

VALIDATING LITHIUM-POLYMER BATTERY DISCHARGE MODELS TO ENSURE UAV FLIGHT SAFETY AND PERFORMANCE

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ABSTRACT

This article proposes a methodology for assessing the adequacy of mathematical models for the discharge characteristics of lithium-polymer batteries (LPABs) used in unmanned aerial vehicles (UAVs). The methodology is based on ISO 5725 standards, focusing on the evaluation of trueness and precision under repeatability and reproducibility conditions. Experimental studies were conducted to collect data on LPAB discharge behavior and surface temperature dynamics across a range of environmental conditions (-20°C to $+50^{\circ}\text{C}$). Key analyses included systematic error evaluation, statistical variance assessment, and the formulation of precision criteria using tools such as pairwise differences, mean square deviation, and statistical tests. The results validate the developed regression and simulation models, confirming their reliability for predicting battery performance under dynamic operational conditions. The study provides a robust framework for UAV battery monitoring, enabling accurate prediction of flight duration and ensuring safe and efficient UAV operation across diverse environmental scenarios.

Keywords

Unmanned Aerial Vehicle (UAV), lithium-polymer battery (LPAB), battery discharge modeling, ISO 5725, flight safety.

List of abbreviations

UAS – unmanned aerial system
UAV – unmanned aerial vehicle
CMS – control and management system
LPAB – lithium polymer accumulator battery
UCM – unit cell modeling
AB – accumulator battery
MSD – mean square deviation
RAES – rechargeable autonomous energy source

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1. INTRODUCTION

The development and improvement of unmanned aerial systems (UAS), particularly unmanned aerial vehicles (UAVs), are at the forefront of modern aviation research. UAVs can be powered through various technologies, including internal combustion engines, rechargeable batteries, solar energy and other emerging innovations still under development. Among these options, electric motors powered by rechargeable batteries are the most widely used due to their efficiency and versatility.

Lithium-polymer batteries (LPABs) are used in many battery systems because of their low self-discharge rate, long service life, and high energy density and capacity. A control and management system (CMS) is required to optimally manage batteries to maintain safety and extend battery life. An effective CMS must be able to assess a battery's "state of health" and "state of charge" using advanced control methods. Most LPAB condition estimation algorithms require battery models that can accurately describe the LPAB voltage response during the current charge/discharge process. Therefore, battery modeling has become an integral part of CMS development.

Accurate battery modeling plays a pivotal role in planning UAV flight tasks, enabling improvements in UAV performance, and fostering advancements in remotely piloted systems. Additionally, precise models are crucial for predicting battery discharge behavior and estimating the remaining operational time – an especially critical factor for UAVs powered by LPABs.

Developing mathematical models of batteries requires preliminary discharge studies of typical batteries and subsequent evaluation of the collected data. Then the experimental results must be compared with simulation outputs to validate the model's accuracy. A model is considered adequate if its simulated characteristics are within the corridor of permissible values of dispersion of LPAB discharge characteristics. Such validation ensures the reliability of the simulation model. By verifying the accuracy of these mathematical models, researchers can reliably forecast UAV battery performance, estimate flight durations, and enhance the overall safety of UAV operations.

In this paper, we present a systematic methodology to assess the adequacy of lithium-polymer battery discharge models for UAV applications using established metrological standards. Employing ISO 5725 guidelines, we quantify both the trueness and precision of our models under repeatability and reproducibility conditions. Our approach involves conducting controlled experimental tests on LPAB samples, analyzing their voltage discharge profiles and surface temperature dynamics over a broad range of environmental conditions. The results are then used to validate the developed regression and simulation models, ensuring that they meet rigorous performance and reliability criteria.

2. VERIFICATION METHODOLOGY

2.1. THE IMPORTANCE OF MODEL VERIFICATION

To verify developed models of battery operation – such as those presented in [1, 2] – it is essential to verify their accuracy against real-world conditions. Achieving this verification requires systematically evaluating the quality of experimental data obtained from testing actual LPABs under various temperatures and operational scenarios. By comparing the measured discharge characteristics against the outcomes predicted by these previously developed models, we can identify permissible deviations and refine regression-based representations of battery behavior. This verification process is guided by ISO 5725-1:2023 standards [3], which define accuracy as encompassing both trueness and precision (in terms of repeatability and reproducibility).

Table 1 summarizes the measured discharge characteristics of a lithium-polymer battery across a wide range of external temperatures. These results serve as key reference points for validating the accuracy and consistency of the developed battery discharge models under diverse environmental conditions.

Table 1. Results of one-time tests of LPAB at different external temperatures.

T _{EXT}	0 m.	1 m.	2 m.	3 m.	4 m.	5 m.	6 m.	7 m.	8 m.
+50°C	12.49	11.32	11.06	10.81	10.77	10.65	10.41	9.38	9.27
+49°C	12.46	11.33	11.06	10.81	10.75	10.63	10.33	9.38	9.32
+48°C	12.52	11.32	11.11	10.93	10.80	10.68	10.48	9.41	9.31
+47°C	12.50	11.32	11.09	10.89	10.77	10.65	10.31	9.41	9.31
+46°C	12.49	11.29	11.07	10.88	10.76	10.63	10.21	9.41	9.34
+45°C	12.50	11.26	11.05	10.88	10.75	10.60	10.14	9.43	9.27
+40°C	12.50	11.15	11.01	10.80	10.70	10.55	10.00	9.38	9.32
+35°C	12.50	11.08	10.91	10.76	10.64	10.44	9.42	9.42	9.31
+30°C	12.50	11.01	10.88	10.74	10.61	10.39	9.43	9.40	9.34
+25°C	12.50	10.95	10.82	10.70	10.57	10.36	9.41	9.40	9.27
+20°C	12.48	10.88	10.77	10.68	10.57	10.37	9.42	9.40	9.32
+15°C	12.50	10.87	10.65	10.58	10.38	10.11	9.40	9.37	9.30
+10°C	12.49	10.86	10.59	10.48	10.29	9.94	9.39	9.37	9.33
+5°C	12.50	10.60	10.50	10.39	10.21	9.82	9.38	9.36	9.32

+4°C	12.49	10.67	10.53	10.36	10.21	10.01	9.46	9.40	9.34
+3°C	12.50	10.66	10.54	10.40	10.24	10.06	9.53	9.35	9.27
+2°C	12.48	10.63	10.51	10.42	10.28	10.07	9.60	9.34	9.32
+1°C	12.49	10.71	10.53	10.44	10.25	10.06	9.56	9.35	9.31
0°C	12.50	10.77	10.56	10.42	10.26	10.03	9.53	9.33	9.31
-1°C	12.49	10.45	10.53	10.40	10.22	10.01	9.50	9.32	9.31
-2°C	12.49	10.75	10.52	10.37	10.19	9.93	9.48	9.33	9.32
-3°C	12.50	10.70	10.49	10.36	10.19	9.92	9.45	9.33	9.32
-4°C	12.49	10.68	10.47	10.35	10.17	9.90	9.45	9.33	9.31
-5°C	12.49	10.67	10.46	10.34	10.17	9.89	9.43	9.32	9.27
-10°C	12.49	10.79	10.46	10.29	10.09	9.77	9.49	9.32	9.32
-15°C	12.48	10.42	10.19	10.02	9.76	9.66	9.43	9.33	9.31
-17°C	12.49	9.83	10.03	10.02	9.82	9.42	9.51	9.32	9.31
-20°C	12.48	9.72	9.83	9.88	9.51	9.63	9.50	9.35	9.34

In the context of ISO 5725-1:2023, the term *accuracy* encompasses both *trueness* and *precision*. *Trueness* refers to the closeness of agreement between the average value obtained from a large series of test results and the accepted reference value, indicating the absence of systematic error. *Precision* denotes the closeness of agreement between independent test results under stipulated conditions, reflecting the extent of random errors. Precision is further divided into *repeatability* (conditions where independent test results are obtained with the same method on identical test items in the same laboratory by the same operator using the same equipment within short intervals of time) and *reproducibility* (conditions where test results are obtained with the same method on identical test items in different laboratories with different operators using different equipment).

In this study, repeatability was assessed by conducting multiple charge/discharge cycles on a single LPAB sample, allowing for the evaluation of variability in discharge characteristics and surface temperature within the same laboratory setting. Reproducibility, in turn, was evaluated by testing multiple LPAB samples with identical passport data across different laboratories, accounting for variations due to manufacturing processes and different testing environments.

By adhering to the definitions and guidelines outlined in ISO 5725-1:2023, this study ensures a rigorous assessment of the trueness and precision of the experimental data, thereby validating the reliability of the developed battery models.

To investigate the discharge characteristics and surface temperature behavior of lithium-polymer batteries (LPABs) under controlled laboratory conditions, experimental studies were conducted on a single battery sample at a fixed ambient temperature of +25°C. Each sample was tested 10 times using a mock-up scheme in a climatic chamber. The collected data provide insight into discharge behavior and thermal dynamics during operation (Tables 2, 3).

Table 2. Measurements of LPAB discharge characteristics of at +25°C.

Meas. No.	0 m.	1 m.	2 m.	3 m.	4 m.	5 m.	6 m.	7 m.	8 m.
1	12.52	11.07	10.88	10.75	10.63	10.48	10.08	9.38	9.37
2	12.48	11.09	10.89	10.76	10.64	10.5	10.18	9.38	9.37
3	12.49	11.1	10.91	10.76	10.65	10.51	10.25	9.38	9.37
4	12.52	11.08	10.87	10.73	10.61	10.46	10.13	9.37	9.36
5	12.48	11.07	10.88	10.75	10.66	10.54	10.21	9.36	9.36
6	12.49	11.08	10.83	10.72	10.62	10.46	9.98	9.36	9.35
7	12.51	10.99	10.82	10.69	10.59	10.47	10.21	9.35	9.34
8	12.49	10.96	10.75	10.62	10.49	10.33	9.97	9.35	9.34
9	12.49	10.97	10.78	10.65	10.55	10.41	10.11	9.36	9.35
10	12.5	11.02	10.85	10.72	10.62	10.49	10.1	9.36	9.36

Table 3. Measurements of LPAB surface temperature of an LPAB at +25°C.

Meas. No.	0 m.	1 m.	2 m.	3 m.	4 m.	5 m.	6 m.	7 m.	8 m.
1	22.6	25.6	31.5	33.5	35.7	36.8	35.8	34.9	34.7
2	22.6	25.6	30.7	33.4	35.5	36.8	35.8	35.2	34.7
3	22.6	25.3	31.5	33.3	35.6	36.6	35.8	35.5	34.7
4	22.5	25.6	31.5	33.4	35.5	36.7	36.5	35.2	34.5
5	22.6	25.6	29.7	33.3	35.6	36.8	35.8	35.1	34.7
6	22.5	25.6	31.5	33.2	35.4	36.8	36.5	35.2	34.7
7	22.6	25.8	31.5	33.4	35.5	36.5	35.8	35.3	34.7
8	22.6	25.6	29.9	33.4	35.6	36.7	35.8	35.2	34.7
9	22.7	25.6	31.5	33.4	35.6	36.8	36.5	35.2	34.6
10	22.6	25.6	29.9	33.4	35.6	36.7	35.8	35.2	34.7

The experimental data are visually represented in Figures 1 and 2.

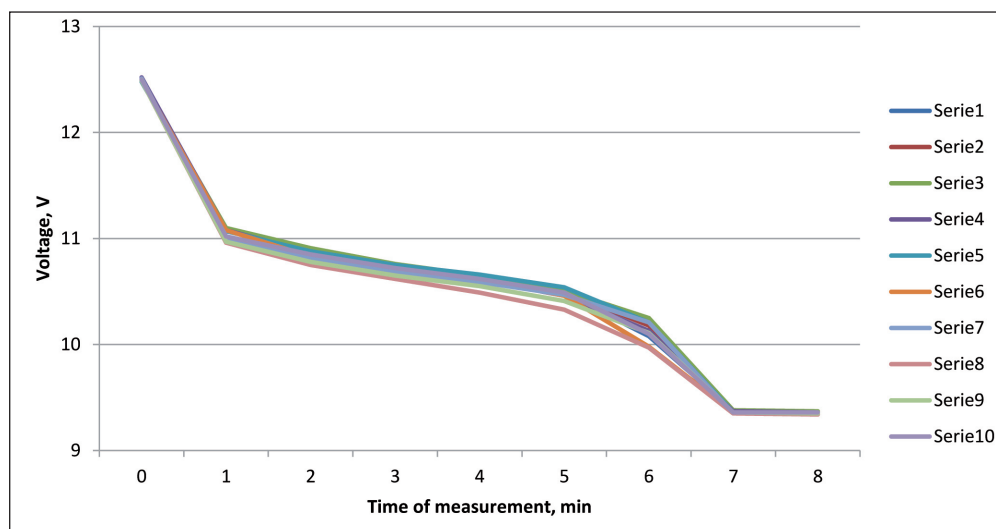


Fig. 1. Family of AB discharge characteristics at +25°C.

2.2. ANALYSIS OF TEST RESULTS UNDER REPEATABILITY CONDITIONS

The analysis of test results under repeatability conditions is a critical step in ensuring the accuracy and reliability of experimental data. The process adheres to ISO 5725-1:2002 guidelines and includes the following steps, as illustrated in Figure 2.

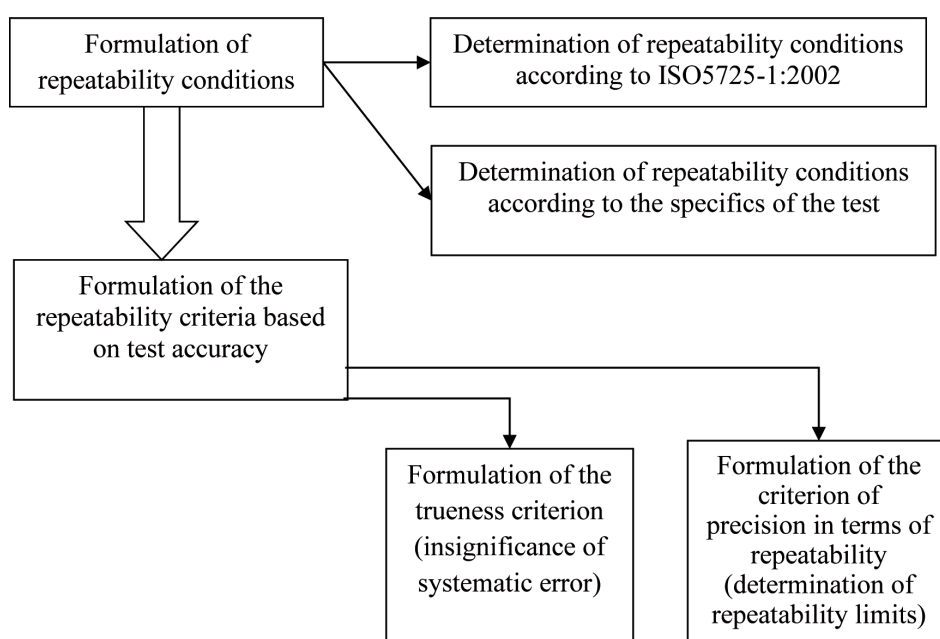


Fig. 2. Algorithm for analyzing results under repeatability conditions.

Formulation of repeatability conditions

According to ISO 3534-1 [4], repeatability and repeatability conditions are defined as follows:

- Repeatability – the precision achieved under repeatability conditions.
- Repeatability conditions – the conditions under which independent measurement or test results are obtained using the same method, on identical test objects, in the same laboratory, by the same operator, using the same equipment, within a short period of time.

In this study, AP discharge characteristics of battery (AB) are measured in certain climatic conditions (at a given temperature). Given the dynamic nature of the discharge process, with voltage measurement at fixed time intervals, it is not feasible to evaluate for a static point over a certain period of time. Hence, the repeatability conditions for this study will be formulated as follows:

- Repeatability conditions – the conditions under which independent results of multiple repetitions in the discharge process are obtained using the same method, on the same research object, in the same laboratory, by the same operator, using the same equipment, at the same time [5, 6].

To verify the developed battery models under repeatability conditions, the trueness and precision of the experimental results were evaluated. This evaluation was performed using a single lithium-polymer AB, observing the repeatability conditions established above. The voltmeter readings were taken at intervals of 1 minute until the engine stopped, 10 times in a row.

Evaluation of trueness under repeatability conditions

To assess the trueness of the results, it is necessary to evaluate the *systematic error* within the data, that is, estimating any drift in the measurement error. This involves analyzing the data set presented in Table 2 to determine whether the results are influenced by systematic deviations. Next, to assess the repeatability, we take the data of the second column of Table 2 (the 0 min. column) and perform a check for the presence of a systematically component error.

Table 4. LPAB discharge characteristics at +25°C obtained at 0 min. from the moment the engine starts.

Measurement No.	0 m.
1	12.52
2	12.48
3	12.49

4	12.52
5	12.48
6	12.49
7	12.51
8	12.49
9	12.49
10	12.5

1) Pairwise differences d_i between consecutive voltage readings were calculated according to the formula:

$$d_i = x_i - x_{i+1}, \quad (1)$$

where x_i represents the obtained voltage value from the data array.

2) The average value of pairwise differences $\bar{d}$ is calculated according to the formula:

$$\bar{d} = \frac{1}{n-1} \cdot \sum_{i=1}^{n-1} d_i \quad (2)$$

where n is the total number of measurements.

3) The average value $\bar{x}$ in a row is calculated according to the formula:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}. \quad (3)$$

4) The average score of the sample mean square deviation (MSD) σ by a number of measurements is calculated as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n-1} (x_i - \bar{x})^2}{n-1}}. \quad (4)$$

The drift is not considered to be significant if the condition $|\bar{d}| < 0.2 \cdot \sigma$ is met.

Therefore, taking into account the obtained result, we can conclude that the systematic error of the obtained results is insignificant. The calculations described above are repeated for each column of Table 2 and for each set temperature value, making it possible to conclude that the obtained results are correct in terms of repeatability [7].

Formation of the criterion of precision in terms of repeatability

Since the accuracy of the obtained results was evaluated for repeatability at each point in time, it is necessary to first generate a series of data (voltage measurements at the same point in time).

The limit of repeatability r – the difference between two measurement results – is used as a measure to assess precision. Since we have a series of 10 results, the first step is to find the difference between two measurement results in all possible combinations, a , i.e. to obtain an array of 45 difference values. The standard deviation for each of them will be $\sigma \cdot \sqrt{2}$.

To consider the difference between two random variables, the critical range coefficient f is used, which depends on the level of probability and the distribution law of a random variable, i.e. $f \cdot \sigma \cdot \sqrt{2}$. For repeatability and reproducibility limits, the confidence probability is 95% and the underlying distribution is considered approximately normal. With this in mind, $f = 1.96$, and for statistical evaluation we will adopt $f \cdot \sigma \cdot \sqrt{2} = 1.96 \cdot \sigma \cdot \sqrt{2} = 2.8 \cdot \sigma$.

Thus, the criterion for assessing the precision of the test results in terms of repeatability is defined as the ratio of the difference between the two measurement results a in all possible combinations with a repeatability limit $r = 2.8 \cdot \sigma_r$. If $a \leq r$, then the results are considered to exhibit precision under the repeatability conditions, otherwise they are deemed to lack precision under the repeatability conditions with the chosen confidence probability [8].

Algorithm for assessing the precision of the obtained data in terms of repeatability:

- 1) First, a series of data is obtained (voltage measurements at the same point in time for 1 sample of LPAB) (Table 5).
- 2) The next step is to find the difference between two measurement results in all possible combinations, yielding an array of 45 values (Table 5).

Table 5. Calculation of discrepancies a .

0.04								
0.03	-0.01							
0	-0.04	-0.03						
0.04	0	0.01	0.04					
0.03	-0.01	0	0.03	-0.01				
0.01	-0.03	-0.02	0.01	-0.03	-0.02			
0.03	-0.01	0	0.03	-0.01	0	0.02		
0.03	-0.01	0	0.03	-0.01	0	0.02	0	
0.02	-0.02	-0.01	0.02	-0.02	-0.01	0.01	-0.01	-0.01

3) We find the average value $\bar{a}$ according to the formula:

$$\bar{a} = \frac{\sum_{i=1}^n a_i}{n}, \quad (5)$$

where a_i is the value of discrepancies and n - is the total number of discrepancies.

4) The average estimate of the sample mean squared deviation of the difference σ_r is calculated:

$$\sigma_r = \sqrt{\frac{\sum_{i=1}^{n-1} (a_i - \bar{a})^2}{n-1}}. \quad (6)$$

5) The value of the repeatability limit is calculated:

$$r = 2.8 \cdot \sigma_r. \quad (7)$$

6) Next each value a is compared with the repeatability limit r . If less than 5% of the obtained values of discrepancies a satisfy the criterion $a \leq r$, then we can assume the test results exhibit precision of in terms of repeatability.

The calculations described above, repeated for each column of Table 3 for each set temperature value, confirm compliance with the conditions of precision of the test results in terms of repeatability throughout the experiment. The findings indicate that the discharge characteristics do not undergo significant changes during multiple charge/discharge cycles. This consistency supports the formulation of a regression model of the discharge characteristics for a specific LPAB sample at a fixed temperature, validating the theoretical models under repeatability conditions.

2.3. ANALYSIS OF TEST RESULTS UNDER REPRODUCIBILITY CONDITIONS

To evaluate the reproducibility of the tests, voltage measurements and discharge characteristic assessments were carried out 10 times for several LPAB samples with identical passport data at a fixed temperature. These tests were performed at a fixed ambient temperature of +25°C to ensure consistency in conditions

The results of these tests for one representative sample, identified as Sample No. 2, are presented in Table 6. This data serves as a basis for analyzing the variability between samples and assessing the reproducibility of the experimental methodology.

Table 6. Discharge characteristics for Sample No. 2 at +25°C.

Meas. No.	0 m.	1 m.	2 m.	3 m.	4 m.	5 m.	6 m.	7 m.	8 m.
1	12.5	11.06	10.89	10.76	10.63	10.48	10.08	9.38	9.37
2	12.48	11.09	10.89	10.77	10.65	10.5	10.17	9.38	9.37
3	12.49	11.09	10.9	10.76	10.65	10.5	10.25	9.38	9.37
4	12.52	11.09	10.88	10.73	10.61	10.47	10.13	9.37	9.36
5	12.48	11.07	10.87	10.76	10.65	10.54	10.19	9.36	9.35
6	12.49	11.08	10.88	10.72	10.63	10.47	9.98	9.36	9.36
7	12.51	11	10.82	10.69	10.58	10.47	10.23	9.36	9.34
8	12.49	10.99	10.78	10.62	10.5	10.37	9.98	9.36	9.34
9	12.49	10.99	10.76	10.65	10.56	10.5	10.13	9.36	9.35
10	12.5	11.01	10.85	10.73	10.62	10.49	10.2	9.37	9.36

The algorithm for analyzing the received data is shown in Fig. 3.

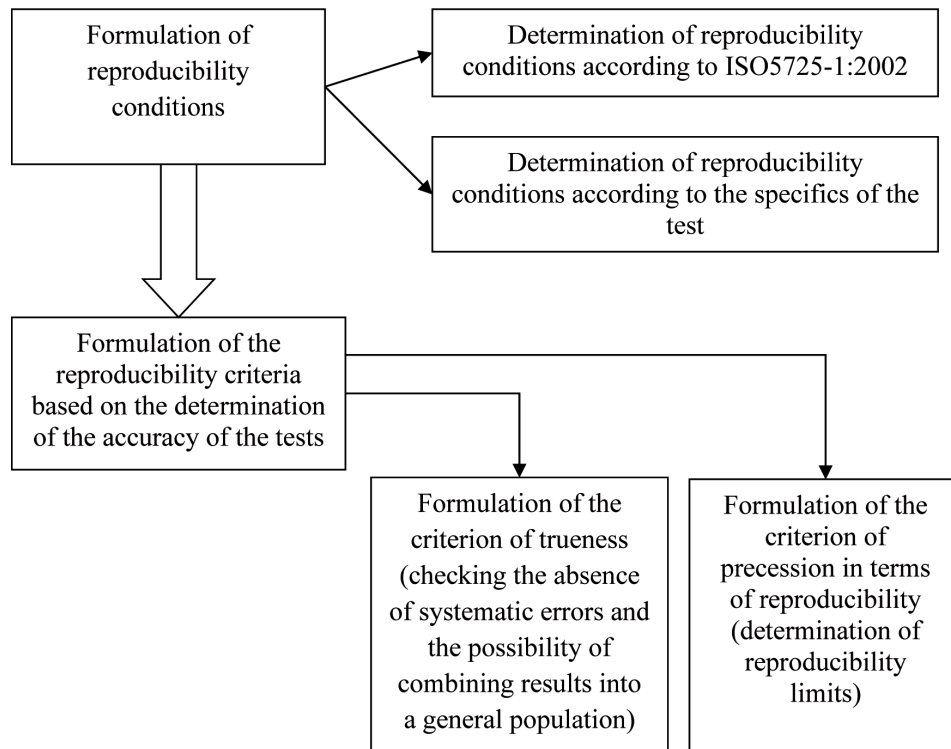


Fig. 3. Algorithm for analyzing results under reproducibility conditions.

Formulation of reproducibility conditions

According to ISO 3534-1, reproducibility and reproducibility conditions are formulated as follows:

- Reproducibility – the precision in terms of reproducibility.
- Reproducibility conditions – the conditions under which the results of measurements or tests are obtained using the same method, on identical objects of research, in different laboratories, by different operators, using different equipment.

In this study, AB discharge characteristics are measured in certain climatic conditions (at a given temperature). Given the dynamic nature of the discharge process, with voltage measurement at fixed time intervals, it is not feasible to evaluate for a static point over a certain period of time. Hence, the reproducibility conditions for this study will be formulated as follows:

- Reproducibility conditions – the conditions under which independent results of multiple repetitions in the discharge process are obtained using the same method, on several research objects identical according to their passport data, in the same laboratory, by the same operator, using the same equipment, at the same time.

Formation of the criterion of trueness in terms of reproducibility

To verify the developed battery models under reproducibility conditions, three sample lithium-polymer ABs with the same passport data were used and the reproducibility conditions established above were observed. The voltmeter readings were taken at intervals of 1 minute until the engine stopped, 10 times in a row. As an example, a comparison of the results for two sample LPAB are given (Table 3, Table 7).

Evaluation of trueness under reproducibility conditions

To assess the trueness of the results, it is necessary to estimate the difference between the average values of each group of measurements (bit characteristics for one sample) and the ratio of the difference between the average scores of sample MSDs. For this, we use data arrays (Table 2, Table 6).

Next, to assess the reproducibility, we take the data of the second column of Table 2 and Table 6 (the 0 min. column) and perform a check for the presence of a systematically component error.

Table 7. LPAB discharge characteristics for Sample No. 2 at +25°C obtained at 0 min. from the moment the engine starts.

Measurement No.	0 m.
1	12.5
2	12.48
3	12.49
4	12.52
5	12.48
6	12.49
7	12.51
8	12.49
9	12.49
10	12.5

- 1) The average value $\bar{x}_j$ by row in each table is calculated according to the formula:

$$\bar{x}_j = \frac{\sum_{i=1}^n x_{ij}}{n}, \quad (8)$$

where x_{ij} represents the voltage values from the table, n is the number of measurements, and j is the sample AB number.

- 2) The obtained average values $\bar{x}_j$ are ranked from largest to smallest:

$$\bar{x}_{j\max} \rightarrow \bar{x}_{j\min}.$$

- 3) The average score of the sample MSD σ_j by a series of measurements for each of the tables is calculated as follows:

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{n-1} (x_{ij} - \bar{x}_j)^2}{n-1}}. \quad (9)$$

- 4) Using Student's criterion, we determine the possibility of combining the obtained data series into one general population, for this we find the value according to the formula:

$$t = \frac{\bar{x}_{j\max} - \bar{x}_{j\min}}{\sqrt{\frac{\sigma_{\max}^2}{n} + \frac{\sigma_{\min}^2}{n}}}, \quad (10)$$

where $\sigma_{\max}^2$ is the sample variance with the maximum mean $\bar{x}_{j\max}$ and $\sigma_{\min}^2$ is the sample variance with the minimum mean $\bar{x}_{j\min}$.

Next, we define the value t_{kr} using the table, setting a significance level of 0.05 and determining the number of degrees of freedom $f = n_1 + n_2 - 2$.

The obtained values t and t_{kr} are compared; if $t \leq t_{kr}$, then we can assume that the obtained data series can be combined into one general population.

5) The obtained values of the average scores of the sample MSD σ_j are ranked from largest to smallest.

$$\sigma_{j\max} \rightarrow \sigma_{j\min}.$$

6) The equality of the obtained variances is checked according to Fisher's test. For this, we find the value of the criterion F_{emp} according to the formula:

$$F_{emp} = \frac{\sigma_{j\max}}{\sigma_{j\min}}. \quad (11)$$

If the obtained criterion value F_{emp} is less than the critical value F_{kr} for a certain level of significance and the corresponding number of degrees of freedom, the variances can be considered equal to each other, and the considered aggregates of measurement results can be combined into one general aggregate. In this way, the correctness of the measurement method will be proven in terms of reproducibility.

According to the results of the experiment, we obtain:

$$F_{emp} = \frac{\sigma_{j\max}}{\sigma_{j\min}}.$$

The critical value of the criterion F_{kr} is chosen from the table, taking into account that the number of degrees of freedom for each row will be $n - 1 = 10 - 1 = 9$, and the chosen significance level of 0.05.

$$F_{kr} = 3.18 \quad F_{emp} = 2.83 \quad \Rightarrow F_{kr} > F_{emp}$$

Therefore, we can conclude that the correctness of the measurement method in terms of reproducibility, based on the considered experimental data, has been proven.

Having carried out the calculations described above for each column of Table 2 and Table 4 and for each set temperature value, we can conclude that the measurement method is correct in terms of reproducibility throughout the entire experiment.

Formulation of the criterion of precision under reproducibility conditions

Since the precision of the obtained results is evaluated for reproducibility at each point in time, here we begin with a series of data (voltage measurements at the same

point in time). The repeatability limit is used as a measure for evaluating R – the difference between two measurement results. Since we have several series of data (measurement results of two samples), which according to the calculations of the previous points can be combined into one general population, the difference between the data will be defined as the difference between each of the results and the average value for the general population. The standard deviation will be $\sigma \cdot \sqrt{2}$.

To consider the difference between two random variables, the critical range coefficient f is again used, which depends on the probability level and the distribution law of the random variable, i.e. $f \cdot \sigma \cdot \sqrt{2}$. For repeatability and reproducibility limits, the confidence probability is again 95% and the underlying distribution is considered approximately normal. With this in mind, $f = 1.96$, and for statistical evaluation we will again adopt $f \cdot \sigma \cdot \sqrt{2} = 1.96 \sigma \cdot \sqrt{2} = 2.8 \cdot \sigma$.

Thus, the criterion for assessing the precision of test results under reproducibility conditions is defined as the ratio of the difference between the measurement results and the general average b_{ij} in all possible combinations with a repeatability limit $R = 2.8 \cdot \sigma_R$. If $b_{ij} \leq R$, then the results are considered to exhibit precision under the reproducibility conditions, otherwise they are deemed to lack precision under the reproducibility conditions with the chosen confidence probability.

Algorithm for assessing the precision of the obtained data in terms of reproducibility:

- 1) First, a series of data is obtained (voltage measurements at the same point in time) (Table 8).
- 2) The next step is to find $\bar{x}$ – the average value for the entire general population of the obtained measurement results according to the formula:

$$\bar{x} = \frac{\sum_{j=1}^k x_j}{k}, \quad (12)$$

where k is the number of AB samples.

- 3) We find the value of the differences b_{ij} :

$$b_{ij} = x_{ij} - \bar{x}. \quad (13)$$

- 4) Next the average estimate of the sample MSD difference σ_R is calculated according to the formula:

$$\sigma_R = \sqrt{\sigma_L^2 + \sigma_r^2}, \quad (14)$$

where σ_r^2 is the arithmetic average of variances for each sample $\sigma_r^2 = \frac{\sum_{i=1}^n \sigma_{ri}^2}{n}$,

σ_L^2 is the intersample variance found by the formula:

$$\sigma_L^2 = \frac{1}{k-1} \cdot \sum_{i=1}^k (\bar{x}_j - \bar{x})^2. \quad (15)$$

5) The value of the limit of reproducibility is calculated:

$$R = 2.8 \cdot \sigma_R. \quad (16)$$

6) Next each value b_{ij} is compared with the limit of reproducibility. If less than 5% of the obtained values of discrepancies b_{ij} satisfy the criterion $b_{ij} \leq R$, then we can assume the test results exhibit precision in terms of reproducibility.

Combining the obtained results into a set and calculating the average characteristic and its uncertainty

In the previous section, the criteria for the possibility of combining the obtained results into one set were given. If there are no systematic errors in several series, that is, the difference between the arithmetic mean and the difference in MSD estimates is permissible, then the series of measurement results can be considered series with the same dispersion [8]. The general average is determined by formula (8). The general scattering of results, i.e. the general variance S^2 is determined by the formula [9, 10]:

$$S^2 = \frac{(n-1)}{(n \cdot k - 1)} \cdot \sum_{j=1}^k S_j^2 + \frac{n}{n \cdot k - 1} \cdot \sum_{j=1}^k (\bar{x}_j - \bar{x})^2, \quad (17)$$

where S_j^2 is the variance of the sample of results for each sample.

The dispersion limit of the discharge characteristic results is determined by the formula:

$$C = 2 \cdot \sqrt{S^2}, \text{ and the scattering limits } \bar{x} \pm C$$

Thus, after performing the calculations for all values by time (Table 2, Table 6), we will obtain the average value, the variance of the total sample and the general dispersion of the results of the discharge characteristics for a temperature of +25°C (Table 8).

Table 8. Overall average value and overall dispersion of voltage measurement results at a temperature of +25°C.

	0 m.	1 m.	2 m.	3 m.	4 m.	5 m.	6 m.	7 m.	8 m.
	12.500	11.050	10.850	10.720	10.610	10.470	10.130	9.370	9.360
S^2	0.00018	0.0023	0.0024	0.0023	0.0024	0.0026	0.0086	0.000108	0.00013
C	0.027	0.095	0.098	0.096	0.097	0.102	0.190	0.021	0.023

These results are graphically represented in Fig. 4.

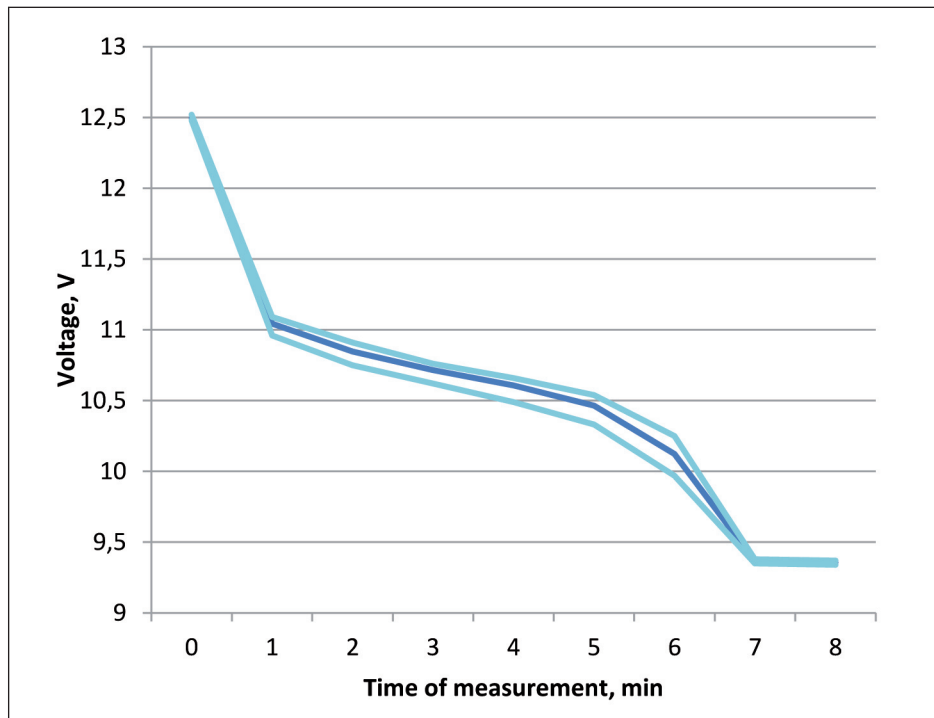


Fig. 4. Plot of overall average value and overall dispersion of discharge characteristic measurements at a temperature of +25°C.

Having carried out similar calculations for the characteristics of the surface temperature of LPAB [11], we will obtain the average value, the variance of the total sample and the general dispersion of the results of the characteristics of the surface temperature of LPAB for an ambient temperature of +25°C (Table 9).

Table 9. Overall average value and the overall dispersion of LPAB surface temperature measurements at an ambient temperature of +25°C.

	0 m.	1 m.	2 m.	3 m.	4 m.	5 m.	6 m.	7 m.	8 m.
$\bar{x}$	22.595	25.595	31.085	33.365	35.520	36.745	36.010	35.200	34.670
S^2	0.00155	0.00682	0.47818	0.00555	0.00800	0.00997	0.10832	0.02105	0.00642
C	0.079	0.165	1.383	0.149	0.179	0.199	0.658	0.290	0.160

These results are graphically represented in Fig. 5.

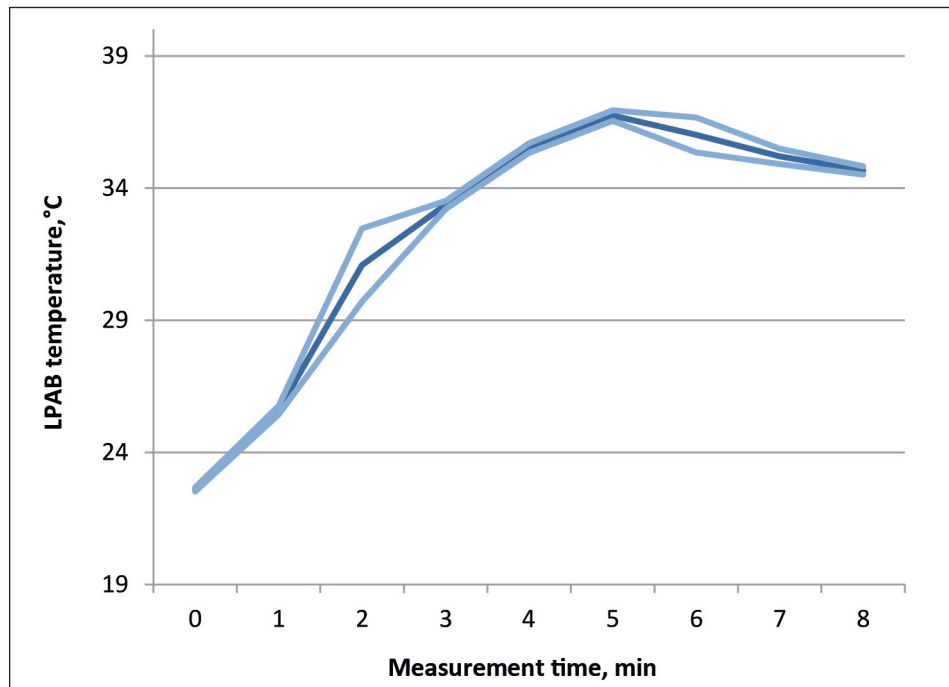


Fig. 5. Plot of overall average value and the overall dispersion of LPAB temperature characteristic measurements at an ambient temperature of +25°C.

After performing the above calculations for other temperature values, we obtain estimated discharge characteristics for the entire specified temperature range [12], graphically illustrated in Fig. 6.

Estimating the characteristics of the surface temperature according to the given algorithm makes it possible to estimate the LPAB surface temperature characteristics for the entire specified temperature range. The results of the calculations are shown in Fig. 7.

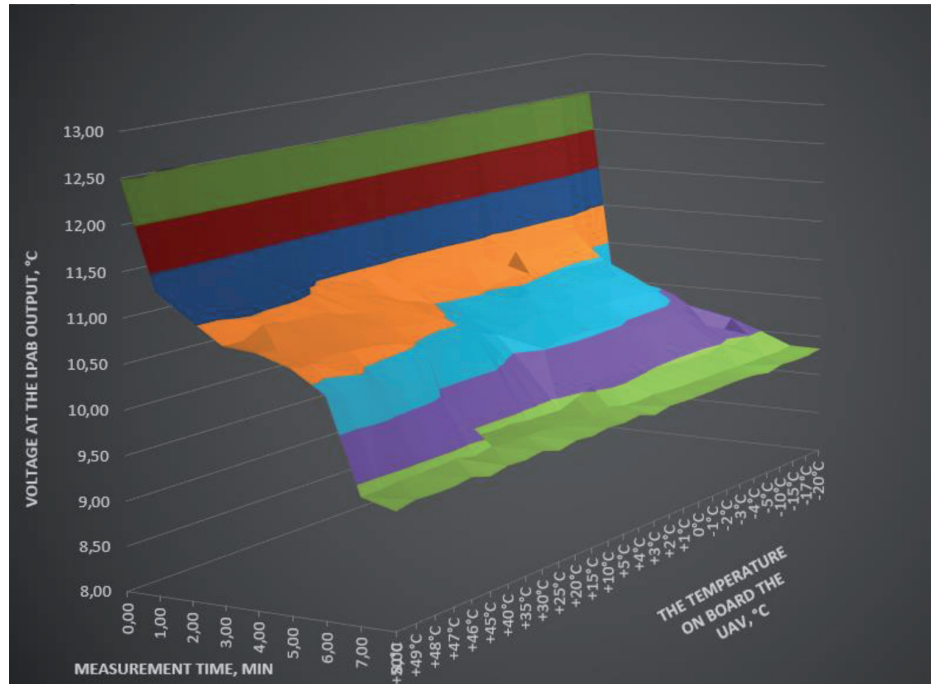


Fig. 6. Plot of overall average value of LPAB discharge characteristic measurements for the temperature range from -20°C to $+50^{\circ}\text{C}$.

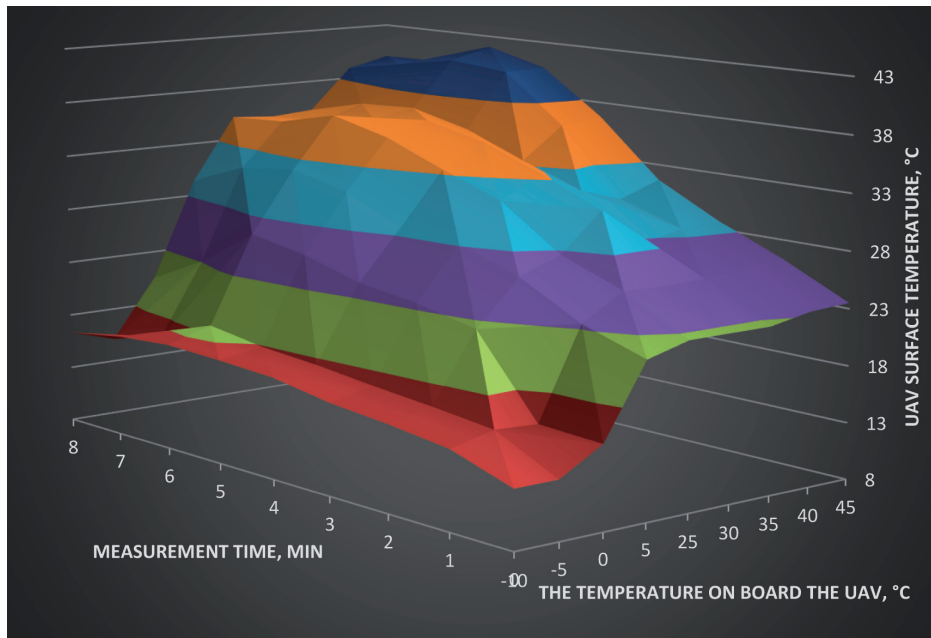


Fig. 7. Plot of the overall average value of LPAB surface temperature characteristic measurements of LPAB for the range of ambient temperatures from -20°C to $+50^{\circ}\text{C}$.

The obtained results indicate that when a specific battery is tested under varying environmental conditions, using measuring devices of the same accuracy class and identical load and temperature parameters, the results remain consistent in terms of accuracy. The reproducibility evaluation of the experimental data confirms that the developed mathematical models of LPAB discharge characteristics are reliable for any battery of this type. This enables the broad application of the proposed method for monitoring the state of the rechargeable autonomous energy source (RAES), tailored to the conditions and operational features of UAVs. These findings validate the mathematical and simulation models for lithium-polymer battery discharge, ensuring their practical usability in UAV operations.

3. CONCLUSIONS

This study presented and implemented a methodology grounded in ISO 5725 standards to verify the trueness and precision of lithium-polymer battery (LPAB) discharge models employed in UAV applications. By examining the experimental data under both repeatability and reproducibility conditions, we confirmed that the developed regression and simulation models accurately represent LPAB discharge behavior and thermal dynamics. These validated models facilitate reliable predictions of UAV flight duration, guide state-of-charge estimations, and ensure dependable operation of the control and management system (CMS) under a wide range of environmental conditions.

This verification framework allows researchers and practitioners to incorporate mathematically sound LPAB models into UAV load simulations, thereby enabling the precise forecasting of flight durations and enhancing overall flight safety. By reducing the risk of control loss and preventing potential aircraft failure, the methodology ensures robust UAV performance even in challenging scenarios. Moreover, the insights gained from this work contribute to refining CMS algorithms and inform the design of more efficient, reliable UAV power systems. Ultimately, this research strengthens our fundamental understanding of LPAB performance and directly supports improved safety, operational versatility, and cost-effectiveness in the expanding domain of unmanned aerial vehicles.

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