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e-mail: a.malec@amdp.strefa.pl**WITTGENSTEIN VERSUS HUME:
THE IS-OUGHT PROBLEM – FINALLY SOLVED**

Abstract. This paper shows that Wittgenstein's situational approach applied to Hume's is-ought problem leads to the conclusion that this famous Hume thesis is false both in its original version (lack of logical value of deontic sentences) and in its modern understanding (logical separation of deontic sentences and sentences about facts).

Keywords: logic, philosophy of language.

What is the law?

The law can be treated not only as a collection of inscriptions, but also as something that is hidden behind this collection. Namely, it can be treated as the set of legal rules where a legal rule is perceived as a specific set of legal events. Thus, the law can be described in terms of **the set theory**:

LEGAL RULES = SETS OF LEGAL EVENTS.

This understanding of the law is a consequence of looking at the domain of law from the perspective of Ludwig Wittgenstein's logical atomism. In his *Tractatus logico-philosophicus*, Wittgenstein presented a picture of the world where the world is a totality of facts (where facts are understood as real configurations of things), and the logical space is a totality of situations (where situations are understood as possible configurations of things). A formal description of this ontology has been provided by Bogusław Wolniewicz¹. His formalism gives a good starting point for defining legal events, including legal acts. Namely, legal events and acts may be understood as sequences of situations:

THE ACTS $\subset$ THE LEGAL EVENTS $\subset$ THE STRINGS OF SITUATIONS.

Thanks to this, legal rules can also be formally defined².

Can the legal norms be true?

One of the most important concepts of legal theory is the concept of legal norm. Legal norms are language expressions that express legal duties. There is a common belief in legal theory that the legal norms are neither true nor false. The reason for this belief is that legal norms do not describe the world, but recommend how to change it. Indeed, the norm 'It is forbidden to trample the lawns' does not describe the condition of the lawns. It does not indicate whether lawns are being trampled on or not. It only recommends not to trample the lawns. Therefore, it is said that rather than being descriptions, legal norms are prescriptions. Thus, the norm 'It is forbidden to trample the lawns' says something similar to the order 'Don't trample the lawns!'.

This view is rooted in the is-ought problem raised by David Hume. According to Hume, moral distinctions cannot be derived from reason:

Reason is the discovery of truth or falsehood. Truth or falsehood consists in an agreement or disagreement either to the real relations of ideas, or to real existence and matter of fact. Whatever, therefore, is not susceptible of this agreement or disagreement, is incapable of being true or false, and can never be an object of our reason³.

Laudable or blameable, therefore, are not the same with reasonable or unreasonable. The merit and demerit of actions frequently contradict, and sometimes controul our natural propensities. But reason has no such influence. Moral distinctions, therefore, are not the offspring of reason⁴.

Thus, according to Hume, moral statements are neither true nor false. As a consequence, it is said that deontic statements, including norms, are neither true nor false.

When, however, legal norms are assumed to be true or false, this truthfulness is usually associated with their legal validity (i.e. with their establishment by the competent authority in accordance with due procedure). A norm is presumed to be true when it is valid, and false when it is not. Thus, the norm 'It is forbidden to trample the lawns' is true when a relevant authority has imposed such a prohibition. On the other hand, this norm is false when such prohibition has not been imposed by this authority. Consequently, a false norm becomes true as soon as it is established, and may later become false again as soon as it is repealed.

Per contra, semantic inquiries prove that in respect of veracity, the legal norms do not differ from the theorems of mathematics. In fact, every declarative sentence, if interpreted, is either true or false. And there is no

doubt that the norms, unlike the orders, are declarative sentences. Instead of the physical world, however, they describe a different world called ‘Popper’s third world’.

Norms and normative expressions

In order to address the question of whether norms can be true or false, it is first necessary to define what a norm is.

In natural language, some sentences are considered to be norms, and others are not. So, let us first look at some sentences we consider to be norms of natural language, and then at some sentences which we do not consider to be such norms. Let us try to map the sentences of both kinds in Wittgenstein’s first order language at the same time. Wittgenstein’s first order language is a first order language where the names refer to solely situations or events⁵.

Example 1

The sentence ‘Killing is prohibited’ is considered to be a norm of natural language, and can be mapped as follows:

$$\forall x_1 (P_1(x_1) \rightarrow \text{FOR}(x_1)),$$

where x_1 is an event, P_1 is read ‘... involves killing a person’. The whole reads as follows: any event that involves killing a person is prohibited.

Example 2

The sentence ‘John Smith is obliged to appear at the Court of Appeal on June 3, 2024’ is considered to be a norm of natural language and can be mapped like this:

$$\text{OBL}(a_5),$$

where ‘ a_5 ’ is an individual constant for the event described above.

Example 3

On the other hand, the sentence ‘If killing is prohibited, then killing by shooting is prohibited’ speaks about a relation between norms, but it is not considered to be a norm of natural language itself. It can be mapped, e.g., as follows:

$$\forall x_1 (P_1(x_1) \rightarrow \text{FOR}(x_1)) \rightarrow \forall x_2 (P_2(x_2) \rightarrow \text{FOR}(x_2)),$$

where x_1 and x_2 are events, P_1 is read ‘... involves killing a person’ and P_2 is read ‘... involves killing by shooting a person’.

Example 4

Similarly, the sentence ‘If killing is prohibited, then the St. Valentine’s Day Massacre is prohibited’ speaks about applying a norm, but it is not considered to be a norm of natural language itself.

It can be mapped, e.g., as follows:

$$\forall x_1 (P_1(x_1) \rightarrow \text{FOR}(x_1)) \rightarrow \text{FOR}(a_1),$$

where x_1 is an event, P_1 is read ‘... involves killing a person’ and ‘ a_1 ’ is the individual constant for the St. Valentine’s Day Massacre.

These examples show that the norms prescribe, prohibit, or permit events, and that such prescriptions, prohibitions or permissions are either unconditional or conditional. Thus, the following schemas of norms can be proposed in Wittgenstein’s first order language:

1) unconditional norms:

$$\text{OBL}(a),$$

$$\text{FOR}(a),$$

$$\text{PER}(a)$$

(where ‘ a ’ is an individual constant for any but determined event),

2) conditional norms:

$$\forall x (\text{DESCRIPTION}(x) \rightarrow \text{OBL}(x)),$$

$$\forall x (\text{DESCRIPTION}(x) \rightarrow \text{FOR}(x)),$$

$$\forall x (\text{DESCRIPTION}(x) \rightarrow \text{PER}(x))$$

(where ‘ x ’ is an individual variable for events, and ‘DESCRIPTION(x)’ stands for a non-deontic predicate or a conjunction of non-deontic predicates).

The schemas described above are the basic ones, but they are not the only possible ones. Clearly, norms based on other schemas are also possible. For example, it may be considered that the following sentence is a norm:

$$P(a) \rightarrow \text{OBL}(a)$$

because it prescribes conditionally an event, although it does not fall under any of the abovementioned schemas.

On the other hand, any expression of our language that contains OBL, FOR, or PER predicates will be called a ‘normative expression’. Consequently, every norm is a normative expression, but there are normative expressions that are not norms, e.g.:

$$\begin{aligned} P(x) &\rightarrow \text{OBL}(a), \\ \text{OBL}(a) &\rightarrow \text{FOR}(x). \end{aligned}$$

These expressions are not norms because they contain a free variable x , and therefore they are not sentences. And solely sentences may be qualified as norms.

In turn, any normative expression without free variables will be called a ‘deontic sentence’.

Thus, every norm is a deontic sentence, but there are deontic sentences that are not norms, e.g.:

$$\begin{aligned} (P_m(a) \rightarrow \text{OBL}(a)) \wedge P_m(a) &\rightarrow \text{OBL}(a), \\ \forall x (P_m(x) \rightarrow \text{OBL}(x)) \wedge P_m(a) &\rightarrow \text{OBL}(a). \end{aligned}$$

The forementioned deontic sentences are not norms because they contain ‘ $\rightarrow$ ’ as well as ‘OBL’ signs in the antecedent, and this is against our understanding of conditional norms.

The following expressions are other examples of deontic sentences that are not norms:

$$\begin{aligned} P_m(a) \wedge P_m(b) &\rightarrow \text{OBL}(a) \vee \text{OBL}(b), \\ \forall x \forall y (P_m(x) \wedge P_n(x) &\rightarrow \text{OBL}(x) \vee \text{OBL}(y)). \end{aligned}$$

The deontic sentences mentioned above are not norms because they contain the ‘ $\vee$ ’ sign in the consequent, and therefore they do not prescribe, prohibit or permit a ‘determined event’.

These terminology findings may be summarized as follows:

NORM $\subset$ DEONTIC SENTENCE $\subset$ NORMATIVE EXPRESSION.

Veracity and falsity of norms

In the first order logic, the following inductive definition of veracity, based on Tarski’s truth conception, is usually accepted:

- 1) ‘ $P(t_1, t_2, \dots, t_n)$ ’ is true $\Leftrightarrow \langle \phi(t_1), \phi(t_2), \dots, \phi(t_n) \rangle \in \phi(P)$,
- 2) ‘ $\forall x \alpha$ ’ is true $\Leftrightarrow$ for any $\phi(x)$ ‘ α ’ is true,
- 3) ‘ $\exists x \alpha$ ’ is true $\Leftrightarrow$ for some $\phi(x)$ ‘ α ’ is true,
- 4) ‘ $\neg \alpha$ ’ is true $\Leftrightarrow$ it is not true that ‘ α ’ is true,
- 5) ‘ $\alpha \wedge \beta$ ’ is true $\Leftrightarrow$ ‘ α ’ is true and ‘ β ’ is true,
- 6) ‘ $\alpha \vee \beta$ ’ is true $\Leftrightarrow$ ‘ α ’ is true or ‘ β ’ is true,

- 7) ' $\alpha \rightarrow \beta$ ' is true $\Leftrightarrow$ it is not true that at the same time ' α ' is true and ' β ' is not true,
- 8) ' $\alpha \leftrightarrow \beta$ ' is true $\Leftrightarrow$ ' α ' is true if and only if ' β ' is true.

The first point of this definition determines the truth condition for simple (atomic) sentences, i.e. the sentences that do not contain logical constants. According to the first point, the simple sentence ' $P(t_1, t_2, \dots, t_n)$ ' is true if and only if the sequence of objects being this sentence terms' interpretations belongs to the set being this sentence predicate's interpretation. For example, if ' t_1 ' means John, ' t_2 ' means Kate, and the predicate ' $P(\dots, \dots)$ ' reads ' $\dots$ and $\dots$ love each other', then the sentence ' $P(t_1, t_2)$ ' is true if and only if the sequence $\langle \text{John}, \text{Kate} \rangle$ belongs to the set of couples who love each other. In other words, the abovementioned sentence is true if and only if John and Kate love each other.

The next points of this definition determine how the truth of a compound sentence depends on the truth of the sentences this compound sentence is made of. For example, the compound sentence ' $\neg\alpha$ ' is true if and only if the sentence ' α ' is false (this is point 4 of the definition). Similarly, the compound sentence ' $\alpha \wedge \beta$ ' is true if and only if the sentence ' α ' is true and the sentence ' β ' is true (this is point 5 of the definition).

Thus, this definition allows us to say about each sentence of our language (which has been interpreted) whether it is true or false. Of course, this definition also applies to norms, both unconditional:

- 1) ' $\text{OBL}(a)$ ' is true $\Leftrightarrow \phi(a) \in \phi(\text{OBL}) \Leftrightarrow \phi(a) \in \mathbf{OBL}$, where $\mathbf{OBL}$ is the set of prescribed events,
- 2) ' $\text{FOR}(a)$ ' is true $\Leftrightarrow \phi(a) \in \phi(\text{FOR}) \Leftrightarrow \phi(a) \in \mathbf{FOR}$, where $\mathbf{FOR}$ is the set of forbidden events,
- 3) ' $\text{PER}(a)$ ' is true $\Leftrightarrow \phi(a) \in \phi(\text{PER}) \Leftrightarrow \phi(a) \in \mathbf{PER}$, where $\mathbf{PER}$ is the set of permitted events,

as well as conditional:

- 4) ' $\forall x (\text{DESCRIPTION}(x) \rightarrow \text{OBL}(x))$ ' is true $\Leftrightarrow$ for any $\phi(x)$ it is not true that at the same time ' $\text{DESCRIPTION}(x)$ ' is true and ' $\text{OBL}(x)$ ' is not true,
- 5) ' $\forall x (\text{DESCRIPTION}(x) \rightarrow \text{FOR}(x))$ ' is true $\Leftrightarrow$ for any $\phi(x)$ it is not true that at the same time ' $\text{DESCRIPTION}(x)$ ' is true and ' $\text{FOR}(x)$ ' is not true,
- 6) ' $\forall x (\text{DESCRIPTION}(x) \rightarrow \text{PER}(x))$ ' is true $\Leftrightarrow$ for any $\phi(x)$ it is not true that at the same time ' $\text{DESCRIPTION}(x)$ ' is true and ' $\text{PER}(x)$ ' is not true.

These equivalences show that the norms are sentences in a logical sense, i.e. every norm (which has been interpreted) is either true or false. Therefore, the claim of legal theory that legal norms are not sentences in the logical sense should rather be read as follows: we the lawyers believe that the legal norms cannot be assessed as true or false because we have not yet learned to interpret them properly⁶.

If, however, everything about the issue of the veracity and falsity of norms is so clear and obvious, why do some researchers claim that norms are neither true nor false?

True norms, false norms, and Popper's third world

The answer to this question is quite simple: the veracity of norms is not related to facts. It is related to **OBL**, **FOR** and **PER** sets of events, and some or even all of these events may be unreal, i.e. may be composed of situations that are not facts.

The question therefore arises of whether such truth conditions for norms do not violate the very concept of truth, which requires that a true sentence shall correspond to reality? After all, the classic, Aristotelian, veracity definition says that a sentence is true if and only if reality is as this sentence says.

Let us examine this issue a bit. Namely, let us consider the sentence 'Kate kissed John'. According to the veracity definition provided in the previous section, this sentence is true if and only if Kate kissed John, i.e. when the fact is that Kate kissed John:

$$\text{'FACT}(a_m)\text{' is true} \Leftrightarrow \phi(a_m) \in \mathbf{FACT}.$$

Since **FACT** is the set of facts, one needs a contact with facts to determine whether the sentence 'FACT(a_m)' is true or false.

Let us now consider the sentence 'It is ordered that Kate kissed John'. According to the veracity definition provided in the previous section, this sentence is true if and only if the event in question belongs to the set of prescribed events:

$$\text{'OBL}(a_m)\text{' is true} \Leftrightarrow \phi(a_m) \in \mathbf{OBL}.$$

Since **OBL** is not the set of facts, one does not need a contact with facts to determine whether the sentence 'OBL(a_m)' is true or false.

In both cases, the veracity criterion is that an object belongs to a set, i.e. this criterion is based on a relation in **the third world of Popper**. The difference between these cases is as follows: in the first case, this set

was defined in strict reference to reality, understood as the totality of facts, and in the second – not.

It should be noted that many sentences share the fate of norms when it comes to the criterion of their veracity. Let us consider the sentence ‘Kate had to kiss John’. According to the veracity definition provided in the previous section, this sentence is true if and only if the event in question belongs to the set of necessary events:

$$\text{‘DET}(a_m)\text{’ is true} \Leftrightarrow \phi(a_m) \in \mathbf{DET}.$$

Also in this case, the veracity criterion is that an object belongs to a set, and this set is defined without strict reference to reality, understood as the totality of facts.

Let us now consider the commonly used sentence ‘Every attorney at law is a lawyer’. According to the veracity definition provided in the previous section, this sentence is true if and only if the set of attorneys is a subset of the set of lawyers:

$$\begin{aligned} \text{‘Every attorney at law is a lawyer’ is true} &\Leftrightarrow \text{for any man } x \text{ it is not} \\ &\text{true that at the same time ‘}x \text{ is an attorney at law’ is true and ‘}x \text{ is} \\ &\text{a lawyer’ is not true} \Leftrightarrow \varphi(\text{attorney at law}) \subset \varphi(\text{lawyer}). \end{aligned}$$

It turns out that this commonly used sentence also tells us nothing about facts. This sentence just states a relation between two sets. So, it states something about Popper’s third world, and compliance with this third world makes this sentence true.

If we think about this for a moment, we will come to the conclusion that all predicates refer to sets, and therefore to abstract objects. Thus, all predicates relate to objects from the third world of Popper. Hence, a compliance with reality, understood as the totality of facts, is the criterion of truth only in the case of sentences concerning situations and events that state that these situations and events are real (are facts).

Thus, if Aristotle-Tarski’s conception of truth is combined with Wittgenstein’s conception of fact, then it turns out that the criterion of truth is not compliance with the reality of Popper’s first world (the totality of facts). This criterion is compliance with the ‘reality’ of Popper’s third world (the world of cultural creations).

When this dependence is learnt, all doubts as to whether norms can be true or false disappear. While norms are incapable of being in compliance with the facts, they can be in compliance with the creations of Popper’s third world.

Therefore, Hume’s famous thesis in its original understanding is not true.

Veracity of norms and the modern understanding of the is-ought problem

The issue of veracity (falsity) of norms shall not be confused with the is-ought problem in its modern understanding. Namely, in contemporary philosophy, the is-ought problem is restated in a narrower way:

Deontic statements are logically separated from non-deontic statements, i.e. deontic statements can not be derived from non-deontic statements (simple Hume's thesis), nor can non-deontic statements be derived from deontic statements (reverse Hume's thesis)⁷.

If this thesis holds, norms are logically separated from statements about facts.

Thus, the is-ought problem may be understood in two ways. First, it may be understood in accordance with Hume's original statement, and then it is about the lack of logical values in normative reasonings. Second, it may be understood in accordance with the contemporary approach, and then it is about logical separation of deontic sentences from sentences about facts.

Of course, these two understandings are different. While the first understanding of the is-ought problem excludes that norms can be either true or false, the second one does not exclude this possibility. Thus, the conclusions of the previous sections are incompatible with the first understanding of this thesis, but are compatible with its second understanding.

To illustrate this compatibility with the contemporary understanding of the is-ought problem, let us return to the 'Kate kissed John' example:

$\text{'FACT}(a_m) \text{' is true} \Leftrightarrow \phi(a_m) \in \mathbf{FACT},$

$\text{'OBL}(a_m) \text{' is true} \Leftrightarrow \phi(a_m) \in \mathbf{OBL}.$

Clearly, the truth of the sentence 'Kate kissed John' depends on the set **FACT**, and the truth of the sentence 'It is ordered that Kate kissed John' depends on the set **OBL**, and these two sets are different. Therefore, it may be the case that the sentences in question are logically separated. Nevertheless, the sentence 'It is ordered that Kate kissed John' has the same right to be true or false as the sentence 'Kate kissed John', when Popper's third world is considered.

However, it is reasonable to call into question the truth of Hume's thesis also in its contemporary understanding. If the veracity of a sentence depends on a set of events, and the veracity of another sentence depends on another

set of events, it does not mean that they must be logically separated. In this respect it is sufficient to notice that the truth of the sentence ‘Kate had to kiss John’ also depends on a set of events which may differ from **FACT**:

$$\text{‘DET}(a_m)\text{’ is true} \Leftrightarrow \phi(a_m) \in \mathbf{DET}.$$

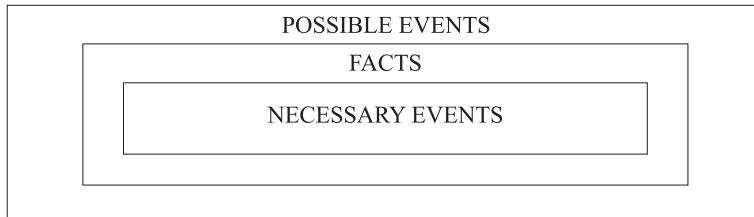
Nonetheless, nobody claims that modal (alethical) statements are logically separated from statements about facts. On the contrary, it is assumed in modal logic that:

$$\begin{aligned} & \Box p \rightarrow p \\ & \text{(if it is necessary that } p \text{ then } p) \end{aligned}$$

as well as that:

$$\begin{aligned} & p \rightarrow \Diamond p \\ & \text{(if } p \text{ then it is possible that } p). \end{aligned}$$

So, it is assumed that there are links between modal and factual statements. These links are expressible with a Venn diagram in the following way:



It cannot be excluded that the **OBL**, **FOR**, and **PER** sets are also related to the **FACT** set in a way, although the nature of this relation is not so clear as it is in the case of the **DET** and **FACT** sets.

Indeed, standard specific axioms for deontic modalities:

$$\text{A1. } \forall x (\text{OBL}(x) \vee \text{PER}(x) \vee \text{FOR}(x) \vee \text{IRR}(x)),$$

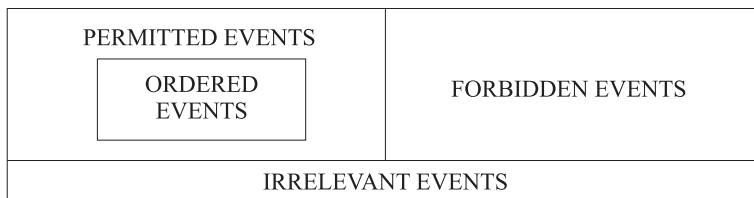
$$\text{A2. } \neg \exists x (\text{PER}(x) \wedge \text{FOR}(x)),$$

$$\text{A3. } \neg \exists x (\text{IRR}(x) \wedge \text{FOR}(x)),$$

$$\text{A4. } \neg \exists x (\text{PER}(x) \wedge \text{IRR}(x)),$$

$$\text{A5. } \forall x (\text{OBL}(x) \rightarrow \text{PER}(x)),$$

lead to the following Venn diagram:

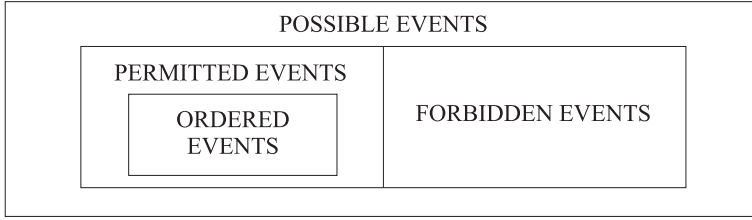


This diagram does not reveal any relation between legal rules and facts.

However, as early as in Roman times, lawyers adopted a fundamental principle of law: *Impossibilia nulla obligatio est* – the impossible is not the subject of duty. This fundamental principle leads to one more specific axiom for deontic modalities:

A6. $\forall x ((\text{OBL}(x) \vee \text{PER}(x) \vee \text{FOR}(x)) \rightarrow \text{POSS}(x))$.

If one takes this axiom into account, then the following Venn diagram can be obtained:



This diagram reveals a direct relation between legal duties (legal rules) and possible events, and – this way – an indirect relation between these duties (rules) and facts.

Thus, while the results of the previous sections do not contradict Hume’s thesis in its modern understanding, the present discussion at least calls it into question.

N O T E S

¹ Bogusław Wolniewicz, *Ontologia sytuacji* (in Polish: *Ontology of situations*), Państwowe Wydawnictwo Naukowe, Warszawa 1985. See also: Bogusław Wolniewicz, *A formal ontology of situations*, *Studia Logica* 41 (1982), pp. 381–413.

² Andrzej Malec, *Introduction to the Semantics of Law*, Springer 2022. This paper is based on one of the threads of the mentioned book.

³ David Hume, *A Treatise of Human Nature*, Penguin Classics 1985, p. 510.

⁴ *Ibidem*.

⁵ For the sake of simplicity, the examples presented below will be limited to events.

⁶ Methods of defining the **OBL**, **FOR**, **PER** sets are given in: Andrzej Malec, *Introduction to the Semantics of Law*, Springer 2022. On the objectivity of norms and values, see also: Andrzej Malec, *Arytmetyka sumienia. Przyczynek do teorii odpowiedzialności słusznościowej* (in Polish: *The Arithmetic of Conscience. A Contribution to the Theory of Equitable Responsibility*), Warszawa 2012. In *the Arithmetic of Conscience*, the theory of acts evaluation is developed. Simple acts and complex acts are distinguished. Then the time aspects of acts evaluation are examined. The concepts of good acts, right acts, and internally moral acts are introduced. Arguments in favour of the objectivity of core values, as well as the possibility of rational acts evaluation are developed.

⁷ Jan Woleński, *Uogólniona teza Hume’a* (in Polish: *A Generalized Hume’s Thesis*), in: I. Bogucka, Z. Tobor (editors) ‘Prawo a wartości. Księga jubileuszowa Profesora Józefa Nowackiego’, Zakamycze Kraków 2003, pp. 293–303.

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