $f^{(2)}$ | Air transport, freight | 1,75E-04 | 6,45E-01 | 2,43E-05 | 2,62E-01 | 1,59E-02 | 1,04E+00 |
| $f^{(3)}$ | Number of flights | 6,52E-05 | 1,16E-01 | 2,70E-05 | 2,15E-01 | 6,57E-03 | 4,43E-01 |
| $f^{(4)}$ | Number of destination points | 6,85E-05 | 1,17E-01 | 1,04E-04 | 7,29E-01 | 9,23E-03 | 6,73E-01 |
| $f^{(5)}$ | International travelers | 6,52E-05 | 4,94E-01 | 1,95E-05 | 1,66E-01 | 2,78E-03 | 1,94E-01 |
| $f^{(6)}$ | Number of Int. destination points | 2,24E-05 | 4,61E-02 | 1,10E-04 | 7,86E-01 | 8,99E-03 | 6,63E-01 |
| $f^{(7)}$ | International number of flights | 3,67E-04 | 5,94E-01 | 2,40E-05 | 1,65E-01 | 5,44E-03 | 3,74E-01 |
| $f^{(8)}$ | ASK International | 2,83E-04 | 3,97E-01 | 2,30E-05 | 3,17E-01 | 8,41E-04 | 6,27E-02 |
| $f^{(9)}$ | ASK (Available seat kilometer) | 1,04E-04 | 1,64E-01 | 2,39E-05 | 3,18E-01 | 1,06E-03 | 7,58E-02 |
| FACTORS | | INDIA | | SINGAPORE | | TURKEY | |
| | | MAPE (%) | | MAPE (%) | | MAPE (%) | |
| | | MDAM wd | MDAM | MDAM wd | MDAM | MDAM wd | MDAM |
| $f^{(1)}$ | Air passengers carried | 2,42E-05 | 1,13E+00 | 5,86E-05 | 1,80E-01 | 3,13E-04 | 2,68E-01 |
| $f^{(2)}$ | Air transport, freight | 2,31E-05 | 3,33E+00 | 2,35E-03 | 7,10E+00 | 1,15E-03 | 7,06E-01 |
| $f^{(3)}$ | Number of flights | 1,99E-05 | 8,77E-01 | 6,50E-05 | 1,66E-01 | 1,24E-03 | 7,24E-01 |
| $f^{(4)}$ | Number of destination points | 7,76E-06 | 3,33E-01 | 6,71E-06 | 9,28E-02 | 5,56E-04 | 3,64E-01 |
| $f^{(5)}$ | International travelers | 2,03E-06 | 2,01E-01 | 6,36E-06 | 5,53E-02 | 4,04E-04 | 5,43E-01 |
| $f^{(6)}$ | Number of Int. destination points | 7,42E-06 | 3,33E-01 | 1,70E-05 | 1,47E-01 | 6,27E-04 | 3,87E-01 |
| $f^{(7)}$ | International number of flights | 2,80E-06 | 4,28E-01 | 6,52E-05 | 1,66E-01 | 5,57E-04 | 3,97E-01 |
| $f^{(8)}$ | ASK International | 7,21E-06 | 6,34E-01 | 8,17E-05 | 2,14E-01 | 9,10E-04 | 6,18E-01 |
| $f^{(9)}$ | ASK (Available seat kilometer) | 1,33E-05 | 6,92E-01 | 8,16E-05 | 2,14E-01 | 9,78E-04 | 6,27E-01 |
| FACTORS | | UK | | USA | | | |
| | | MAPE (%) | | MAPE (%) | | | |
| | | MDAM wd | MDAM | MDAM wd | MDAM | | |
| $f^{(1)}$ | Air passengers carried | 2,46E-06 | 1,76E+00 | 1,66E-04 | 1,15E-01 | | |
| $f^{(2)}$ | Air transport, freight | 6,26E-08 | 2,25E-01 | 2,57E-04 | 7,25E-01 | | |
| $f^{(3)}$ | Number of flights | 3,09E-07 | 5,33E-01 | 1,94E-04 | 1,33E-01 | | |
| $f^{(4)}$ | Number of destination points | 4,10E-07 | 2,80E-01 | 2,56E-05 | 2,02E-02 | | |
| $f^{(5)}$ | International travelers | 2,68E-07 | 3,22E-01 | 3,45E-04 | 2,64E-01 | | |
| $f^{(6)}$ | Number of Int. destination points | 4,75E-07 | 3,98E-01 | 4,39E-04 | 3,79E-01 | | |
| $f^{(7)}$ | International number of flights | 4,47E-07 | 6,29E-01 | 2,70E-04 | 2,14E-01 | | |
| $f^{(8)}$ | ASK International | 3,27E-07 | 1,29E-01 | 1,70E-04 | 1,25E-01 | | |
| $f^{(9)}$ | ASK (Available seat kilometer) | 3,15E-07 | 1,22E-01 | 2,16E-04 | 1,62E-01 | | |

Modeling performance was measured using Mean Absolute Percentage Error (MAPE) in equation (4.1). Modeling graphs for the first four factors belonging to each country are given in Figure 1– Figure 4. Table 2 and figures clearly show that all aviation factors between the years 2014-2019 have been modeled well by both methods, but MDAM wd has smaller MAPE values than MDAM method. While $\mathrm{v}(i)$ represents a real value, $\tilde{\mathrm{v}}(i)$ represents the data modeled with MDAM wd and MDAM methods. $t-z+1$ is the number of sample data; since a value of $l=2$ is used, data between 2011 and 2019 are modeled, and thus $t-z+1$ value is taken as 2 in the current study.

$$MAPE = \frac{1}{t-z+1}\sum_{i=z}^{t}\left|\frac{\mathrm{v}(i)-\tilde{\mathrm{v}}(i)}{\mathrm{v}(i)}\right| \times 100. \quad (4.1)$$

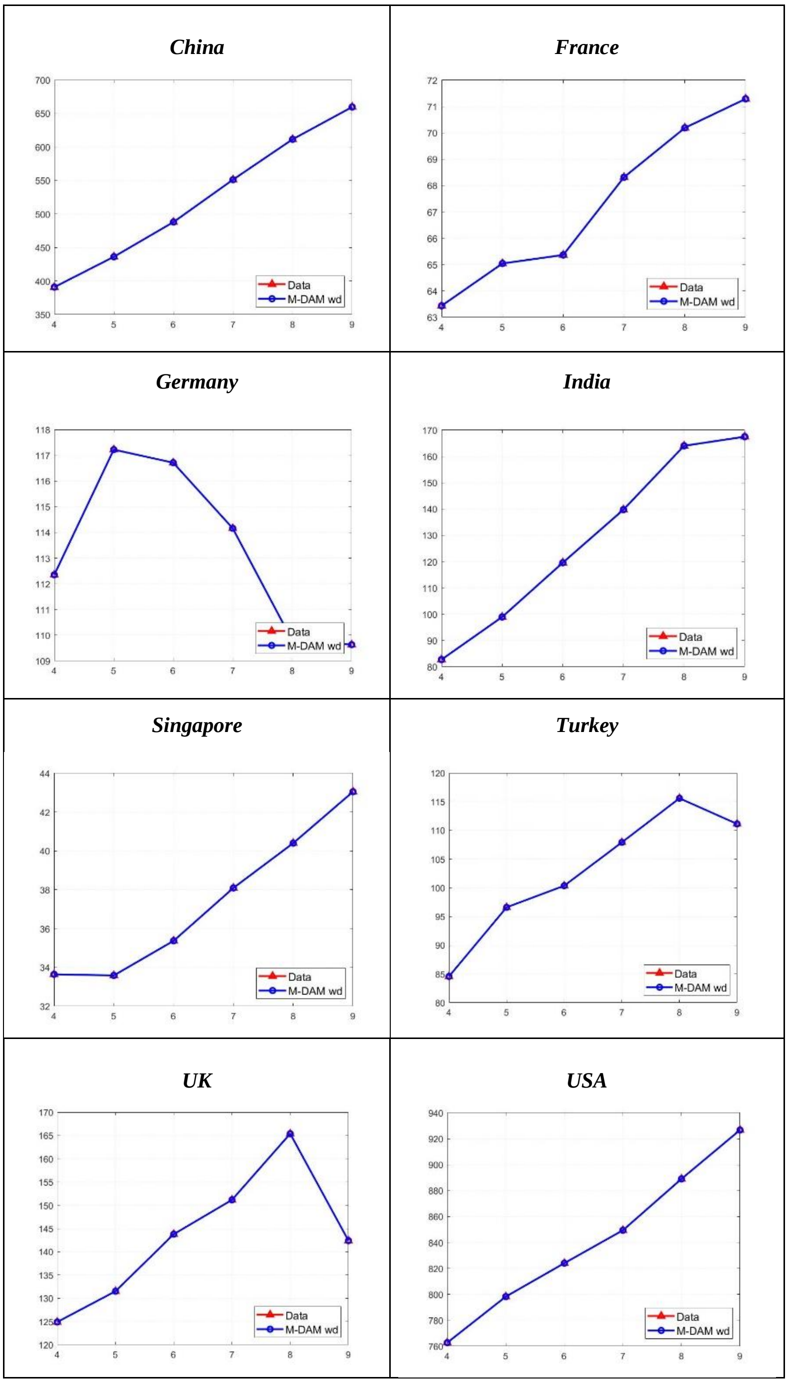


*Figure 1*. Modeling graphs between the years 2014 and 2019 for Air transport passengers carried (million)
(Axis x: years, Axis y: values)

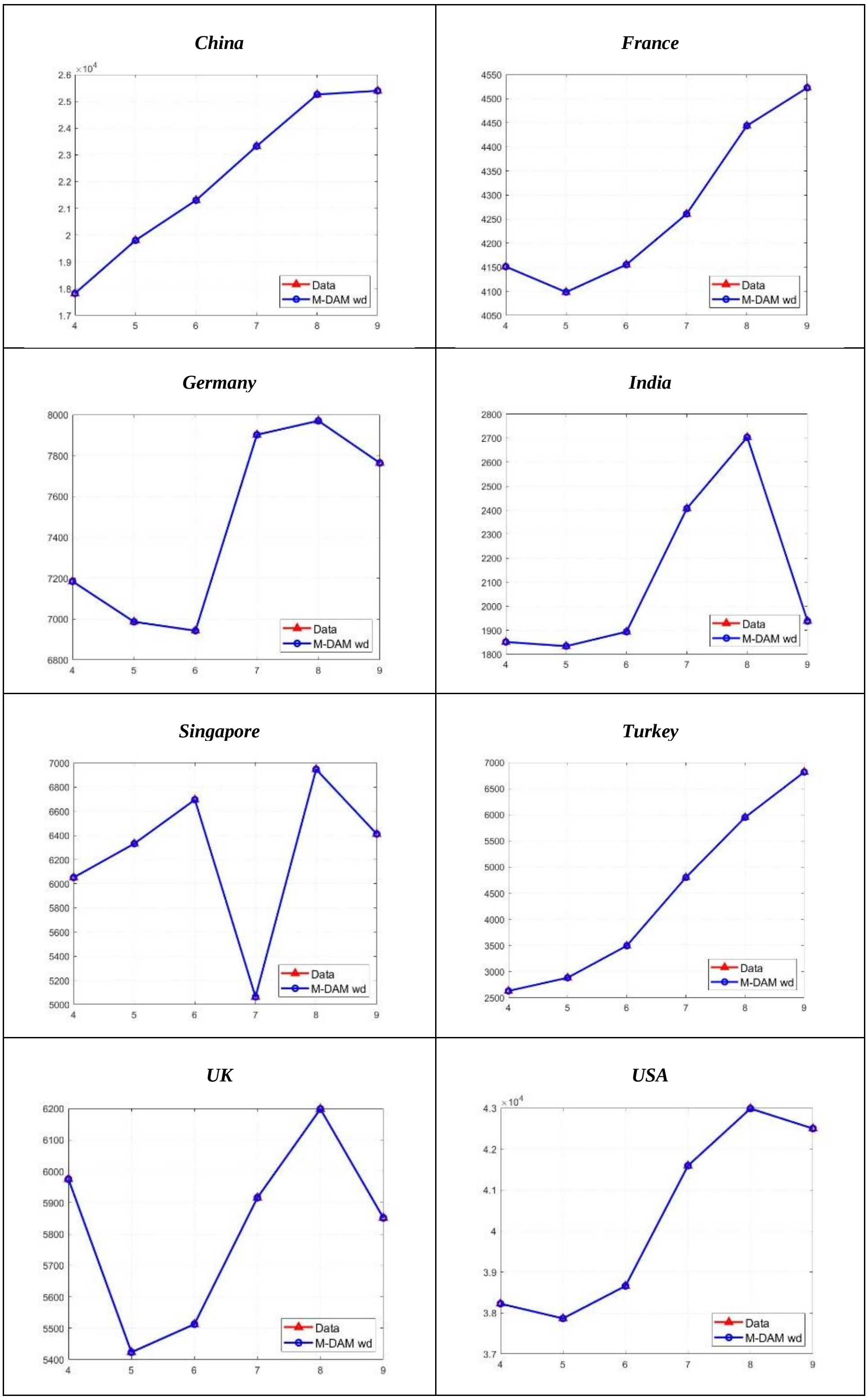


*Figure 2.* Modeling graphs between the years 2014 and 2019 for Air transport freight (million ton-km).
(Axis x: years, Axis y: values)

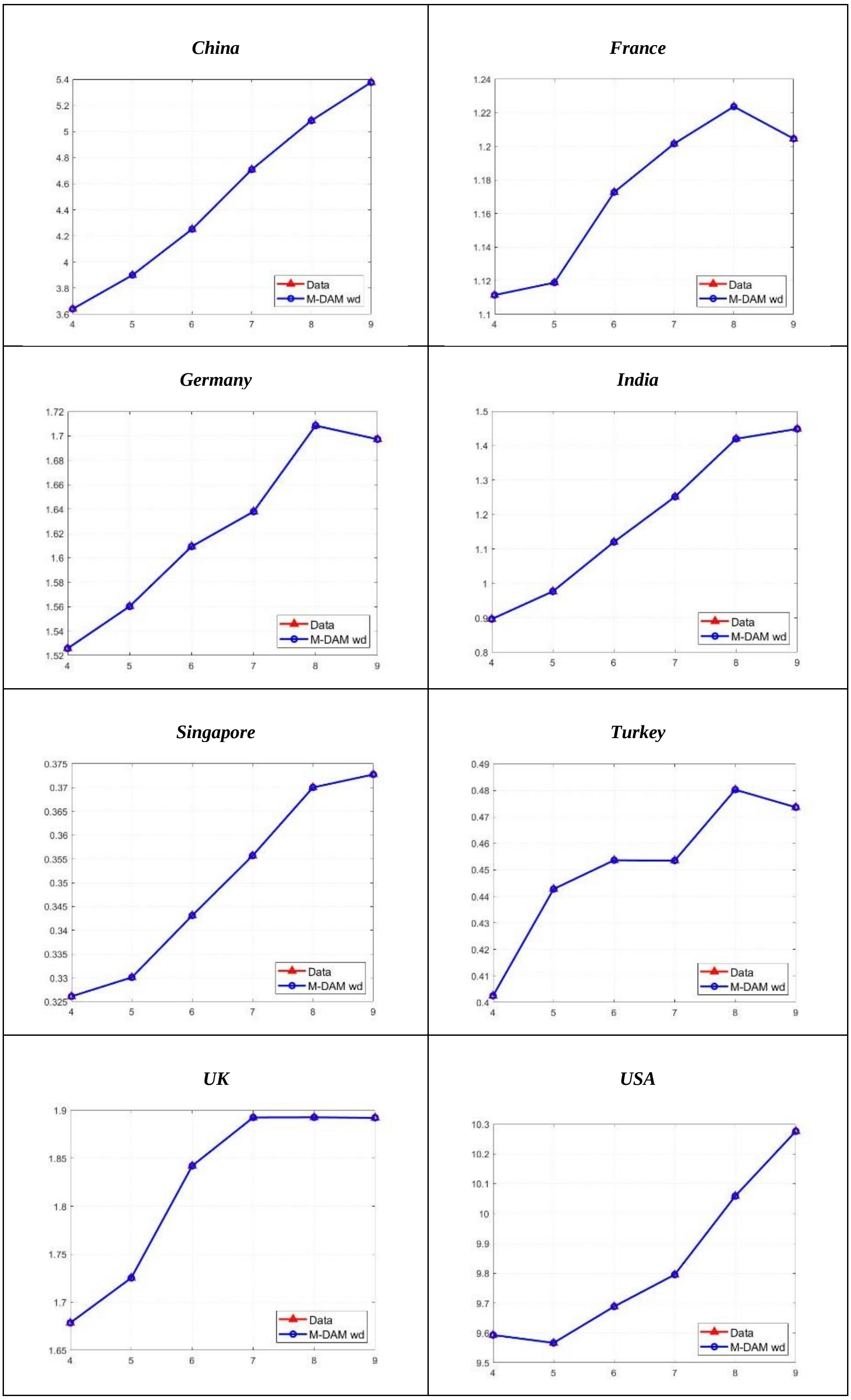


*Figure 3.* Modeling graphs between the years 2014 and 2019 for the Number of flights (thousand).
(Axis x: years, Axis y: values)

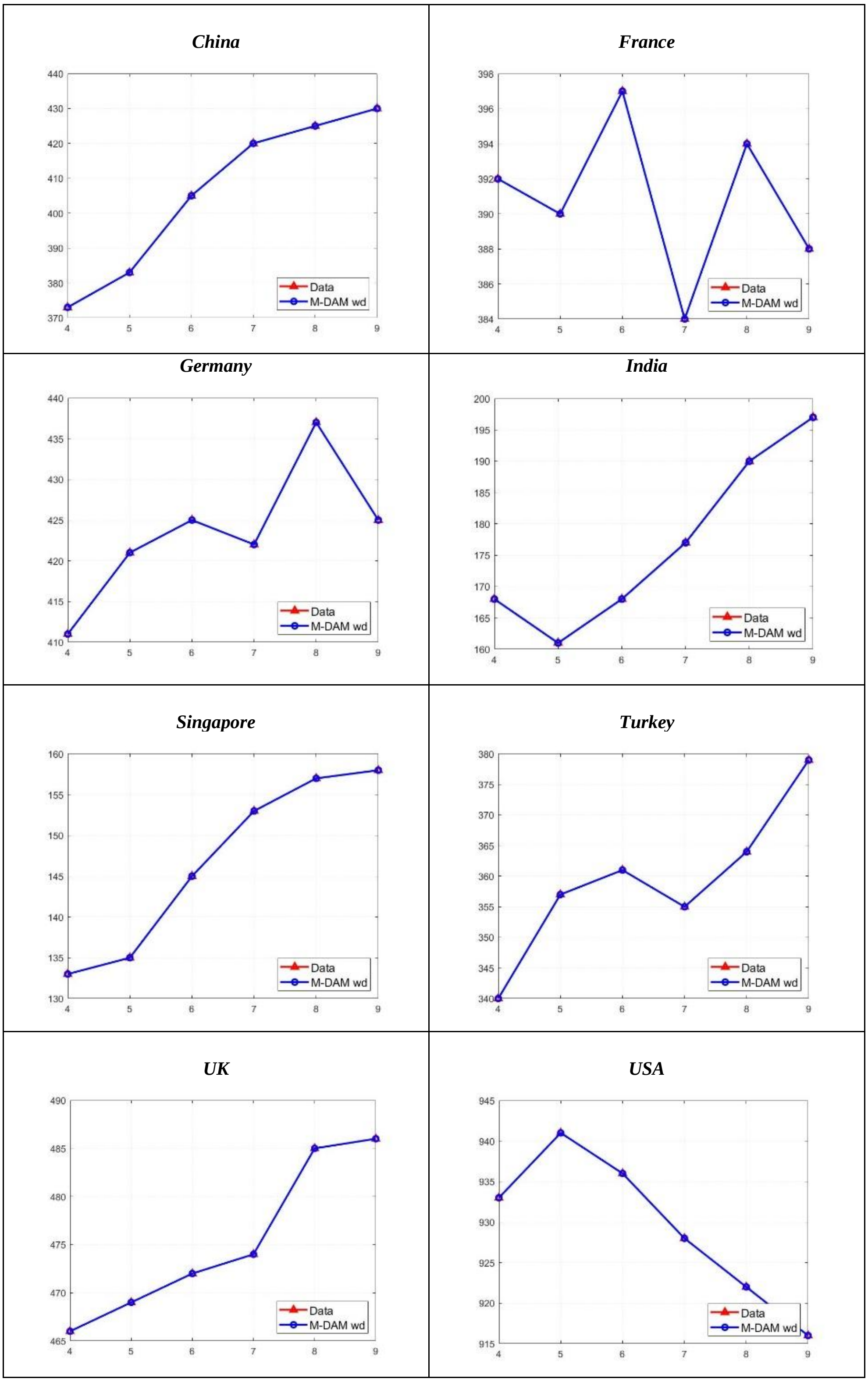


*Figure 4.* Modeling graphs between the years 2014 and 2019 for the Number of destination points (airport).
(Axis x: years, Axis y: values)

### 3.2 Prediction Results

Prediction accuracy is unquestionably important, and better predictions can be made for data that remains stable. Because aviation data is unstable it is more difficult to find accurate results. Table 3 displays the 2019 prediction results and error values (MAPE) for all countries and factors using the prediction method with and without derivatives. The prediction results are shown as MAPE values in Table 3 for all countries with $M = 2\ and\ l = 2$. According to the tables, the UK's International number of flights factor was predicted with the least amount of error, at 0.02 percent for the prediction method with derivative. When prediction values are considered in terms of predictability of countries, the UK takes first place; Germany is second, the USA is third, France is fourth, China is fifth, Singapore is sixth, Turkey is seventh, and India is last for year 2019. As a result, the United Kingdom appears to be the most predictable country, while India appears to be the least predictable country for all factors.

**Table 3.** Reel, prediction, and MAPE values for the year 2019 ( $M = 2\ and\ l = 2$).

| | CHINA FACTORS | REEL | PRED wd | PRED | MAPE (%) wd | MAPE (%) |
|---|---|---|---|---|---|---|
| $f^{(1)}$ | **Air passengers carried** | 659,6 | 671,71 | 683,1189 | 1,83 | 3,56 |
| $f^{(2)}$ | **Air transport, freight** | 25394 | 24665,83 | 27495,98 | 2,87 | 8,27 |
| $f^{(3)}$ | **Number of flights** | 5,4 | 5,73 | 5,562956 | 6,67 | 3,48 |
| $f^{(4)}$ | **Number of destination points** | 430 | 432,98 | 434,4221 | 0,69 | 1,03 |
| $f^{(5)}$ | **International travelers** | 37,8 | 39,40 | 40,10546 | 4,37 | 6,23 |
| $f^{(6)}$ | **Number of Int. dest. points** | 194 | 195,88 | 196,2961 | 0,97 | 1,18 |
| $f^{(7)}$ | **International number of flights** | 970,3 | 1002,51 | 973,0167 | 3,32 | 0,28 |
| $f^{(8)}$ | **ASK International** | 701,5 | 634,52 | 733,5403 | 9,55 | 4,56 |
| $f^{(9)}$ | **ASK (Available seat kilometers)** | 1577 | 1523,55 | 1644,196 | 3,40 | 4,25 |
| | **FRANCE FACTORS** | **REEL** | **PRED wd** | **PRED** | **MAPE (%) wd** | **MAPE (%)** |
| $f^{(1)}$ | **Air passengers carried** | 71,29 | 68,43 | 74,63 | 4,00 | 4,69 |
| $f^{(2)}$ | **Air transport, freight** | 4522,59 | 4200,98 | 4660,64 | 7,11 | 3,05 |
| $f^{(3)}$ | **Number of flights** | 1,2 | 1,20 | 1,29 | 0,67 | 6,75 |
| $f^{(4)}$ | **Number of destination points** | 388 | 389,25 | 397,84 | 0,32 | 2,54 |
| $f^{(5)}$ | **International travelers** | 121,5 | 113,51 | 124,63 | 6,58 | 2,57 |
| $f^{(6)}$ | **Number of Int. dest. points** | 332 | 337,23 | 337,45 | 1,58 | 1,64 |
| $f^{(7)}$ | **International number of flights** | 947,8 | 939,52 | 1014,00 | 0,87 | 6,99 |
| $f^{(8)}$ | **ASK International** | 448,62 | 426,99 | 454,67 | 4,82 | 1,35 |
| $f^{(9)}$ | **ASK (Available seat kilometers)** | 468,34 | 444,03 | 475,48 | 5,19 | 1,53 |
| | **GERMANY FACTORS** | **REEL** | **PRED wd** | **PRED** | **MAPE (%) wd** | **MAPE (%)** |
| $f^{(1)}$ | **Air passengers carried** | 109,63 | 112,55 | 125,14 | 2,66 | 14,14 |
| $f^{(2)}$ | **Air transport, freight** | 7763,62 | 7570,73 | 5607,18 | 2,48 | 27,78 |
| $f^{(3)}$ | **Number of flights** | 1,7 | 1,65 | 1,68 | 2,73 | 0,90 |
| $f^{(4)}$ | **Number of destination points** | 425 | 430,46 | 461,39 | 1,28 | 8,56 |
| $f^{(5)}$ | **International travelers** | 160,97 | 155,09 | 135,91 | 3,66 | 15,57 |
| $f^{(6)}$ | **Number of Int. dest. points** | 393 | 397,18 | 424,26 | 1,06 | 7,95 |
| $f^{(7)}$ | **International number of flights** | 1455,81 | 1417,25 | 1408,46 | 2,65 | 3,25 |
| $f^{(8)}$ | **ASK International** | 574,68 | 550,88 | 523,62 | 4,14 | 8,88 |
| $f^{(9)}$ | **ASK (Available seat kilometers)** | 589,71 | 568,66 | 540,08 | 3,57 | 8,42 |
| | **INDIA FACTORS** | **REEL** | **PRED wd** | **PRED** | **MAPE (%) wd** | **MAPE (%)** |
| $f^{(1)}$ | **Air passengers carried** | 167,5 | 152,06 | 193,16 | 9,22 | 15,32 |
| $f^{(2)}$ | **Air transport, freight** | 1938,23 | 2164,75 | 3270,45 | 11,69 | 68,73 |
| $f^{(3)}$ | **Number of flights** | 1,45 | 1,21 | 1,71 | 16,56 | 17,75 |
| $f^{(4)}$ | **Number of destination points** | 197 | 172,44 | 235,92 | 12,47 | 19,75 |
| $f^{(5)}$ | **International travelers** | 46,43 | 44,34 | 57,08 | 4,52 | 22,92 |
| $f^{(6)}$ | **Number of Int. dest. points** | 97 | 89,53 | 119,16 | 7,70 | 22,85 |
| $f^{(7)}$ | **International number of flights** | 381,5 | 314,27 | 481,11 | 17,62 | 26,11 |
| $f^{(8)}$ | **ASK International** | 268,46 | 274,68 | 338,80 | 2,32 | 26,20 |
| $f^{(9)}$ | **ASK (Available seat kilometers)** | 428,93 | 378,80 | 518,98 | 11,69 | 20,99 |

*Continuation of Table 3*

| | SINGAPORE FACTORS | REEL | PRED wd | PRED | MAPE (%) wd | MAPE (%) |
|---|---|---|---|---|---|---|
| $f^{(1)}$ | **Air passengers carried** | 43,05 | 40,31 | 24,36 | 6,36 | 43,41 |
| $f^{(2)}$ | **Air transport, freight** | 6411,99 | 6105,87 | 13173,78 | 4,77 | 105,46 |
| $f^{(3)}$ | **Number of flights** | 0,37 | 0,36 | 0,31 | 2,70 | 17,19 |
| $f^{(4)}$ | **Number of destination points** | 158 | 151,53 | 119,97 | 4,09 | 24,07 |
| $f^{(5)}$ | **International travelers** | 7,91 | 7,33 | 5,72 | 7,32 | 27,75 |
| $f^{(6)}$ | **Number of Int. dest. points** | 156 | 152,18 | 123,51 | 2,45 | 20,83 |
| $f^{(7)}$ | **International number of flights** | 186,37 | 184,60 | 154,33 | 0,95 | 17,19 |
| $f^{(8)}$ | **ASK International** | 149,27 | 131,27 | 99,04 | 12,06 | 33,65 |
| $f^{(9)}$ | **ASK (Available seat kilometers)** | 298,53 | 278,80 | 198,08 | 6,61 | 33,65 |
| | **TURKEY FACTORS** | **REEL** | **PRED wd** | **PRED** | **MAPE (%) wd** | **MAPE (%)** |
| $f^{(1)}$ | **Air passengers carried** | 111,13 | 116,25 | 147,76 | 4,60 | 32,96 |
| $f^{(2)}$ | **Air transport, freight** | 6815,51 | 6401,49 | 4402,77 | 6,07 | 35,40 |
| $f^{(3)}$ | **Number of flights** | 0,47 | 0,49 | 0,75 | 3,69 | 57,85 |
| $f^{(4)}$ | **Number of destination points** | 379 | 372,45 | 579,84 | 1,73 | 52,99 |
| $f^{(5)}$ | **International travelers** | 65,67 | 57,48 | 85,32 | 12,47 | 29,92 |
| $f^{(6)}$ | **Number of Int. dest. points** | 327 | 318,84 | 499,34 | 2,49 | 52,70 |
| $f^{(7)}$ | **International number of flights** | 597,82 | 562,36 | 850,31 | 5,93 | 42,24 |
| $f^{(8)}$ | **ASK International** | 296,04 | 275,14 | 410,31 | 7,06 | 38,60 |
| $f^{(9)}$ | **ASK (Available seat kilometers)** | 331,72 | 316,54 | 465,30 | 4,58 | 40,27 |
| | **UK FACTORS** | **REEL** | **PRED wd** | **PRED** | **MAPE (%) wd** | **MAPE (%)** |
| $f^{(1)}$ | **Air passengers carried** | 142,39 | 153,05 | 171,29 | 7,48 | 20,29 |
| $f^{(2)}$ | **Air transport, freight** | 5851,19 | 5839,65 | 6767,48 | 0,20 | 15,66 |
| $f^{(3)}$ | **Number of flights** | 1,89 | 1,91 | 2,15 | 0,76 | 13,50 |
| $f^{(4)}$ | **Number of destination points** | 486 | 488,38 | 533,35 | 0,49 | 9,74 |
| $f^{(5)}$ | **International travelers** | 228,3 | 225,52 | 245,91 | 1,22 | 7,71 |
| $f^{(6)}$ | **Number of Int. dest. points** | 423 | 415,31 | 460,30 | 1,82 | 8,82 |
| $f^{(7)}$ | **International number of flights** | 1596,61 | 1596,25 | 1793,54 | 0,02 | 12,33 |
| $f^{(8)}$ | **ASK International** | 826,71 | 809,99 | 887,38 | 2,02 | 7,34 |
| $f^{(9)}$ | **ASK (Available seat kilometers)** | 838,84 | 820,48 | 901,82 | 2,19 | 7,51 |
| | **USA FACTORS** | **REEL** | **PRED wd** | **PRED** | **MAPE (%) wd** | **MAPE (%)** |
| $f^{(1)}$ | **Air passengers carried** | 926,74 | 872,11 | 922,91 | 5,89 | 0,41 |
| $f^{(2)}$ | **Air transport, freight** | 42498,26 | 40829,66 | 43626,22 | 3,93 | 2,65 |
| $f^{(3)}$ | **Number of flights** | 10,28 | 9,89 | 10,65 | 3,77 | 3,67 |
| $f^{(4)}$ | **Number of destination points** | 916,00 | 927,39 | 1002,51 | 1,24 | 9,44 |
| $f^{(5)}$ | **International travelers** | 143,97 | 138,47 | 142,39 | 3,82 | 1,10 |
| $f^{(6)}$ | **Number of Int. dest. points** | 254,00 | 263,56 | 283,41 | 3,76 | 11,58 |
| $f^{(7)}$ | **International number of flights** | 1651,94 | 1626,54 | 1727,07 | 1,54 | 4,55 |
| $f^{(8)}$ | **ASK International** | 1620,70 | 1590,50 | 1631,39 | 1,86 | 0,66 |
| $f^{(9)}$ | **ASK (Available seat kilometers)** | 3030,03 | 2926,58 | 3028,75 | 3,41 | 0,04 |

The most predictable factors for China were air transport passengers carried, number of int. destination points, and number of destination points. The least predictable factors for China were international travelers, the number of flights, and ASK international.

The most predictable factors for France were the number of destination points, number of flights, and international number of flights. The least predictable factors for France were ASK (Available seat kilometers), international travelers, and air transport freight.

The most predictable factors for Germany were the number of int. destination points, number of destination points, and air transport freight. The least predictable factors for Germany were ASK (Available seat kilometers), international travelers and ASK international.

ASK international, international travelers, and the number of international destination points were the most predictable factors for India. The least predictable factors for India were the number of destination points, number of flights, and international number of flights.

The most predictable factors for Singapore were the international number of flights, number of international destination points, and number of flights. The least predictable factors for Singapore were ASK (Available seat kilometers), international travelers and ASK international.

The most predictable factors for Turkey were the number of destination points, the number of international destination points, and the number of flights. The least predictable factors for Turkey were air transport freight, ASK international, and international travelers.

The international number of flights, air transport freight, and number of destination points were the most predictable factors for the UK. The least predictable factors for the UK were ASK international, ASK (Available seat kilometers) and air passengers carried.

The most predictable factors for the USA were the number of destination points, the international number of flights, and ASK international. The least predictable factors for the USA were international travelers, air transport freight, and air passengers carried.

## 4. Conclusion

The current study aimed to model the data, and make future predictions with a novel method. Therefore, differential equations obtained by modeled data were generalized using a fractional derivative method, and a novel, competent, and flexible approach was proposed. The dataset was modeled with the least margin of error with the least squares method, and other necessary goals were examined afterwards.

In this paper, the efficiency of multi-input deep assessment system with first order derivative (MDAM wd), and multi-input deep assessment system (MDAM) models in simulating air-transport data are compared and contrasted. It is concluded that a multi-input deep assessment system with first order derivative model (MDAM wd) is far superior not only in modeling capabilities but also in applicability to predict such data.

Accurate results were obtained for modeling aviation factors in the current study, such that the maximum error was found to be 1,59E-02 %, for Germany’s air transport freight factor. Regarding prediction, 90.278 % of the errors were smaller than 10% for a total of 72 predictions shown in Table 3. The result of the proposed method is quite satisfactory and gives better results compared to the previous non-derivative method.

Moreover, the most predictable country was found to be the UK; in second was Germany, the third was the USA, the fourth was France, the fifth was China, the sixth was Singapore, the seventh was Turkey, and the last was India for 2019. The most predictable factors were the international number of flights and air transport freight of the UK, and the number of destination points in France. The least predictable factors were international travelers from Turkey, and number of flights, and the international number of flights to Singapore.

In summary, a new mathematical model for the literature is proposed by first developing a multi-input deep assessment system using fractional calculus. In terms of application, in the current study, air transport passengers carried, air transport freight, number of flights, number of destination points (airport), international travelers, number of international destination points (airport), international number of flights, ASK (Available seat kilometers) international, and ASK data between 2011-2019 years for eight world’s busiest airport owner countries were chosen. The data were modeled, and prediction studies were conducted and assessed. As shown by the modeling results in Table 2, modeling can be performed with near-zero error.

The new MDAM wd method can be applied to both strategic projects that civil aviation authorities can carry out over the course of months or years, as well as tactical operations that can be implemented daily in reaction to dynamic developments. For future studies, using this model for daily and monthly evaluation with the addition of the function’s second and higher order derivatives would be an improvement to our multi input deep assessment system (MDAM wd) model. While the current study has assessed each country by itself, the future study will explore the effects of transportation factors of one country on other nations, and the degree to which a change in one country can affect another.

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## 6. Data Availability

Matlab codes for MDAM wd modeling, and prediction will be made available on request.

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