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Investigation of the Effect of Dilation Angle of Soil on the Performance of Grouted Tieback Under Axial Static Load by Analytical Approach

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Abstract
Studies on how non-associated flow rules affect the performance of grouted tieback using the upper bound of limit analysis theory are relatively rare. In this paper, the expression of the collapse load of a single, inclined, rough grouted tieback subjected to axial static load placed in a frictional soil was determined by constructing a failure mechanism by combining a non-associated flow rule in the upper bound limit analysis technique and the linear Mohr-Coulomb yield criterion. The effects of variation of dilation angle, the soil friction angle, the grouted tieback's angle of inclination, the bonded length, and the diameter of bonded length on the ultimate load were investigated. The results obtained from the present study clearly indicated that the dilation angle, the soil friction angle, the grouted tieback's angle of inclination, and the bonded length all significantly affect the collapse load but the bonded length's diameter has no effect on the critical load.

Keywords: axial static load, bonded length, diameter of bonded length, dilation angle, performance of grouted tieback

1 Introduction

Grouted tiebacks are tension members broadly used in different civil engineering practices as a soil reinforcement technique either temporarily or permanently because their application is an efficient, attractive, and economical design solution. These devices are used to control the deformation, and the displacement of engineered structures and to improve the bearing capacity of the soil. Grouted tiebacks are able to transfer the applied tensile load from the structure to the surrounding ground at greater depth through the friction mobilized at their interfaces.

These structural elements can be installed in an inclined position or vertically depending on the load orientation or the type of structure requiring support. The determination of the performance of grouted tiebacks is very important in the design and the dimensioning of these tension members.

However, their performance is not easily determined because it is affected by several parameters including the properties of the surrounding soil, the installation technique, the orientation of the grouted tiebacks, their diameter, and their length, the method of interaction between the surrounding soil and grouted tiebacks.

Numerous studies have been conducted by various researchers to compute the static uplift capacity of grouted tiebacks placed in frictional soils; the majority of past research has been based on empiricism . Some scholars [1, 2] proposed an expression to estimate the skin friction along the bonded length of grouted tiebacks. Other authors [3, 4, 5] suggested a formulation to calculate the critical load of grouted tieback. In addition, the failure mechanism was developed theoretically using the upper bound of limit analysis and linear Mohr-Coulomb failure criterion by [6, 7, 9, 12] to investigate the collapse load of single vertical grouted tieback under uplift tensile force and they considered the influence of the length of grouted tieback and the friction angle of the soil on the ultimate load. The researchers [11] also evaluated the ultimate pullout capacity which is expressed according to the bearing capacity factor of a single inclined grouted tieback subjected to axial static loading based on the kinematic approach of limit analysis and the linear Mohr-Coulomb yield criterion. They identified the effect of the angle of slope, the inclination of grouted tieback and the friction angle of the soil on the non-dimensional factor. The authors [13] compare the performance of inclined and vertical grouted tiebacks subjected to axial static loading placed in three sandy soils using the upper bound limit analysis method to investigate the variation of the bonded length and the diameter of the bonded length on the ultimate pullout capacity. Several researchers have used this theorem extensively to come up with solid solutions to a variety of geotechnical engineering problems [14, 15, 16, 17, 18].

However, the existing studies are based on the associated flow rule to determine the performance of grouted tieback under axial static loading using the upper bound theorem of limit analysis; the dilatancy impact was not taken into account in any investigation.

The objective of the study described in this work is:
- To incorporate the non-associated flow rule in the upper bound theorem of limit analysis and also to find the influence of dilation angle (ψ) on the pullout capacity of single, inclined, rough grouted tieback subjected to axial static load placed in frictional soil .
- To examine the variation of limit load with changes in the dilation angle, the soil friction angle, the grouted tieback's angle of inclination, the bonded length and the diameter of bonded length.

2 Problem Definition and Principal Hypotheses

The upper bound limit analysis approach combined with linear Mohr-Coulomb failure criterion must contain the non-associated flow rule in order to examine the impact of the dilation angle (ψ) on the collapse load of inclined , single rough grouted tieback subjected to axial static load (F) placed in sloping ground (homogeneous , isotropic , dry , frictional soil medium which is defined through its unit weight (γ) and friction angle (φ)) ; the sloping surface and the grouted tieback are inclined respectively at an angle (μ) and (λ) with the horizontal plane. The grouted tieback's interaction with the surrounding soil mass is expected to have a higher friction angle (δ) than the soil itself (φ) [19].

3 Including the Non-Associated Flow Rule in the Formulation of Upper Bound of Limit Analysis

According to the plasticity theory, if the dilation angle (ψ) is not equal to the friction angle (φ), the frictional material obeys a non-associated flow rule; (η) represents the dilative coefficient which is expressed according to the Equations (1); the dilative coefficient of frictional material can range in value from 0 to 1. The material follows the associated flow rule when the dilation angle (ψ) and friction angle (φ) are the same; however, this rule was unable to predict the plastic behavior of soil. Consequently, non-associated plasticity must be employed.

$$\eta = \frac{\Psi}{\varphi} \tag{1}$$

The researchers [20] proposed the following expressions to take dilation into consideration:

$$\eta = \frac{cos(\varphi)\,.\,cos\,(\Psi)}{1 - sin(\varphi)\,.\,sin\,(\Psi)} \tag{2}$$

$$\tan(\varphi^{*}) = \tan(\varphi)\,.\,\frac{cos(\varphi)\,.\,cos\,(\Psi)}{1 - sin(\varphi)\,.\,sin\,(\Psi)} \tag{3}$$

$$c^{*} = \mathrm{c}.\frac{cos(\varphi)\,.\,cos\,(\Psi)}{1 - sin(\varphi)\,.\,sin\,(\Psi)} \tag{4}$$

where the modified cohesion and modified friction angle are denoted by (c^{*}) and (φ^{*}) respectively. Equations (3) and (4) are used in the current work to introduce a non-associated flow rule in the upper bound approach of limit analysis for examining the impact of dilation angle on the ultimate load of grouted tiebacks.

4 Determination of the Failure Mechanism

According to the specifications of the upper bound theorem of limit analysis, a failure mechanism is constructed in which the soil is subjected to the linear Mohr-Coulomb failure criterion and non-associated flow rule as indicated in Figure 1. This mechanism permitted the calculation of the critical load in the case of axial static loading. The collapse mechanism is composed of a triangular rigid block (*AMN*) which is defined with the kinematic variable (β); where (*OM*) is the total length of the grouted tieback, (*AM*) and, (*NM*) are the linear rupture surfaces.

It is believed that the triangular wedge (*AMN*) moves with velocity (V_1), making an angle (Ψ) with the linear rupture surfaces (*AM*), and (*NM*). The grouted tieback is assumed to move with velocities (V_0) and (V_{01}) which is the relative velocity at the soil-grouted tieback interface. The direction of the relative velocity (V_{01}) between the triangles (*AMO*) and (*OMN*) is at an angle (Ψ) with the contact surface.

At the ultimate failure state, it is supposed that the grouted tieback and the triangular rigid block (*AMN*) having weight (W_{AMN}) move with the velocity (V_0) in the inclined direction as a single rigid unit. Following the velocity hodograph, all the velocities can be represented in terms of (V_0), as indicated in Figure 2 below.

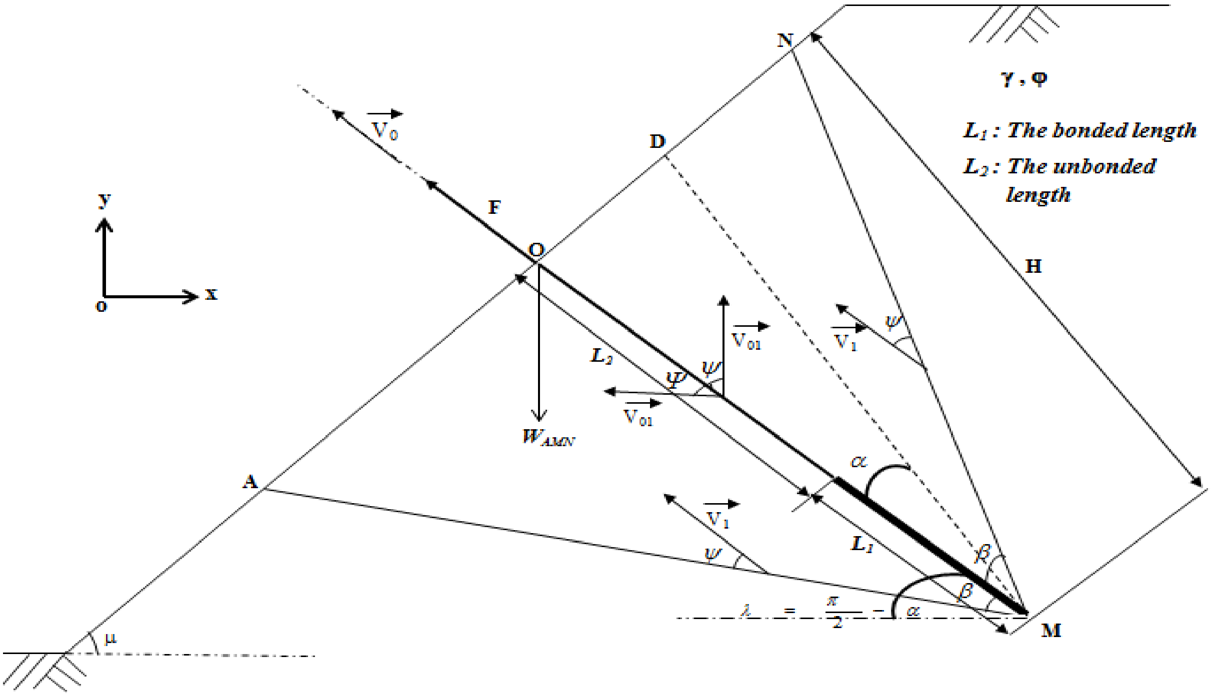

Figure 1: Failure mechanism at ultimate load for rough inclined grouted tieback in frictional soil under axial static loading according to non-associated flow rule and linear Mohr-Coulomb yield criterion

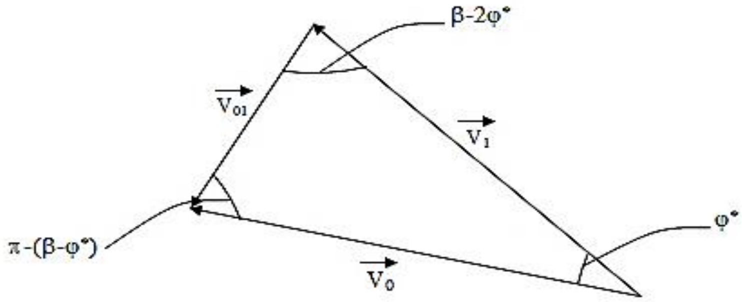

Figure 2: Velocity hodograph

4.1 Problem Formulation

The current study will suggest a number of formulations predicated on the upper bound theorem of limit analysis to determine the collapse load of an inclined , single rough grouted tieback subjected to axial static load (F) placed in sloping ground (frictional soil). Based on the geometric relationships of the kinematically admissible velocity hodograph, as illustrated in Figure 2 above; the following expressions of the velocities can be derived:

The velocity of the block:

$$V_0 = \frac{V_0 . sin(\beta - \varphi^*)}{sin(\beta - 2\varphi^*)} \quad (5)$$

The relative velocity:

$$V_{01} = \frac{V_0 . sin(\varphi^*)}{sin(\beta - 2\varphi^*)} \quad (6)$$

The internal energy dissipation rate owing to the soil cohesion (E_1) and the internal energy dissipation rate of the interface soil-grouted tieback (E_2) are added together to form the overall internal energy dissipation rate:

$$E_1 = \frac{c^* . \pi . H^2 . V_0 . sin(2\beta) . sin(\beta - \varphi^*) . cos(\varphi^*)}{2 . sin(\beta - 2\varphi^*) . \cos(\beta - \alpha) . \cos(\alpha + \beta) . \sin(\beta - 2\varphi^*)} \quad (7)$$

$$\alpha = \frac{\pi}{2} - \lambda \quad (8)$$

where (λ) is the grouted tieback's inclination.

$$E_2 = \frac{c^* . \pi . d . L_1 . V_0 . sin(\varphi^*) . cos(\varphi^*)}{sin(\beta - 2\varphi^*)} \quad (9)$$

$$E_{Total} = E_1 + E_2 \quad (10)$$

$$E_{Total} = \frac{c^* . \pi . H^2 . V_0 . sin(2\beta) . sin(\beta - \varphi^*) . cos(\varphi^*)}{2 . sin(\beta - 2\varphi^*) . \cos(\beta - \alpha) . \cos(\alpha + \beta) . \sin(\beta - 2\varphi^*)} + \frac{c^* . \pi . d . L_1 . V_0 . sin(\varphi^*) . cos(\varphi^*)}{sin(\beta - 2\varphi^*)} \quad (11)$$

The rigid block's weight can be expressed mathematically as follows:

$$W_{AMN} = \frac{\pi . \gamma . L . H^2 . \tan(\beta) . [tan(\beta - \alpha) + tan(\alpha + \beta)]}{6} \quad (12)$$

where (L) is the grouted tieback's total length.

The following expression can be used to calculate the work done by the external force:

$$T = F . V_0 \quad (13)$$

The expression for computing the work of the gravitational force is provided below:

$$T_{AMN} = W_{AMN}.V_1.\cos(\pi - \alpha) \quad (14)$$

By equating the rate of work performed by external forces to the rate of internal energy dissipation, the value of the limit load (*F*) is obtained:

$$\mathrm{F} = \frac{c^*.\pi.H^2.sin(2\beta).sin(\beta-\varphi^*).cos(\varphi^*)}{2.sin(\beta-2\varphi^*).\cos(\beta-\alpha).\cos(\alpha+\beta).\sin(\beta-2\varphi^*)} + \frac{c^*.\pi.d.L_1.sin(\varphi^*).cos(\varphi^*)}{sin(\beta-2\varphi^*)} + \frac{\pi.\gamma.\mathrm{L}.H^2.\tan(\beta).[tan(\beta-\alpha)+tan(\alpha+\beta)].\sin(\beta-\varphi^*).\cos(\pi-\alpha)}{6.sin(\beta-2\varphi^*)} \quad (15)$$

The internal energy dissipation rate through the friction of granular soil is equal to zero for a rough grouted tieback ($\delta > \varphi$), hence the expression of the collapse load (*F*) can be written as:

$$\mathrm{F} = \frac{\pi.\gamma.\mathrm{L}.H^2.\tan(\beta).[tan(\beta-\alpha)+tan(\alpha+\beta)].\sin(\beta-\varphi^*).\cos(\pi-\alpha)}{6.sin(\beta-2\varphi^*)} \quad (16)$$

5 Results and Discussion

Using MATHCAD software, the current study's optimizations were carried out. The following sets of parameters' results were computed: The range of the soil's angle of friction (φ) was 30°-40° with an interval of 2°, the dilation angle (ψ) was equal to {0, 0.25φ, 0.5φ, 0.75φ, φ}, the angle of inclination of grouted tieback (λ) took on three different values {10°, 15°, 20°}, the bonded length (L_{bonded}) was varied from 6 m to 10 m with an interval of 2 m and three values were considered of the diameter of bonded length ($d_{bonded\ length}$): {0.20 m, 0.25 m, 0.30 m}. Knowing that the angle of inclination of the sloping surface; the soil's unit weight and the unbonded length of grouted tieback are respectively as follows: (μ = 45°, γ = 20 kN/m^3, $L_{unbonded}$ = 14 m); they were kept constant.

5.1 The Effect of Combination of Non-Associated Flow Rule With the Soil Friction Angle and the Grouted Tieback's Angle of Inclination

The variation of limit load (*F*) using different dilative coefficient values (η) for changes in soil friction angle (φ) and the grouted tieback's angle of inclination of (λ) are demonstrated in (Figures 3, 4, and 5). The figures show that as the dilative coefficient (η), the friction angle of the soil (φ), and the angle of inclination of grouted tieback (λ) increase, the collapse load (F) increases as well.

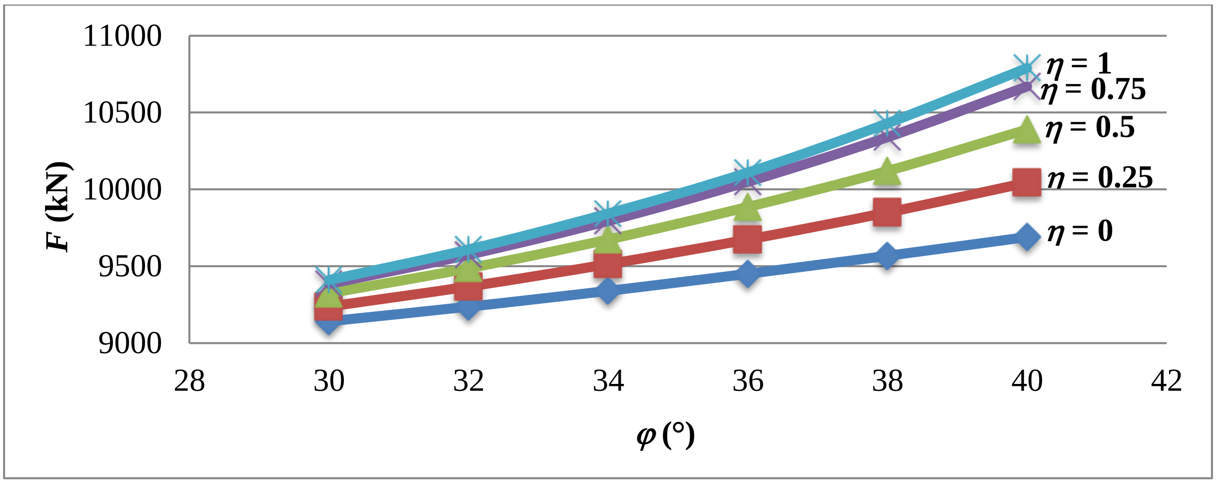

Figure 3: The variation of ultimate load with (φ) and (η) for (λ = 10°)

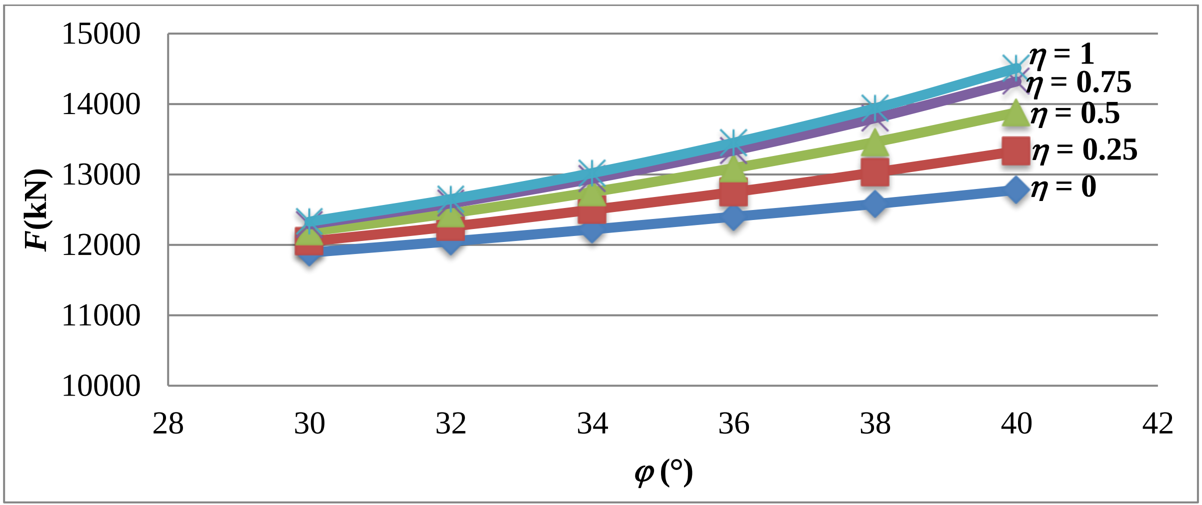

Figure 4: The variation of ultimate load with (φ) and (η) for (λ = 15°)

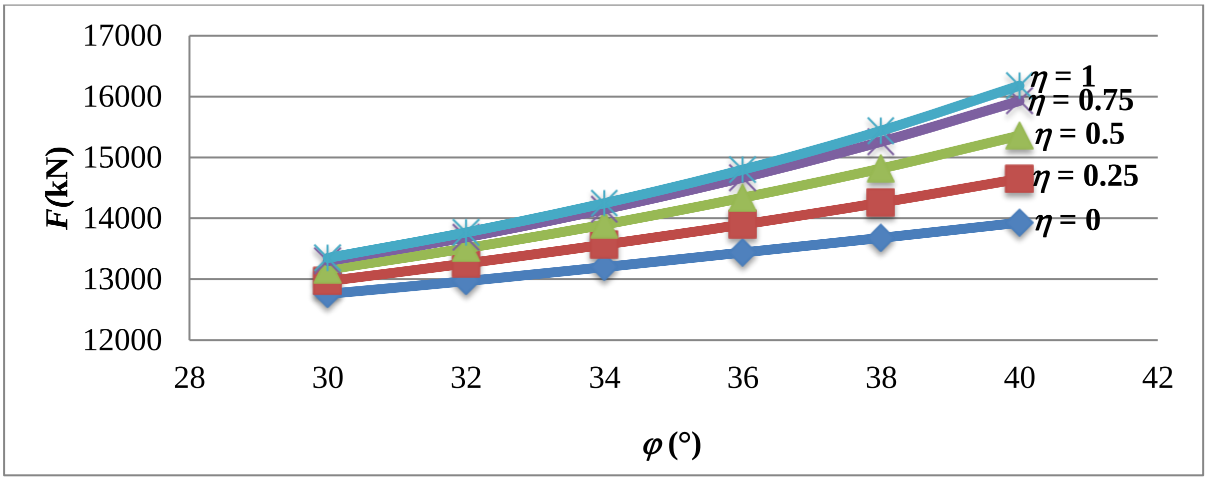

Figure 5: The variation of ultimate load with (φ) and (η) for (λ = 20°)

5.2 The Effect of Combination of Non-Associated Flow Rule With the Soil Friction Angle and the Bonded Length

In this section, a parametric analysis is done to determine how the non-associated flow rule, the soil friction angle (φ), and the bonded length (L_{bonded}) affect the grouted tieback's limit load (F). (Figures 6, 7 and 8) present a variation on critical load (F) against the friction angle of the soil (φ) under different dilative coefficients (η) and different bonded lengths (L_{bonded}). It can be found that the non-associated flow rule, the soil friction angle (φ), and the bonded length (L_{bonded}) have a significant effect on the ultimate load (F).

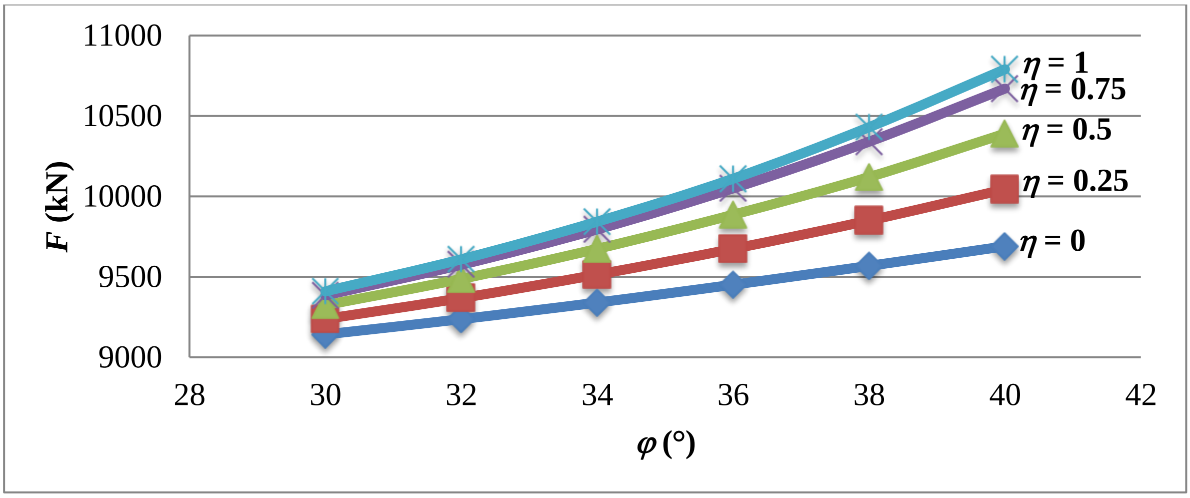

Figure 6: The variation of ultimate load with (φ) and (η) for (L_{bonded} = 6 m)

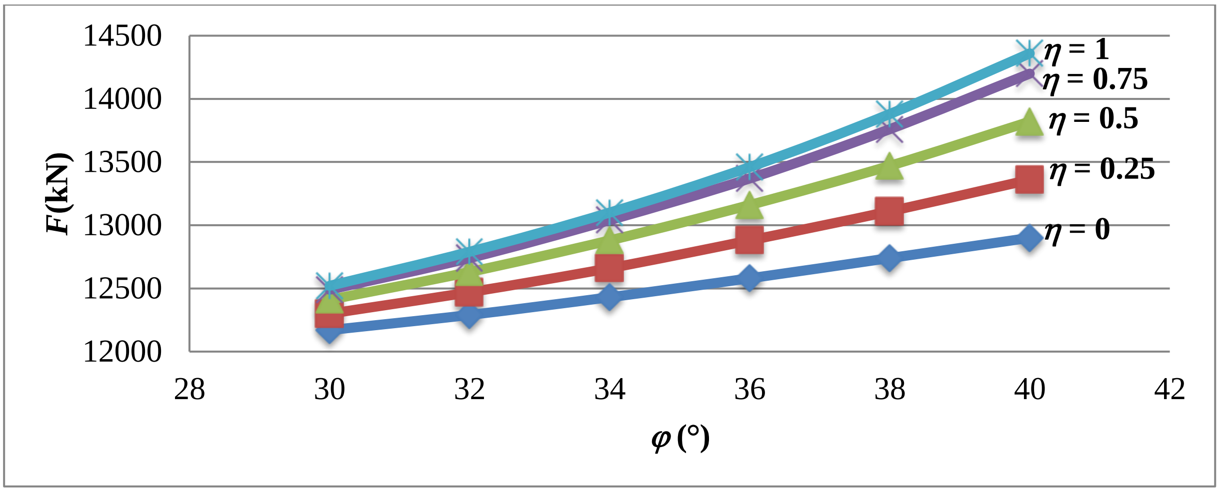

Figure 7: The variation of ultimate load with (φ) and (η) for (L_{bonded} = 8 m)

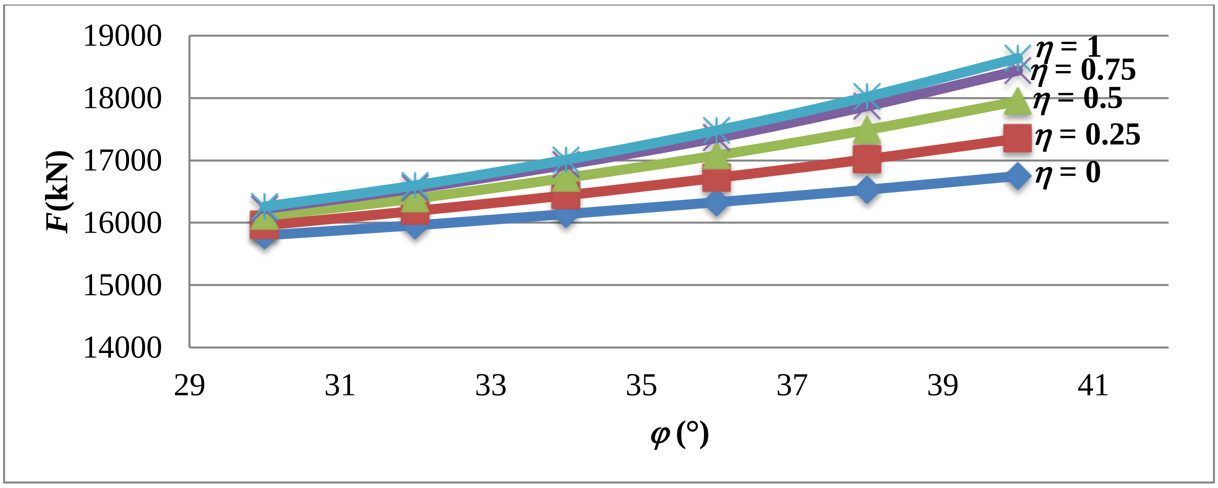

Figure 8: The variation of ultimate load with (φ) and (η) for (L_{bonded} = 10 m)

5.3 The Effect of Combination of Non-Associated Flow Rule With the Soil Friction Angle and the Bonded Length's Diameter

The influence of the non-associated flow rule and the bonded length's diameter of ($d_{bonded\ length}$) with different angles of friction of the soil (φ) on the ultimate load (F) are presented in (Figures 9, 10, and 11). The collapse load (F) is shown to increase with increasing soil angle of friction (φ), dilative coefficient (η), and maintain constant with increasing diameter of bonded length ($d_{bonded\ length}$).

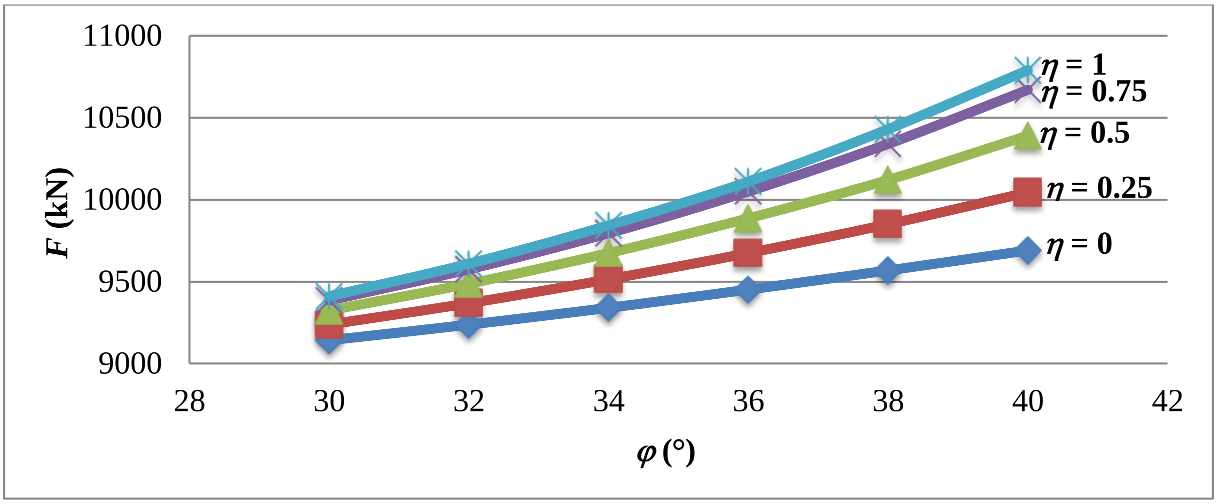

Figure 9: The variation of ultimate load with (φ) and (η) for ($d_{bonded\ length}$ = 0.20m)

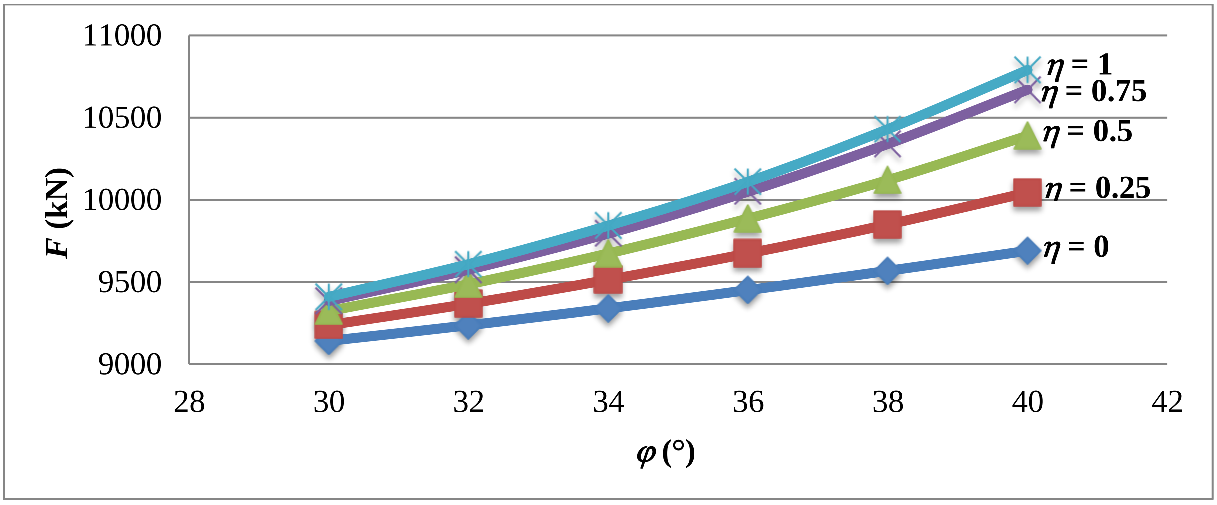

Figure 10: The variation of ultimate load with (φ) and (η) for ($d_{bonded\ length}$ = 0.25m)

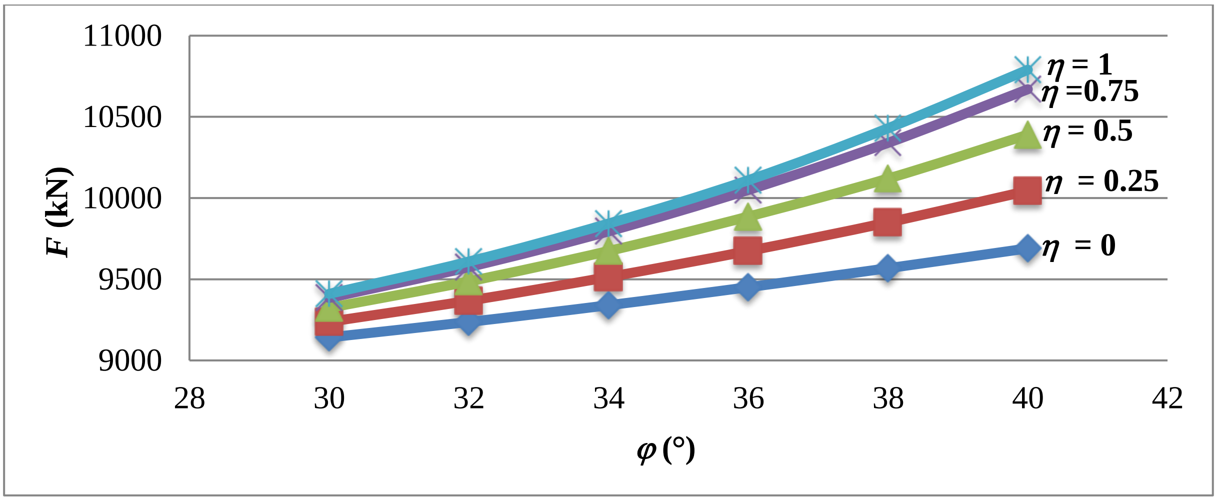

Figure 11: The variation of ultimate load with (φ) and (η) for ($d_{bonded\ length}$ = 0.30m)

6 Conclusion

A series of formulations using the upper bound theorem was utilized to determine the collapse load of a single, inclined rough grouted tieback under axial static loading placed in cohesionless soil. Effects of dilation angle, soil friction angle, grouted tieback's angle of inclination, the bonded length, and the diameter of bonded length were considered in this study. The main conclusions are recapitulated as follows:

- When employing the non-associated flow rule the collapse load (F) of grouted tieback is lower than when using the associated flow rule;

- The ultimate load (F) increases substantially with the increase in the dilation angle (ψ). Therefore, the dilatancy of geotechnical materials should be taken into account while designing the grouted tiebacks.

- Also, the soil angle of friction (φ), the grouted tieback's inclination (λ), and the bonded length (L_{bonded}) affect significantly the collapse load but the stability of grouted tieback is unaffected by the bonded length's diameter ($d_{bonded\ length}$).

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